Forward Kinematics

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Google DeepMind

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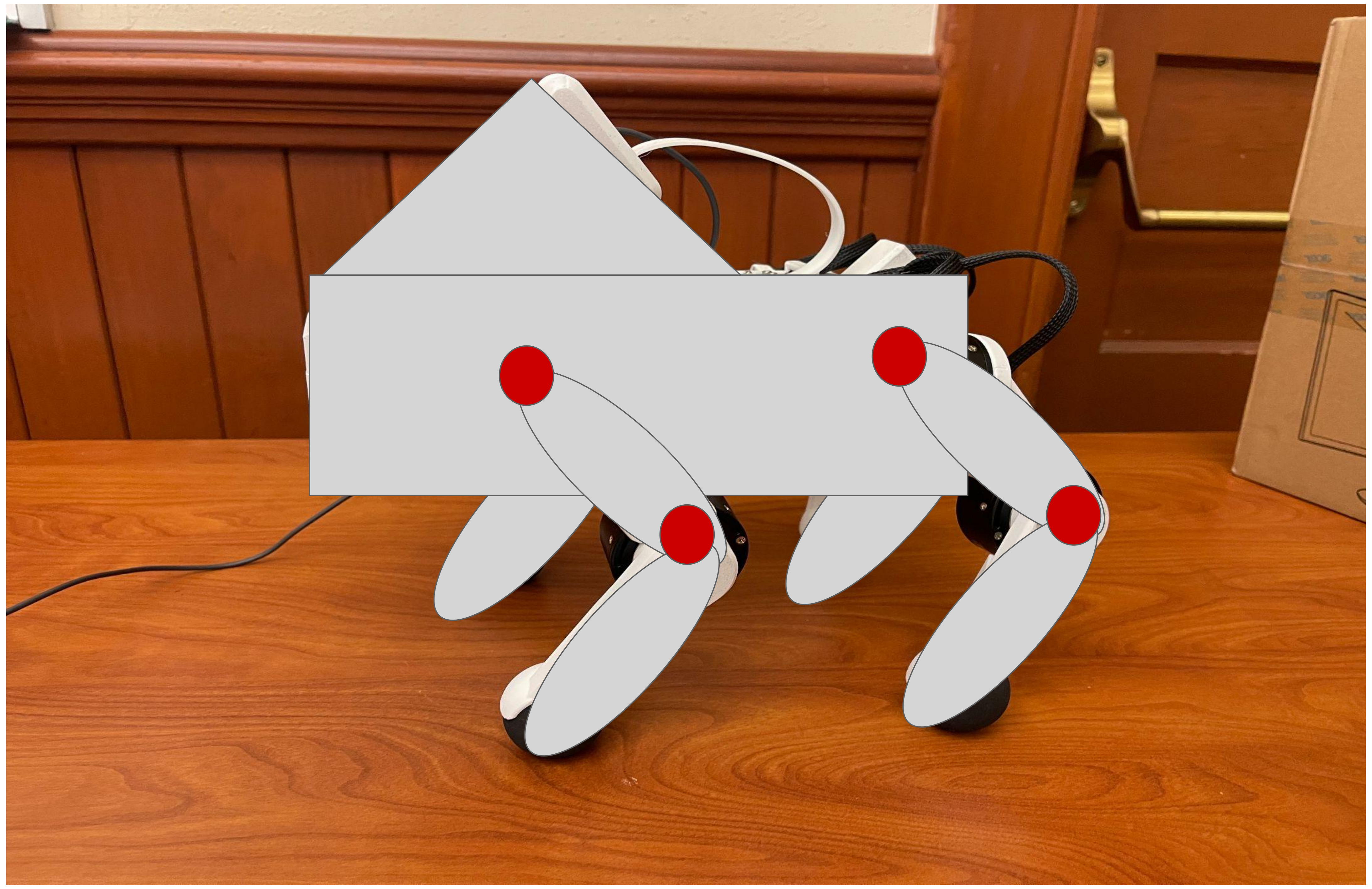
Goal

What is Forward Kinematics (FK)?

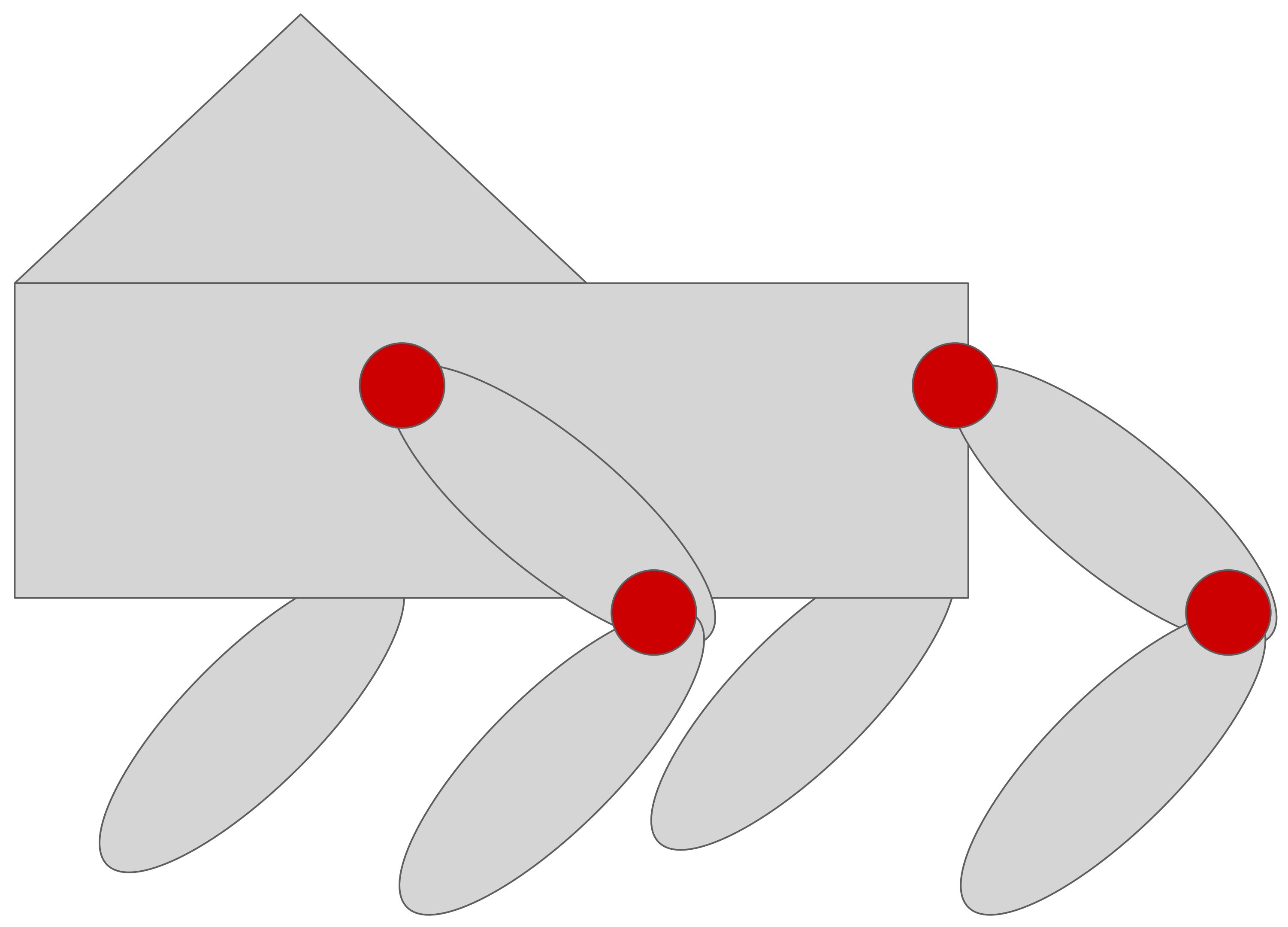
Why is FK important in robotics?

How to calculate FK?

Problem Statement

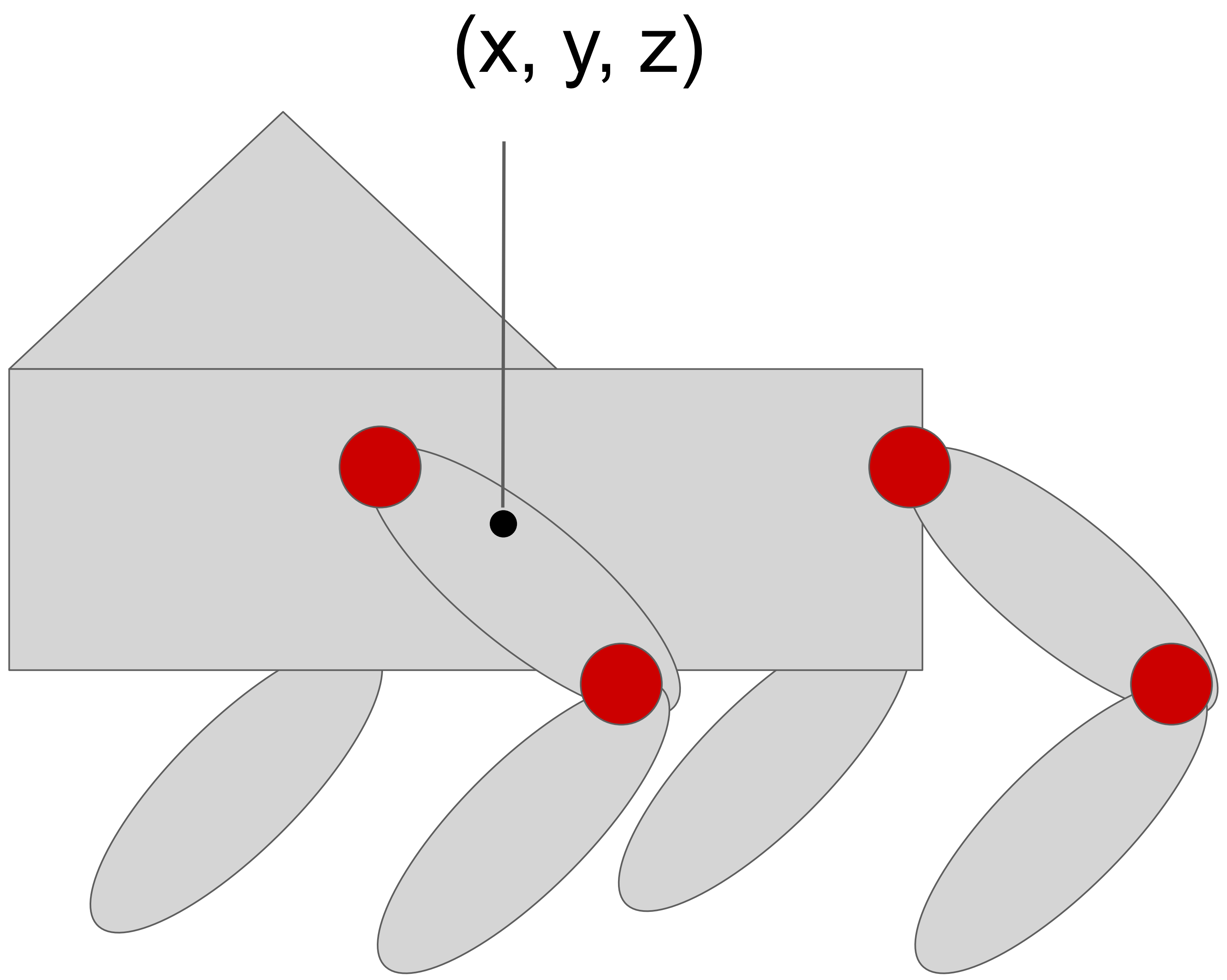


Problem Statement



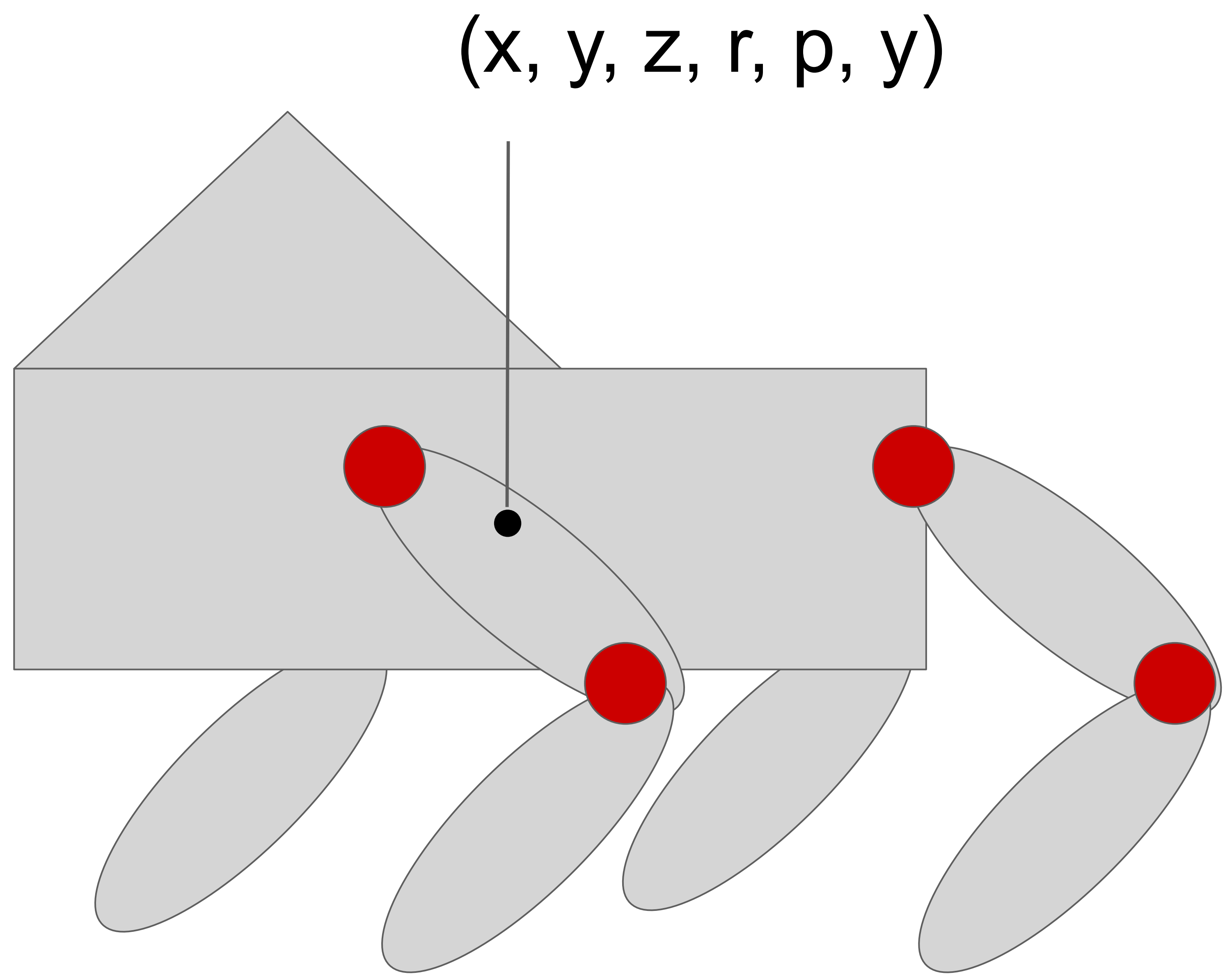
Pose: $\mathbf{q}$ = ?

Problem Statement



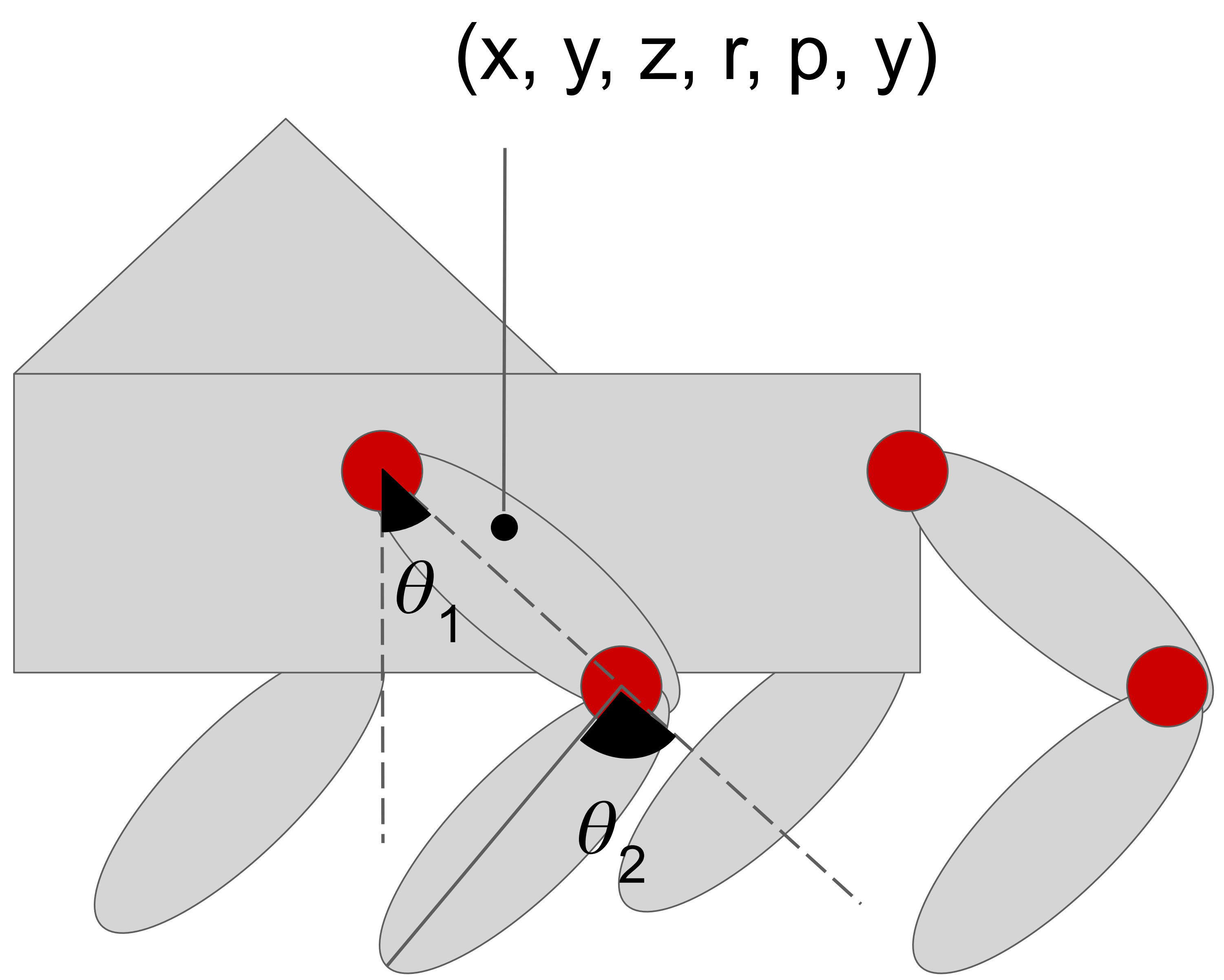
Pose: $\mathbf{q}$ = [x, y, z, …

Problem Statement



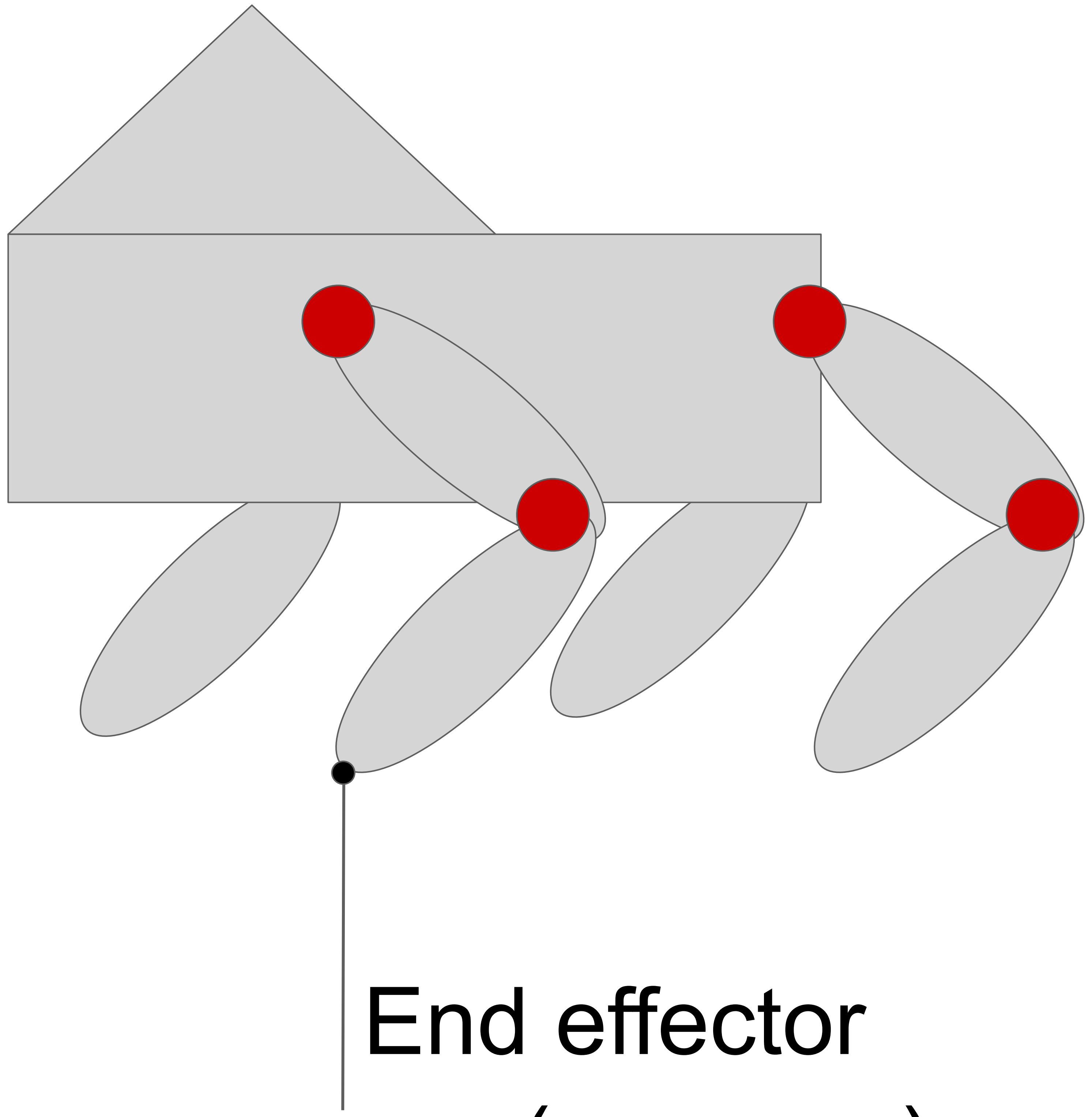
Pose: $\mathbf{q}$ = [x, y, z, r, p, y, …]

