CMDA 3605 Final Project Part 2

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Question 1

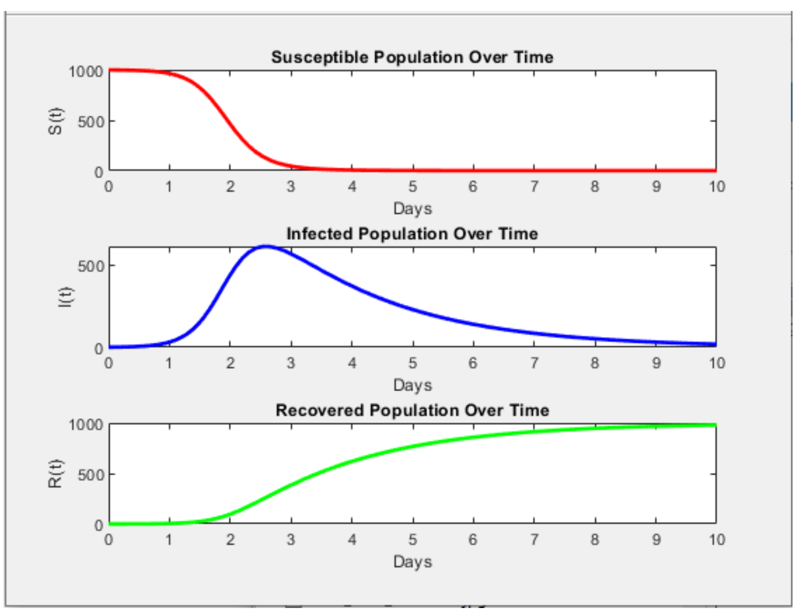
$$\frac{dS}{dt} = -\beta S(t)I(t)$$

$$\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t)$$

$$\frac{dR}{dt} = \gamma I(t)$$

Question 2

The graph models an outbreak. It starts with 999 susceptible people and jumps to a peak between days 2 and 3. By day 4 there are no more susceptible people so the infected population slowly starts to decrease as people recover.



Question 3

Divide $\frac{dI}{dt}$ by $\frac{dS}{dt}$:

$$\frac{dI}{dS} = \frac{\beta \cdot S \cdot I - \gamma \cdot I}{\beta \cdot S \cdot I}$$

Move dS over:

$$\frac{dI}{dS} = -1 + \frac{\gamma}{\beta S}$$

Integrate both sides:

$$dI = (-1 + \frac{\gamma}{\beta S})dS$$

Integrate:

$$I + c_1 = -S + \frac{\gamma}{\beta} \ln S + c_2$$

Combine constants:

$$K = -S + \frac{\gamma}{\beta} \ln S - I$$

Question 4

We take the model at the two given points and set them equal to eachother, as K is a constant.

For the point (S_0, I_0):

$$K = -S_0 + \frac{\gamma}{\beta} \ln S_0 - I_0$$

For the point $(S_\infty, 0)$:

$$K = -S_\infty + \frac{\gamma}{\beta} \ln S_\infty$$

Set equal:

$$(-S_0 + \frac{\gamma}{\beta} \ln S_0 - I_0) - (-S_\infty + \frac{\gamma}{\beta} \ln S_\infty) =$$
$$- S_0 + \frac{\gamma}{\beta} \ln S_0 - I_0 + S_\infty - \frac{\gamma}{\beta} \ln S_\infty$$

Combining like terms:

$$S_\infty - S_0 + \frac{\gamma}{\beta}(\ln S_0 - \ln S_\infty) - I_0 = 0$$

Simplify:

$$\frac{\gamma}{\beta}(\ln S_0 - \ln S_\infty) = S_0 - S_\infty + I_0$$

Simplify:

$$\frac{\gamma}{\beta} = \frac{S_0 - S_\infty + I_0}{\ln S_0 - \ln S_\infty}$$

Question 5

We use the same logic as for question 4, but with $S_m.I_m$.

