



Shape As Points

A Differentiable Poisson Solver

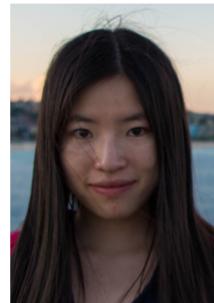
Songyou Peng



Chiyu "Max" Jiang



Yiyi Liao



Michael Niemeyer

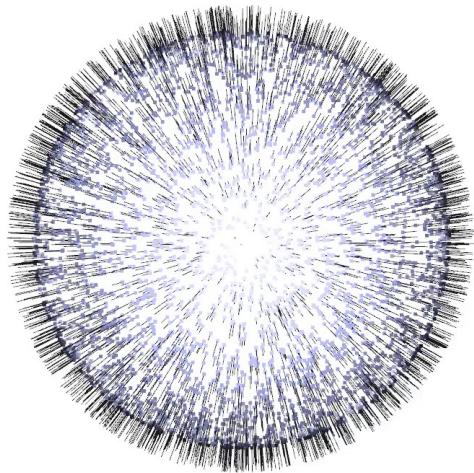


Marc Pollefeys

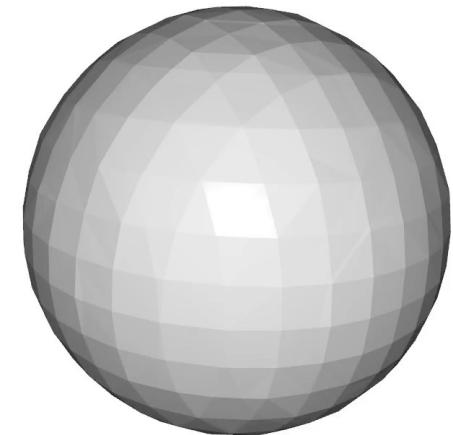


Andreas Geiger





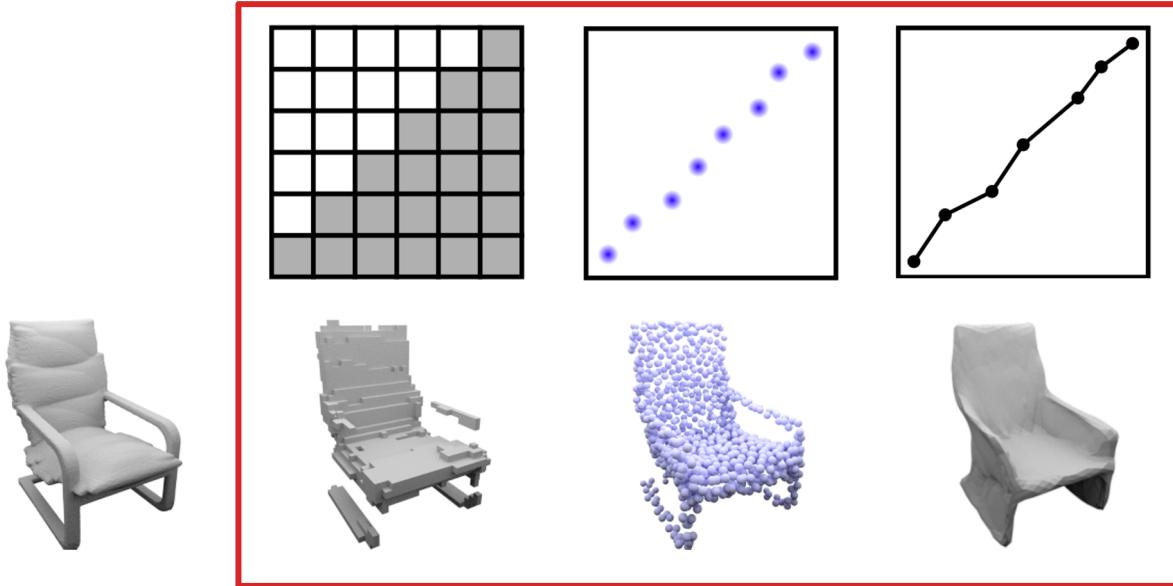
Shape As Points
(SAP)



Duality between **oriented point clouds** and **3D dense geometry**

What is a good 3D shape representation?

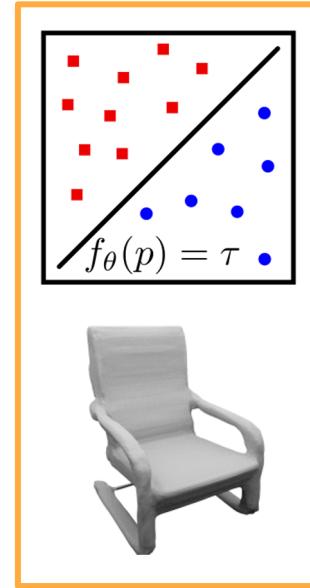
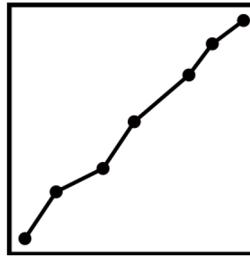
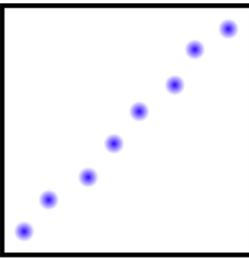
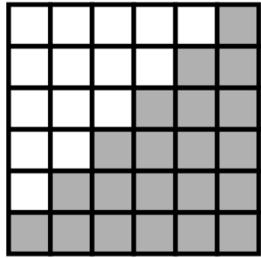
3D Shape Representations