Problem Statement



Pose: $\mathbf{q}$ = [x, y, z, r, p, y, θ_1, …, θ_{12}]

Problem Statement

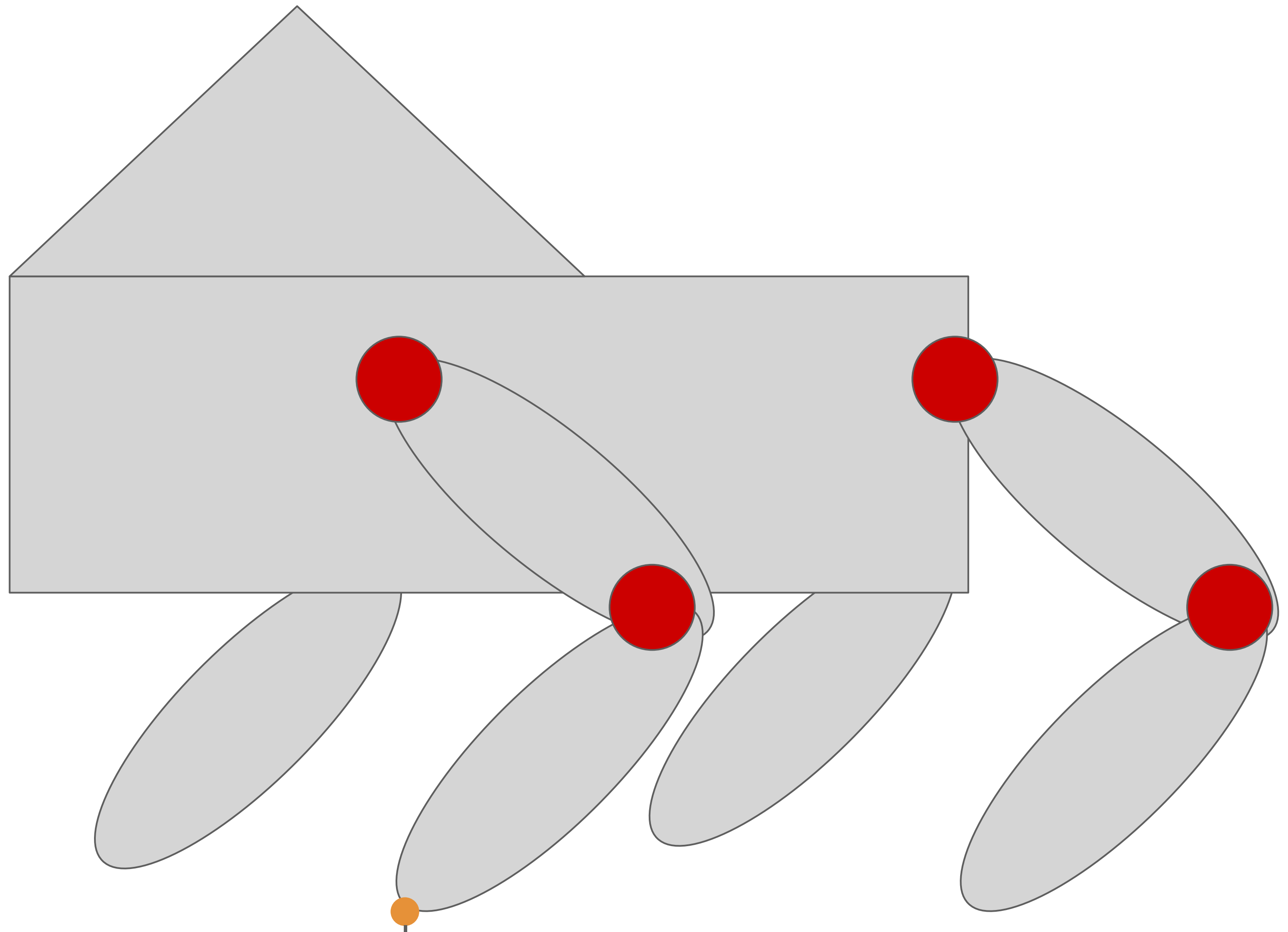


End effector
$\mathbf{x} = (x_e, y_e, z_e)$

Pose: $\mathbf{q}$

Position: $\mathbf{x} = (x_e, y_e, z_e)$

Problem Statement



End effector
$\mathbf{x} = (x_e, y_e, z_e)$

$\mathbf{x} = FK(\mathbf{q})$

$\mathbf{q} = IK(\mathbf{x})$

Why Forward Kinematics?

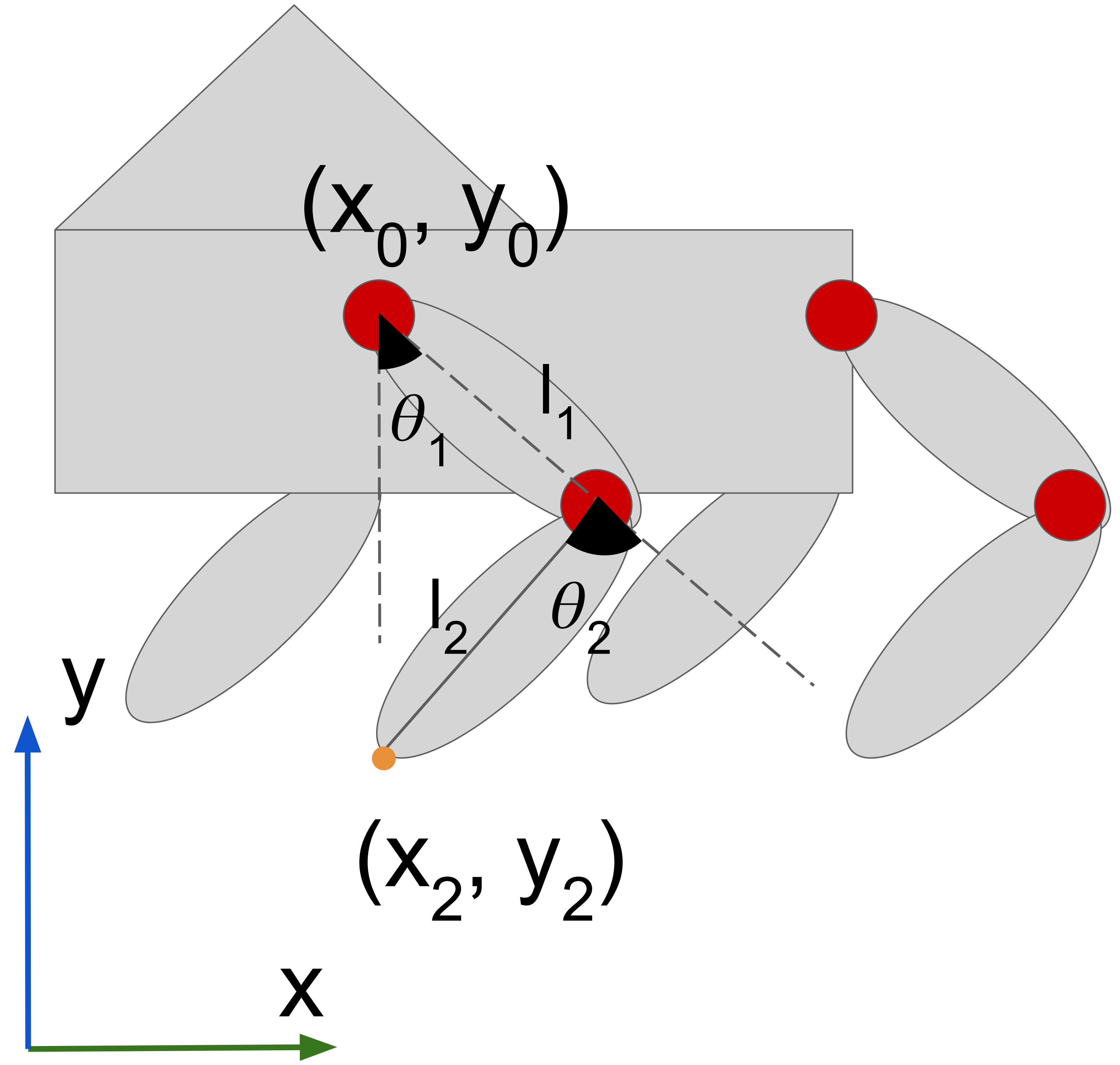
Legs as Manipulator: Pushing Quadrupedal Agility Beyond Locomotion, Cheng et al. ICRA 2023

Learning Agile Soccer Skills for a Bipedal Robot with Deep Reinforcement Learning, Haarnoja et al. 2023

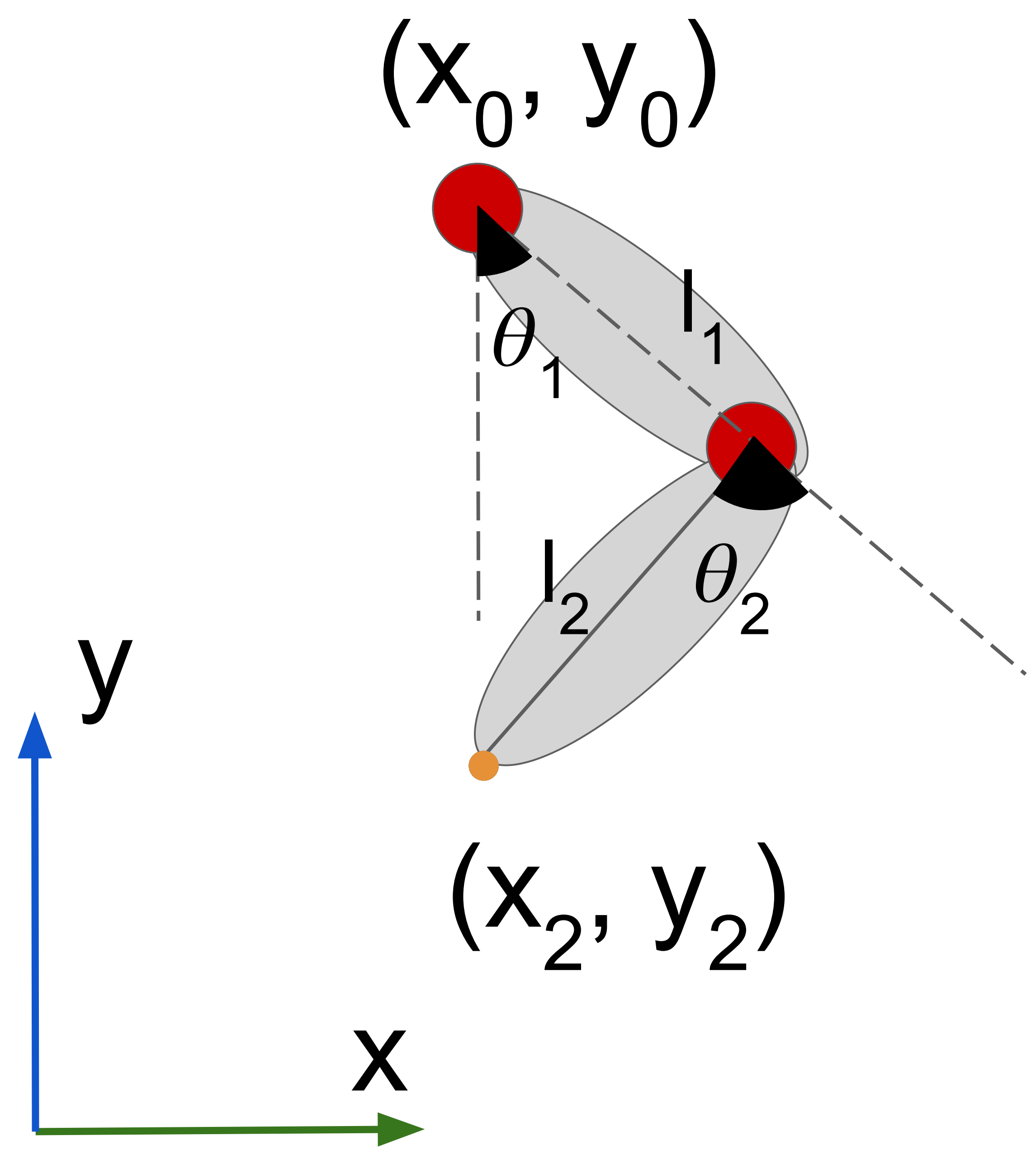
Learning Fine-Grained Bimanual Manipulation with Low-Cost Hardware, Zhao et al. RSS 2023

- Knowing end effector positions are critical for robotic tasks
- Onboard sensors only provide (motor) joint angles

