Project Final Report

Bond Investment Applications

ISOM3900 - Decision Analytics

Introduction

Bond investment is a big area in the financial industry that is considered a lower risk investment compared with equity investment. Similar to a differential asset investment strategy, a diversified portfolio will spread out the specific risks of bonds. The most important decision for the investor to make is to design an optimized portfolio, which has a maximum expected return in a preferred risk level. Hence, we are interested in applying the decision analytics model or techniques in lecture to construct the optimized portfolio. **Linear programming** is always used in portfolio construction as the aim of the portfolio construction is to maximize the expected profit, which is a linear function. Besides, we would also like to apply the **Markov chain** technique in the bond investment strategy, which is an innovative application in bond investment strategy. **Different investment strategies** would be used and compared with each other. The report is composed of eight guiding questions and a summary section to include the key deliveries of the report finding.

Decision Problems Design Methodology

Firstly, we would like to introduce the raw dataset we used and the data preprocessing process which transform the data to be preferable for the later analysis.

1. Describe the dataset. How to prepare and normalize the data? How is the credit quality of the bond?

Then, we would apply the Markov chain concept to generate some investment insights from the data set.

2. Assume that we have invested in a perpetual bond (non-expiring bond). What is the long-term probability that the bond will acquire the credit rating of 'AAA'?
3. As an investment manager you will have to advise your clients the probability of invested non-junk bonds ('B' or above) turning into junk bonds ('CCC' or below) in 30 years. What are the probabilities?

Thirdly, we would apply the Linear programming and Markov chain technique together to construct the optimized portfolio, evaluate the portfolio performances and explain the advantages of applying Markov chain in portfolio construction.

4a. Use Linear Programming to optimize the bond portfolio, achieving the highest 5-year coupon return with the consideration of initial risk (t=0).

4b. Use LP to optimize the bond portfolio, achieving the highest 5-year coupon return with the consideration of initial risk (t=0) and estimated future risk in the overall investment period (t=1-4) through the application of Markov chain.

After that, we would like to apply different risk level investment strategies, including the aggressive and conservative strategy, and compare the performance with the strategy mentioned previously in Q4.

5. Suppose instead an aggressive manager wants to maximize the coupon rate with the low quality bond (-1SD of your portfolio's credit rating), optimize the portfolio for this manager.
6. Suppose a conservative manager wants to maximize the coupon rate with the high quality bond (+2SD of the whole bond list's credit rating), optimize the portfolio for this manager. Compare your strategy with this manager.
7. Explain which strategy is the best performing strategy among questions 4 - 6?

Lastly, we would apply the dynamic investment strategy with the application of Markov chain, and compare the performance with the portfolio in Q4.

8. Suppose you are allowed to adjust your portfolio each year (dynamic strategy). Your portfolio is required to have an average credit rating of +1SD in every adjustment. Compare the dynamic strategy to the previous strategies.

1. Describe the dataset. How to prepare and normalize the data? How is the credit quality of the bond?

<u>Description</u>

The raw dataset has 11 columns, which includes:

Index, S&P, Moody's, Fitch, Seniority, Min. Size, Issuer, CPN, Indicative Offer, YTM, and Risk Rating.

The following table shows the definitions of these columns:

#	*Column*	*Definition*
1	Index	Index of the bond (180 bonds in total).
2	S&P	Credit risk rating from S&P.
3	Moody's	Credit risk rating from Moody's.
4	Fitch	Credit risk rating from Fitch.
5	Seniority	Seniority means the priority to get back your asset and associate cash flow when the bond goes default. There are 6 rankings in total, starting from the most secured bond: First lien/Mortgage, Senior Secured, Junior Secured, Senior Unsecured, Subordinated, Junior Subordinated.
6	Min. Size	Minimum capital size to invest in a bond.

7	Issuer	The bond issuer.
8	CPN	Coupon rate of the bond.
9	Indicative Offer	The current bond price.
10	YTM	Yield to maturity, which is the anticipated total return anticipated on a bond if it is held until it matures.
11	Risk Rating	Credit risk rating from the bank. There are 6 rankings in total, level 1 is the most secured while level 6 is the most unsecured.

Normalization

First, we normalize the three credit ratings columns (S&P, Moody's, and Fitch). It is because the credit risk ratings are in letters (e.g. 'AAA'), and we cannot do calculations based on letters. Therefore, we turn the credit rating based on the 8-point scale, which was done on Excel VBA.

	S&P	Moody's	Fitch
8	AAA	Aaa	AAA
7	AA+ AA AA-	Aa1 Aa2 Aa3	AA+ AA AA-
6	A+ A A-	A1 A2 A3	A+ A A-
5	BBB+ BBB BBB-	Baa1 Baa2 Baa3	BBB+ BBB BBB-
4	BB+ BB BB-	Ba1 Ba2 Ba3	BB+ BB BB-
3	B+ B B-	B1 B2 B3	B+ B B-
2	CCC+ CCC CCC-	Caa1 Caa2 Caa3	CCC+ CCC CCC-
1	CC SD D	Ca C	CC C DDD DD D

normalized credit risk rating

The reason for turning the credit risk rating into an 8-point scale is to fit in the ratings with the given transition matrix table. To interpret the table, assume we hold a bond with the credit rating of 'AAA'. In the coming year the probability of the bond rated 'AAA' is 91.93%. Furthermore, the probability of the bond rated 'AA' is 7.46%.

	AAA	AA	A	BBB	BB	B	CCC	D	sum
AAA	0.9193	0.0746	0.0048	0.0008	0.0000	0.0000	0.0000	0.0000	0.9995
AA	0.6400	0.9181	0.0676	0.0060	0.0012	0.0012	0.0003	0.0000	1.6344
A	0.0700	0.0227	0.0917	0.0512	0.0025	0.0025	0.0001	0.0004	0.2411
BBB	0.0400	0.0270	0.0556	0.8788	0.0102	0.0102	0.0017	0.0024	1.0259
BB	0.0400	0.0010	0.0061	0.0775	0.0790	0.0790	0.0111	0.0101	0.3038
B	0.0000	0.0010	0.0028	0.0046	0.8280	0.8280	0.0396	0.0545	1.7585
CCC	0.1900	0.0000	0.0037	0.0075	0.1213	0.1213	0.6045	0.2369	1.2852
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000

transition matrix (raw)

This is the original transition table from *Siu et al (2005) [1]*. It shows the probability of how a bond will change its credit rating annually, which was derived by historical data). However, the table shown above have certain issues to be solved:

1. The sum of probabilities are not equal to 1.0
   - This may be possible due to external factors, such as being delisted; however, for simplicity, we assume that the bond can only be rated ranging from 'AAA' to 'D' with probabilities of 1.0.
2. 'D' credit risk bond never changes its state annually
   - 'D-to-D' probability is 1.0
     - This is known as an absorbing state, which will cause an issue in calculating the long-term steady state of each credit rating for the Markov chain.

