BUYING A HOUSE

Debbie and George Calvert are thinking of making an offer to purchase a house in Shaker Heights, Ohio. Both George and Debbie saw the house this morning and fell in love with it. The asking price for the house is $400,000, and it has been on the market for only one day. Their broker told them that there were more than 20 potential buyers who saw the house that day. She also added that another broker told her that an offer on the house was going to be presented by that broker this afternoon. Their broker has advised them that if they decide to make an offer on the house, they should offer very close to the asking price of $400,000. She also added that if there are competing offers on the house that are close in value, then it is common practice for the seller to ask the potential buyers to submit their final offers the following day.

Trying to be objective about this decision, Debbie has decided to construct a decision tree to help her with this decision. She has assumed that the "fair market value" of the house under consideration is $400,000. She has assigned an "emotional value" of $10,000 if she and George are successful in purchasing the house. That is, whereas the fair market value of the house is $400,000, the house is worth $410,000 to Debbie and George. Thus, if they were to be successful in purchasing the house for $390,000, the value of this outcome would be $20,000. Of course, if they were not successful in purchasing the house, the value of this outcome would be simply $0. Debbie has also assigned a probability of 0.30 that they will be the only bidders on the house.

Debbie has decided to consider making one of only three offers: $390,000, $400,000, or $405,000. She estimates that if they are the only bidders, the probability that an offer of $390,000 is accepted is 0.40, the probability that an offer of $400,000 is accepted is 0.60, and the probability that an offer of $405,000 is accepted is 0.90.

If, however, there are other bidders, Debbie assumes that the seller will ask them to submit a final offer the following day. In such a scenario, she will then have to rethink what to do: She can withdraw her offer, submit the same offer, or increase her offer by $5,000. She feels that in the event of multiple bids, the probability that an offer of $390,000 is accepted is 0.20, the probability that an offer of $395,000 is accepted is 0.30, the probability that an offer of $400,000 is accepted is 0.50, the probability that an offer of $405,000 is accepted is 0.70, and the probability that an offer of $410,000 is accepted is 0.80.

Assignment:

(a) Structure Debbie and George's problem as a decision tree.

(b) Solve for Debbie and George's optimal decision strategy.

(c) Find the expected value of perfect information about whether they will be the only bidders on the house.

Eric Agency, Inc.

Mr. Eric Wong operates a real estate agency that specializes in selling properties that are hard to sell. He is approached one day by a prospective client who has three properties he wishes to sell. The client indicates the prices he wishes to receive for these properties as follows:

Property	**Price** ($000s)
A	$ 2,500
B	5,000
C	10,000

Eric would receive a commission of 4% on any of the properties he is able to sell.

The client laid down the following conditions for an exclusive listing:

> Eric, you have to sell the property A first. If you can't sell it within a month, the entire deal is off--no commission and no chance to sell the other properties. If you sell property A within a month, then I'll give you the commission for it and the option of (a) stopping at this point; or (b) trying to sell either B or C next under the same conditions (i.e., sell within a month or no commission on the second property and no chance to sell the third property). If you succeed in selling the first two properties, you will also have the option of selling the third.

After the client has left, Eric proceeds to analyze the proposal to determine whether or not to accept it. He figures his selling costs and his chances of selling each property at the set prices to be:

Property	**Cost** ($000s)	**Probability of Sale**
A	$80	.7
B	20	.6
C	40	.5

He believes that sale of a particular property would not make it any more or less likely that the two remaining properties could be sold. Selling costs would have to be incurred whether or not a particular property is sold but could be avoided by deciding not to attempt to sell the property. Assume that Eric has the option of not selling any property for the client.

Questions:

1. Draw a decision tree to find Eric's best choices.
2. Find the expected values of perfect information, respectively, about (i) whether A can be sold at the set price; (ii) whether B can be sold at the set price; (iii) whether C can be sold at the set price. In each case, assume that the perfect information is revealed before Eric makes any decision.

Brainy Business

(Adapted from the case "Brainy Business" in Hillier and Lieberman Book[1])

Charlotte Rothstein, CEO, major shareholder, and founder of Cerebrosoft, is contemplating the decision she faces regarding her company's newest proposed product, Brainet. This has been a particularly difficult decision. Brainet might catch on and sell very well. However, Charlotte is concerned about the risk involved. In this competitive market, marketing Brainet also could lead to substantial losses. Should she go ahead anyway and start the marketing campaign? Or just abandon the product? Or perhaps buy additional marketing research information from a local market research company before deciding whether to launch the product? She has to make a decision very soon.

