## LINEAR LEAST GOUARES (LLS)

- FINDING SOL, FOR LINEAR REGRESSION

(BATCH SETTING)

BY SOLVING THE QUADRADIC EQUATIONS

 $(x\bar{w}-\bar{y})^{T}(x\bar{w}-\bar{y})$ 

n-IN ROW
IS INSTANCEN

$$= \left(\overline{w}^{T}X^{T} - \overline{g}\right)^{T} \left(X\overline{w} - \overline{g}\right)$$

= - w'x'xw-wxy-yxw+5'y

XTX W = XTG NORMAL EQUATIONS

XW=Y MIGHT NOT HAVE SOLUTION

BUT NORMAL EQATIONS ALWAYS HAVE SOLUTION (Why?)

$$X^{T}X\overline{w}^{X} = X^{T}y$$

$$\overline{w}' = (X^{T}X)^{T}X^{T}y$$

$$dxI dxNNd dxNNxI$$

PROBLEM: XTX MIGHT NOT HAVE FULL RANK

$$v(X^{T}X) = v(X)$$

FIX 1: REGULARIZE

nif ( > 11w112 + 11 × w - g 112 ) & REGRESSION

ALWAYS

EULL RANK

Secrement to the second	
	FIX7; PSEUDO INVERSE
- Paris	DIGRESSION
	MATRIX DECOMPOSITION:
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SAULATOODSSÄRSOOMI	ui's ARE ORTHOGONAL:

$$u_i \cdot u_j = \begin{cases} 0 & \text{if } i \neq j \\ i & \text{if } i \end{cases}$$

Usut

= U Z si eiei U

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Uj IS EIGENVEC j

(Z si ui ui) uj = Z si ui (ui uj)

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EIGENVALUE

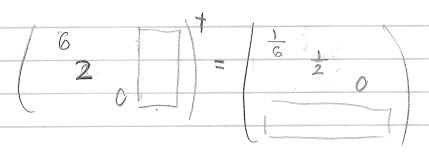
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## PSEUDO INVERSE OF X

1 INVERSE OF NON-ZERO ELMT'S OF DIAL

LEAVEO'S AS 0'S



CHECK NORMAL EQUATIONS !

VS'U'USV VS'U'Ğ

dxd dxN Nxd dxN NxN NxI

$$= \bigvee_{\substack{S \\ X}} S^{T} U^{T} S^{T}$$

CONNECTION BETWEEN

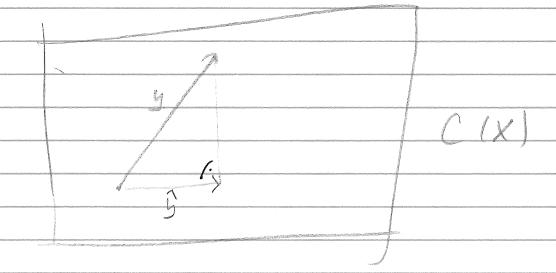
FIX1 & FIX2

 $\lim_{X \to 0} (X^T X + \lambda I)^{-1} X^T = X^+$ 

min || Xw-y||<sup>2</sup>
w

G COLUMN SPACE OF X C(X)

OPT 9 19 CLOSEST VECTOR TO 4 THAT LIES IN C(X)



G is PROJECTION OF Y ONTO C(X)