For the first issue, we normalized the probability to let the summation of probabilities be 1. For example, the normalized probability of 'AAA'-to-'AAA' bonds will be 0.9193/0.9995 = 0.9198. For the second issue, we turn the 'D'-to-'D' probability from 1 to 0.9999 while the remaining 0.000014 will be equally distributed to the remaining states. By giving 'D'-to-'D' it changes the state to a non-absorbing state but the difference is negligible.

The normalized table is shown as follows:

	AAA	AA	A	BBB	BB	B	CCC	D	sum
AAA	0.9198	0.0746	0.0048	0.0008	0.0000	0.0000	0.0000	0.0000	1.0000
AA	0.3916	0.5617	0.0414	0.0037	0.0007	0.0007	0.0002	0.0000	1.0000
A	0.2903	0.0942	0.3803	0.2124	0.0104	0.0104	0.0004	0.0017	1.0000
BBB	0.0390	0.0263	0.0542	0.8566	0.0099	0.0099	0.0017	0.0023	1.0000
BB	0.1317	0.0033	0.0201	0.2551	0.2600	0.2600	0.0365	0.0332	1.0000
B	0.0000	0.0006	0.0016	0.0026	0.4709	0.4709	0.0225	0.0310	1.0000
CCC	0.1478	0.0000	0.0029	0.0058	0.0944	0.0944	0.4704	0.1843	1.0000
D	0.000014	0.000014	0.000014	0.000014	0.000014	0.000014	0.000014	0.99990	1.0000

transition matrix (normalized)

From the above transition table, there are eight states. We define 'AAA' = 8, 'AA' = 7 … 'D' = 1 and now S&P, Moody's and Fitch ratings are normalized as 8-point scales as well, which can fit S&P, Moody's and Fitch ratings into this transition table.

By using the normalized transition table, we can calculate the expected credit ratings over years using matrix multiplication. The following table shows the probability that the initial bond with AAA credit rating changes its credit rating over 5 years.

	8	7	6	5	4	3	2	1
	AAA	**AA**	**A**	**BBB**	**BB**	**B**	**CCC**	**D**
0	1	0	0	0	0	0	0	0
1	91.98%	7.46%	0.48%	0.08%	0.00%	0.00%	0.00%	0.00%
2	87.66%	11.10%	0.94%	0.27%	0.01%	0.01%	0.00%	0.00%
3	85.26%	12.88%	1.25%	0.55%	0.03%	0.03%	0.00%	0.00%
4	83.85%	13.73%	1.45%	0.86%	0.05%	0.05%	0.01%	0.01%
5	82.96%	14.13%	1.57%	1.17%	0.07%	0.07%	0.01%	0.02%

To calculate the expected credit rating (for AAA bond) in year 1, the formula is 8*91.98% + 7*7.46% + 6*0.48% + … + 1*0.00% = 7.91.

Using similar methods, we can calculate the expected credit rating for all bonds over 5 years, the table (Row: rating; Column: Year) is shown as follows:

Rating\Year	1	2	3	4	5
8	7.91	7.86	7.83	7.80	7.78
7	7.34	7.51	7.60	7.64	7.66
6	6.40	6.61	6.73	6.83	6.90
5	5.18	5.37	5.56	5.73	5.89
4	4.40	4.69	4.93	5.14	5.32
3	3.40	3.80	4.14	4.42	4.65
2	3.01	3.56	3.87	4.07	4.21
1	1.00	3.56	3.87	4.07	4.21
0	0.00	0.00	0.00	0.00	0.00

rating-year table

Note that there are bonds that do not receive ratings from all three bond rating agencies. For those bonds without ratings, we would value it as zero. We name this table as RY (Rating-Year) table. According to the RY table, we can find out the average expected credit risk rating from three evaluators (S&P, Moody's and Fitch) for every year.

Since there are some bonds that do not receive ratings from all three bond rating agencies, we would need special treatment for calculating average. For example, if a bond is only evaluated by two bond rating agencies, then the formula should be the sum of the expected credit risk rating divided by 2. We decided to use AVERAGEIF() function in Excel to calculate the average score excluding values with zero.

Since every bond will have different expected credit risk every year, the process of calculating the average will be done by 5 times if we want to know the average expected credit risk of the bond within the first 5 years.

At last, we normalized the "Seniority" column into a 6-point scale. We do not need to put seniority rating into the average credit rating in the same scale since their meaning is not identical. Credit rating is "the risk that the bond will go default" while seniority rating is "given the bond is default, what is my priority to get back my asset and cash flow".

Here are some statistical figures regarding the average credit rating of all bonds:

	Average	*Minimum*	*Maximum*	*Std. Dev*
Year 0	5.36	4.33	7.00	0.45
Year 1	5.61	4.66	7.34	0.53
Year 2	5.81	4.91	7.51	0.54
Year 3	5.97	5.14	7.60	0.51
Year 4	6.12	5.34	7.64	0.48

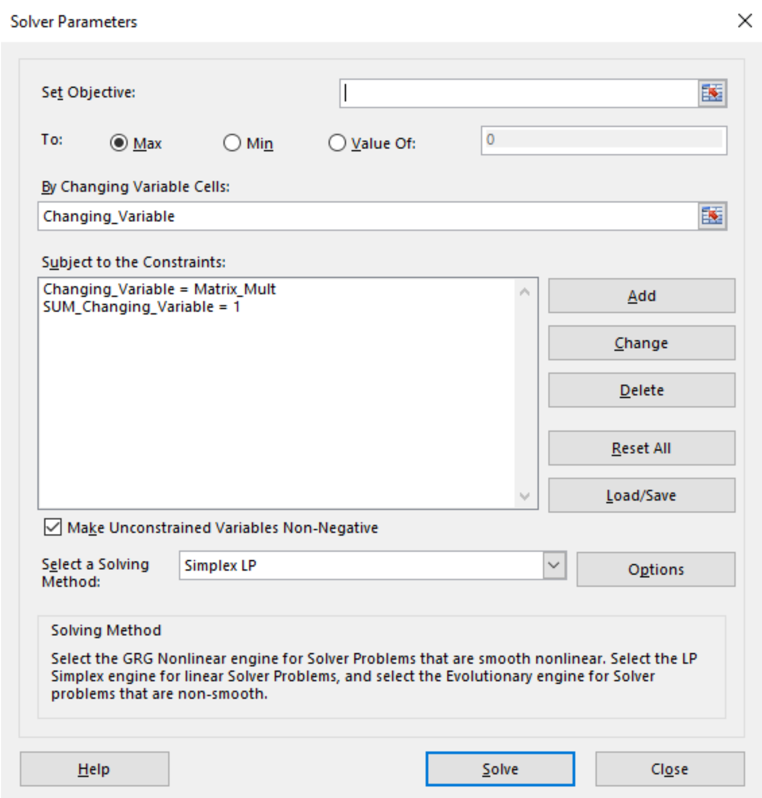
The reason behind the trend of the credit risk rating is because except AAA credit risk probability is decreasing over time, all other credit risk state probability is increasing over time while high quality bond rating's increasing rate is higher than low quality bond, so overall the bond has an increasing trend.