Traditional Explicit Representations

- + Fast inference
- Discrete

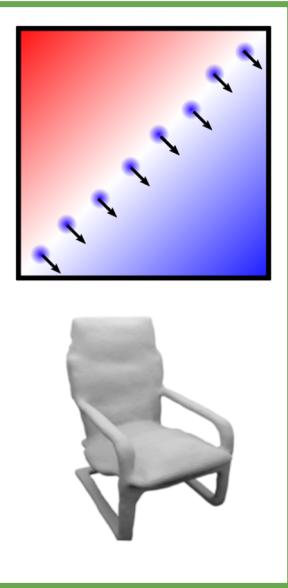
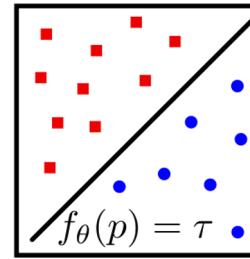
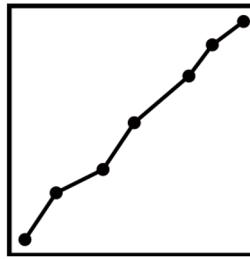
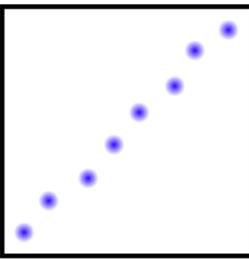
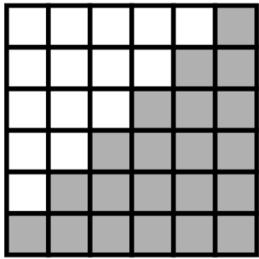
3D Shape Representations



Neural Implicit Representations

- + Continuous, watertight
- Slow inference
- Difficult to initialize

3D Shape Representations



Shape As Points (SAP) - Hybrid Representation

- + Discrete \Rightarrow Continuous
- + Fast inference
- + Easy initialization, topology-agnostic

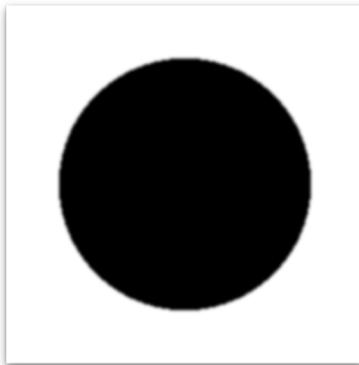
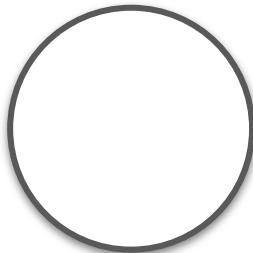
Method

Differentiable Poisson Solver

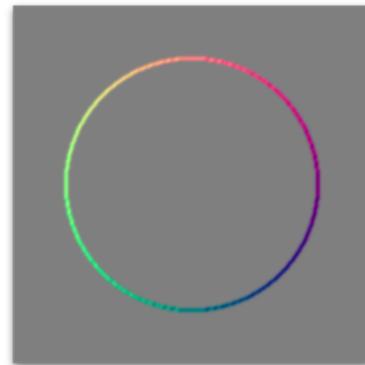


Intuition of Poisson Equation

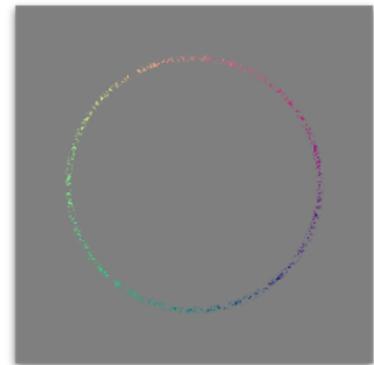
$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$

 χ

Shape

 $\nabla \chi$

Gradient

 \mathbf{v}

Point Normals

Our Poisson Solver

$$\nabla^2 \chi := \nabla \cdot \nabla \chi = \nabla \cdot \mathbf{v}$$

- **Discretization** allows to invert the divergence operator

$$\chi = (\nabla^2)^{-1} \nabla \cdot \mathbf{v}$$

- **Spectral methods** to solve the Poisson equation

- Derivatives of signals in spectral domain are computed analytically
- Fast Fourier Transform (FFT) are **highly optimized on GPUs/TPUs**
- Only **25-line code**

$$\tilde{\mathbf{v}} = \text{FFT}(\mathbf{v}) \quad \rightarrow \quad \tilde{\chi} = \tilde{g}_{\sigma,r}(\mathbf{u}) \odot \frac{i\mathbf{u} \cdot \tilde{\mathbf{v}}}{-2\pi\|\mathbf{u}\|^2} \quad \rightarrow \quad \chi' = \text{IFFT}(\tilde{\chi})$$

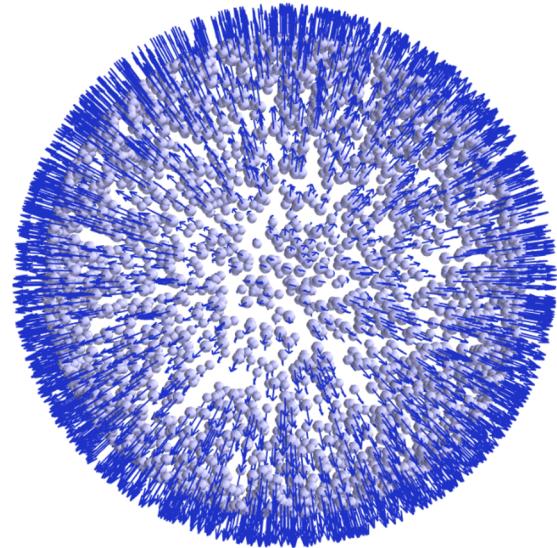
Surface Reconstruction from Unoriented Point Clouds

