Homework 7

I. Suppose that $X = \sigma Z$ has a mixed normal distribution, where (1) σ is a random variable that takes values 1 and 4 with equal probabilities 0.5; (2) $Z \sim N(0,1)$; and (3) σ and Z are independent. Compute the mean, variance and kurtosis of X.

(Hint: $E(Z) = 0,\ E(Z^2) = 1,\ E(Z^4) = 3.$)

II. Suppose the monthly returns X_t (in percentages) of a portfolio follows a GARCH Model:

$$X_t = \sigma_t W_t,$$

where W_t are i.i.d. standard normals, and

$$\sigma_t^2 = 0.2 + 0.09X_{t-1}^2 + 0.9\sigma_{t-1}^2.$$

Recall that the VaR at level α of a portfolio is the negative of the α-quantile of the return distribution.

(1). If $X_t = 1.5$ and $\sigma_t^2 = 0.5$, find the (conditional) 2.5% VaR for the next month.

(2). Suppose instead we model monthly returns as i.i.d. normals with mean zero and variance given by the unconditional variance implied by the GARCH model. Find the 2.5% VaR under the i.i.d. normal assumption (this approach is commonly used in practice).

(3). Compare the two VaR's obtained in Parts (1) and (2). Comment.

III. Download the dataset `HSI_2021.csv` from the course website, which contains the Hang Seng Index values in 2021. Conduct the following analysis parallel to what we did in class for SP500 Index in 2021.

1. Draw the time series plots of the index values and the returns, comment;
2. Draw the acf plots of returns and squared returns, comment;
3. Determine the order of ARCH using both pacf and aic;
4. Fit GARCH(1,1) to the returns (excluding the last 5 entries), check whether the mean μ is significant at 5% level;
5. Refit GARCH(1,1) without mean to the returns, and conduct model diagnostics;
6. Give the one-day-ahead volatility forecast;
7. Under conditional normal assumption, estimate the one-day-ahead 1%-VaR and 1%-expected shortfall.