Overall the bond's credit quality is quite high, which makes sense because banks will not offer junk bonds to customers while most customers are not willing to take such a high risk to invest in a junk bond.

2. Assume that we have invested in a perpetual bond (non-expiring bond). What is the long-term probability that the bond will acquire the credit rating of 'AAA'?

Most bonds mature and expire in 30 years, so to calculate the long-term probability of a bond's credit rating, we will have to apply Markov chains to determine the credit risk. A transition matrix of credit risk will represent the likelihood of future evolution of ratings.

Use the transition matrix (normalized) table above. We can employ Excel Matrix Multiplication and Solver to determine the steady-state. To set up the calculation, we create a row of changing variables cells (with arbitrary variables), and another row with Matrix Multiplication - using the function {=MMULT([changing variable cells], [normalized table]}. Lastly, we have a cell that contains the sum of the changing cells. Solver is set up with the specification below:



constraints:

1. sum of the changing variable cells = 1

2. changing variable cells is = to matrix multiplication cells

Finally, we arrive with the conclusion below of the steady-state of each of the credit ratings. To interpret the table below, the first row is the label of the credit rating and the second row is the long-term probability of the bond acquiring a certain credit rating. For long-term and perpetual bonds, 13.90% will acquire the rating of ‘AAA’.

AAA	AA	A	BBB	BB	B	CCC	D
13.90%	2.49%	0.34%	0.76%	0.06%	0.06%	0.01%	82.37%

Since most bonds expire in 30 years, it would be unrealistic that the bond reaches a steady-state of the credit risk of ‘D’. In reality most bonds maintain good ratings in the period of 30 years.

3. As an investment manager you will have to advise your clients the probability of invested non-junk bonds (‘B’ or above) turning into junk bonds (‘CCC’ or below) in 30 years. What are the probabilities?

Because most bonds mature and expire in 30 years, we can assume this is the “short-term”. To find the short-term probability of the bond credit ratings, instead of using steady-state calculations, we have to use sequential Markov chains - up to 30 years. To calculate this, we will be using the transition matrix (normalized) table above and matrix multiplication.

At year 0, we can determine which credit rated non-junk bond to purchase (‘AAA’, ‘AA’, ‘A’, ‘BBB’, ‘BB’, or ‘B’). Then we set up a 30 year period for the bond. Starting from year 1 we will be using matrix multiplication to calculate the yearly changes in the transition probabilities - using the function {=MMULT([previous row transition probability], [normalized table])}. A complete table should look like this:

	AAA	AA	A	BBB	BB	B	CCC	D
0	0	0	0	0	0	0	0	0
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
5	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
6	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
7	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
8	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
9	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
10	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
11	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
12	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
13	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
14	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
15	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
16	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
17	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
18	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
19	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
20	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
21	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
22	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
23	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
24	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
25	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
26	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
27	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
28	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
29	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
30	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

To initialize the table, we will have to give a starting value for year 0 for each non-junk bond. For example, to determine the probability of an ‘AAA’ bond turning into ‘CCC’ or ‘D’, we give ‘AAA” an initial value of 1, while the other values are 0.

	AAA	AA	A	BBB	BB	B	CCC	D
0	1	0	0	0	0	0	0	0
1	91.98%	7.46%	0.48%	0.08%	0.00%	0.00%	0.00%	0.00%
2	87.66%	11.10%	0.94%	0.27%	0.01%	0.01%	0.00%	0.00%
3	85.26%	12.88%	1.25%	0.55%	0.03%	0.03%	0.00%	0.00%
4	83.85%	13.73%	1.45%	0.86%	0.05%	0.05%	0.01%	0.01%
5	82.96%	14.13%	1.57%	1.17%	0.07%	0.07%	0.01%	0.02%
6	82.35%	14.31%	1.64%	1.47%	0.09%	0.09%	0.01%	0.03%
7	81.89%	14.38%	1.69%	1.75%	0.11%	0.11%	0.02%	0.05%
8	81.53%	14.39%	1.73%	2.01%	0.13%	0.13%	0.02%	0.06%
9	81.22%	14.39%	1.76%	2.24%	0.14%	0.14%	0.02%	0.08%
10	80.96%	14.37%	1.78%	2.45%	0.16%	0.16%	0.03%	0.10%
11	80.73%	14.35%	1.80%	2.63%	0.17%	0.17%	0.03%	0.13%
12	80.52%	14.32%	1.81%	2.80%	0.18%	0.18%	0.03%	0.15%
13	80.33%	14.30%	1.82%	2.94%	0.19%	0.19%	0.03%	0.18%
14	80.16%	14.28%	1.83%	3.08%	0.20%	0.20%	0.04%	0.21%
15	80.00%	14.26%	1.84%	3.19%	0.21%	0.21%	0.04%	0.24%
16	79.86%	14.24%	1.85%	3.30%	0.22%	0.22%	0.04%	0.27%
17	79.73%	14.22%	1.86%	3.39%	0.23%	0.23%	0.04%	0.30%
18	79.61%	14.20%	1.87%	3.48%	0.23%	0.23%	0.04%	0.34%
19	79.50%	14.19%	1.87%	3.55%	0.24%	0.24%	0.04%	0.37%
20	79.39%	14.17%	1.88%	3.62%	0.24%	0.24%	0.04%	0.40%
21	79.30%	14.16%	1.88%	3.68%	0.25%	0.25%	0.04%	0.44%
22	79.21%	14.15%	1.89%	3.73%	0.25%	0.25%	0.04%	0.47%
23	79.13%	14.14%	1.89%	3.78%	0.26%	0.26%	0.05%	0.51%
24	79.05%	14.13%	1.89%	3.82%	0.26%	0.26%	0.05%	0.55%
25	78.98%	14.11%	1.90%	3.85%	0.26%	0.26%	0.05%	0.58%
26	78.91%	14.10%	1.90%	3.89%	0.26%	0.26%	0.05%	0.62%
27	78.85%	14.10%	1.90%	3.92%	0.27%	0.27%	0.05%	0.66%
28	78.79%	14.09%	1.90%	3.94%	0.27%	0.27%	0.05%	0.70%
29	78.73%	14.08%	1.90%	3.97%	0.27%	0.27%	0.05%	0.74%
30	78.67%	14.07%	1.91%	3.99%	0.27%	0.27%	0.05%	0.78%

the probability of ‘AAA’ bond transitioning to ‘CCC’ and ‘D’ in 30 years is 0.05% and 0.78% respectively

This process will be repeated five times for the remaining non-junk bonds to determine the 30 year transition probability.

The transition probability is compiled in the table below:

Non-Junk Credit Rating	Transition % to CCC	Transition % to D
AAA	0.05%	0.78%
AA	0.05%	1.06%
A	0.07%	3.48%
BBB	0.09%	6.09%
BB	0.08%	16.69%
B	0.08%	22.42%

The probabilities in the table show the total transition probability of a 30 year bond into ‘CCC’ and ‘D’ rating.