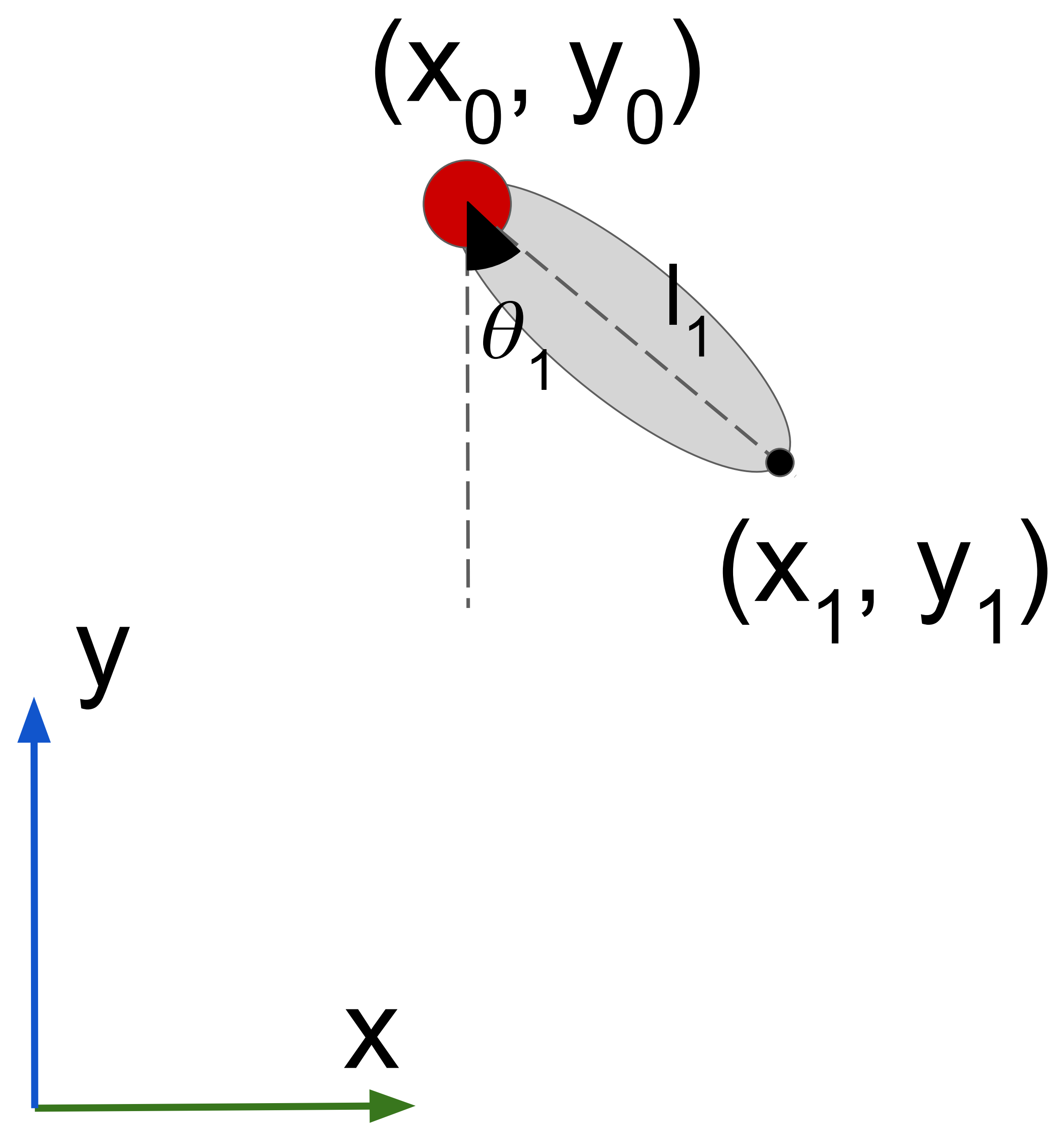
A Simple Example



A Simple Example



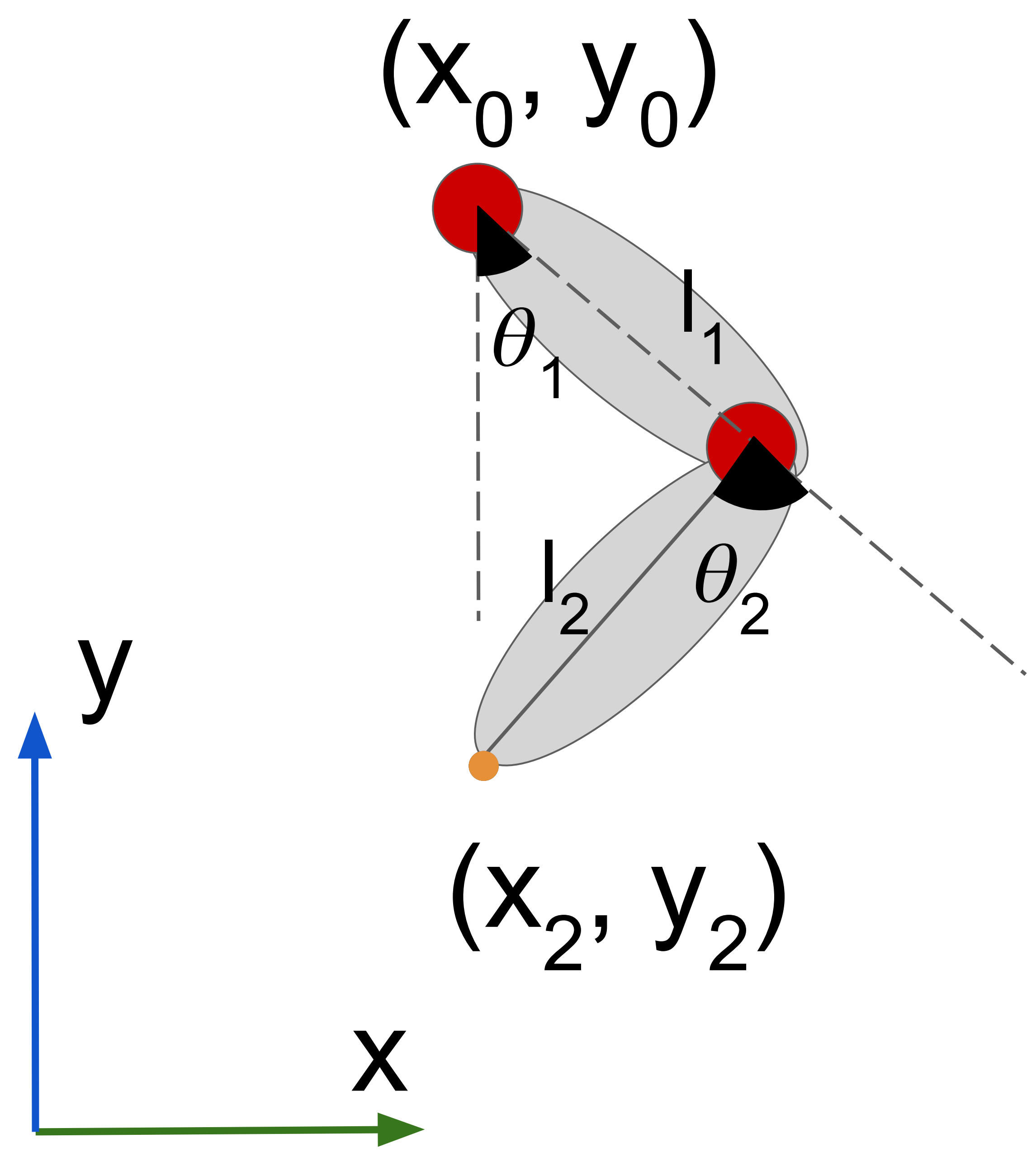
A Simple Example



$$x_1 = l_1 \sin(\theta_1) + x_0$$

$$y_1 = -l_1 \cos(\theta_1) + y_0$$

A Simple Example



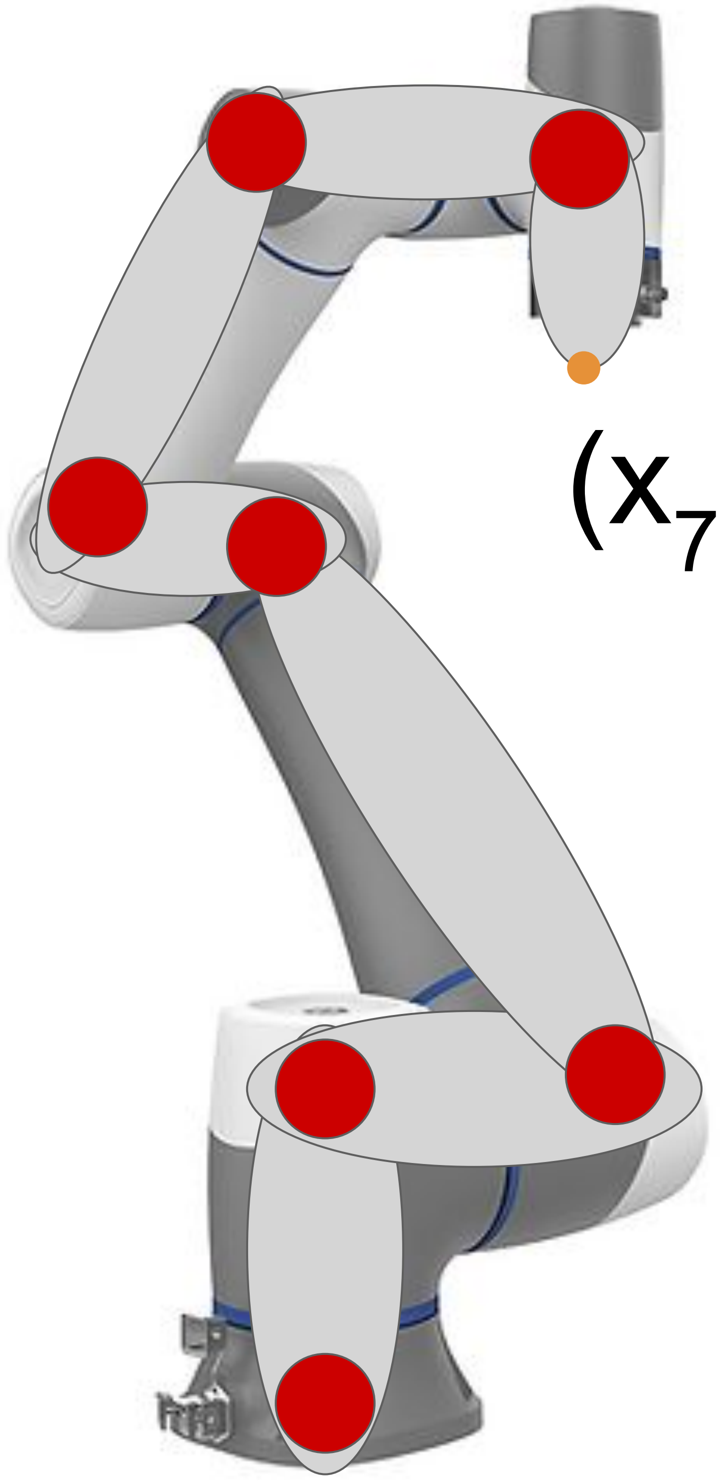
$$x_1 = l_1 \sin(\theta_1) + x_0$$

$$y_1 = -l_1 \cos(\theta_1) + y_0$$

$$x_2 = x_1 - l_2 \sin(\theta_2 - \theta_1) = l_1 \sin(\theta_1) - l_2 \sin(\theta_2 - \theta_1) + x_0$$

$$y_2 = y_1 - l_2 \cos(\theta_2 - \theta_1) = -l_1 \cos(\theta_1) - l_2 \cos(\theta_2 - \theta_1) + y_0$$

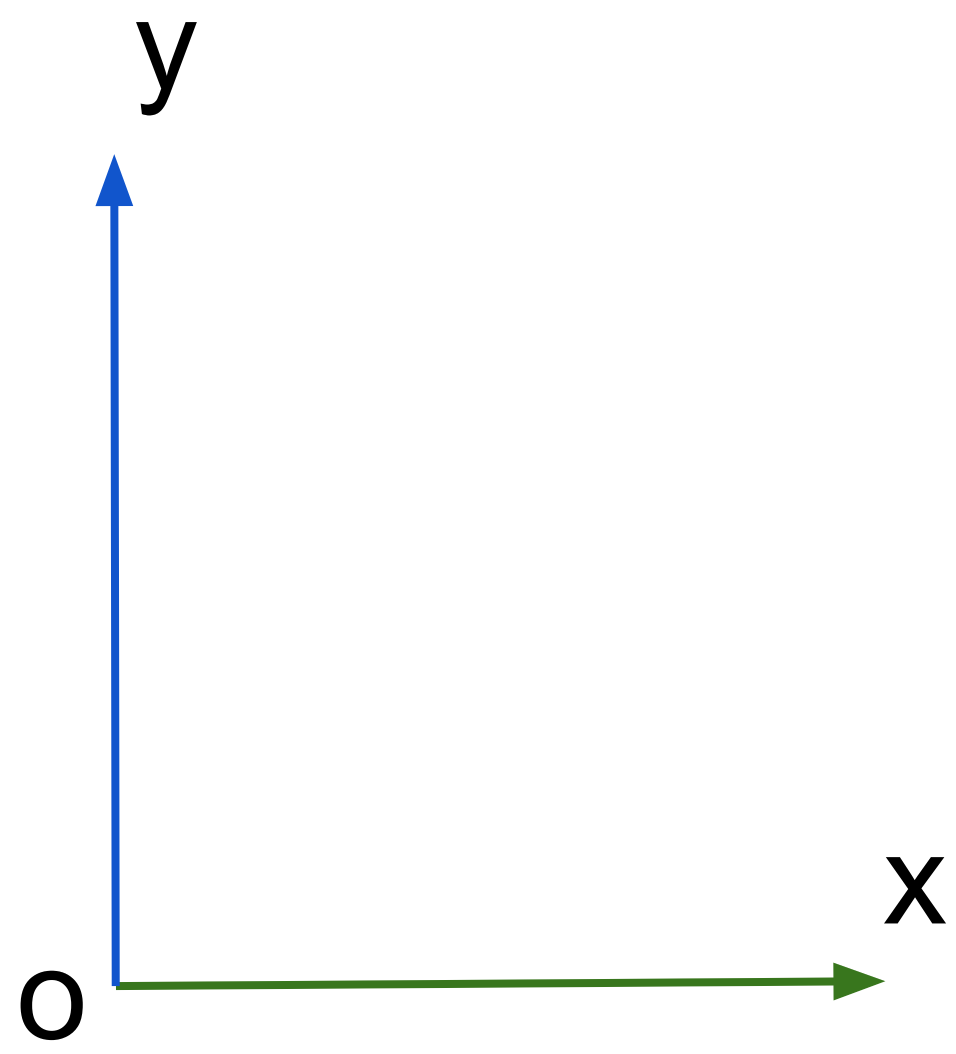
A Complex Example



$(x_7, y_7) = ?$

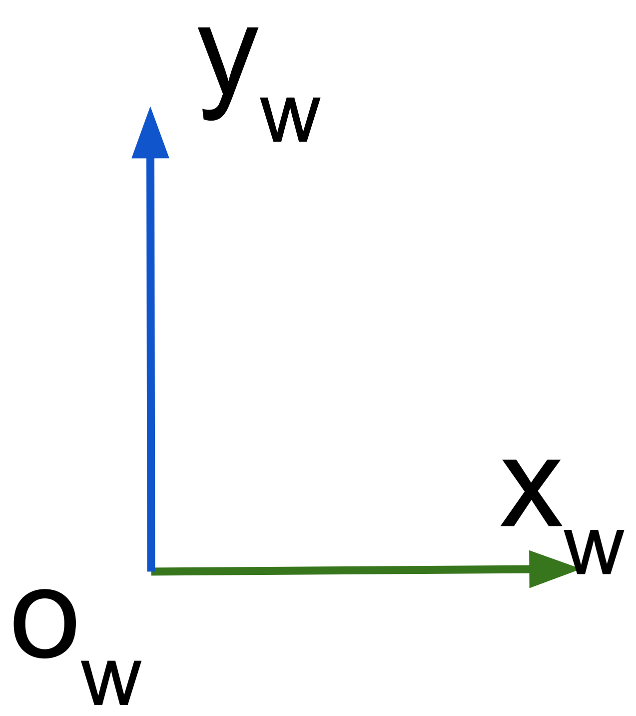
Coordinate Systems

Coordinate system: Origin and Axes



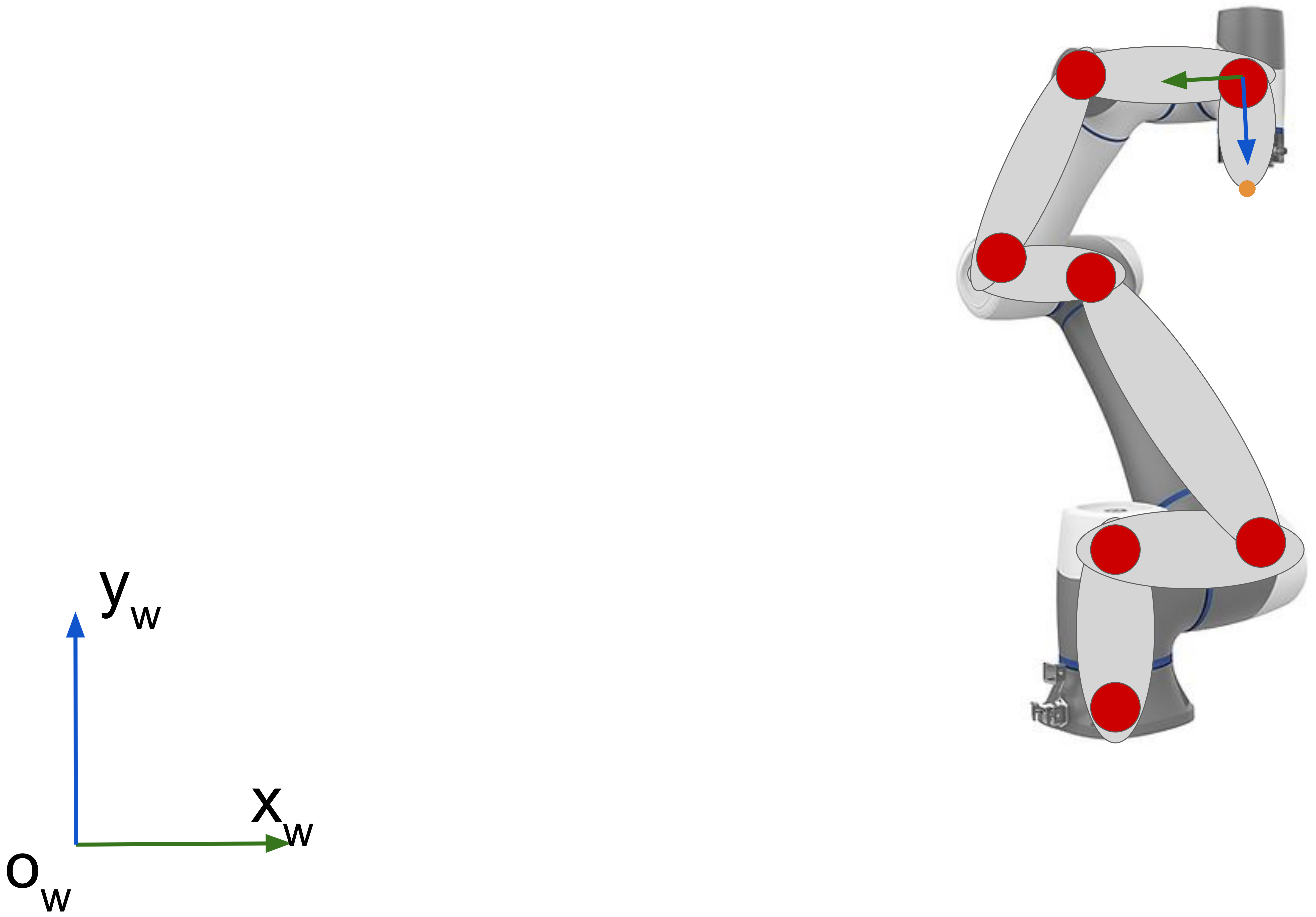
Coordinate Systems

World coordinate: An absolute coordinate system that is fixed in space



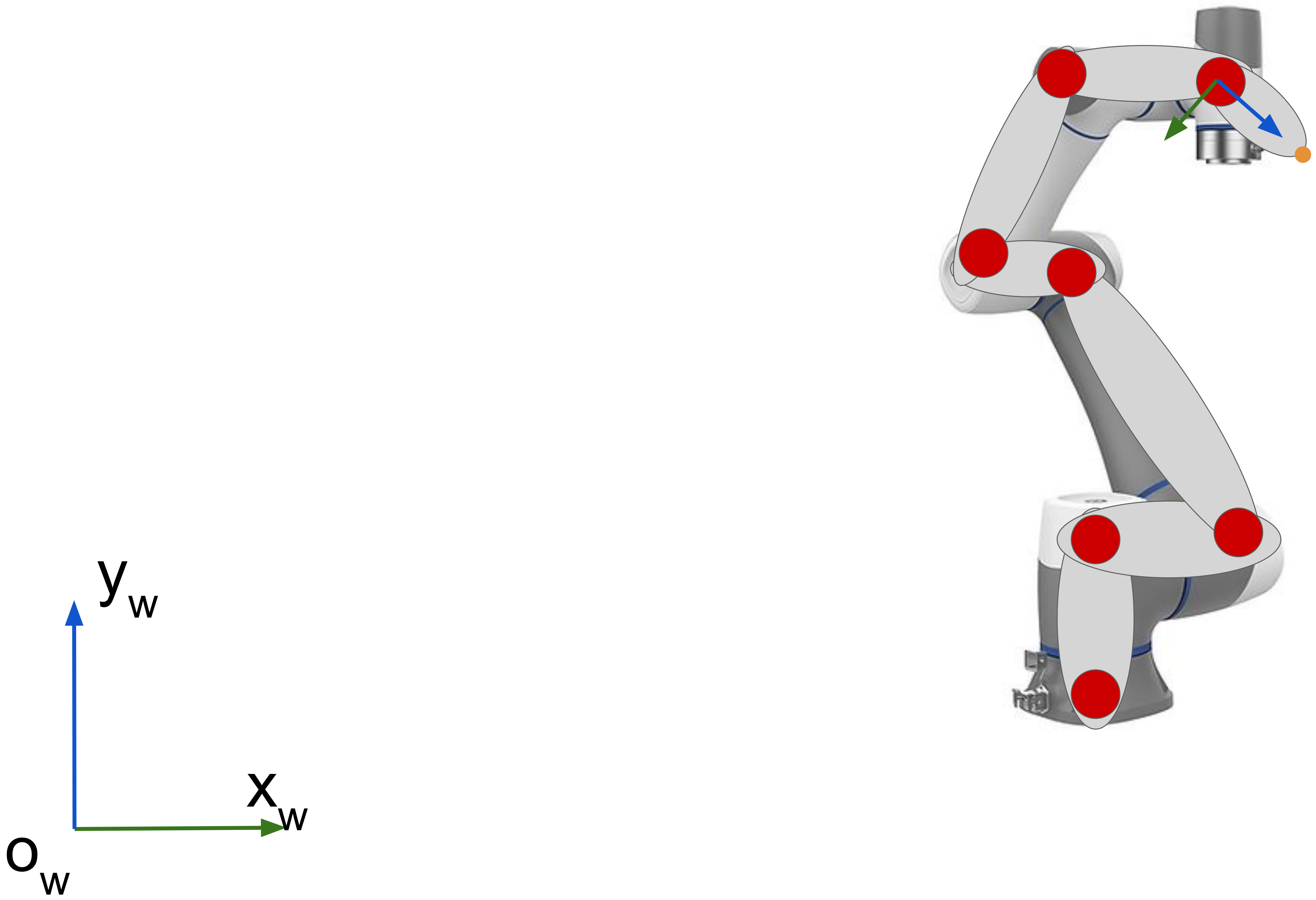
Coordinate Systems

Local coordinate: A coordinate system that is attached to a robot link



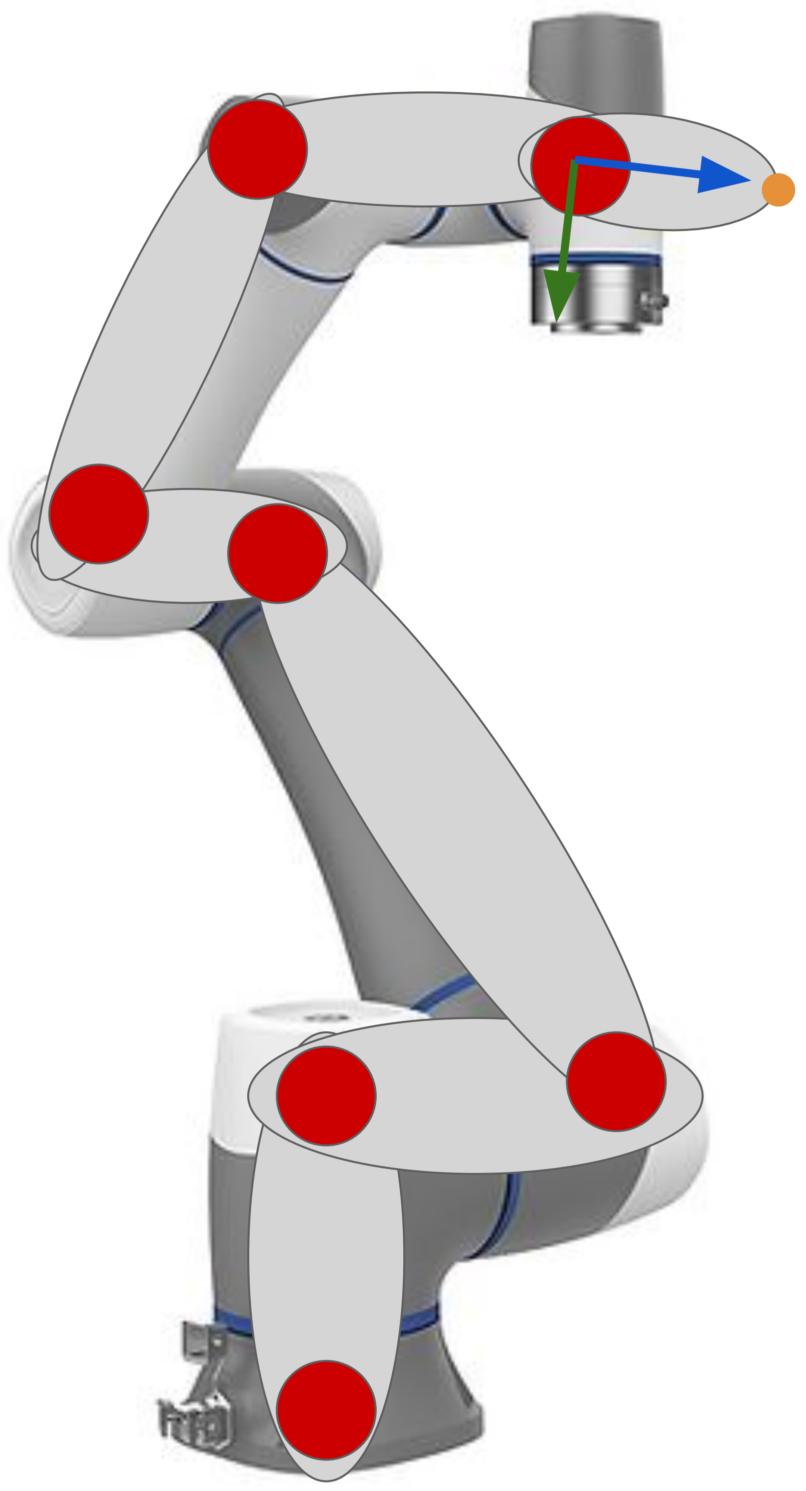
Coordinate Systems

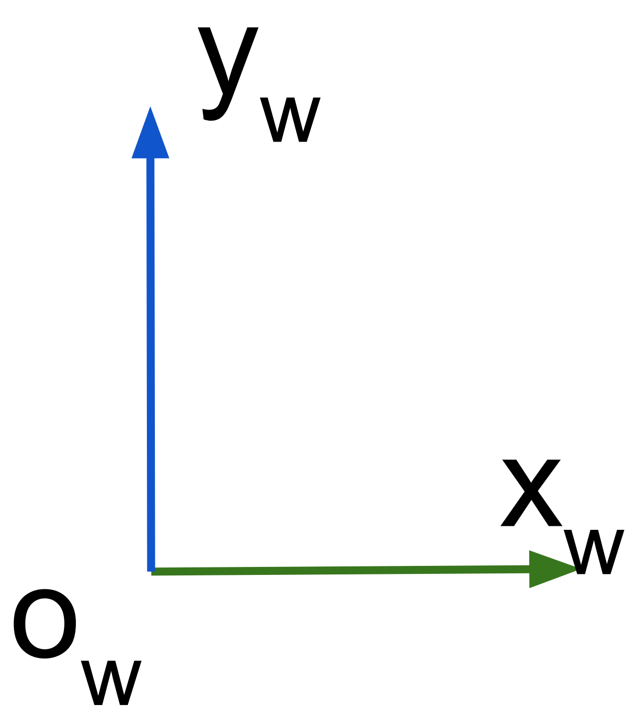
Local coordinate: A coordinate system that is **attached to a robot link**



