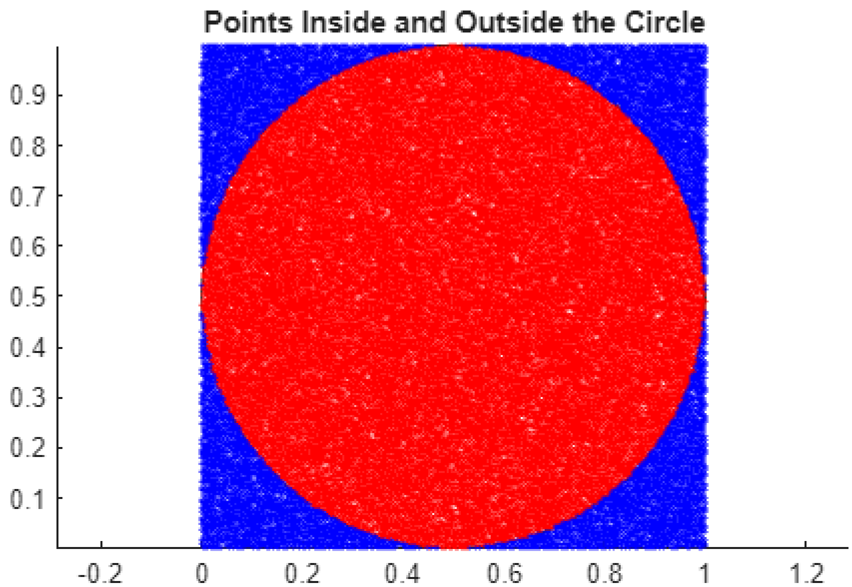
Question 1

We begin by writing a script that uses a *for* loop to generate the value of π.

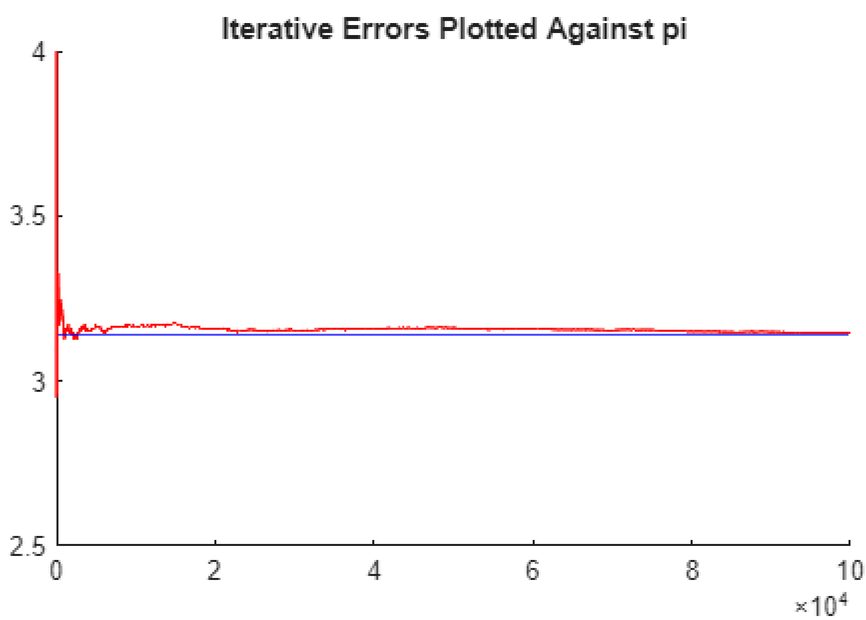
```
clear

% This gives us a table of random points, correspondent to the number of
% points we want to generate. Additionally, we create a vector of the
% distance of points from (0.5, 0.5), and a vector for the iterated values
% of pi
numPoints = 100000;
genPoints = [rand(numPoints, 2)];

distPoints = zeros(numPoints,1);
iterPi = zeros(numPoints,1);

% Our for loop calculates the iterates of our Monte Carlo simulation for
% plotting convergence. This can alternatively be done with vectorized
% operations, eliminating the need for a for loop
for i = 1:numPoints
    distPoints(i) = dist(genPoints(i,:));
    iterPi(i) = calcPi(distPoints(1:i), i);
end

% Here, we display the approximate value of pi
fprintf('Approximate value of pi: %.15f\n', iterPi(end));
```

```
Approximate value of pi: 3.147240000000000
```

```

% We would now like to plot the circle and points
theta = linspace(0,pi*2, 100);
xCircle = 0.5 + 0.5*cos(theta);
yCircle = 0.5 + 0.5*sin(theta);
% Now plot the circle and the generated points
figure;
hold on;
fill(xCircle, yCircle, 'white', 'FaceAlpha', 0.9); % Circle
scatter(genPoints(distPoints <= 0.5, 1), genPoints(distPoints <= 0.5, 2), 1, 'r');
% Points inside the circle
scatter(genPoints(distPoints > 0.5, 1), genPoints(distPoints > 0.5, 2), 1, 'b'); %
Points outside the circle
axis equal;
title('Points Inside and Outside the Circle');
hold off;
```




```
% Plotting iterative errors
figure;
hold on;
plot(1:numPoints, iterPi, 'red');
yline(pi,'blue');
ylim([2.5 4]);
title('Iterative Errors Plotted Against pi')
hold off;
```