4a. Use Linear Programming to optimize the bond portfolio, achieving the highest 5-year coupon return with the consideration of initial risk (t=0).

Average the non-zero ratings only

Initial risk (t=0)

	S&P_norm	Moody's_norm	Fitch_norm	Average_risk
	0	0	6	6.00
	0	5	5	5.00
	0	0	6	6.00
No rating	0	6	6	6.00
	5	0	0	5.00

Firstly, the initial risk level is computed by taking the average of the risk ratings from the 3 individuals risk evaluators. Higher score means a better risk level, and the 0 score means there is no rating from the specific evaluator. We will only take the average of the non-zero value. The initial risk level is represented as an "Average_risk" column.

	A	I	AE	AF	AG	AH	AI	AJ	AK	AL	AM
1	Index	Average_risk	Seniority_norm	Min. Capital	Issuer	CPN	Indicative Offer	Per Lot Size	YTM	Lot	Expected Coupon Value in 5 Years
2	1	5.00	3	200,000	WESTWOOD GRP HOLD LTD	4.875%	101.59	1969	0.74%	0.00	$ -
3	2	5.00	3	200,000	YUEXIU REIT MTN CO	4.750%	101.59	1969	0.85%	0.00	$ -
4	3	5.00	3	200,000	FAR EAST HORIZON LTD	2.200%	100.32	1994	1.69%	0.00	$ -
5	4	6.00	3	200,000	HUARONG LEASING MGT HK	2.200%	100.87	1983	1.13%	0.00	$ -
6	5	5.00	3	200,000	SINO OCEAN LAND IV	2.500%	100.63	1988	1.58%	0.00	$ -

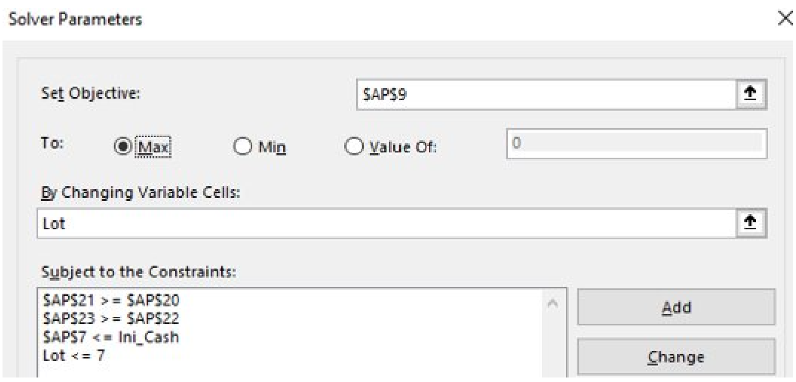
We used the bond_list sheet to construct the portfolio. The sheet includes the information of the bonds, such as the issuer, coupon payment CPN, etc. The initial risk level "Average_risk" column is included. We also added the "Lot" and "Expected Coupon Values in 5 Years" to the dataset, which means the specific buying quantity of the bond in the portfolio and the specific expected return in 5 years. Hence, we can start to build the portfolio template to apply solver.

Portfolio 1	
Optimized Portfolio	
Initial Cash	5000000
Invest Cost	=SUMPRODUCT(Indicative_Offer,Minimum_Size,Lot)
Total Coupon value	=SUM(Expected_Coupon_Value_5y)
Average credit risk	=SUMPRODUCT(Average_risk,Lot)/SUM(Lot)
Average seniority	=SUMPRODUCT(Seniority_norm,Lot)/SUM(Lot)
Credit Rating Statistic	
average	=AVERAGE(Average_risk)
sd	=STDEV.P(Average_risk)
Target total credit risk	=SUM(Lot)*(AP14+AP15)
Actual total credit risk	=SUMPRODUCT(Average_risk, Lot)
Target total seniority	=4*SUM(Lot)
Actual total sceniority	=SUMPRODUCT(Seniority_norm, Lot)

*This table is used to transform the non-linear constraint to fit to a LP solver, can be ignored

The portfolio template is illustrated above. The "Initial Cash" cell is the budget we have to construct the portfolio, the "Invest Cost" cell is the money we have spent in the portfolio, the "Total Coupon value" cell can be viewed as the portfolio expected return, the "Average credit risk" cell can be viewed as the portfolio risk, and the "Average seniority" is another quality control variable. It is notable that "Total Coupon value" and the "Average credit risk" are the most important variable of the portfolio template as they indicate the return and risk of the portfolio, which will be used to indicate the portfolio performance and strategy comparison.

The Credit risk statistic is used for computing the minimum risk level of the portfolio needed to achieve, which is a constraint in the later solver setting. The variables in the grey table are used to transform the non-linear functions like averaging involved in the solver into the linear functions in order to enable the simple_LP application to construct the portfolio. The grey table does not have significant meaning, which can be ignored. After constructing the portfolio template, we can move on to the solver setting to construct the optimized portfolio.



The setting of the solver is explained below:
objective: maximizing the return of the portfolio.
changing variable: the specific buying quantity of the bonds in the bond_list sheet.
constraint 1: risk >= +1SD of the entire bond list's risk rating, which is a risk control.
constraint 2: seniority level is <= 4, which is a quality control.
constraint 3: cost spent <= 5 million budget.
constraint 4: Each bond <= 7 lots in order to diversify the risk.

Portfolio 1		
Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	1,557,619.86
Average credit risk	$	5.81
Average seniority	$	4.00

Optimized Portfolio components			
Issuer	**Lot**	**Value**	**Weighting**
COUNTRY GARDEN HLDGS	7.00	1400559	28.0%
CITIC LTD	6.08	1216063	24.3%
KING POWER CAPITAL LTD	5.92	1183715	23.7%
ALPHABET INC	7.00	700163	14.0%
COUNTRY GARDEN HLDGS	2.50	499500	10.0%

After using the LP through solver, the optimized portfolio statistic is indicated above, which has 1,557,619 expected coupon return and 5.81 risk level respectively. The optimized portfolio is composed of 5 bonds. The bond from the COUNTRY GARDEN HLDGS accounts for the largest proportion of the portfolio with 28% weighting.