For the point (S_0, I_0):

$$K = -S_0 + \frac{\gamma}{\beta}\ln S_0 - I_0$$

For the point (S_m, I_m):

$$K = -S_m + \frac{\gamma}{\beta}\ln S_m - I_m$$

Set equal:

$$-S_0 + \frac{\gamma}{\beta}\ln S_0 + I_0 = -S_m - \frac{\gamma}{\beta}\ln S_m - I_m$$

Take I_m out

$$I_m = -S_m + \frac{\gamma}{\beta}\ln S_m - I_0 + S_0 - \frac{\gamma}{\beta}lnS_0$$

Swap out $\frac{\gamma}{\beta}$ for S_m

$$I_m = -\frac{\gamma}{\beta} + \frac{\gamma}{\beta}\ln\frac{\gamma}{\beta} + I_0 + S_0 - \frac{\gamma}{\beta}S_0$$

Question 6

1. Look at data on the third row of the graph. S_∞ is 19 as it ends with 19 students not getting sick.

$$S_3 = 738, I_3 = 25, S_\infty = 19$$

2. I have the formula for $\frac{\gamma}{\beta}$ in question 4, so just flip the formula

$$\frac{\gamma}{\beta} = \frac{S_0 - S_\infty + I_0}{\ln S_0 - \ln S_\infty}$$

$$\frac{\beta}{\gamma} = \frac{\ln S_0 - \ln S_\infty}{S_0 - S_\infty + I_0}$$

$$\frac{\beta}{\gamma} = \frac{ln\frac{760}{19}}{760 - 19 + 3}$$

$$\frac{\beta}{\gamma} = 0.004958$$

3. Use mean time infected to find $\frac{1}{\gamma}$

$$2.1 = \frac{1}{\gamma}$$

$$\gamma = \frac{1}{2.1}$$

Now use that to find β using part 2

$$0.0049568 = \frac{\beta}{\gamma}$$

$$\beta = 0.004958 * \frac{1}{2.1} = 0.00236$$

4. Use formula from question 5

$$I_{max} = I_m = -\frac{\gamma}{\beta} + \frac{\gamma}{\beta} \ln \frac{\gamma}{\beta} + I_0 + S_0 - \frac{\gamma}{\beta} S_0$$

$$\frac{\gamma}{\beta} = \frac{\frac{1}{2.1}}{0.00236} = 201.7756$$

Plug in values: I_0 is 3, so S_0 is 760(763-3)

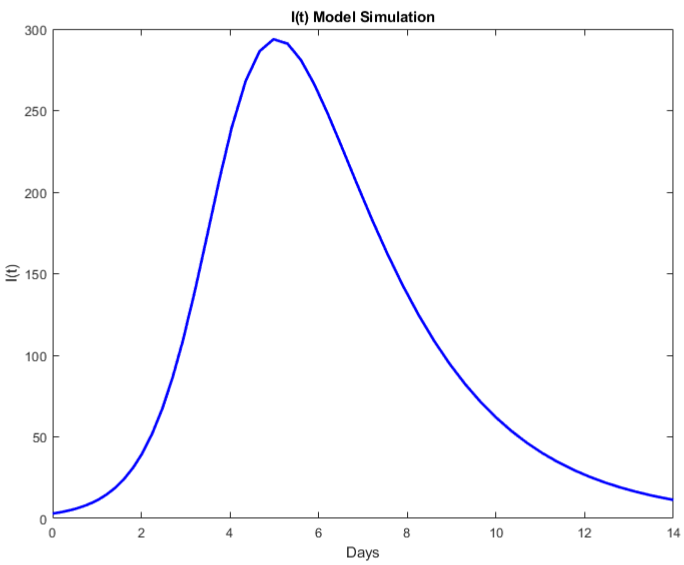
$$-201.7756 + 201.7756 \ln 201.7756 + 3 + 760 - 201.7756 \ln 760$$

Calculate

$$I_m = 293.637$$

Yes, max is 296 in the set so it about checks out. Most likely off by 2.4 due to rounding error or calculator.

5. The graph seems to fit the model pretty well. It starts at 3 people, then slowly jumps up to just over 300 at day 6, then goes down to around 7.



Question 7

The values for γ and β I got were 4.454651e-01 and 2.223842e-03 respectively. This is pretty close to the numbers in question 6C, with the γ being off by about 0.04 and the β being off by about 0.142