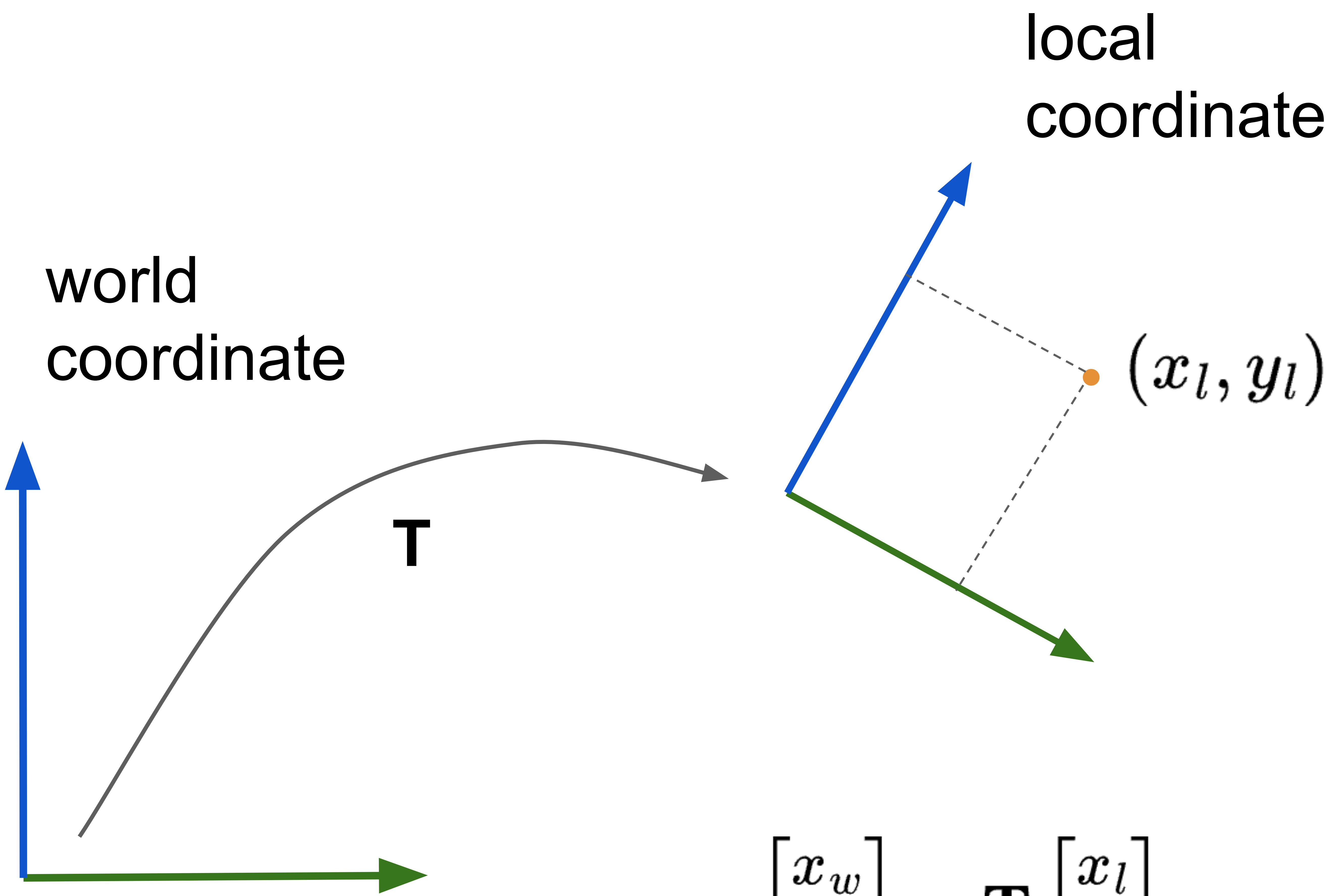
Coordinate Systems

Quiz: Does the end-effector coordinate change in local vs. world coordinate?



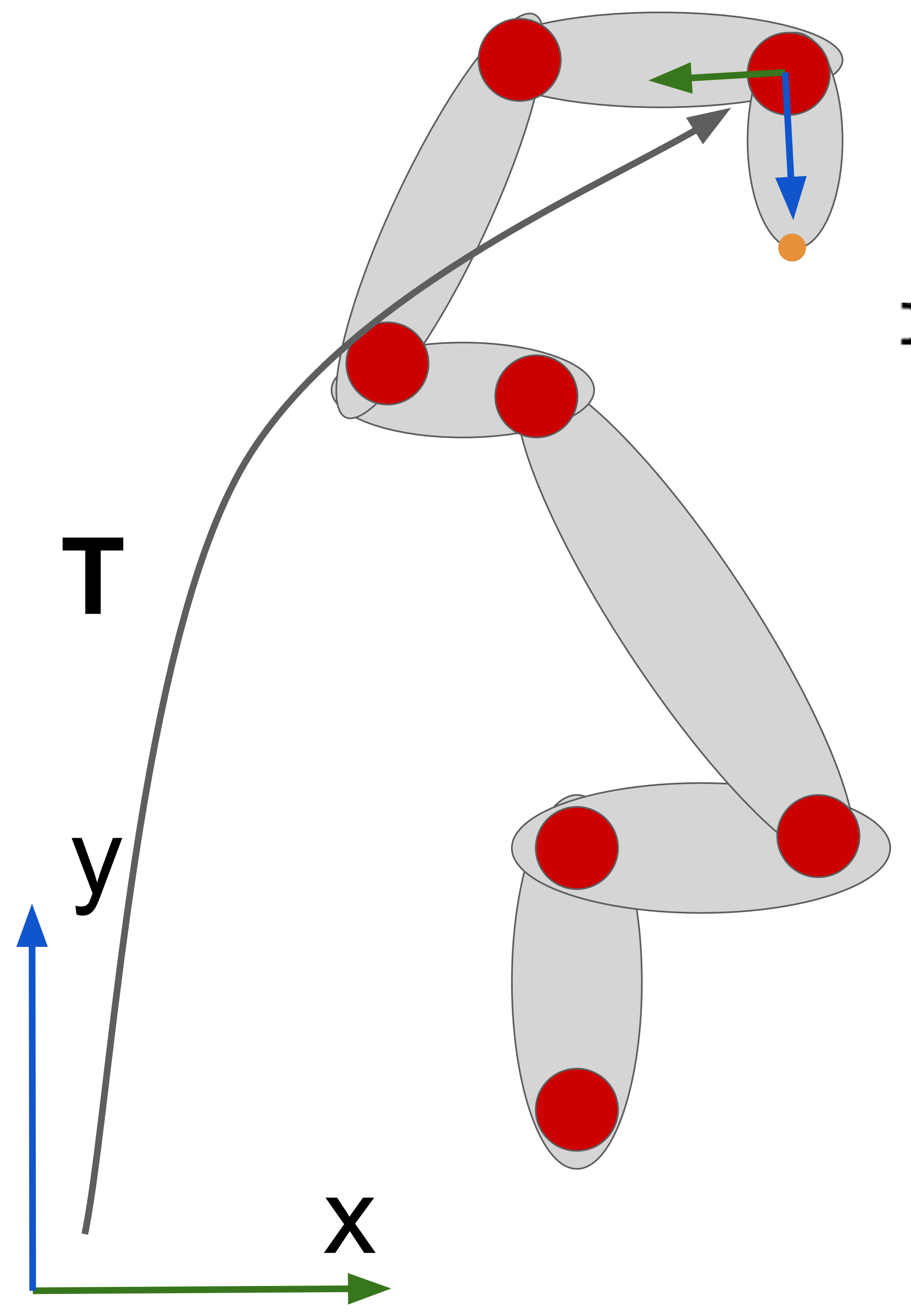


Transformation between Coordinate Systems



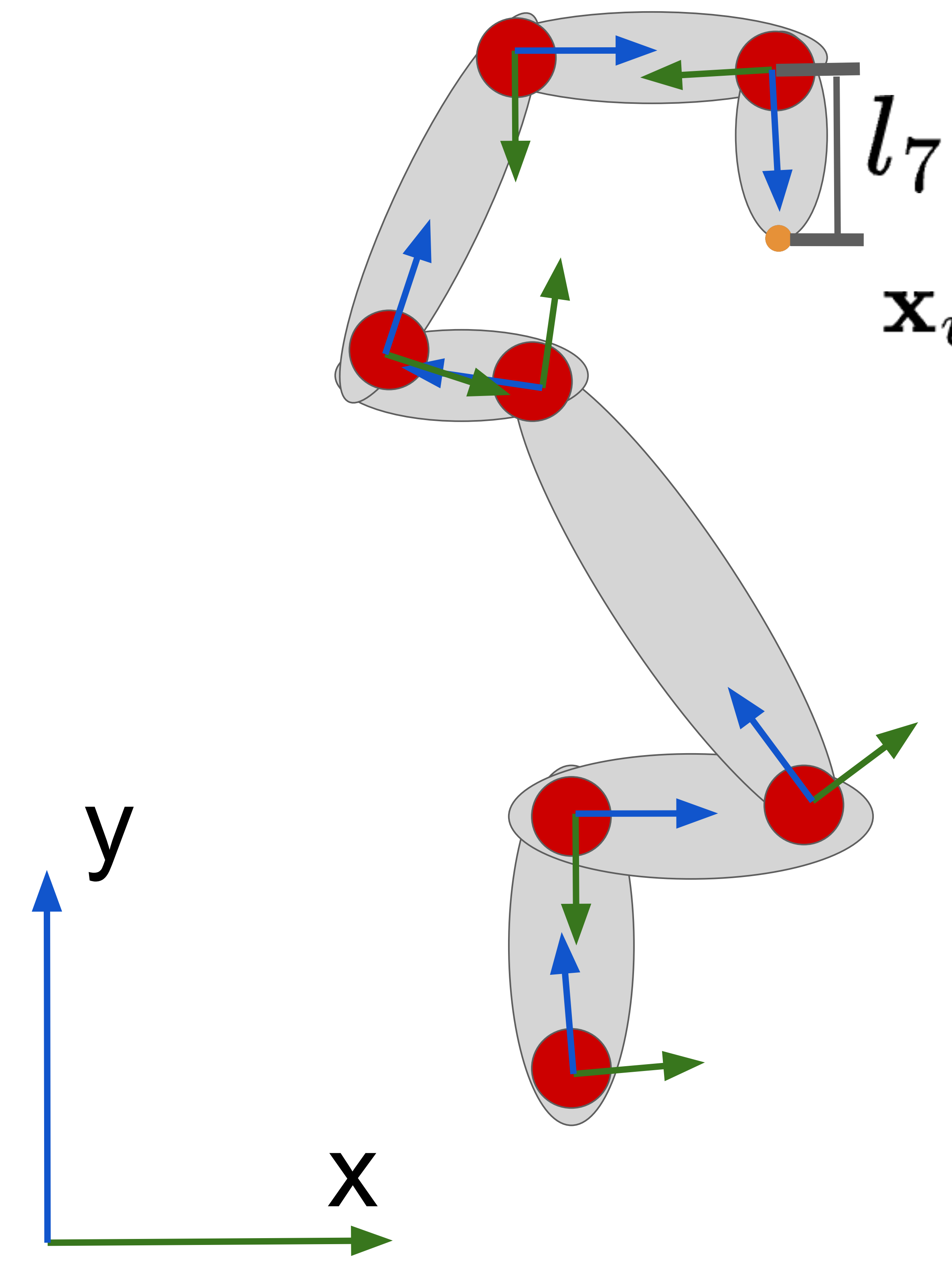
$$\begin{bmatrix} x_w \\ y_w \end{bmatrix} = \mathbf{T} \begin{bmatrix} x_l \\ y_l \end{bmatrix}$$

General Approach



$$\mathbf{x}_w = \mathbf{T}\,\mathbf{x}_l$$

General Approach



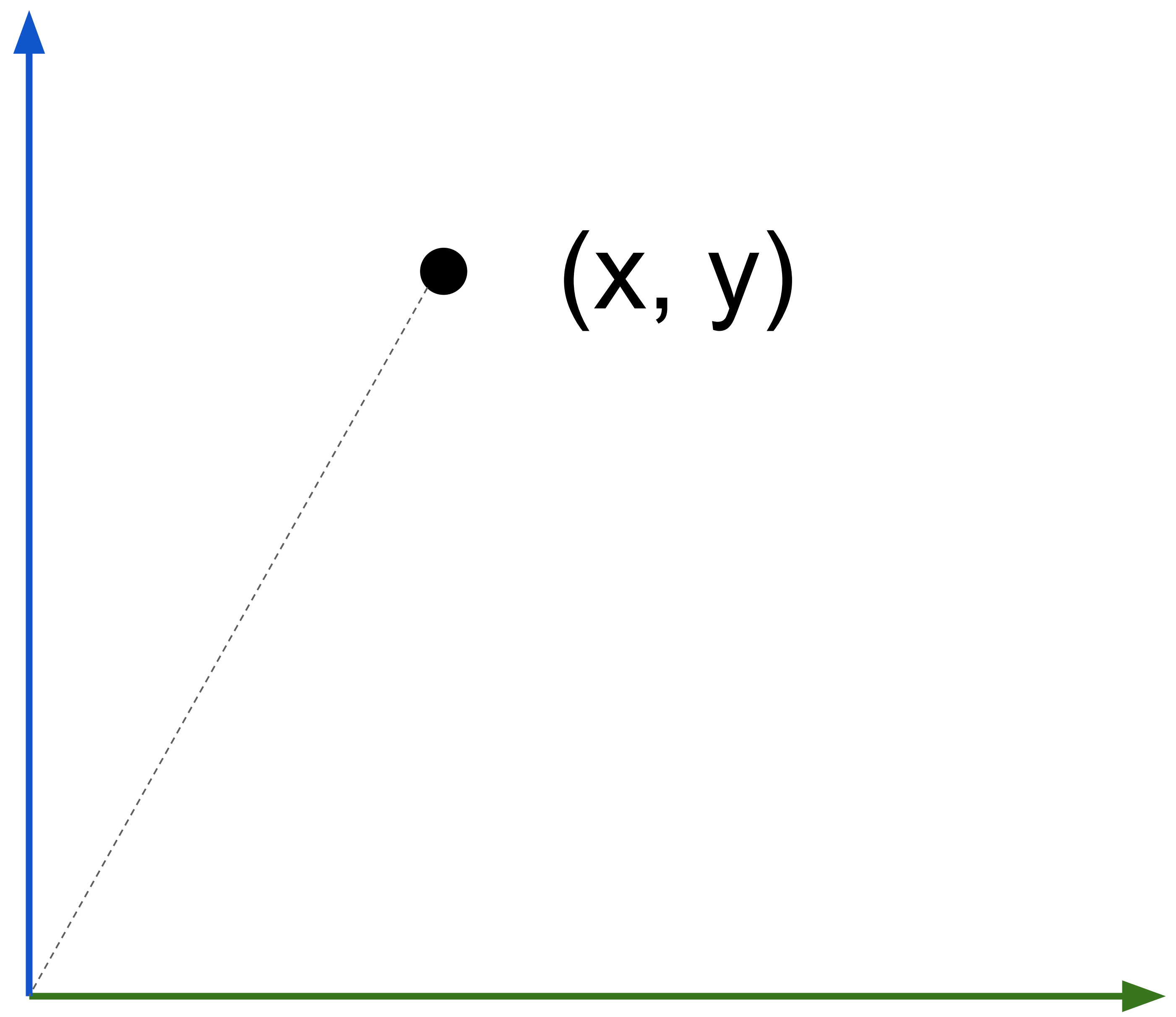
$$\mathbf{x}_w = \mathbf{T}\,\mathbf{x}_l$$

$$= T_{w\to j_1}\ T_{j_1\to j_2}\ T_{j_2\to j_3}\ T_{j_3\to j_4}\ T_{j_4\to j_5}\ T_{j_5\to j_6}\ T_{j_6\to j_7}\ \mathbf{x}_l$$

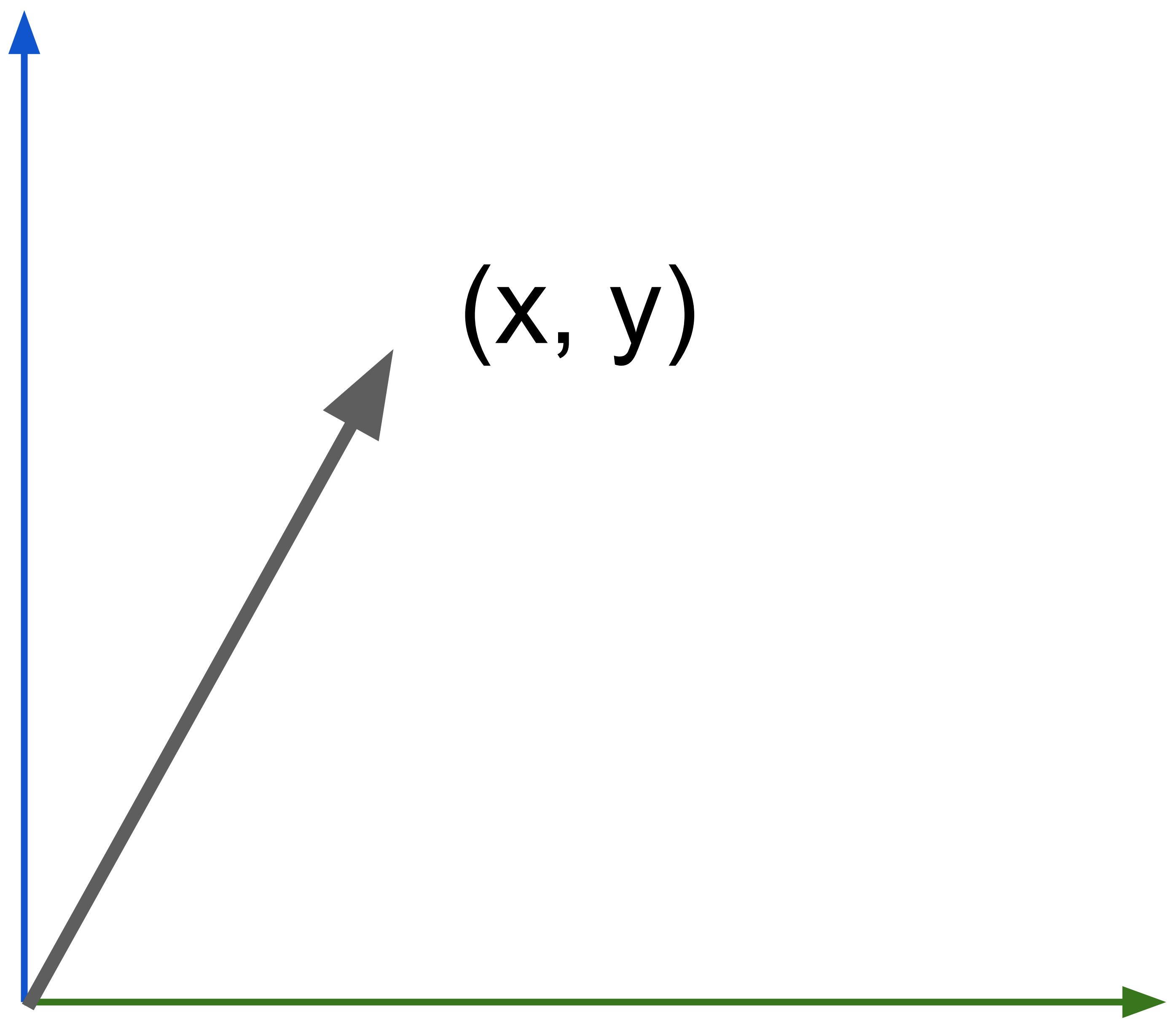
$$= T_{w\to j_1}\ T_{j_1\to j_2}\ T_{j_2\to j_3}\ T_{j_3\to j_4}\ T_{j_4\to j_5}\ T_{j_5\to j_6}\ T_{j_6\to j_7} \begin{bmatrix} 0 \\ l_7 \\ 1 \end{bmatrix}$$