4b. Use LP to optimize the bond portfolio, achieving the highest 5-year coupon return with the consideration of initial risk (t=0) and **estimated future risk in the overall investment period (t=1-4) through the application of Markov chain.**

Rating\Year	1	2	3	4	5
8	7.91	7.86	7.83	7.80	7.78
7	7.34	7.51	7.60	7.64	7.66
6	6.40	6.61	6.73	6.83	6.90
5	5.18	5.37	5.56	5.73	5.89
4	4.40	4.69	4.93	5.14	5.32
3	3.40	3.80	4.14	4.42	4.65
2	3.01	3.56	3.87	4.07	4.21
1	1.00	3.56	3.87	4.07	4.21
0	0.00	0.00	0.00	0.00	0.00

rating-year table

(2) (2)

S&P_norm	Moody's_norm	Fitch_norm	Average_risk	S&P_1st	Moody's_1st	Fitch_1st	Average_risk_1st	S&P_2nd	Moody's_2nd	Fitch_2nd	Average_risk_2nd
0	5	5	5.00	0.00	5.18	5.18	5.18	0.00	5.37	5.37	5.37
5	5	0	5.00	5.18	5.18	0.00	5.18	5.37	5.37	0.00	5.37
5	0	0	5.00	5.18	0.00	0.00	5.18	5.37	0.00	0.00	5.37

(1) (1)

Taking the *rating-year table* computed from Markov chain in Q1, the risk ratings from the individual evaluators are adjusted accordingly to the future years (arrow 1). Then, we can take the average of that year's updated individual ratings to get the risk level of that specific year (arrow 2), like what we did in computing the initial risk level in part a. "Average_risk_1st" column means the average rating of the bonds in the next 1 year.

(3)

Average_risk	Average_risk_1st	Average_risk_2nd	Average_risk_3rd	Average_risk_4th	Average_risk_5y
5.00	5.18	5.37	5.56	5.73	5.37
5.00	5.18	5.37	5.56	5.73	5.37
5.00	5.18	5.37	5.56	5.73	5.37
6.00	6.40	6.61	6.73	6.83	6.51
5.00	5.18	5.37	5.56	5.73	5.37

By applying the same computation, we can get the average rating of the next 4 years of the bonds, which cover the whole investment period. And we will take the average of the 5 years risk average (arrow 3), indicated as **"Average_risk_5y"**, which can be interpreted as the **risk in the overall investment period (t=0-4).**

Portfolio 2	
Optimized Portfolio	
Initial Cash	5000000
Invest Cost	=SUMPRODUCT(Indicative_Offer,Minimum_Size,Lot)
Total Coupon value	=SUM(Expected_Coupon_Value_5y)
Average credit risk	=SUMPRODUCT($\boxed{\text{Average_risk_5y,}}$Lot)/SUM(Lot)
Average seniority	=SUMPRODUCT(Seniority_norm,Lot)/SUM(Lot)
Credit Rating Statistic	
average	=AVERAGE(Average_risk_5y)
sd	=STDEV.P(Average_risk_5y)
Target total credit risk	=SUM(Lot)*(AP14+AP15)
Actual total credit risk	=SUMPRODUCT(Average_risk_5y, Lot)
Target total seniority	=4*SUM(Lot)
Actual total sceniority	=SUMPRODUCT(Seniority_norm, Lot)

*This table is used to transform the non-linear constraint to fit to a LP solver, can be ignored

The only difference with part a is we are now using the overall risk level in the whole investment period (t=0-4) instead of the initial risk level (t=0) to construct the portfolio. The risk constraint in the solver becomes +1SD risk level in the whole investment period instead of the initial risk level. Beside this risk consideration difference, the other setting in the solver which applies LP to construct the optimized portfolio is exactly the same as part a.

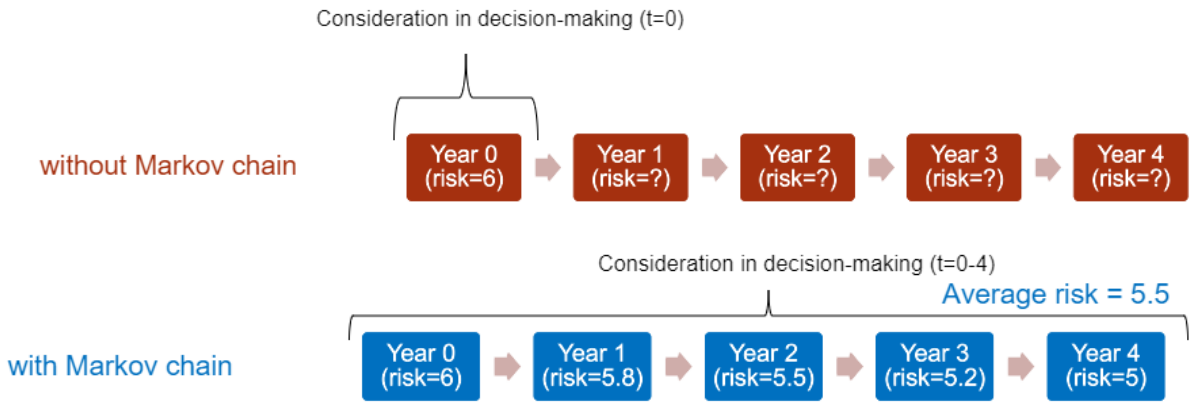
Portfolio 2		
Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	1,526,302.20
Average credit risk	$	6.29
Average seniority	$	4.00

The new optimized portfolio statistic is indicated above. The coupon return and the risk level of the optimized portfolio are 1,517,122 and 5.96 respectively. The optimized portfolio 2 has 30k lower expected coupon return but 0.48 better risk level than portfolio 1

Optimized Portfolio components			
Issuer	**Lot**	**Value**	**Weighting**
COUNTRY GARDEN HLDGS	7.00	1400559.16	28.0%
KING POWER CAPITAL LTD	7.00	1400724.78	28.0%
ALPHABET INC	7.00	700163.10	14.0%
CITIC LTD	4.44	887837.96	17.8%
COUNTRY GARDEN HLDGS	2.64	527266.70	10.5%
APPLE INC	0.83	83448.30	1.7%

Issuer	Porfolio 1 Weighting	Porfolio 2 Weighting	Change
COUNTRY GARDEN HLDGS	28.5%	28.0%	-0.5%
KING POWER CAPITAL LTD	24.1%	28.0%	3.9%
ALPHABET INC	14.2%	14.0%	-0.2%
CITIC LTD	24.7%	17.8%	-7.0%
COUNTRY GARDEN HLDGS	10.2%	10.5%	0.4%
APPLE INC	0.0%	1.7%	1.7%

The optimized portfolio component for portfolio 2 and the difference with the portfolio 1 are shown above. We can see the component does change significantly and include the new bond from APPLE INC in the new portfolio 2.



<u>What are the advantages to apply the Markov chain in bond investment?</u>
Originally we can only consider the initial risk level of the bonds to construct the portfolio. For example, in the 5 year investment period, we use the initial risk of the bond at t=0 to make a decision, however, the risk level in t=1-4 years might change significantly, which makes the initial risk being unable to represent the overall risk level in the whole investment period. Hence, the optimized portfolio is only valid in the initial timestamp but not the whole investment period as it does not take account of the future risk level. The optimized portfolio risk is less able to achieve the +1SD in the overall investment period.

With the use of Markov chain, the transition table of the risk rating provides us the estimation of the future risk level based on the initial risk level. Hence, we can consider the **risk in the whole investment period** to construct the portfolio instead of the initial timestamp risk level only. With the future risk consideration, the portfolio construction is more **forward-looking and comprehensive with better risk control**. The optimized portfolio risk is more able to achieve the +1SD in the overall investment period.