1. SAP for **Optimization-based** 3D Reconstruction
2. SAP for **Learning-based** 3D Reconstruction

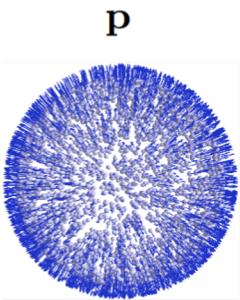
SAP for Optimization-based 3D Reconstruction

Pipeline - Forward Pass

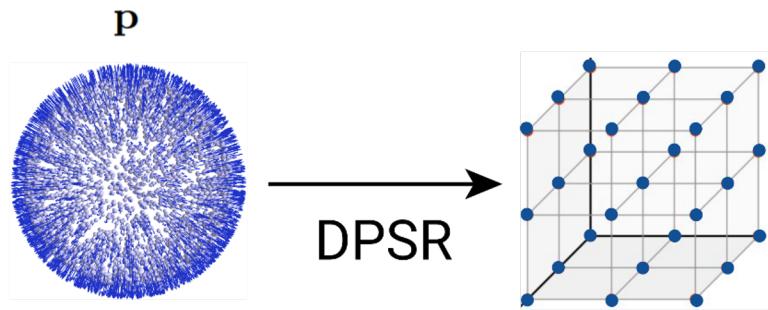
Input an initial oriented point cloud
(noisy / incomplete observations)