Homogeneous Coordinate

Point



Vector

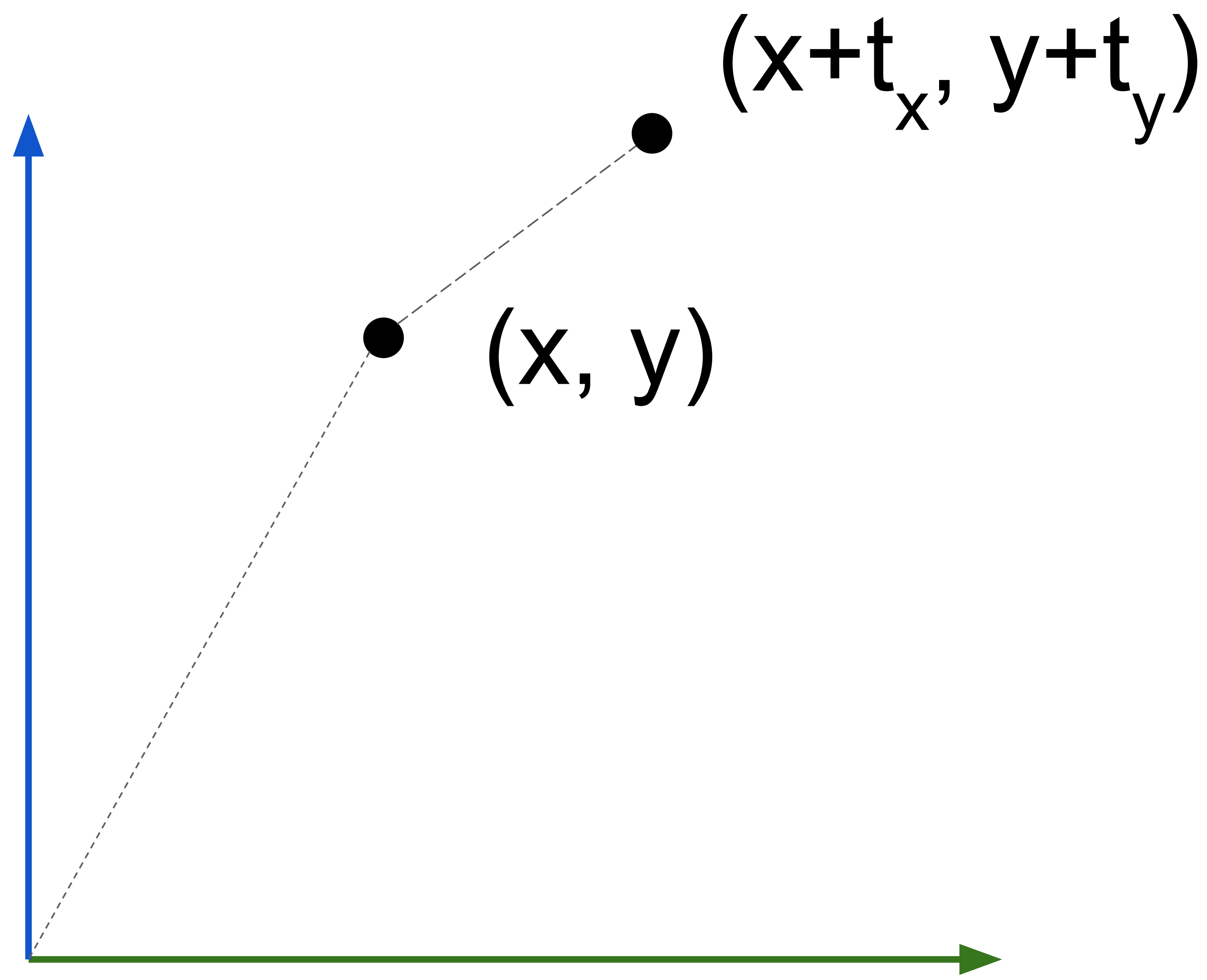


Homogeneous Coordinate:

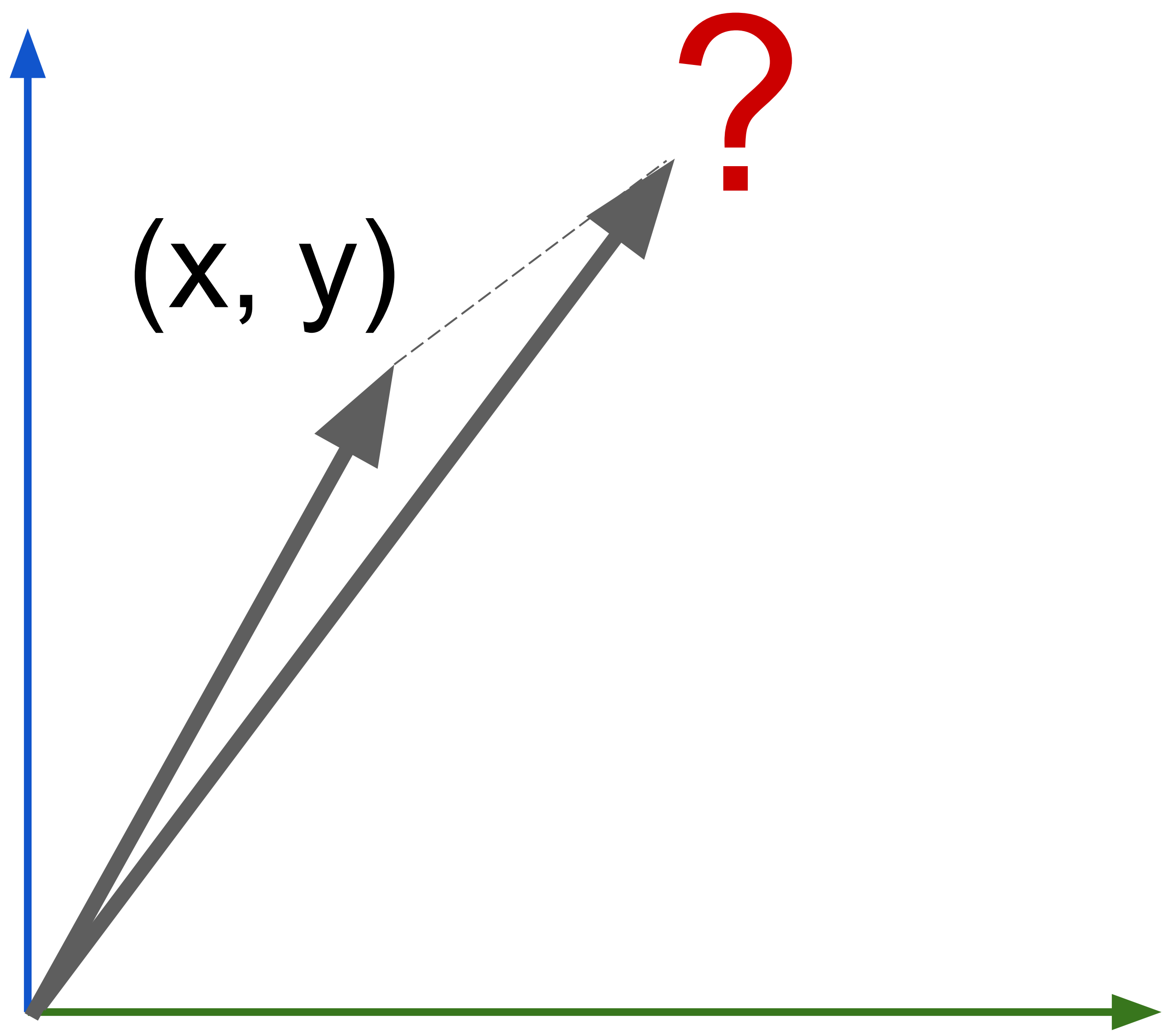
$$\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Transformation Matrix

Translation Matrix



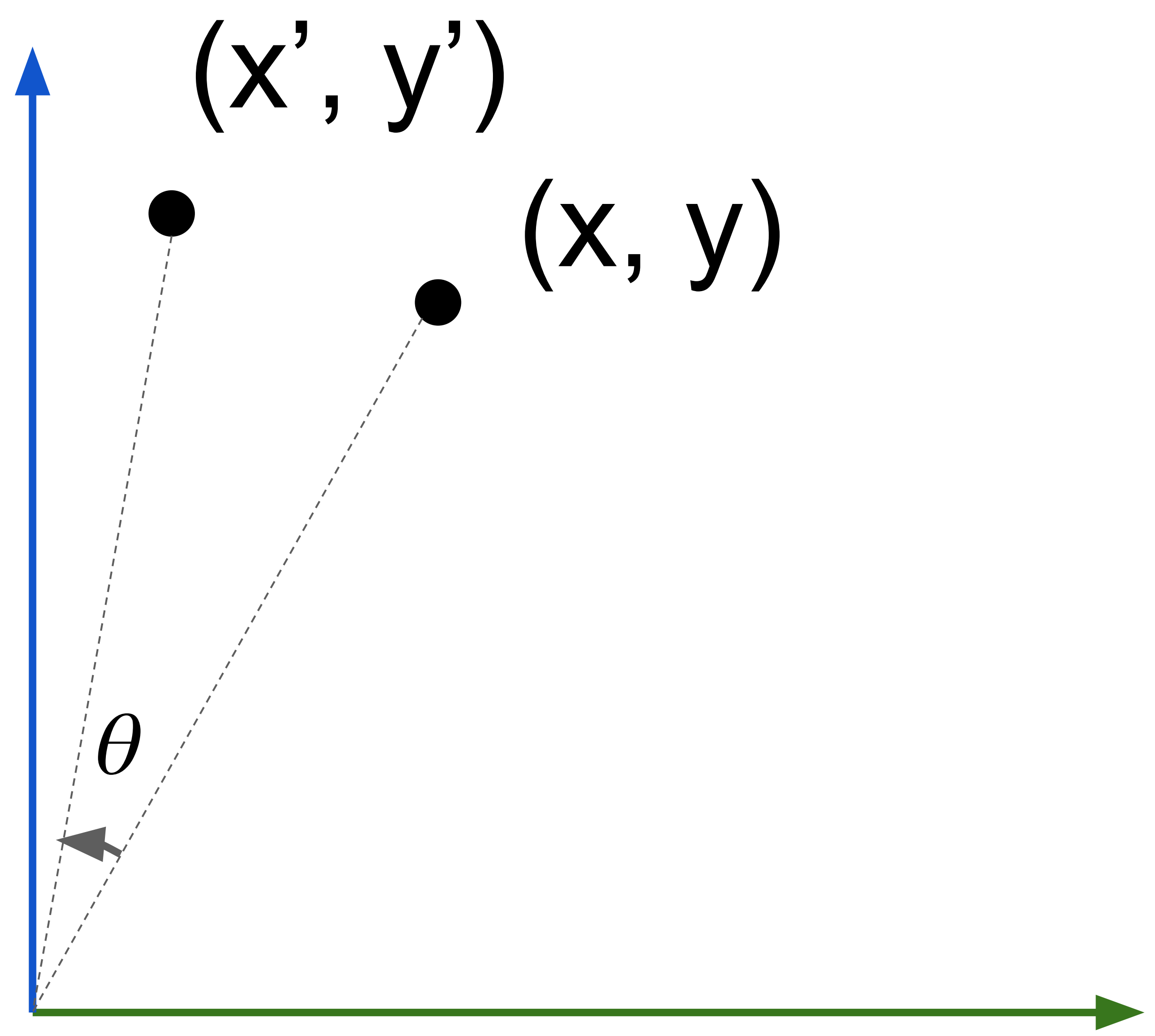
$$\begin{bmatrix} x+t_x \\ y+t_y \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} x \\ y \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

Transformation Matrix

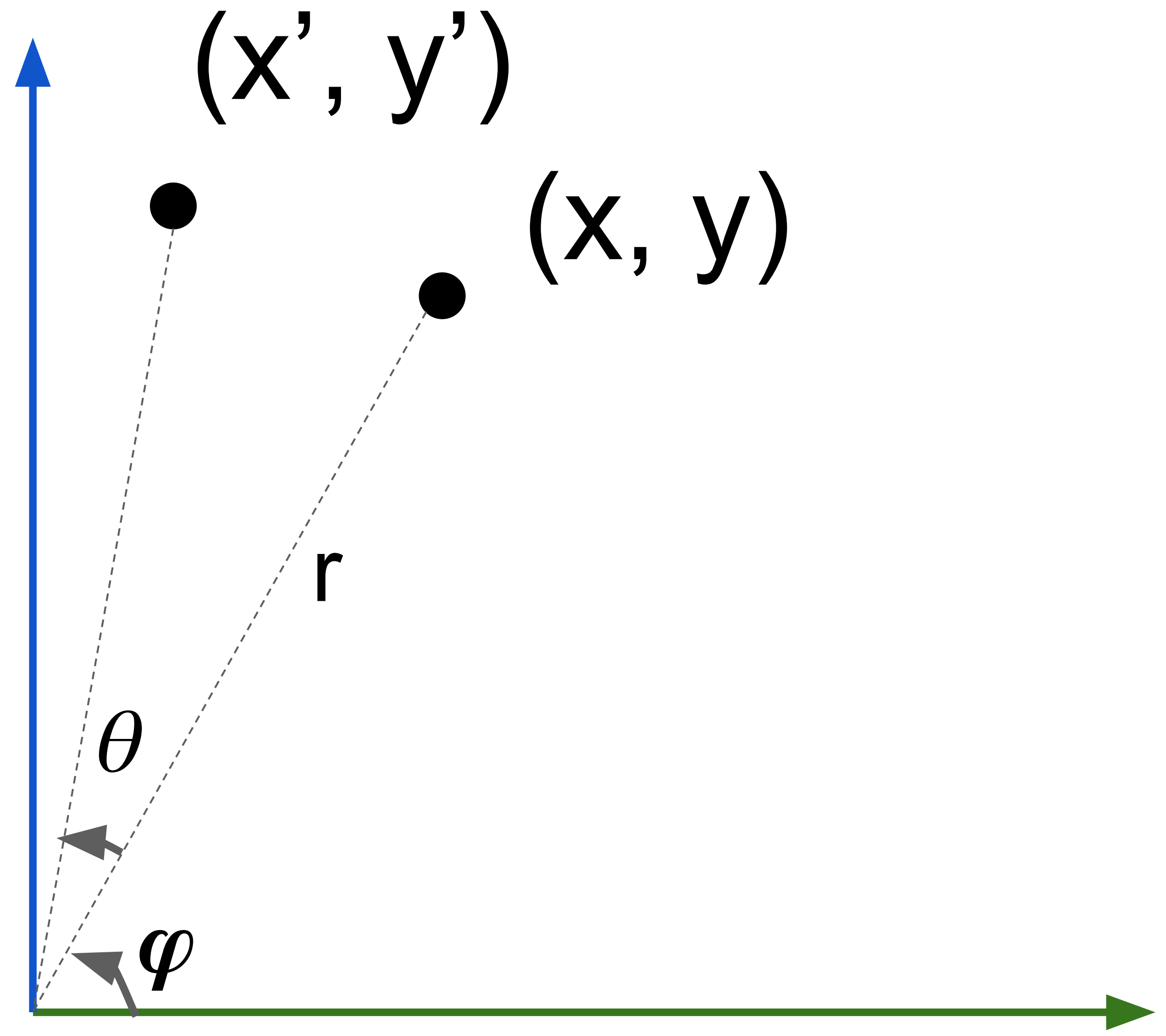
Rotational Matrix



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Transformation Matrix

Rotational Matrix



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\underline{x = r\cos(\phi)}$$

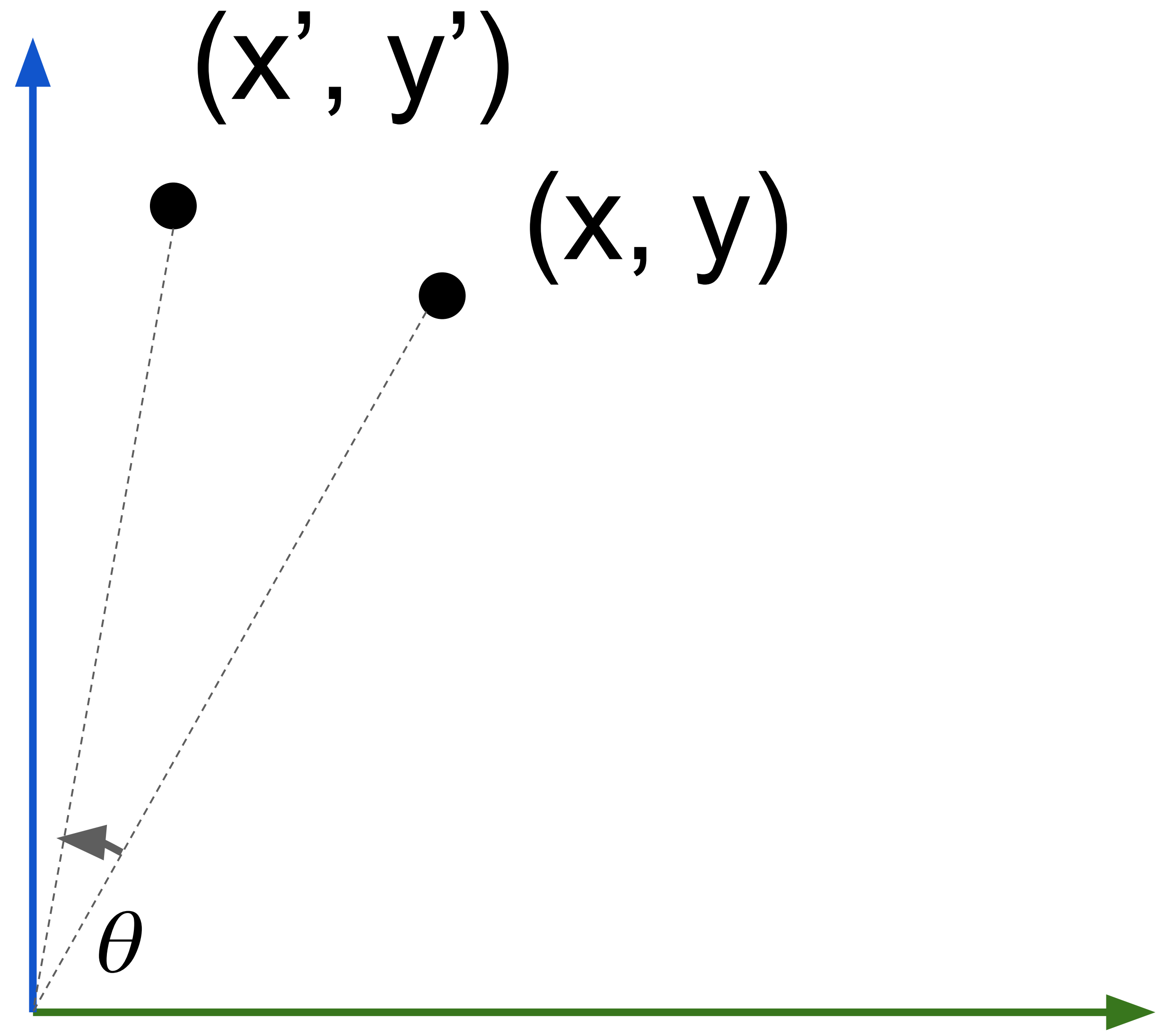
$$\underline{y = r\sin(\phi)}$$

$$\begin{aligned} x' &= r\cos(\phi+\theta) \\ &= \underline{r\cos(\phi)}\cos(\theta) - \underline{r\sin(\phi)}\sin(\theta) \\ &= x\cos(\theta) - y\sin(\theta) \end{aligned}$$