In conclusion, the advantage of Markov chain in bond investment is it can bring an **estimation of the future risk levels** of the bond based on its initial risk level in order to optimize our decision making.

5. Suppose instead an aggressive manager wants to maximize the coupon rate with the low quality bond (-1SD of your portfolio's credit rating), optimize the portfolio for this manager.

Generally, we perceive that higher risk is associated with greater probability of higher return. Therefore, suppose there is an aggressive manager instead, his investment decision would be investing in low-quality bonds, which the average risk of them is higher. To be specific, the average credit rating of the selected bonds should be at least 1 SD lower than that of the whole bond list.

As explained in Q4, we would apply Markov Chain in finding the optimized portfolio because of the aforementioned advantages. Thus, we obtain the average risk of these 5 years by taking average of the credit risks from t0 to t4. The average credit rating is 5.77 and the standard deviation is 0.52. Therefore, for his selected bonds, the average credit rating of them should be at least 1 SD lower than that of the whole bond list, which is 5.77-0.52 = 5.25.

Credit Rating Statistic	
average	5.77
sd	0.52
-1SD	5.25

Then, we can start to use Excel Solver to obtain the optimal portfolio.

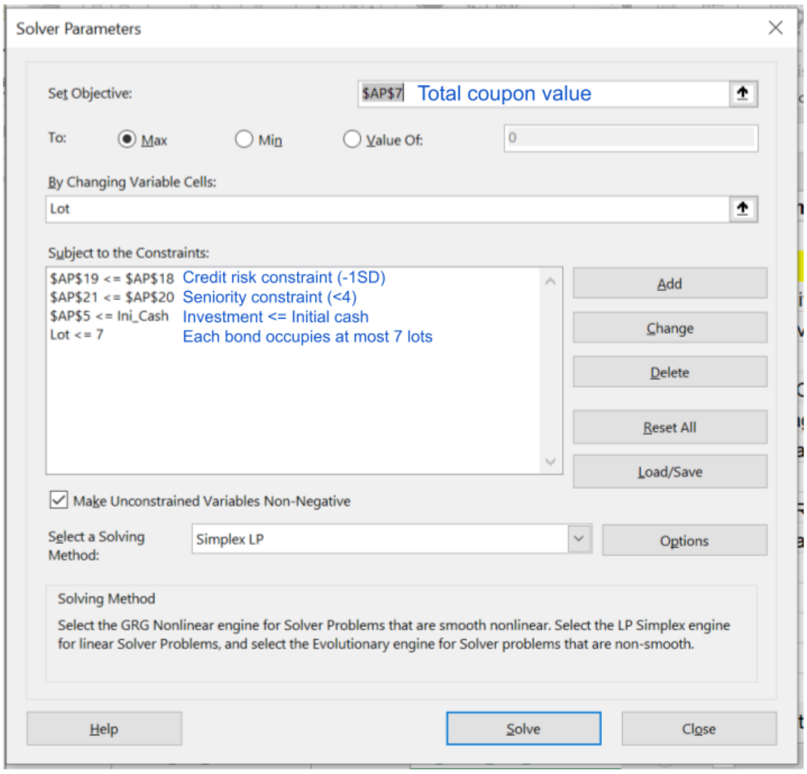
The objective is to maximize the total coupon value that he is expected to receive in 5 years time, by selecting which bonds to invest in.

Assume that the discount rate is zero and the bond price remains unchanged within 5 years and the payoff is the expected coupon value in 5 years.

While subject to the following constraints:

1. Initial cash is 5 million
2. Each bond can occupy at most 7 lots
3. The average credit rating should be at least **-1SD** of the entire bond list's credit rating (the credit rating is 5-years later after the markovian process)
4. The average seniority level is **lower** than 4
5. The portfolio will not be adjusted within 5 years

The solver setup would be similar to Q4, except two of the constraints above, 3. credit rating constraint and 4. seniority constraints are different.



Then, we can run the solver to obtain the following optimized portfolio. With an initial cash of $5,000,000, the total coupon value we are expected to receive in 5 years time is $1,689,614. Compared with the general risk strategy in Q4, the aggressive strategy has a higher return by taking a higher risk.

Optimized Portfolio	
Initial Cash	$ 5,000,000.00
Invest Cost	$ 5,000,000.00
Total Coupon value	$ 1,689,614.58
Average credit risk	5.25
Average seniority	4.00

With these 6 bonds selected.

Index	Issuer	CPN	Indicative Offer	Per Lot Size	YTM	Lot	Expected Coupon Value in 5 Years
18	BI HAI CO LTD	6.250%	96.47	2074	9.26%	1.28	$ 80,016.27
32	CREDIT SUISSE AG	6.500%	113.89	1757	1.23%	7.00	$ 455,238.26
39	COUNTRY GARDEN HLDGS	8.000%	108.68	1841	5.00%	7.00	$ 560,223.66
77	COUNTRY GARDEN HLDGS	7.250%	112.71	1775	4.55%	5.17	$ 375,217.07
85	VALE OVERSEAS LIMITED	6.250%	123.4	811	1.89%	7.00	$ 218,919.31
106	KRAFT HEINZ FOODS CO	4.625%	115.34	868	2.53%	2.07	$ 47,876.22

6. Suppose a conservative manager wants to maximize the coupon rate with the high quality bond (+2SD of the whole bond list's credit rating), optimize the portfolio for this manager. Compare your strategy with this manager.

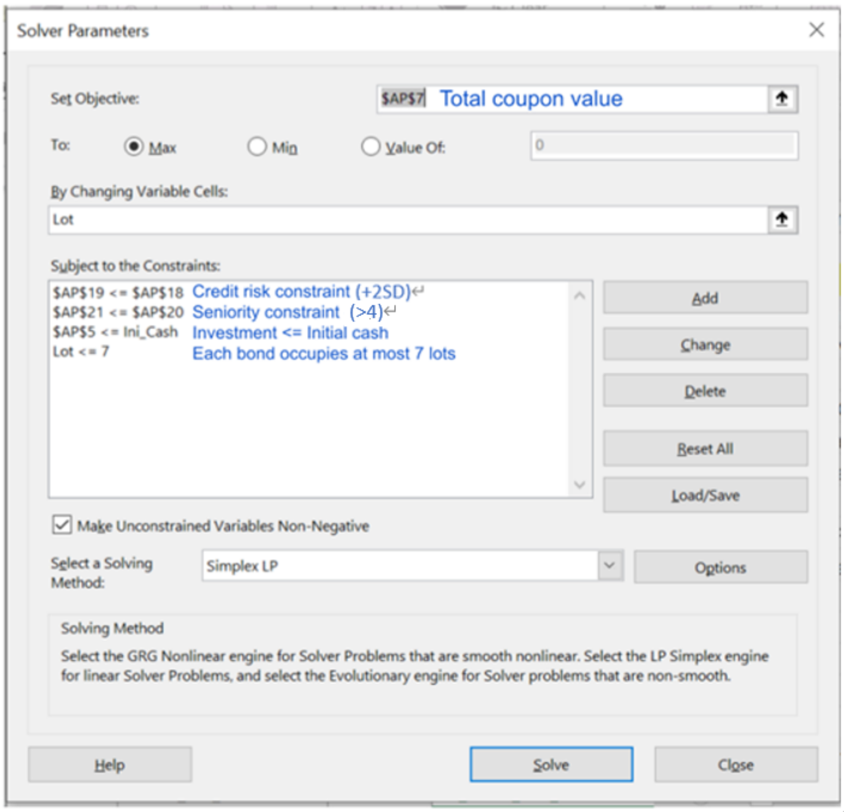
When the manager is conservative, his/her investment decision would be choosing a high quality bond with high credit rating, assuming 2SD higher than the entire bond list. As the portfolio's average credit rating is 5.77 with a standard deviation of 0.52, the credit rating of the chosen bond must be higher than 6.81 (= 5.77 + 2 *0.52).