Required Functions

```
% We write a function to determine which points are in the circle and
% additionally calculate the approximate value of pi.
```

```
function approxPi = calcPi(d,n)
    insideCircle = nnz(d(:) <= 0.5);
    % display(insideCircle)
    approxPi = (insideCircle / n) * 4;
end

% The distance function under the l^2 norm
function d = dist(v)
    d = abs(sqrt(sum((v-0.5).^2,2)));
end
```

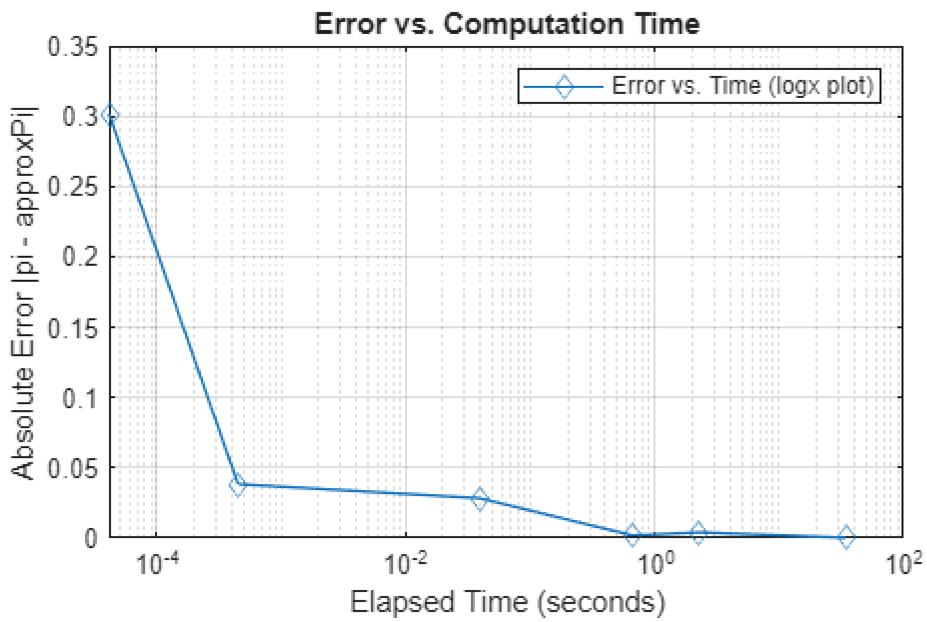
A function version of the for loop is written below, without comments. This is specifically for precision vs time analysis.

```
% Precision vs cost analysis

% The point list represents the entries for numPoints that we use.
% The results matrix stores the approximate value of pi and the elapsed
% time.
pointList = [100, 1000, 10000, 50000, 100000, 250000];
results = zeros(length(pointList), 2);

% Our for loop loops through every value in the point list, evaluating the
% function for each entry.
for i = 1:length(pointList)
    numPoints = pointList(i);

    [approxPi, elapsedTime] = monteCarloPiAnalysis(numPoints);

    results(i, 1) = approxPi;
    results(i, 2) = elapsedTime;

    % We have a progress display to ensure our code is running
    fprintf('Completed analysis for %d points.\n', numPoints);
end
```

```
Completed analysis for 100 points.
Completed analysis for 1000 points.
Completed analysis for 10000 points.
Completed analysis for 50000 points.
Completed analysis for 100000 points.
Completed analysis for 250000 points.
```

```

% This calculus the absolute error against pi
absError = abs(pi-results(:,1));

% Plotting iterative errors against computational time
figure;
semilogx(results(:, 2), absError, '-d');
grid on;
```

```
title('Error vs. Computation Time');
xlabel('Elapsed Time (seconds)');
ylabel('Absolute Error |pi - approxPi|');
legend('Error vs. Time (logx plot)');
```