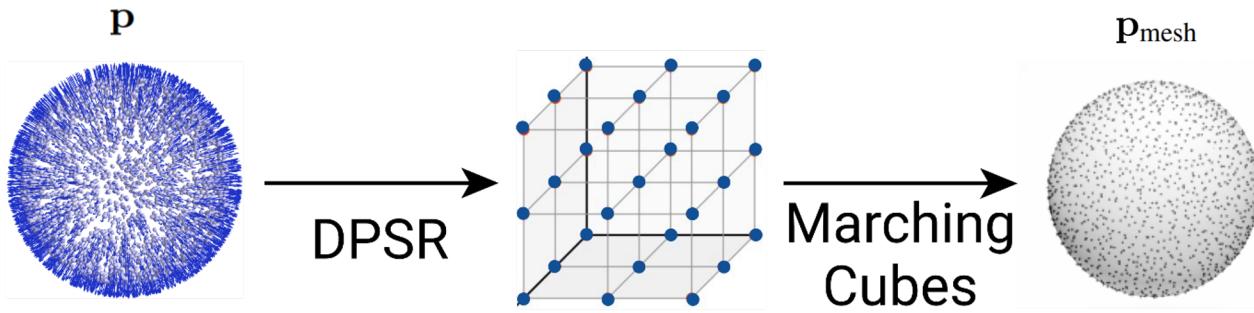
Pipeline - Forward Pass



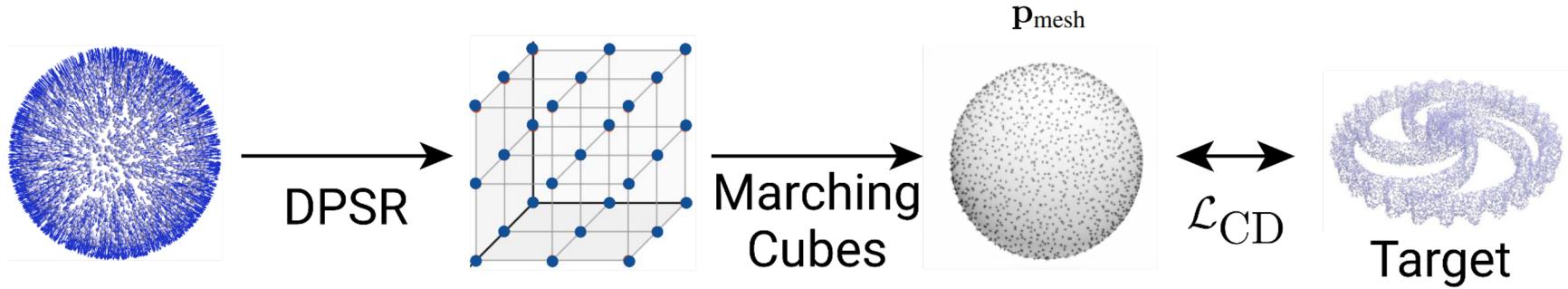
Pipeline - Forward Pass



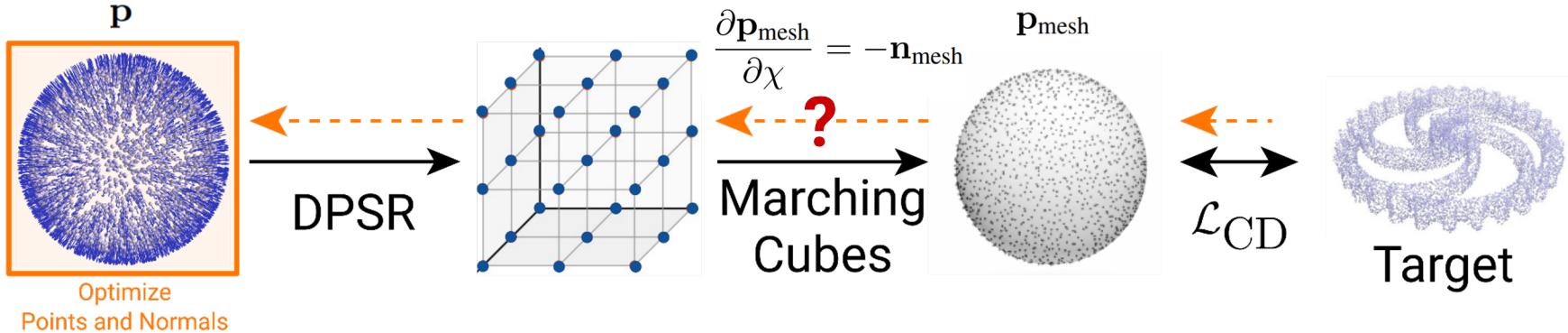
Pipeline - Forward Pass



Pipeline - Forward Pass

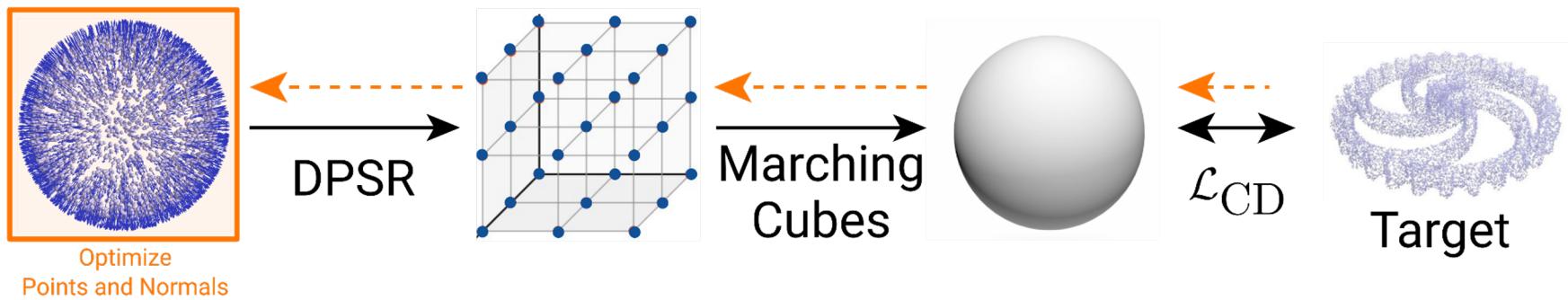


Pipeline - Backward Pass

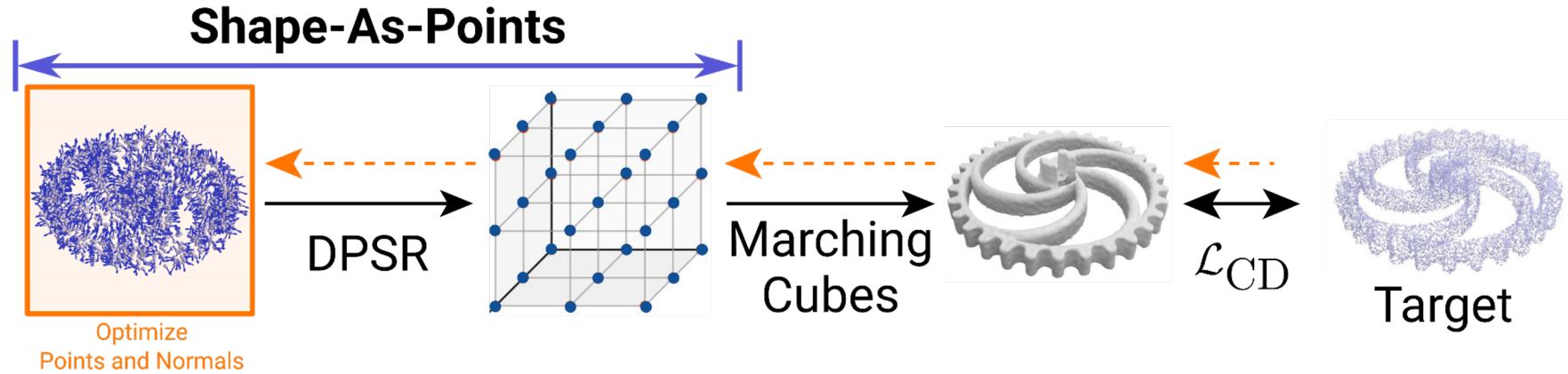