After calculating the credit rating constraint, we can use Excel Solver to obtain our optimal portfolio together with other constraints:

1. Initial cash is 5 million.
2. Each bond can occupy at most 7 lots.
3. The average credit rating should be **+2SD** of the entire bond list's credit rating.
4. The average seniority level is **greater** than 4.
5. The portfolio will not be adjusted within 5 years.

Assume that the discount rate is zero and the bond price remains unchanged within 5 years and the payoff is the expected coupon value in 5 years.

The following is the solver setup with the constraints:



From running the solver, we get this optimized portfolio.

Index	Issuer	CPN	Indicative Offer	Per Lot Size	YTM	Lot	Expected Coupon Value in 5 Years
30	CITIC LTD	6.800%	111.65	1792	1.24%	4.35	$ 296,201.04
39	COUNTRY GARDEN HLDGS	8.000%	108.68	1841	5.00%	7.00	$ 560,223.66
53	KING POWER CAPITAL LTD	5.625%	114.87	1742	1.69%	7.00	$ 393,953.84
77	COUNTRY GARDEN HLDGS	7.250%	112.71	1775	4.55%	2.64	$ 191,134.18
86	ALPHABET INC	1.998%	107.9	927	0.59%	7.00	$ 69,946.29
121	APPLE INC	2.200%	107.99	927	1.24%	1.00	$ 11,011.74

We are expected to receive a total coupon value of $1,511,459.02 with an initial cash of $5,000,000 in this optimized portfolio.

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	1,511,459.02
Average credit risk	$	6.30
Average seniority	$	4.00
Credit Rating Statistic		
Average		5.77
SD		0.52
+2SD		6.81

7. Explain which strategy is the best performing strategy among questions 4 - 6?

Q4 General risk

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	1,557,619.86
Average credit risk		5.81
Average seniority		4.00

Q5 Aggressive (High risk)

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	1,689,614.58
Average credit risk		5.25
Average seniority		4.00

Q6 Conservative (low risk)

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	1,511,459.02
Average credit risk	$	6.30
Average seniority	$	4.00

The above suggested three strategies to optimize the bond portfolio. The first strategy (question 4) is a general risk diversified strategy. The second (question 5) is an aggressive strategy that aims at high return and bears higher risk at the same time, while the third one (question 6) is a conservative strategy that manager is choosing low risk, low return bonds.

Comparing these three strategies, the **aggressive strategy has the highest coupon return** ($1,689,614.58), followed by the general strategy ($1,557,619.86) and lastly the conservative strategy ($1,511,459.02). However, the **conservative strategy has the best risk level** (6.3), followed by the general strategy (5.81) and lastly the conservative strategy (5.25).

It seems that the aggressive strategy is the best performing strategy since it gives us the highest return by just looking at the total coupon value, Yet, the three portfolios are actually all the best with the highest return in their own risk levels. An aggressive strategy generates the highest return but with the worst risk level. On the other hand, a conservative strategy generates the lowest return but with the best risk level. It is suggested that **investors should choose the strategy according to their personal risk acceptance** so that it will be the best performing strategy in his/her risk tolerance level.

8. Suppose you are allowed to adjust your portfolio each year (dynamic strategy). Your portfolio is required to have an average credit rating of +1SD in every adjustment. Compare the dynamic strategy to the previous strategies.

There are assumptions that there is no adjustment cost of the portfolio, the coupon rate and discount rate is zero and the bond price remains unchanged within 5 years.

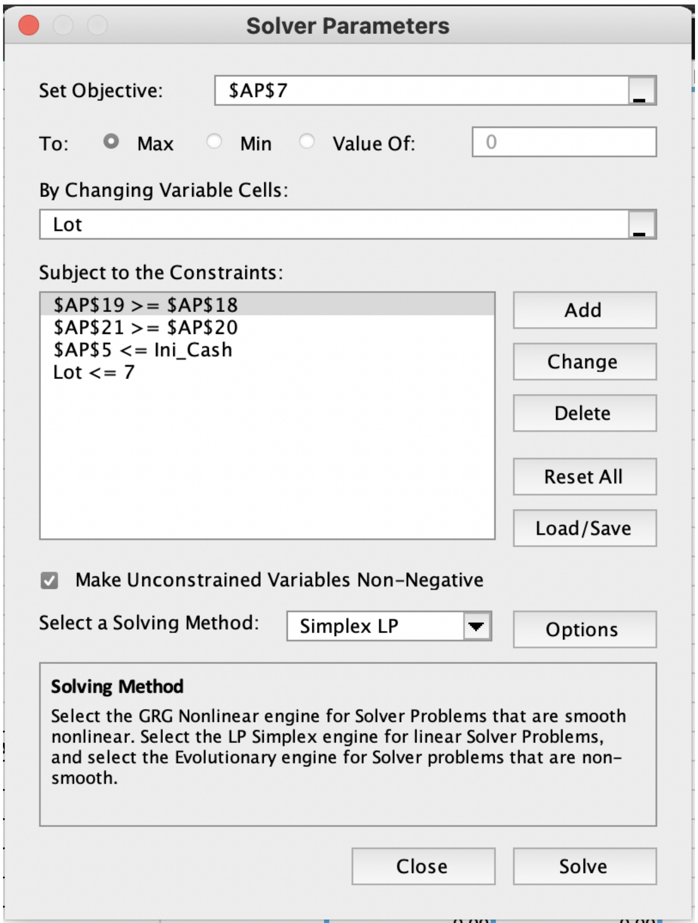
According to Markov Process, the credit rating will be changed every year. Therefore, it is reasonable to change the bond portfolio year by year to maintain the overall credit rating of the bond portfolio across different years.The average rating of the bond across the year is shown by the *rating-year table.*

Rating\Year	1	2	3	4	5
8	7.91	7.86	7.83	7.80	7.78
7	7.34	7.51	7.60	7.64	7.66
6	6.40	6.61	6.73	6.83	6.90
5	5.18	5.37	5.56	5.73	5.89
4	4.40	4.69	4.93	5.14	5.32
3	3.40	3.80	4.14	4.42	4.65
2	3.01	3.56	3.87	4.07	4.21
1	1.00	3.56	3.87	4.07	4.21
0	0.00	0.00	0.00	0.00	0.00

The next step is to calculate the mean and the standard deviation of bond for each year's bond credit rating. The statistics are summarized as follows, like what has been calculated during Question 1:

	Average	*Minimum*	*Maximum*	*Std. Dev*
Year 0	5.36	4.33	7.00	0.45
Year 1	5.61	4.66	7.34	0.53
Year 2	5.81	4.91	7.51	0.54
Year 3	5.97	5.14	7.60	0.51
Year 4	6.12	5.34	7.64	0.48

After that, by utilizing linear programming, various constraints are used to calculate payoff per year, which is the same as the setting in Q4.