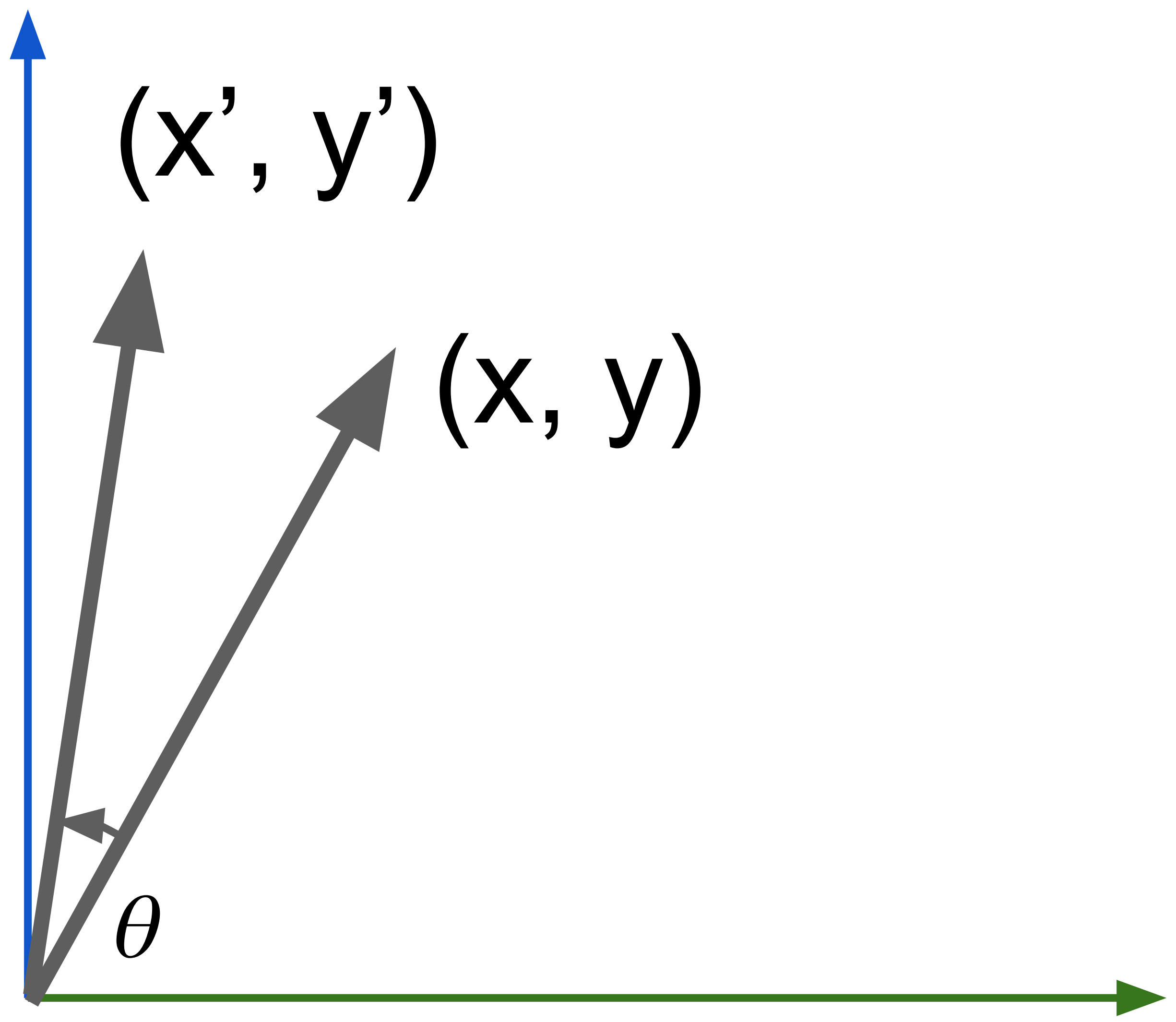
$$\begin{aligned} y' &= r\sin(\phi+\theta) \\ &= r\sin(\phi)\cos(\theta) + r\cos(\phi)\sin(\theta) \\ &= x\sin(\theta) + y\cos(\theta) \end{aligned}$$

Transformation Matrix

Rotational Matrix



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$



$$\begin{bmatrix} x' \\ y' \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

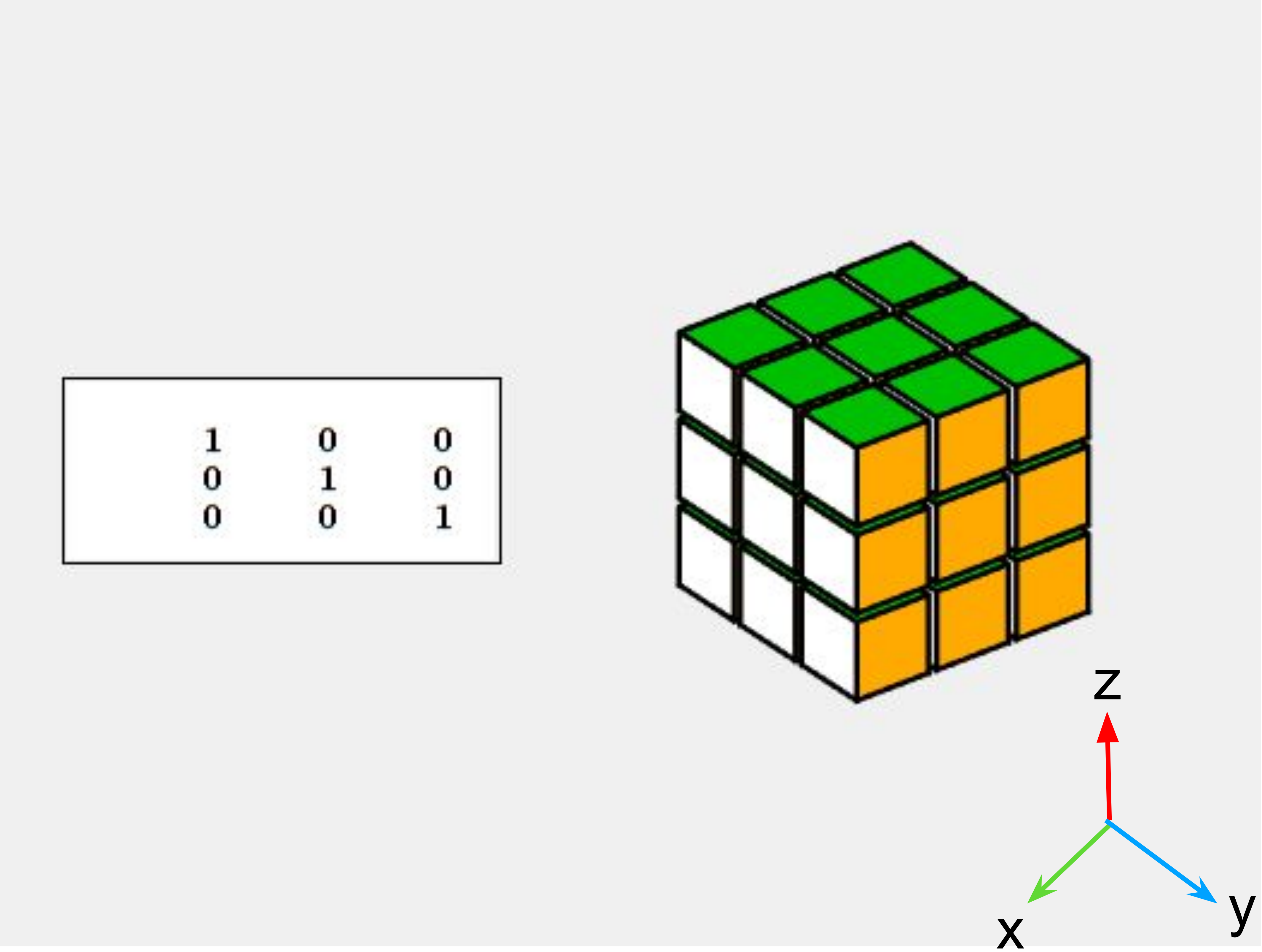
Transformation Matrix

Rotational Matrix in 3D

$$\mathrm{Rot}_x(\theta_x) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x & 0 \\ 0 & \sin\theta_x & \cos\theta_x & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

$$\mathrm{Rot}_y(\theta_y) = \left(\begin{array}{ccc|c} \cos\theta_y & 0 & \sin\theta_y & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta_y & 0 & \cos\theta_y & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$

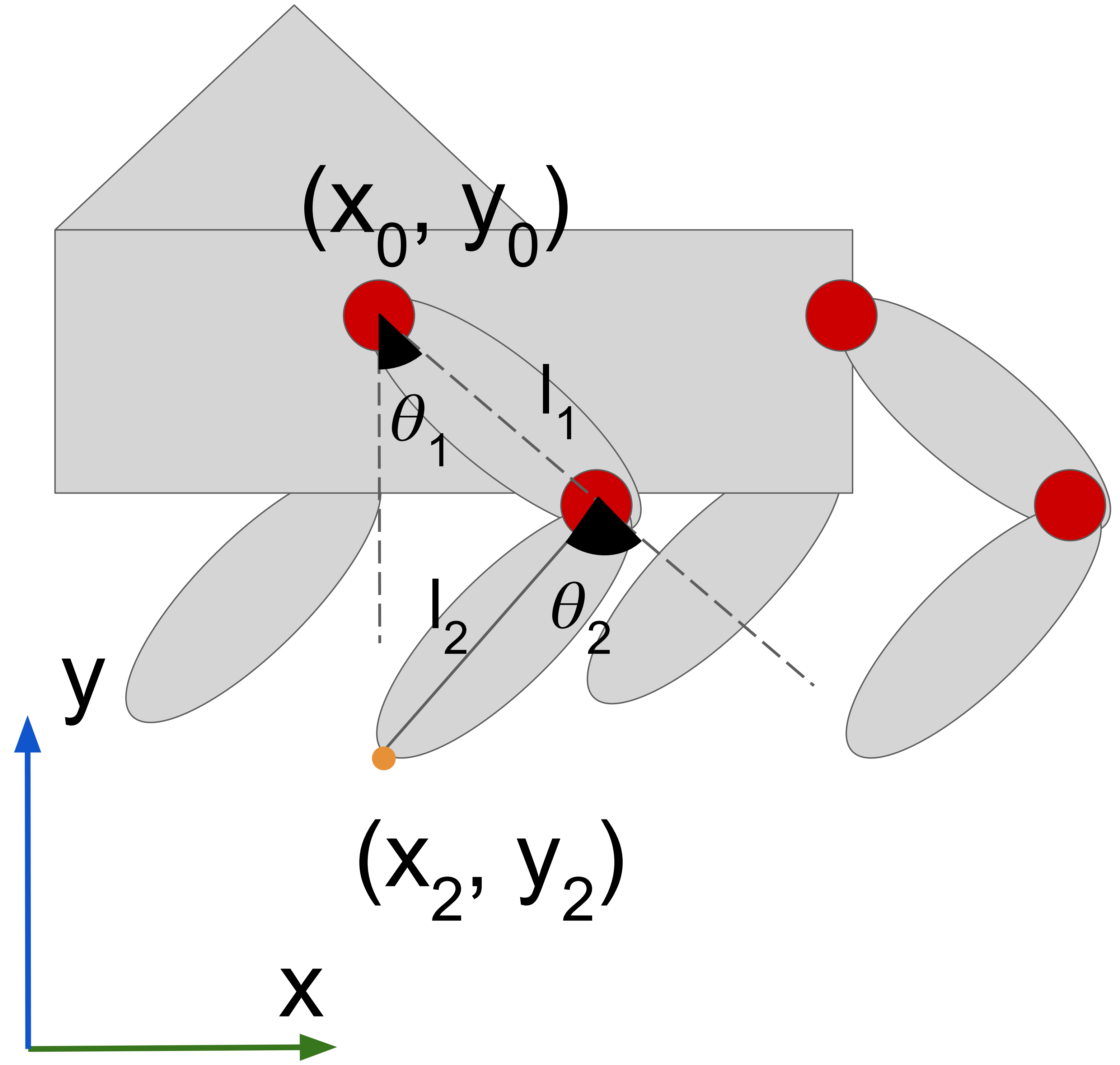
$$\mathrm{Rot}_z(\theta_z) = \left(\begin{array}{ccc|c} \cos\theta_z & -\sin\theta_z & 0 & 0 \\ \sin\theta_z & \cos\theta_z & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right)$$



Quiz

$T_1T_2 == T_2T_1$?

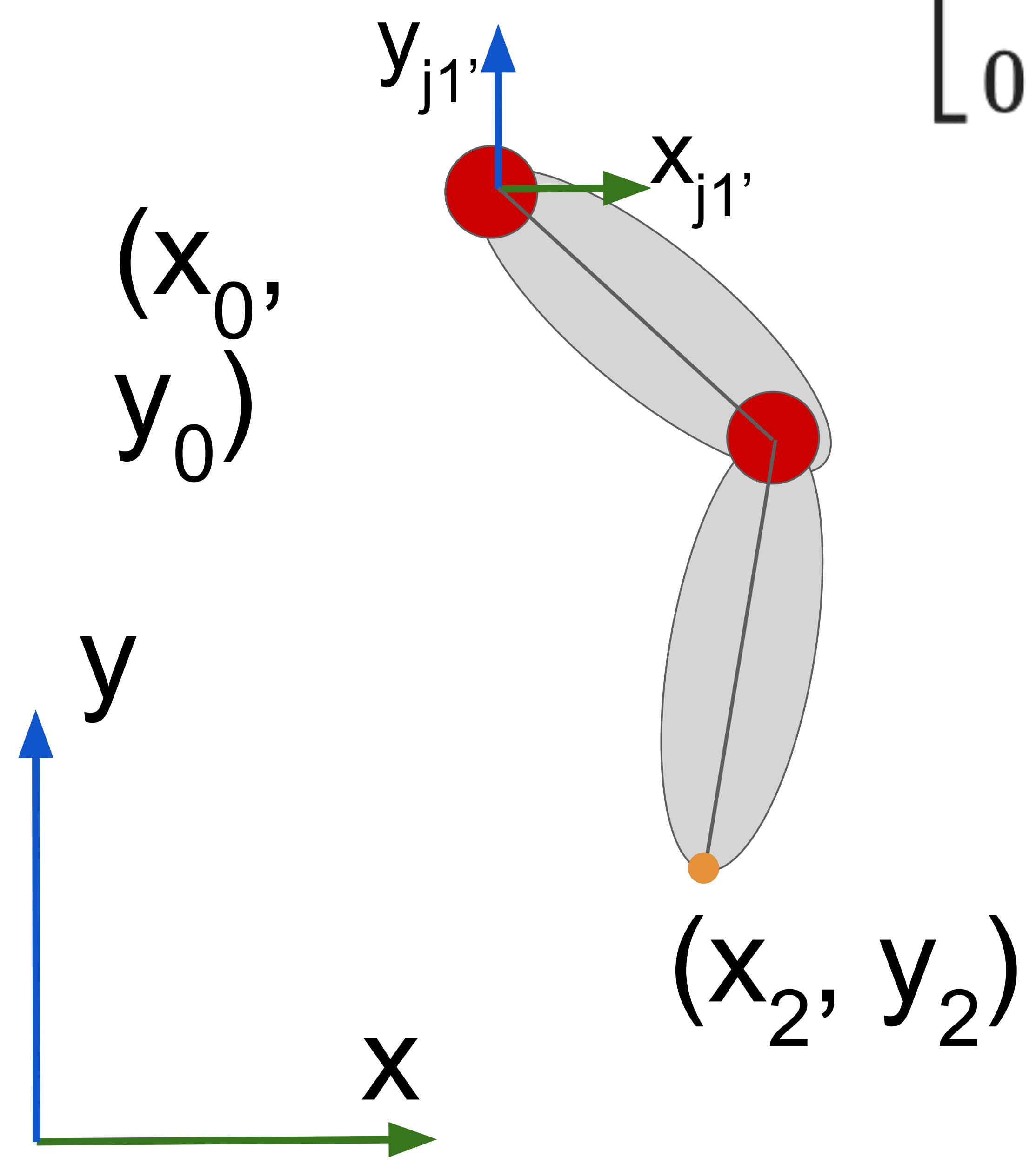
A Simple Example



General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}}$$

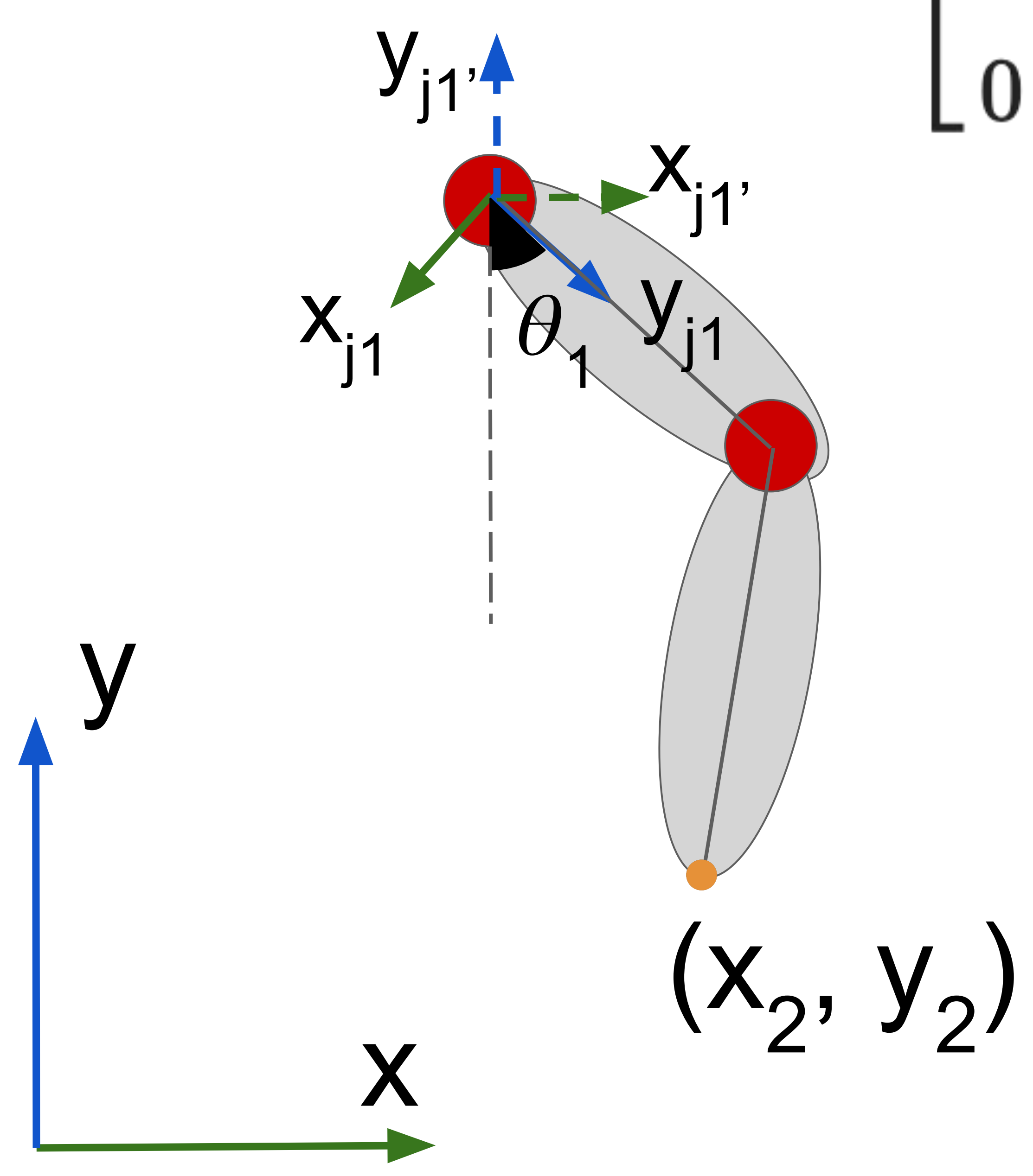
$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}$$