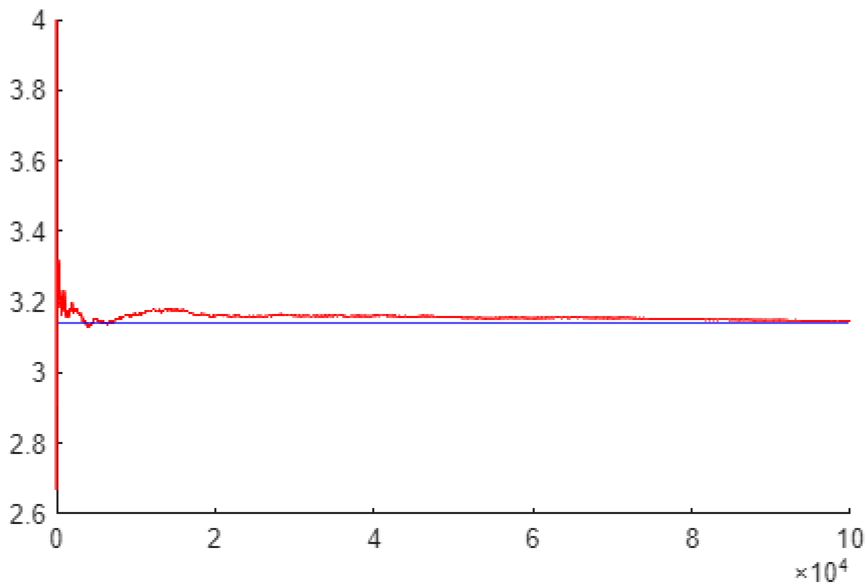



```
% Function begins here
function [approxPi, elapsedTime] = monteCarloPiAnalysis(numPoints)
    startTime = tic;
    genPoints = rand(numPoints, 2);
    distPoints = zeros(numPoints, 1);
    iterPi = zeros(numPoints, 1);

    for i = 1:numPoints
        distPoints(i) = sqrt((genPoints(i,1) - 0.5)^2 + (genPoints(i,2) - 0.5)^2);
        pointsInside = sum(distPoints(1:i) <= 0.5);
        iterPi(i) = 4 * pointsInside / i;
    end

    approxPi = iterPi(end);
    elapsedTime = toc(startTime);
end
```

Now with matrix operations exclusively:

```
numPoints = 100000;
genPoints = [rand(numPoints, 2)];

distPoints = sqrt(sum((genPoints-0.5).^2,2));
insideCircle = nnz(distPoints(:) <= 0.5);
```

```
approxPi = (insideCircle / numPoints) * 4;

pointIndices = (1:numPoints)';
cumulativeInside = cumsum(distPoints(:)<= 0.5);
iterPi = (cumulativeInside ./ pointIndices) * 4;

figure;
hold on;
plot(pointIndices,iterPi,'red');
yline(pi, 'blue');
hold off
```




```
fprintf('Approximate value of pi: %.15f\n', approxPi);
```

```
Approximate value of pi: 3.146360000000000
```