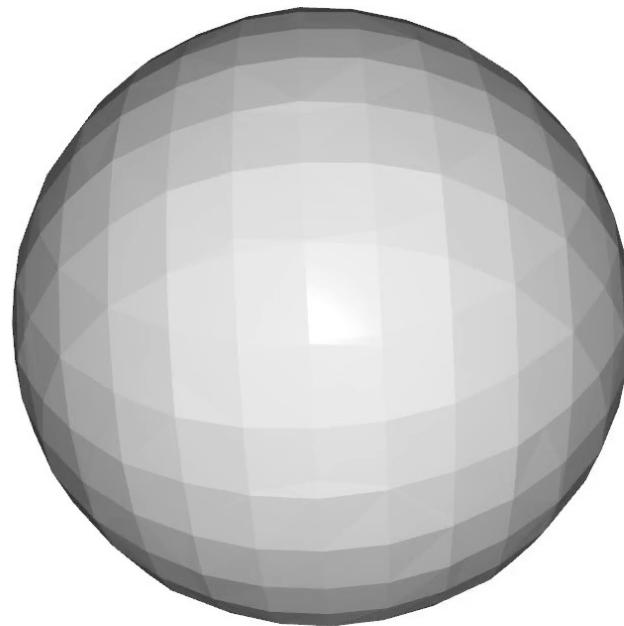
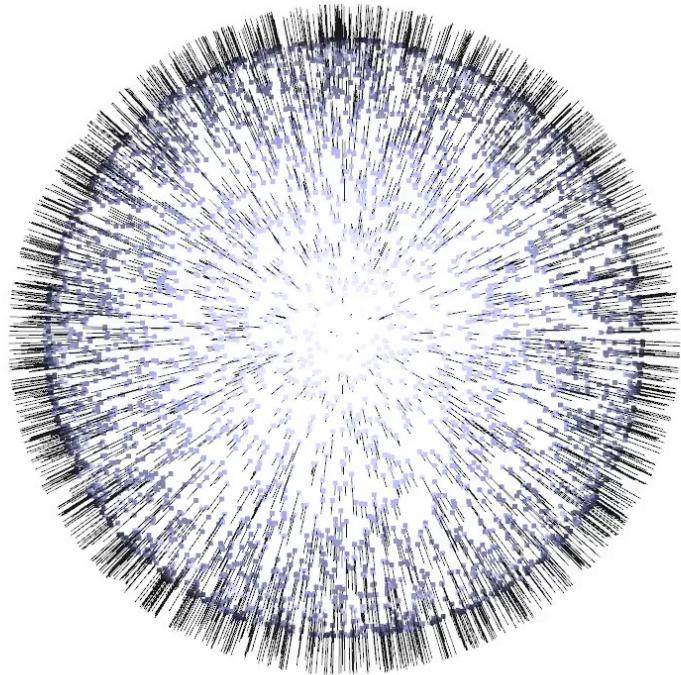
$$\frac{\partial \mathcal{L}_{\text{CD}}}{\partial \mathbf{p}} = \frac{\partial \mathcal{L}_{\text{CD}}}{\partial \mathbf{p}_{\text{mesh}}} \frac{\partial \mathbf{p}_{\text{mesh}}}{\partial \chi} \frac{\partial \chi}{\partial \mathbf{p}}$$