General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1}$$

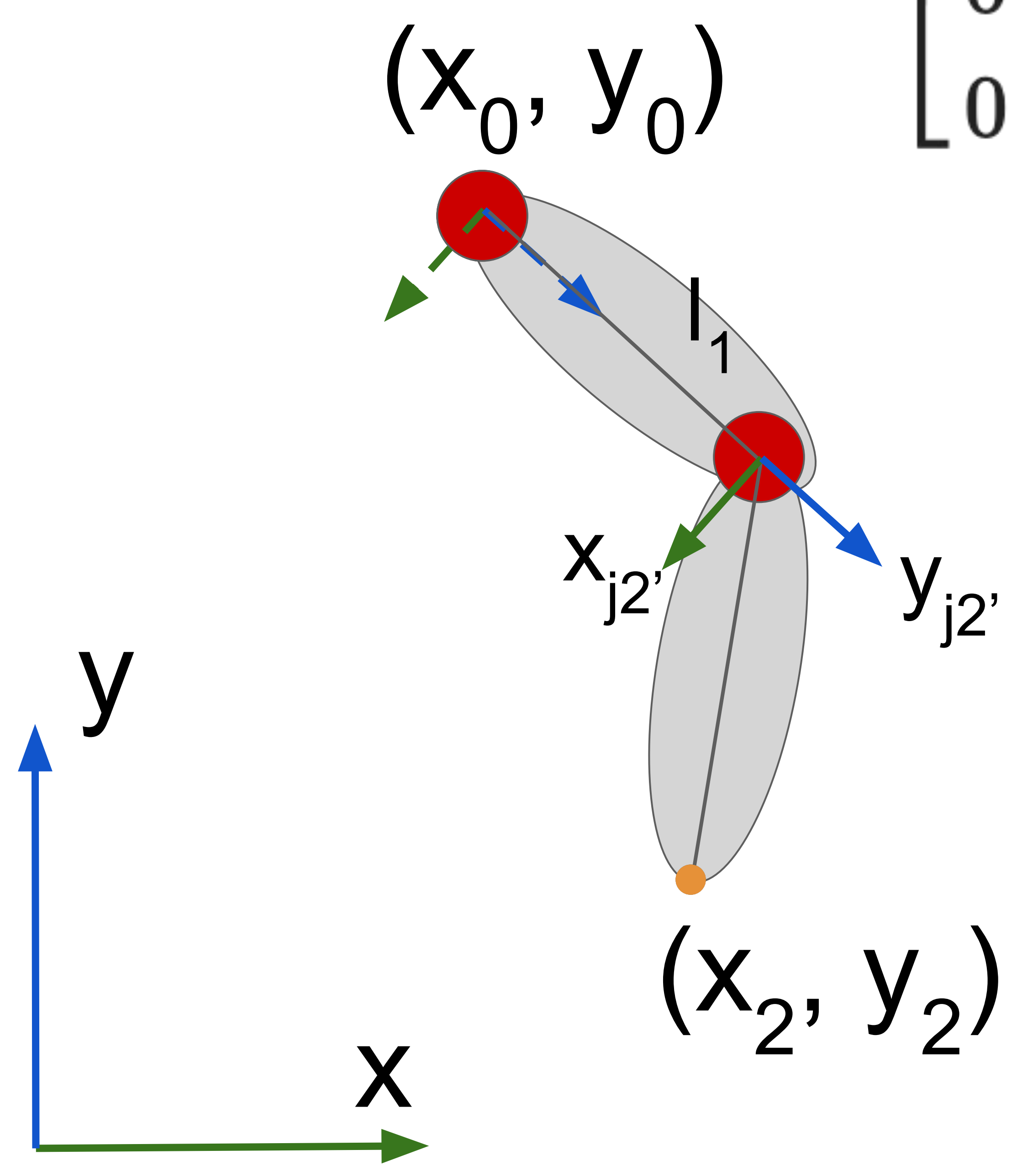
$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi + \theta_1) & -\sin(\pi + \theta_1) & 0 \\ \sin(\pi + \theta_1) & \cos(\pi + \theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1} T_{j_1 \to j_{2'}}$$

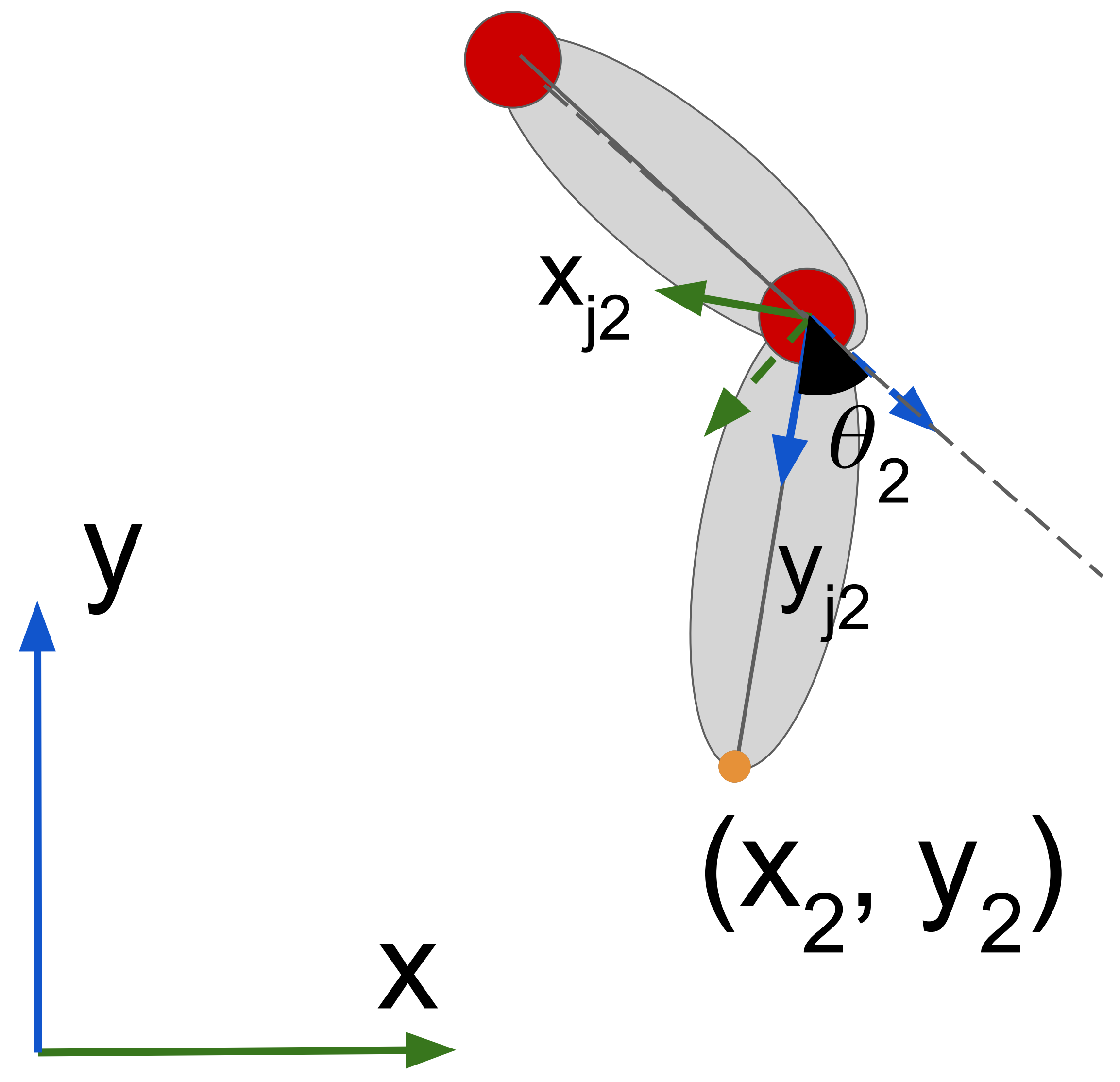
$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix}$$



General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1} T_{j_1 \to j_{2'}} T_{j_{2'} \to j_2}$$

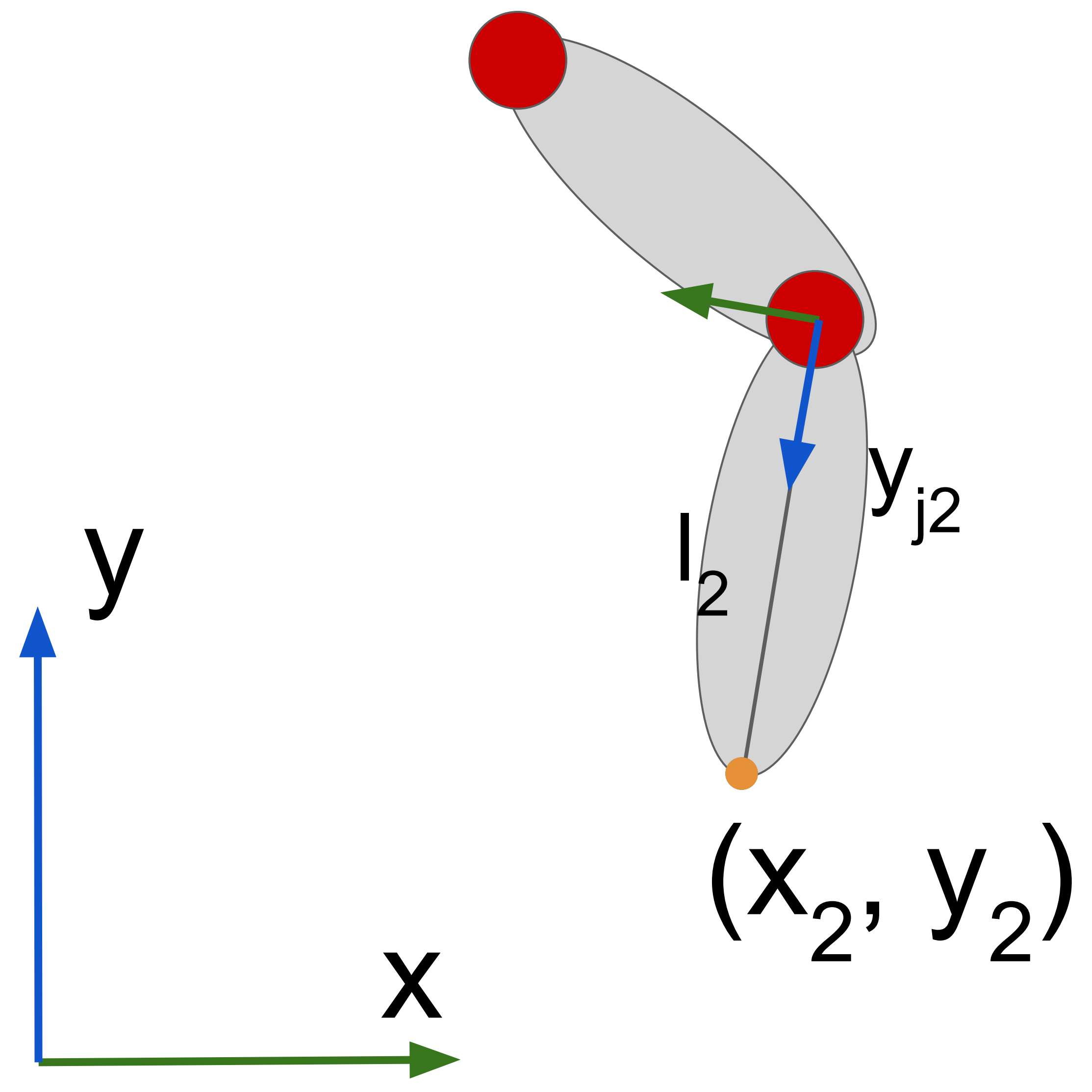
$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1} T_{j_1 \to j_{2'}} T_{j_{2'} \to j_2} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$



General Approach

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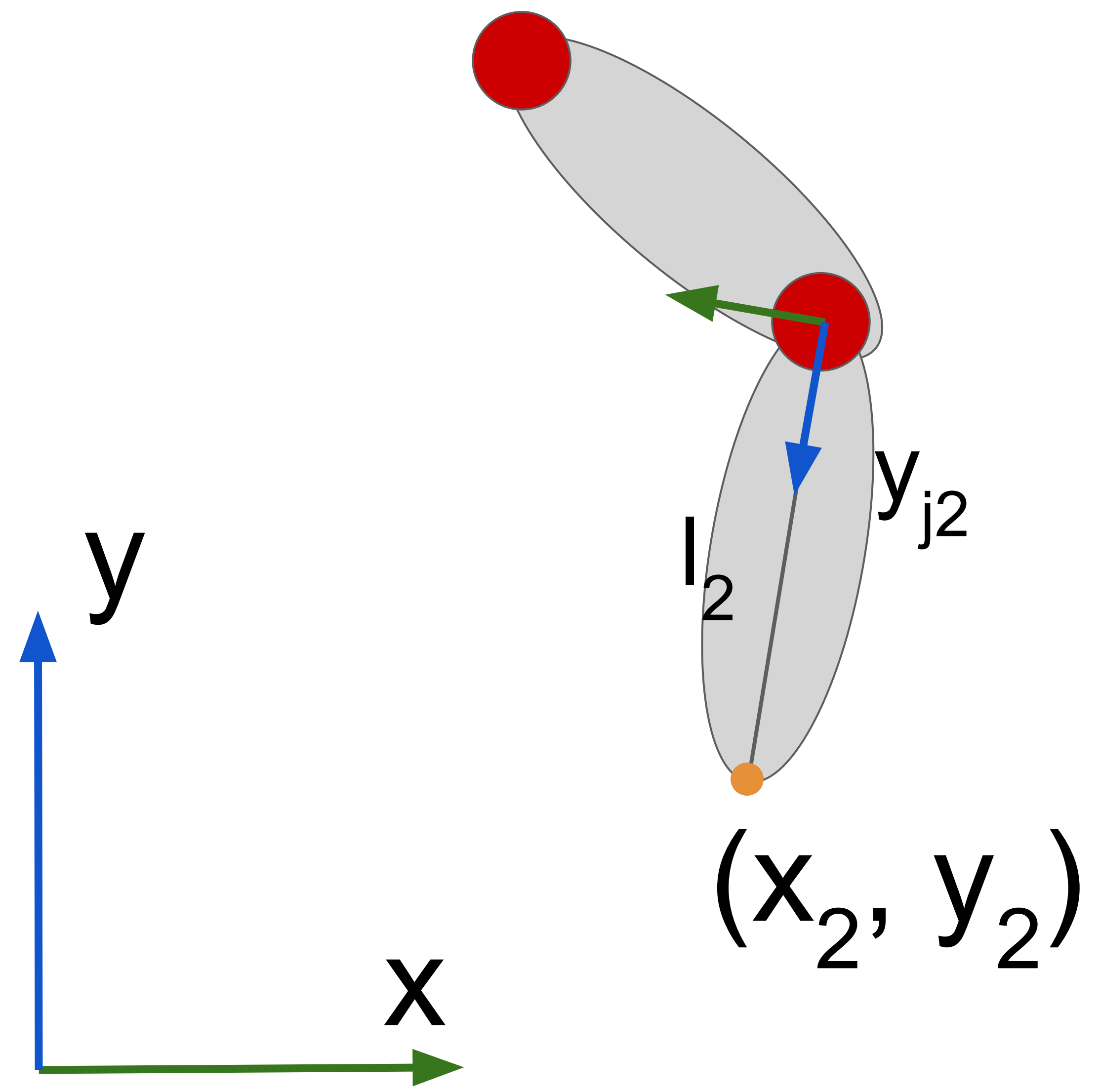
$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

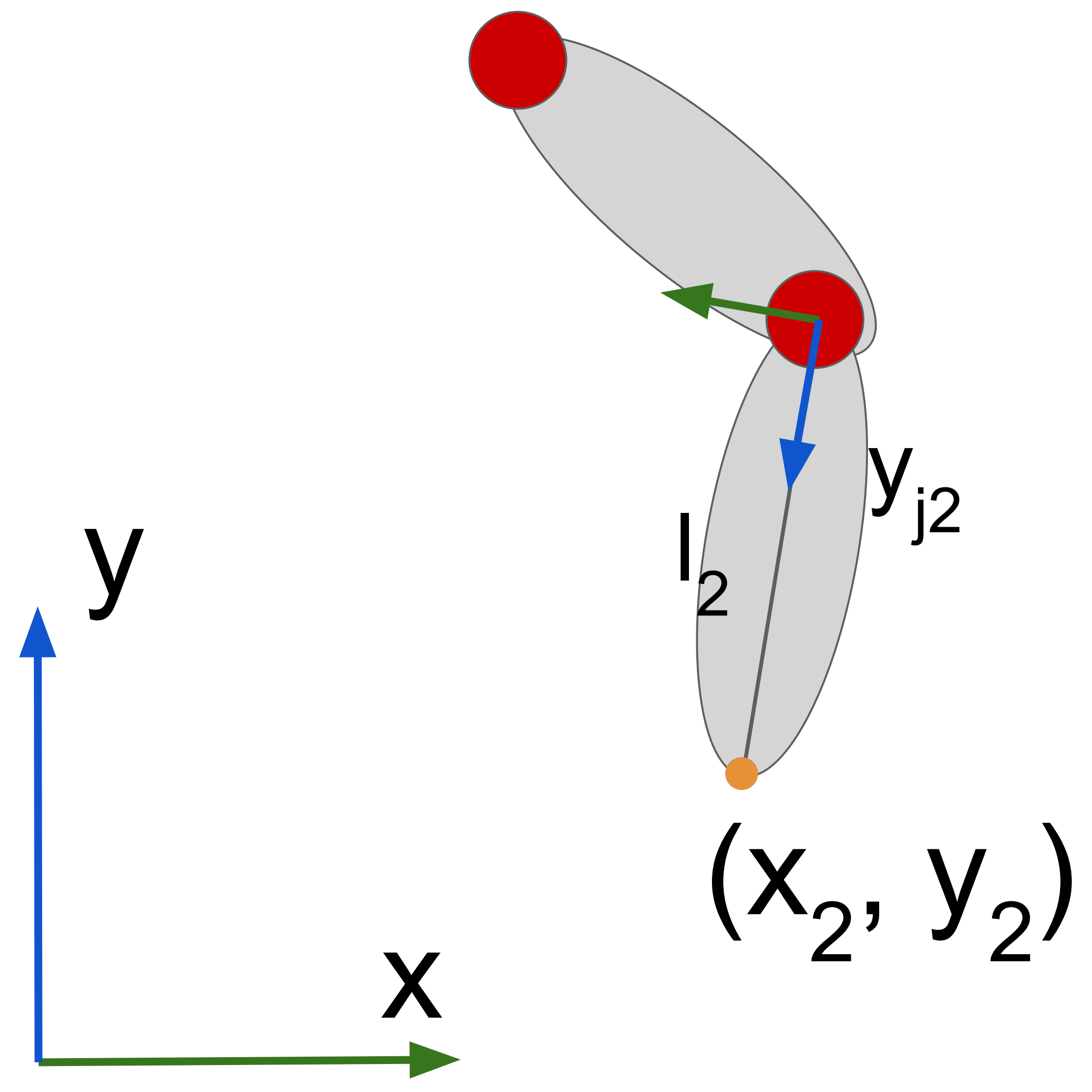


$$= \begin{bmatrix} -l_2 \cos\theta_1 \sin\theta_2 + l_2 \sin\theta_1 \cos\theta_2 + l_1 \sin\theta_1 + x_0 \\ -l_2 \sin\theta_1 \sin\theta_2 - l_2 \cos\theta_1 \cos\theta_2 - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$

General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1} T_{j_1 \to j_{2'}} T_{j_{2'} \to j_2} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$



$$= \begin{bmatrix} -l_2 \cos\theta_1 \sin\theta_2 + l_2 \sin\theta_1 \cos\theta_2 + l_1 \sin\theta_1 + x_0 \\ -l_2 \sin\theta_1 \sin\theta_2 - l_2 \cos\theta_1 \cos\theta_2 - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$

$$\overset{?}{=} \begin{bmatrix} -l_2 \sin(\theta_2 - \theta_1) + l_1 \sin\theta_1 + x_0 \\ -l_2 \cos(\theta_2 - \theta_1) - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$

General Approach

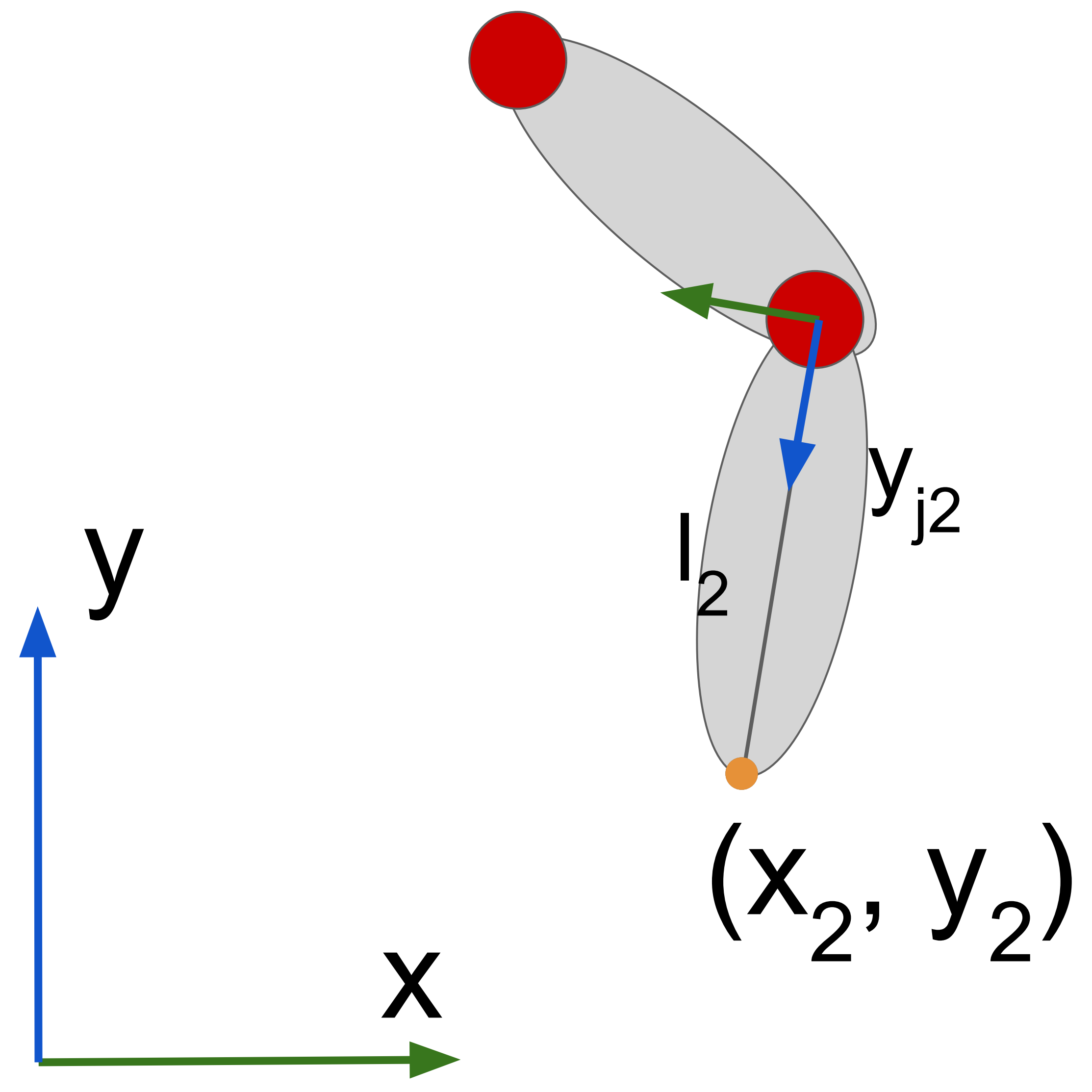
$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1} T_{j_1 \to j_{2'}} T_{j_{2'} \to j_2} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi + \theta_1) & -\sin(\pi + \theta_1) & 0 \\ \sin(\pi + \theta_1) & \cos(\pi + \theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 \cos\theta_1 \sin\theta_2 + l_2 \sin\theta_1 \cos\theta_2 + l_1 \sin\theta_1 + x_0 \\ -l_2 \sin\theta_1 \sin\theta_2 - l_2 \cos\theta_1 \cos\theta_2 - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 \sin(\theta_2 - \theta_1) + l_1 \sin\theta_1 + x_0 \\ -l_2 \cos(\theta_2 - \theta_1) - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$, because

$$\boxed{\begin{aligned} \sin(\theta_2 - \theta_1) &= \sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1 \\ \cos(\theta_2 - \theta_1) &= \cos\theta_2 \cos\theta_1 + \sin\theta_2 \sin\theta_1 \end{aligned}}$$