Question 2

In the following section, we adopt the above script to use a while loop to achieve a desired level of precision

```
clear

% Initial assumptions needed to bootstrap the while loop
batchSize = 500;
totalPoints = 0;
totalInside = 0;
tolerance = 0.001;
piCurrent = 10;
piPrevious = 0;
numIter = 0;
```

```
while abs(piCurrent - piPrevious) > tolerance
    piPrevious = piCurrent;

    % This generates a new list of points every iteration to check distance on
    newPoints = rand(batchSize, 2);
    % Checking distance of our list of points
    insideCircle = nnz(dist(newPoints) <= 0.5);

    % We count the number of totalPoints
    totalPoints = totalPoints + size(newPoints,1);
    % We count the number of inside points with totalInside
    totalInside = totalInside + insideCircle;
    piCurrent = (totalInside/totalPoints) * 4;

    % Iteration counter
    numIter = numIter + 1;
end


display(piCurrent);
```

```
piCurrent =
3.1328
```

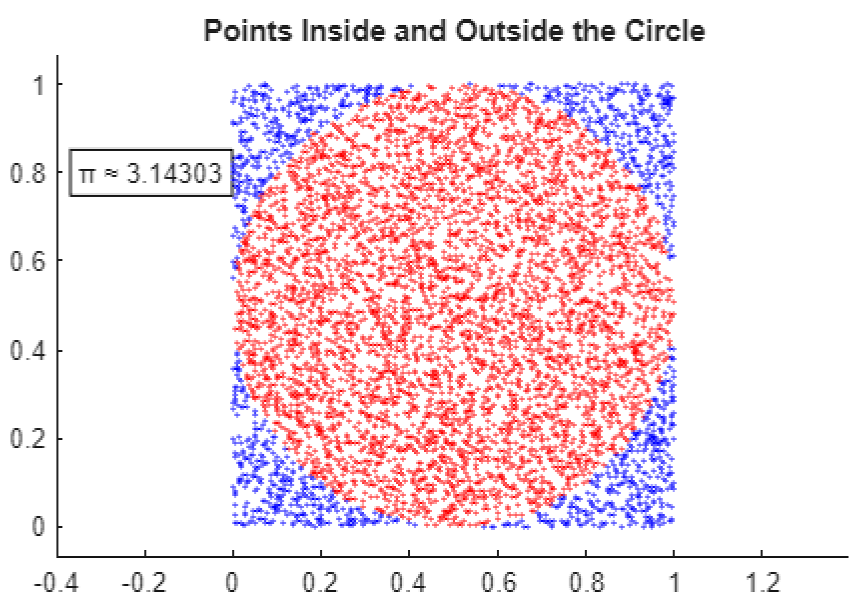
```
display(totalPoints);
```

```
totalPoints =
2500
```

We are now asked to turn it into a function that takes user input for a certain *tolerance*. Precision as defined by convergence to pi is hard to define without more advanced statistical methods.

```
function desiredPi = calcPiTol(precision)
    % Bootstrapping variables
    batchSize = 200;
    totalPoints = 0;
    totalInside = 0;
    piCurrent = 10;
    piPrevious = 0;
    numIter = 0;

    figure;

    % Filler needed for updating text on plot
    updateText = text(-0.35, 0.8, '0', 'EdgeColor', 'black');
    while abs(piCurrent - piPrevious) > precision
        piPrevious = piCurrent;
        % This generates a new list of points every iteration to check distance on
        newPoints = rand(batchSize, 2);
        % Checking distance of our list of points
        insideCircle = nnz(dist(newPoints) <= 0.5);
```

```
        % We count the number of totalPoints
        totalPoints = totalPoints + size(newPoints,1);
        % We count the number of inside points with totalInside
        totalInside = totalInside + insideCircle;
        piCurrent = (totalInside/totalPoints) * 4;

        % Drawing figure
        hold on;
        xlim([-0.4, 1.4])
        scatter(newPoints(dist(newPoints) <= 0.5, 1), newPoints(dist(newPoints) <=
0.5, 2), 1, 'r'); % Points inside the circle
        scatter(newPoints(dist(newPoints) > 0.5, 1), newPoints(dist(newPoints) >
0.5, 2), 1, 'b'); % Points outside the circle
        axis equal;
        title('Points Inside and Outside the Circle');
        drawnow;
        piString = sprintf('π ≈ %.5f', piCurrent);
        set(updateText, 'String', piString);
        hold off;

        % Iteration counter
        numIter = numIter + 1;

        pause(0.25)
    end
    desiredPi = piCurrent;
end

calcPiTol(0.0001)
```




```
ans =
3.1430
```