Pipeline

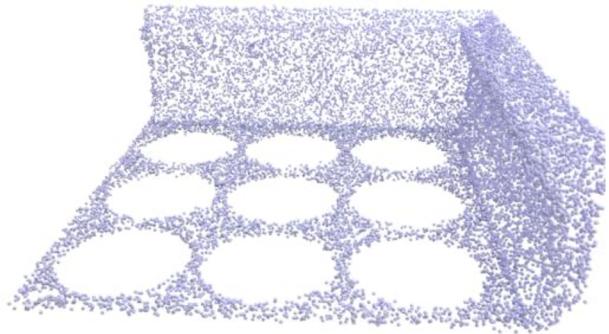


Pipeline





Comparison

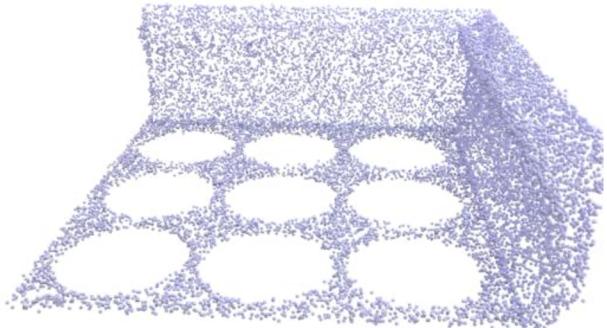


Unoriented Point Clouds

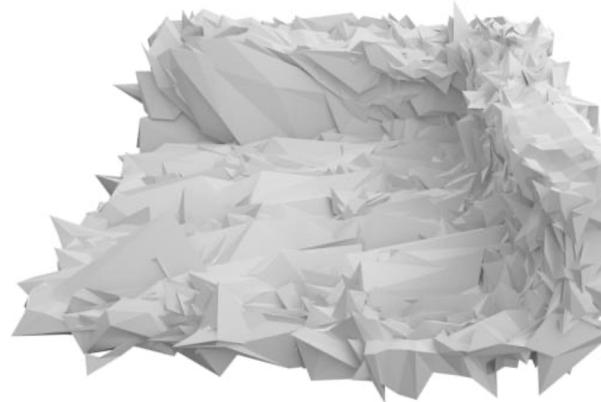


GT Mesh

Comparison



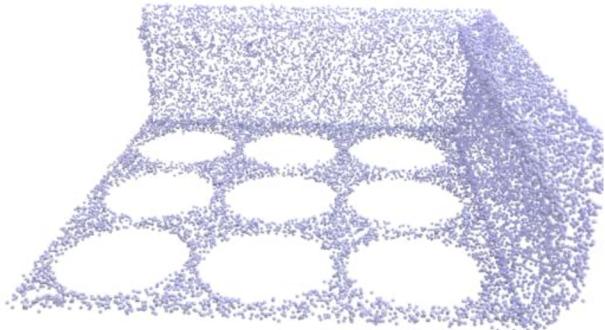
Unoriented Point Clouds



Point2Mesh

Runtime: 62 mins

Comparison



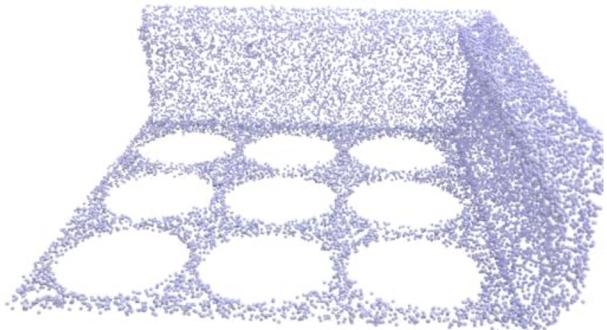
Unoriented Point Clouds



IGR

Runtime: 30 mins

Comparison



Unoriented Point Clouds



SAP

Runtime: ~6 mins

Comparison



SPSR

Runtime: ~9 sec



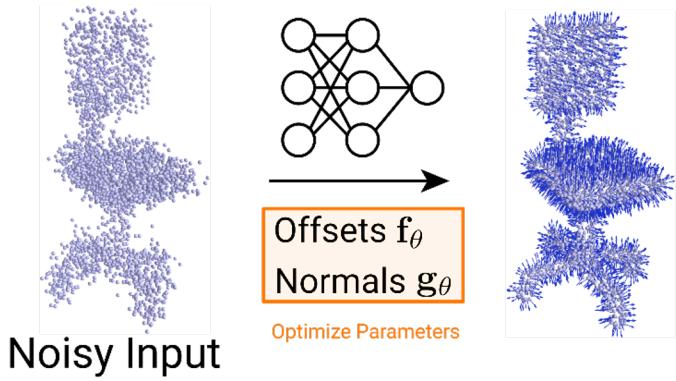
SAP

Runtime: ~6 mins

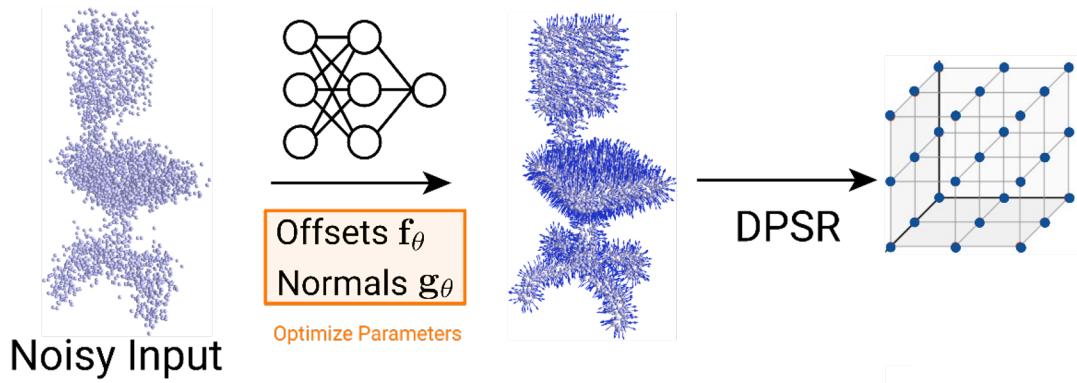
Can we further leverage the **differentiability** of the Poisson solver
for **deep neural networks**?