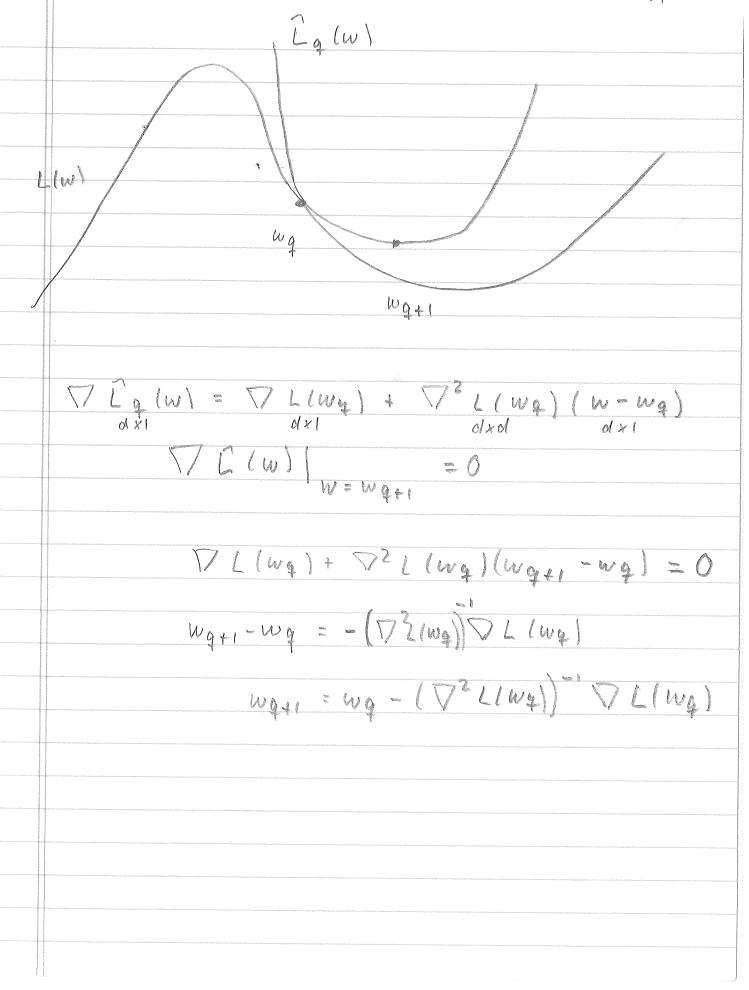
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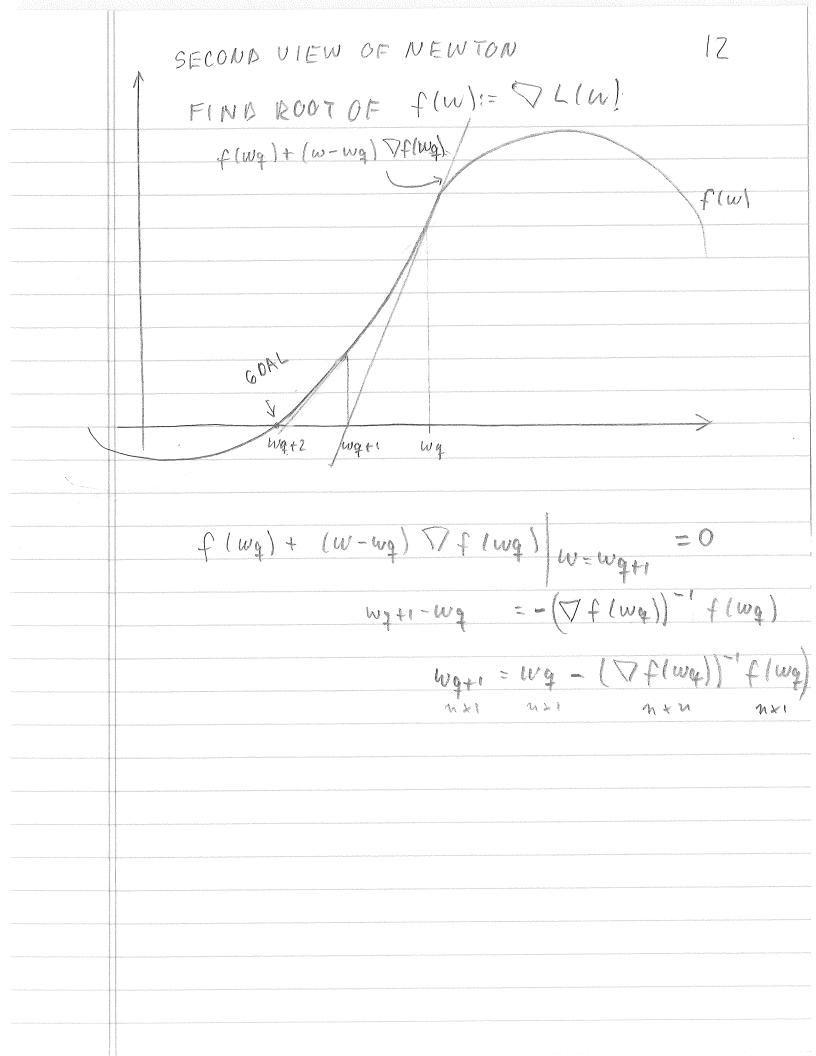
NON-ZERO EIGEN
- Zuihi VAL BREGMAN PROJURT 11 112