The first constraint is set for controlling the average credit rating to be +1SD.
The second constraint is set for controlling the seniority level to higher than 4.
The third constraint is for limiting the investing cost to be lower than the initial capital.
The fourth constraint is for limiting the lot size per bond to be fewer than 7.

After setting up the constraints, the solver is run for five times. Each time the average credit risk and standard deviation are updated according to which year that is being calculated.

Then the expected coupon values for each year are summed up to get the total payoff.

Year	Pay-offs in each Year
Year 0	$ 311,523.97
Year 1	$ 304,747.14
Year 2	$ 303,156.52
Year 3	$ 293,681.76
Year 4	$ 291,467.03
Total Payoff	1,504,576.42

*Each year's detailed LP results are shown in the appendix.

By comparing the payoff between the dynamic strategy and the portfolio 1 and 2 in Q4, the **dynamic portfolio has a lower expected return**, which performs 3.41% lower than portfolio

1 ($1,557,619.86) and 0.82% ($1,517,122.89) lower than portfolio 2. Therefore, in terms of payoff, it is more profitable to keep the same portfolio across 5 years.

In terms of average bond credit risk, the bond crediting risk generated by the dynamic portfolio is (5.81+6.15+6.16+6.54+6.67)/5) = 6.27, which is 7.92% lower than portfolio 1 (5.81) and similar to portfolio 2 (6.29). The dynamic portfolio has a better bond credit.

Why does the dynamic strategy have a lower expected return comparatively?

Credit rating	Mean	SD	Above 1 SD
Year 0	5.357	0.450	5.807
Year 1	5.614	0.533	6.147
Year 2	5.810	0.538	6.348
Year 3	5.974	0.512	6.487
Year 4	6.118	0.478	6.596

Year	Pay-offs in each Year
Year 0	$ 311,523.97
Year 1	$ 304,747.14
Year 2	$ 303,156.52
Year 3	$ 293,681.76
Year 4	$ 291,467.03

Referring to the above statistics, it can be seen that the average credit rating for each year is becoming higher year by year. Therefore, if we modify the portfolio to fit the increasing average rating per year, it will generate less pay off because bonds with a higher credit rating will associate with a lower coupon rate, which is proved in Q7. Therefore, the portfolio will have no option to include more bonds with lower coupon rates but higher credit rating bonds to meet the new requirement of average credit rating +1SD per year. As a result, the portfolio generates lower coupon value.

In conclusion, the dynamic portfolio optimization strategy makes use of the linear programming and Markov chain technique, which is similar to the Q4b strategy. Although it is not proved to have a higher expected return, the dynamic adjustment still provides the solid flexibility to construct the portfolio according to the risk estimation in every year.

Summary

The key deliveries of the report are summarized below:

i. The application of Markov chain provides us the estimation of the overall investment period risk, which can replace the initial risk level in the decision making to construct the portfolio. It achieves a more forward looking and comprehensive portfolio with better risk control (Q4b).

ii. The expected return of the portfolio is positively correlated to its risk. The aggressive strategy achieves the highest return by taking more risk, and the conservative strategy achieves the lowest return by taking less risk. Investors should design the portfolio construction strategy according to their personal risk acceptance (Q5-7).

iii. The dynamic portfolio construction strategy achieves a lower expected return compared with the previous portfolio in Q4 surprisingly. The reason may be aligned with the improving risk level in each year, which lowers the expected return of the portfolio in every year afterward. However, it still offers flexibility in portfolio construction to best respond to the risk estimation in every year (Q8).

In conclusion, linear programming techniques allow the construction of the optimized portfolio with the highest expected return in different constraints like risk. By adjusting the constraints setting, we can experiment and test different investment strategies and compare the result. Moreover, the application of the Markov chain provides the estimation of the future risk level, which makes our decision making in bond investment more forward looking and comprehensive. We are excited to apply more decision analytics techniques or models in bond investment strategy to fight against the dynamic financial market in the near future.

References

Siu, T.K., Ching, W.K., Fung, E.S., Ng, M.K. (2005). On a multivariate Markov chain model for credit risk measurement. *Quantitative Finance, 5*, 543-556

Table 1. The transition probability table from Standard & Poor's (1999).

Asset/Year	AAA	AA	A	BBB	BB	B	CCC	D
AAA	0.9193	0.0746	0.0048	0.0008	0.0004	0.0000	0.0000	0.0000
AA	0.6400	0.9181	0.0676	0.0060	0.0006	0.0012	0.0003	0.0000
A	0.0700	0.0227	0.9169	0.0512	0.0056	0.0025	0.0001	0.0004
BBB	0.0400	0.0270	0.0556	0.8788	0.0483	0.0102	0.0017	0.0024
BB	0.0400	0.0010	0.0061	0.0775	0.8148	0.0790	0.0111	0.0101
B	0.0000	0.0010	0.0028	0.0046	0.0695	0.8280	0.0396	0.0545
CCC	0.1900	0.0000	0.0037	0.0075	0.0243	0.1213	0.6045	0.2369
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

Appendix

Question 8 LP results for each year.

Year 0

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	311,523.97
Average credit risk	$	5.81
Average seniority	$	4.00
Credit Rating Statistic		
average	$	5.36
sd	$	0.45

Year 1

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	304,747.14
Average credit risk	$	6.15
Average seniority	$	4.00
Credit Rating Statistic		
average	$	5.61
sd	$	0.53

Year 2

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	303,156.52
Average credit risk	$	6.16
Average seniority	$	4.00
Credit Rating Statistic		
average	$	5.81
sd	$	0.53

Year 3

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	293,681.76
Average credit risk	$	6.54
Average seniority	$	4.00
Credit Rating Statistic		
average	$	5.96
sd	$	0.59

Year 4

Optimized Portfolio		
Initial Cash	$	5,000,000.00
Invest Cost	$	5,000,000.00
Total Coupon value	$	291,467.03
Average credit risk	$	6.67
Average seniority	$	4.00
Credit Rating Statistic		
average	$	6.10
sd	$	0.56