SAP for Learning-based 3D Reconstruction

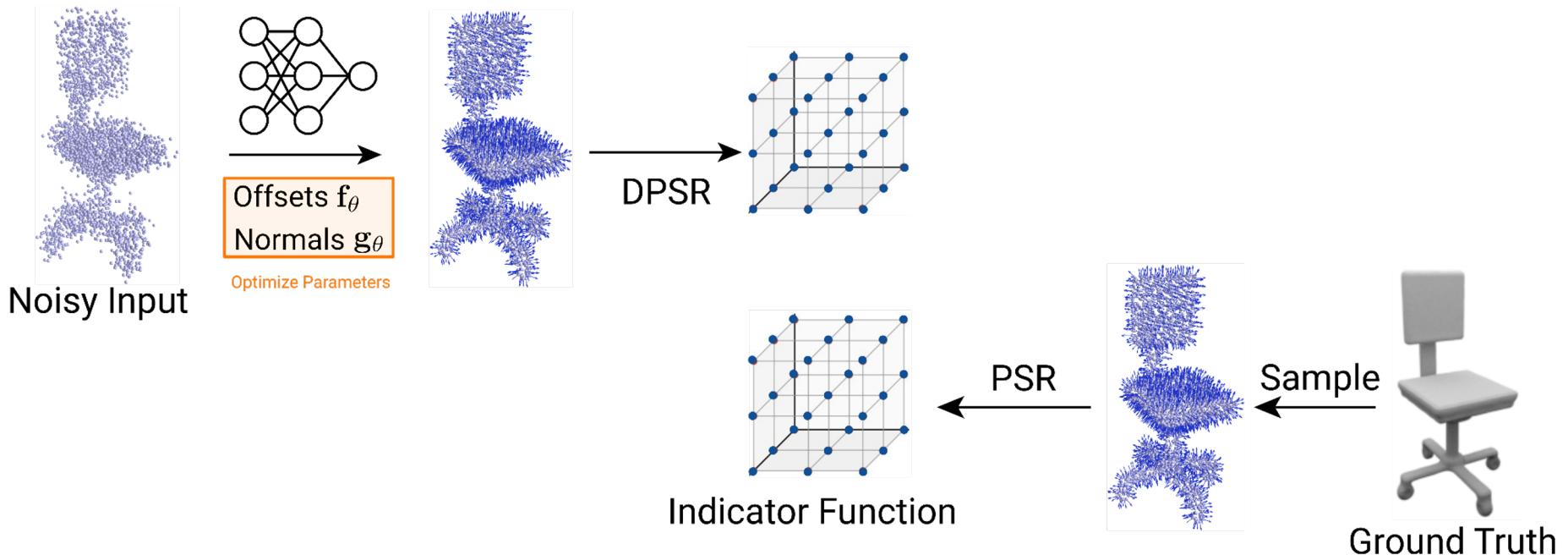
Learning-based Pipeline



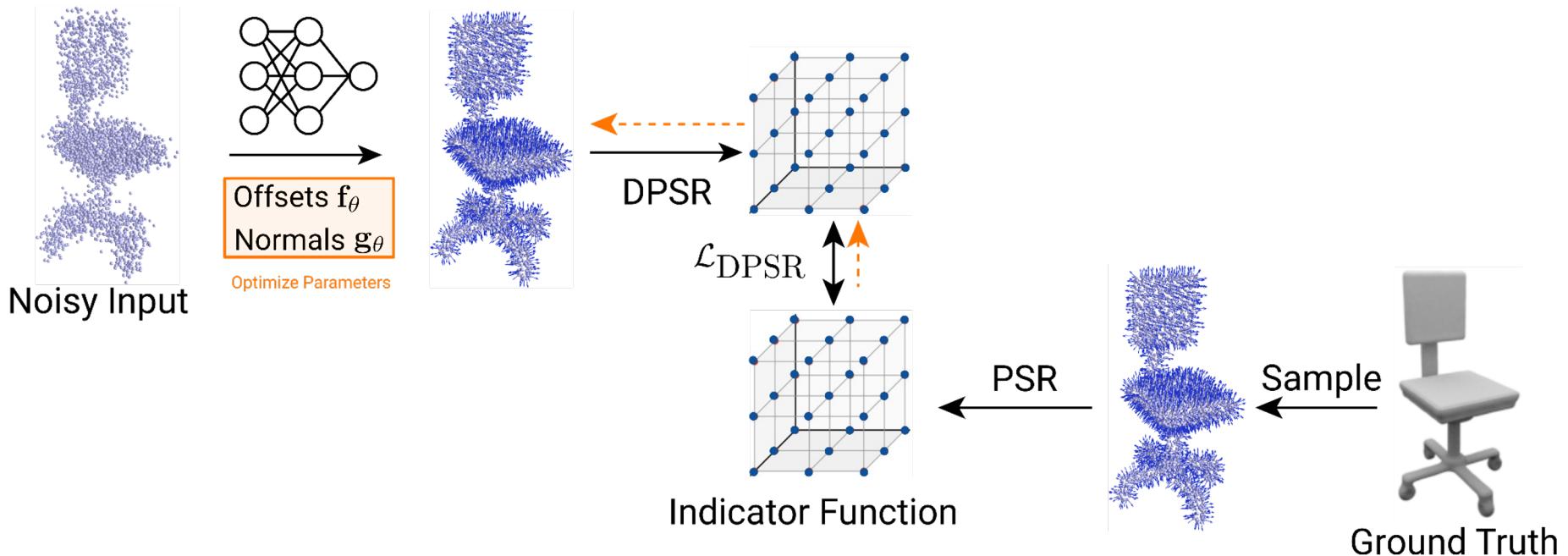
Learning-based Pipeline



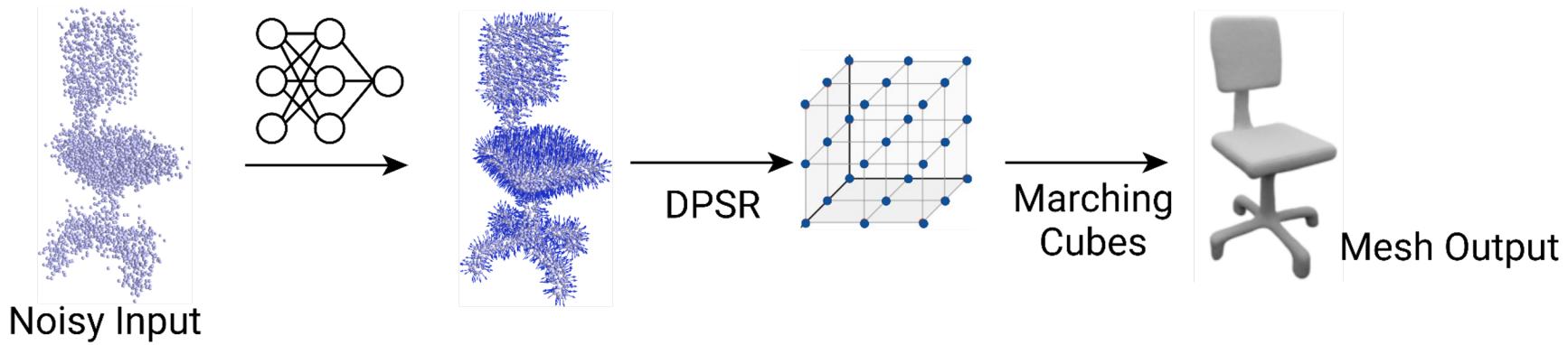
Learning-based Pipeline