Later in the afternoon, Charlotte meets with Reggie Ruffin, a jack of all trades and the production manager. Reggie has a solid track record in his field and Charlotte wants his opinion on the Brainet project.

"Well. Charlotte, quite frankly, I think that there are three main factors that are relevant to the success of this project: competition, units sold, and cost—ah, and, of course, our pricing. Have you decided on the price yet?"

"I am still considering which of the three strategies would be most beneficial to us. Selling for $50.00 and trying to maximize revenues—or selling for $30.00 and trying to maximize market share. Of course there is still your third alternative; we could sell for $40.00 and try to do both."

At this point, Reggie focuses on the sheet of paper in front of him. "And I still believe that the $40.00 alternative is the best one. Concerning the costs, I checked the records; basically we have to amortize the development costs we incurred for Brainet. So far we have spent $800,000 and we expect to spend $5 per unit for support after sales."

Reggie next hands a report to Charlotte. "Here we have some data on the industry. I just received that yesterday, hot off the press. Let's see what we can learn about the industry here." He shows Charlotte some of the highlights. Reggie then agrees to compile the most relevant information contained in the report and have it ready for Charlotte the following morning. It takes him long into the night to gather the data from the pages of the report, but in the end he produces three tables, one for each of the three alternative pricing strategies. Each table shows the corresponding probability of various amounts of sales given the level of competition (high, medium, or low) that develops from other companies.

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[1] *Introduction to Operations Research*, by F. Hillier and G. Lieberman, 9th edition, Case 15.1, pp720-722

TABLE 1 Probability distribution of unit sales, given a high price ($50)

	Level of Competition		
Sales	**High**	**Medium**	**Low**
50,000 units	0.2	0.25	0.3
30,000 units	0.25	0.3	0.35
20,000 units	0.55	0.45	0.35

TABLE 2 Probability distribution of unit sales, given a medium price ($40)

	Level of Competition		
Sales	**High**	**Medium**	**Low**
50,000 units	0.25	0.30	0.40
30,000 units	0.35	0.40	0.50
20,000 units	0.40	0.30	0.10

TABLE 3 Probability distribution of unit sales, given a low price ($30)

	Level of Competition		
Sales	**High**	**Medium**	**Low**
50,000 units	0.35	0.40	0.50
30,000 units	0.40	0.50	0.45
20,000 units	0.25	0.10	0.05

The next morning, Charlotte meets with Reggie again in her office. When Reggie enters the office, he immediately bursts out: “Guys, I just spoke to our marketing research company. They say that they could do a study for us about the competitive situation for the introduction of Brainet and deliver the results within a week.”

“How much do they want for the study?”

“I knew you'd ask that, Charlotte. They want $10,000, and I think it's a fair deal.”

“Do we have any data on the quality of the work of this marketing research company?”

“Yes, I do have some reports here. After analyzing them, I have come to the conclusion that the predictions of the marketing research company are pretty good: Given that the competition turned out to be high, they predicted it correctly 80 percent of the time, while 15 percent of the time they predicted medium competition and 5 percent of the time they

predicted low competition in that setting. Given that the competition turned out to be medium, they predicted high competition 15 percent of the time, medium competition 80 percent of the time, and low competition 5 percent of the time. Finally, for the case of low competition, the numbers were 90 percent of the time a correct prediction, 7 percent of the time a 'medium' prediction and 3 percent of the time a 'high' prediction.”

Charlotte feels that all these numbers are too much for her. “Don't we have a simple estimate of how the market will react?”

"Some prior probabilities, you mean? Sure, from our past experience, the likelihood of facing high competition is 20 percent, whereas it is 70 percent for medium competition and 10 percent for low competition” says Reggie, his numbers always ready when needed.

All that is left to do now is to sit down and make sense of all this…

Charlotte has to decide what to do with Brainet. Should they launch the product? If so, at what price? Should they do the market research?

Questions:

1. For the initial analysis, ignore the opportunity of hiring the marketing research company. Formulate the decision problem in a decision tree. What is Charlotte’s optimal decision if she uses Bayes’ decision rule?

2. Now consider the possibility of doing the marketing research. Develop the corresponding decision tree. Should Cerebrosoft pay $10,000 for the marketing research? What is the overall optimal policy?