Recall 60 update for minimizing
$L(w) = \sum_{n} L_{yn}(w \cdot x_n)$
Wate = ma - NZL'yn (mg *xn) Xn
Motivation:
water = arginf(±11 w-wall² + y \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \
10 = 0 10 = wq+1
Wg+1-Wq+NZLyn(Wg+1-Xn) Xn
Wgt1 = wy = y Z Lyn (wgti Xu) Xn
APPROXIMATION DE Lyn (wg. Xn) Xn
EXPLICIT UPPATE
IMPLICIT USES GRADIENT OF LOSS AT CURRENT ITERATION EXPLICIT " LAST "

	PRECISE WAY TO MOTIVATE EXPLICIT UPDATE:  water = arguif (Nw-wgll2+y (L(wg)+ (w-wg) ALIwg))					
	INERTIA TERM FIRST GROEK APPROX.					
	NEENEN BECAUSE OF BATCH LOSS					
	LOSS LINEAR AT WY					
	BATCH GD. US STOCHASTIC GD BASED ON SINGLE EXAMPLE					
TIME	I ITERATION & I PASS					
	1					
	converges facter					
	becomse it uses					
	"more recent" gradients					
	IDEA: WHY NOT USE ZND ORDER APPROXIM.					
	OF LOSS					
Control of the contro						
To the second se						

 $L(w) \approx L(wq) + (w-wq)^{T} \nabla L(wq)$ + 2 (W-wg) 572 L(wg) (W-wg) quadradic approx. of L(w) at wa [ q (w) Water = argunin ( q (w) (1)WITH INERTIA TERM : wy+1 = argmin 11w-wg112+ y Lg(w) (Z) (1) SIMPLER FOCUS ON (1) FIRST





	NEWTON W. REGULARIZATION
	$w_{q+1} = wif \left(\frac{2}{2}   w-w_q  ^2 + L(w_q) + (w-w_q)^T \nabla L(w_q) + \frac{1}{2}   w-w_q ^T \nabla^2 L(w_q) (w-w_q)$
	> (w-wg) + 7L(wg) + 72L(wg) (w-wg)
	$\nabla u \mid w = wq + 1 = 0$
	$\nabla L(wq) + (\nabla^2 L(wq) + \lambda I)(wq+, -wq) = 0$
	Wate = wa - (72 L(ma) + xI) TL(ma)
	k ,,
The state of the s	L'LS = I STEP OF NEWTON
	RIDGE REGRESSION
	= 1 STEP OF REG. NEWTON
et een neem en de	

	NEWTON-RAPSON FOR LOGISTIC REGRESSION
	BATCH LOGS L(W) = Z Lyn(w. Xu)
	WHERE Ly(a) = ln(1+ea) - ya
	$\nabla L(w) = X^{T}(\hat{g} - y) = \sum_{n} x_{n} (\hat{g}_{n} - g_{n})$
X HAS	EXAMPLE AS ROWS (S.) (S.)
	$\nabla_{w} \times (\delta(w \cdot x) - y)$
	$= x \times 6'(a)$ = $x \times 6(a) (1-6(a))$
	a a a a a a a a a a a a a a a a a a a
	$5(a) = \frac{e^{\alpha}}{1 + e^{\alpha}}$ $5'(a) = \frac{e^{\alpha}(1 + e^{\alpha}) - e^{2\alpha}}{(1 + e^{\alpha})^2}$
	= 5 (a) ((-6 (a))

DPL(w)  $= \bigvee X^{T}(\widehat{y}-y)$ = 17 Z ×n (gn-yn) = 2 xx 6(wxn) (1-6(wxn)) 4. (1-Gu) LINEAR REGR. 772 L(w) = XTX R depends on weight vector

$$w_{q+1} = w_{q} - (X^{T}R_{q}X)^{-1}X^{T}(\hat{y}_{q} - y)$$

$$= (X^{T}R_{q}X)^{-1}(X^{T}R_{q}X w_{q} - X^{T}(\hat{y}_{q} - y))$$

$$= (X^{T}R_{q}X)^{-1}X^{T}R_{q}Z_{q}$$

-> CALLED ITERATED REWEIGHTED

MANY NUMERICAL IGSUES

EXPLICITY GD USES OLD GRADIENTS

NEWTON " + HESSIANS

IMPLICIT GD USES CURRENT GRADIENTS

Conjecture: IMPLICIT GB BEATS NEWTON