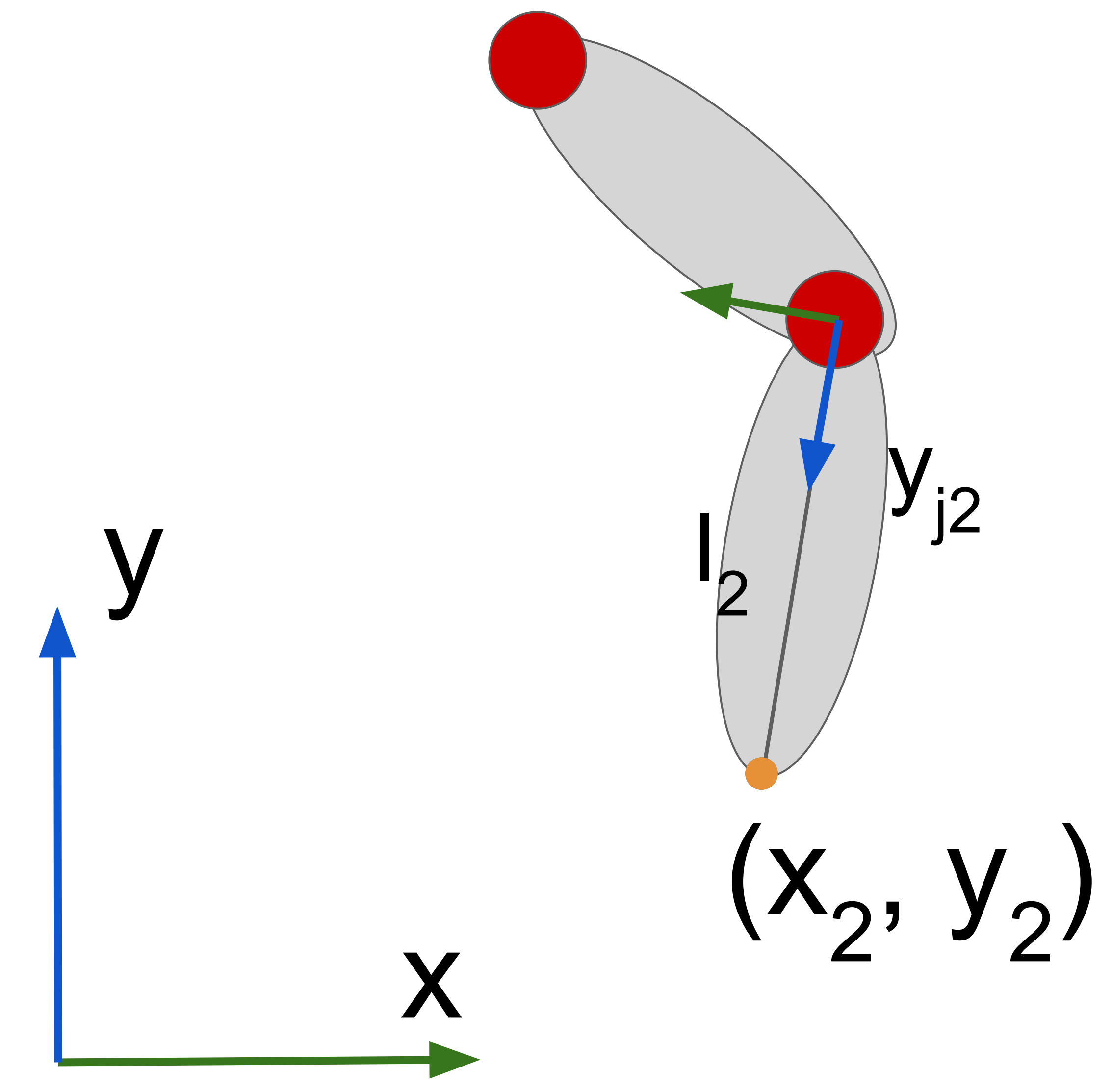


General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w\to j_{1'}} T_{j_{1'}\to j_1} T_{j_1 \to j_{2'}} T_{j_{2'}\to j_2} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \underline{-l_2 \cos\theta_1 \sin\theta_2 + l_2 \sin\theta_1 \cos\theta_2} + l_1 \sin\theta_1 + x_0 \\ -l_2 \sin\theta_1 \sin\theta_2 - l_2 \cos\theta_1 \cos\theta_2 - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$

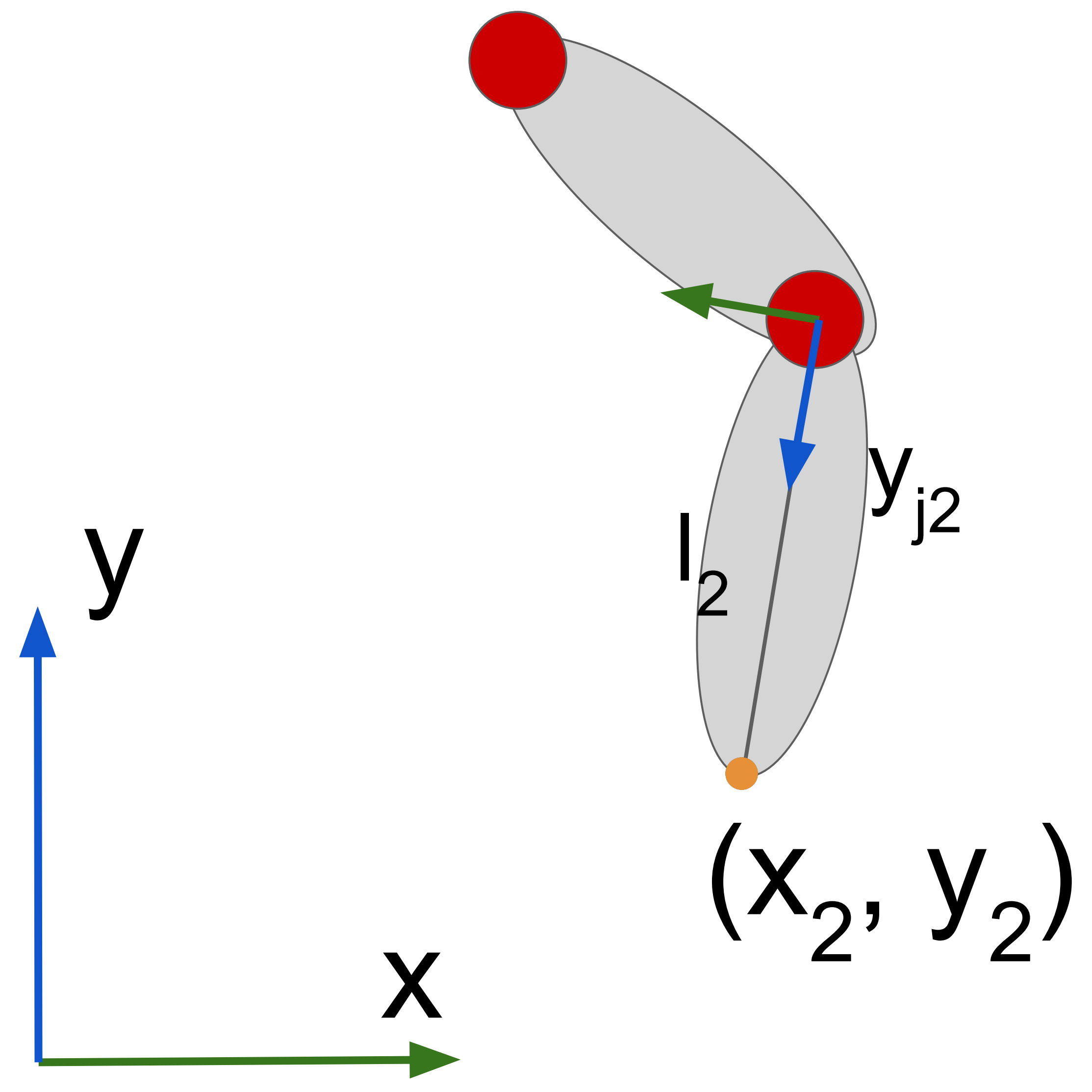


$$= \begin{bmatrix} \underline{-l_2 \sin(\theta_2 - \theta_1)} + l_1 \sin\theta_1 + x_0 \\ -l_2 \cos(\theta_2-\theta_1) - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$, because $$\boxed{\begin{aligned} \sin(\theta_2-\theta_1) &= \sin\theta_2\cos\theta_1 - \cos\theta_2\sin\theta_1 \\ \cos(\theta_2-\theta_1) &= \cos\theta_2\cos\theta_1 + \sin\theta_2\sin\theta_1 \end{aligned}}$$

General Approach

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = T_{w \to j_{1'}} T_{j_{1'} \to j_1} T_{j_1 \to j_{2'}} T_{j_{2'} \to j_2} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\pi+\theta_1) & -\sin(\pi+\theta_1) & 0 \\ \sin(\pi+\theta_1) & \cos(\pi+\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_2) & -\sin(-\theta_2) & 0 \\ \sin(-\theta_2) & \cos(-\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ l_2 \\ 1 \end{bmatrix}$$



$$= \begin{bmatrix} -l_2 \cos\theta_1 \sin\theta_2 + l_2 \sin\theta_1 \cos\theta_2 + l_1 \sin\theta_1 + x_0 \\ \underline{-l_2 \sin\theta_1 \sin\theta_2 - l_2 \cos\theta_1 \cos\theta_2} - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 \sin(\theta_2 - \theta_1) + l_1 \sin\theta_1 + x_0 \\ \underline{-l_2 \cos(\theta_2 - \theta_1)} - l_1 \cos\theta_1 + y_0 \\ 1 \end{bmatrix}$$, because $\boxed{\begin{aligned} \sin(\theta_2 - \theta_1) &= \sin\theta_2 \cos\theta_1 - \cos\theta_2 \sin\theta_1 \\ \cos(\theta_2 - \theta_1) &= \cos\theta_2 \cos\theta_1 + \sin\theta_2 \sin\theta_1 \end{aligned}}$

Quiz

Is FK a linear transformation?

Pseudo-code

```
Function FK2D(l[], theta[], x, y):
    // Input:
    // l[]: Array of body length
    // theta[]: Array of joint angles
    // x, y: The offset of 1st joint in the world coordinate

    // Output:
    // pos: position of the end effector

    T = TranslationMatrix(x, y) * RotationMatrix(theta[0])
    for i = 1 to n-1:
        Trans = TranslationMatrix(0, l[i - 1])
        Rot = RotationMatrix(theta[i])
        T = T * Trans * Rot
    pos = T * (0, l[n-1], 1)
    return pos[0:2]
```

Considerations about Speed

$$\mathbf{x}_w = \mathbf{T}_1\mathbf{R}_1 \ldots \mathbf{T}_n\mathbf{R}_n\mathbf{x}_l$$

Time complexity?

Considerations about Speed

$$\mathbf{x}_w = \mathbf{T}_1\mathbf{R}_1 \ldots \mathbf{T}_n\mathbf{R}_n\mathbf{x}_l$$

Time complexity?

Affine transformation: combine translation and rotation

$$\mathbf{A} = \mathbf{T}(x, y) * \mathbf{R}(\theta) = \begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & x \\ \sin\theta & \cos\theta & y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{x}_w = \mathbf{A}_1 \ldots \mathbf{A}_n\mathbf{x}_l$$

Considerations about Speed

$$\mathbf{x}_w = \mathbf{T}_1\mathbf{R}_1 \ldots \mathbf{T}_n\mathbf{R}_n\mathbf{x}_l$$

Time complexity?

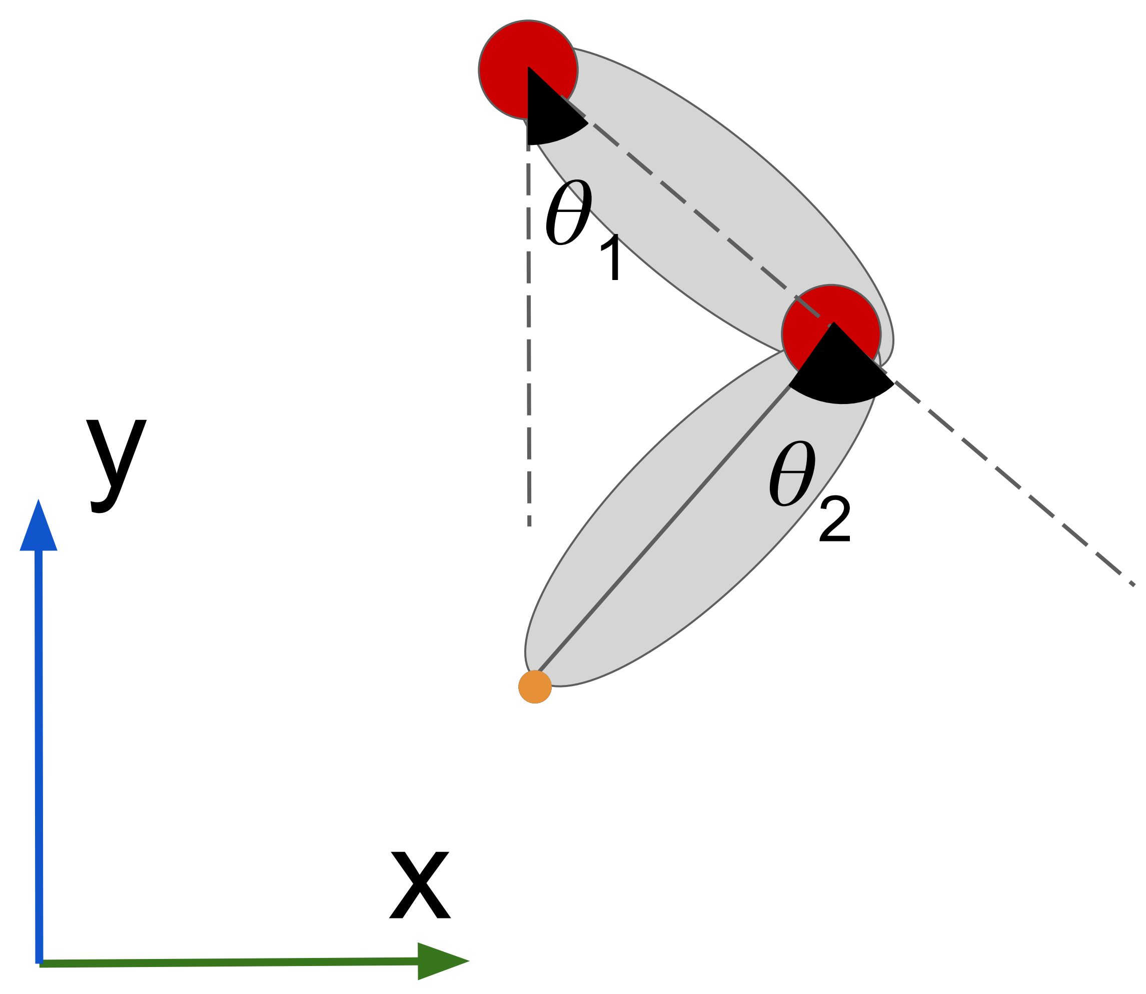
Affine transformation: combine translation and rotation

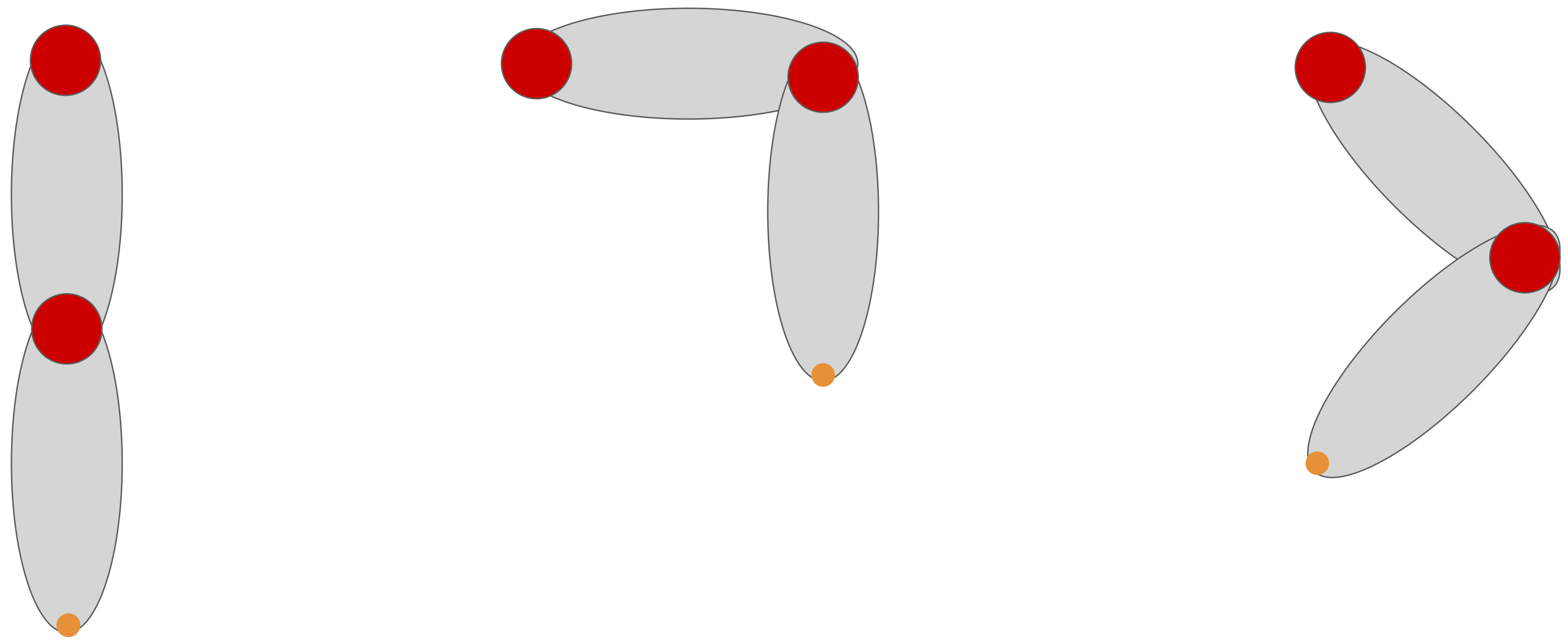
Multiply from the right!

$$\mathbf{x}_w = (\mathbf{A}_1(\ldots(\mathbf{A}_n\mathbf{x}_l)))$$

Debug

- Start with fewer links
- Do NOT just guess or try random corrections
- Write unit tests





(a) θ_1=0; θ_2=0 (b) θ_1=π/2; θ_2=-π/2 (c) θ_1=π/4; θ_2=-π/2

Summary

FK and IK are widely used in robotics, computer graphics, and many engineering applications

FK is a nonlinear mapping from joint angles to the position of end-effectors in the world coordinate

FK can be calculated efficiently via iteratively performing transformations (matrix multiplications) from one joint to the next

Office Hour

Time: This Friday 3-4 pm

Location: Outside of Karen’s office (Coda E356)

Topic: Anything about AI and Robotics