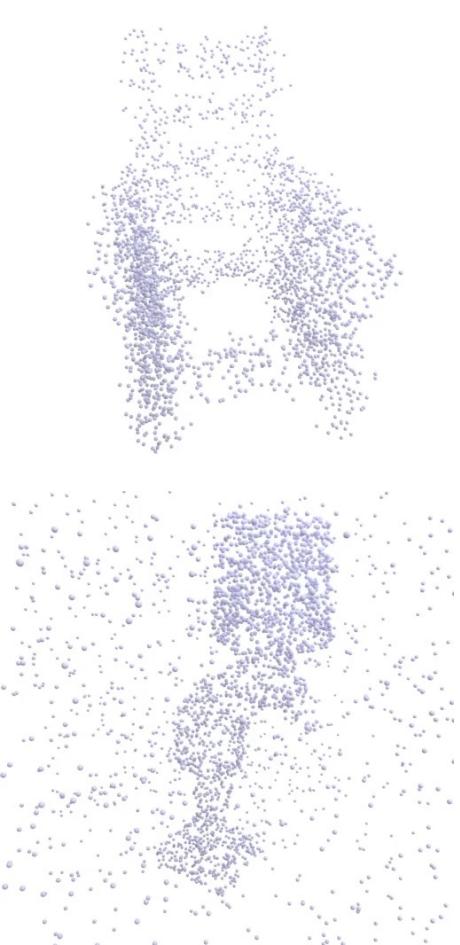
Learning-based Pipeline



Learning-based Pipeline



Results

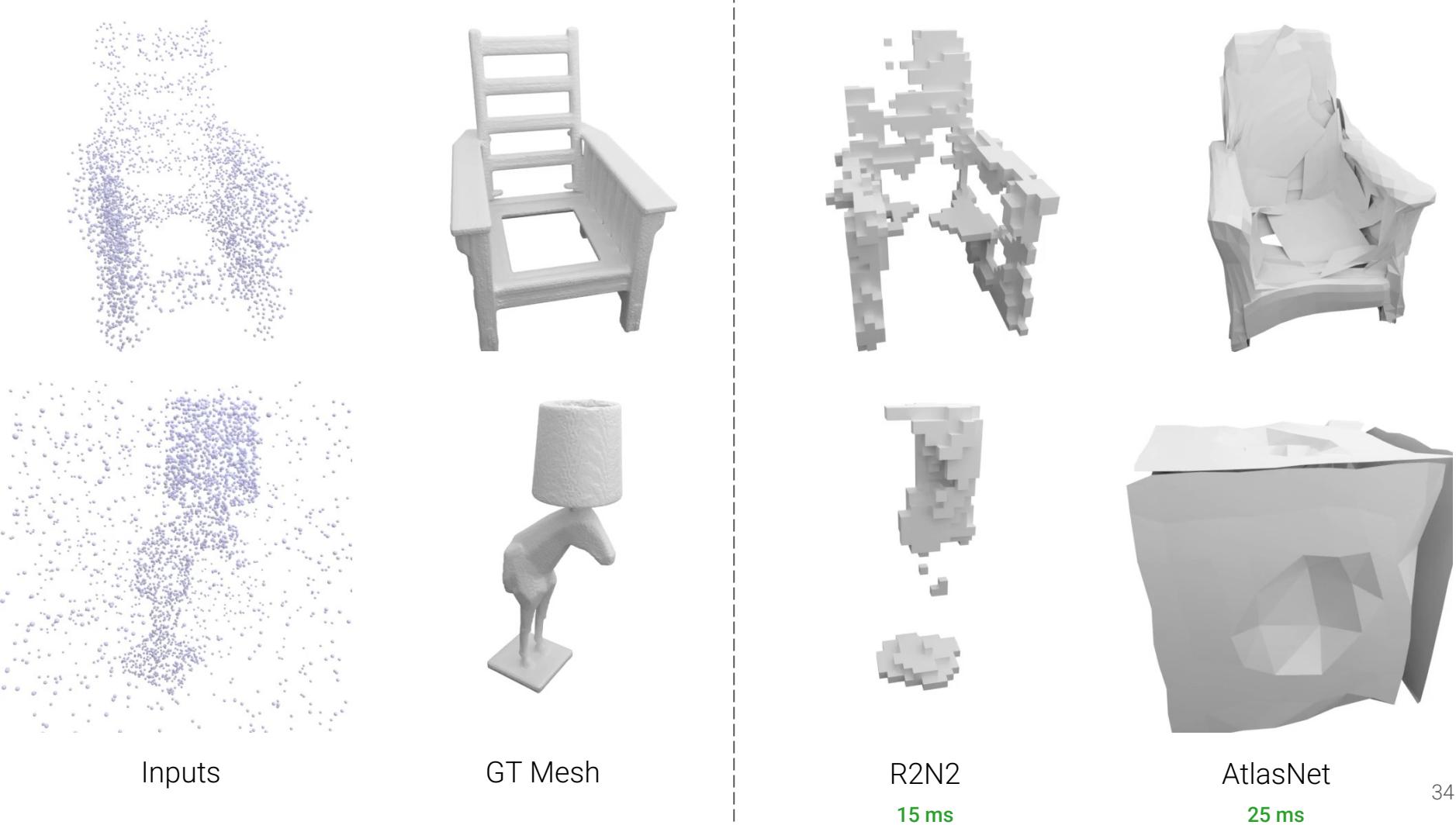


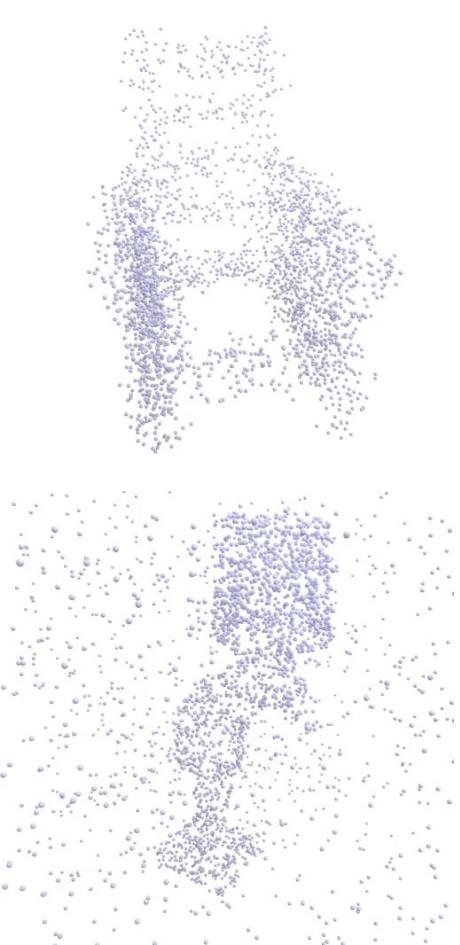
Inputs



GT Mesh







Inputs

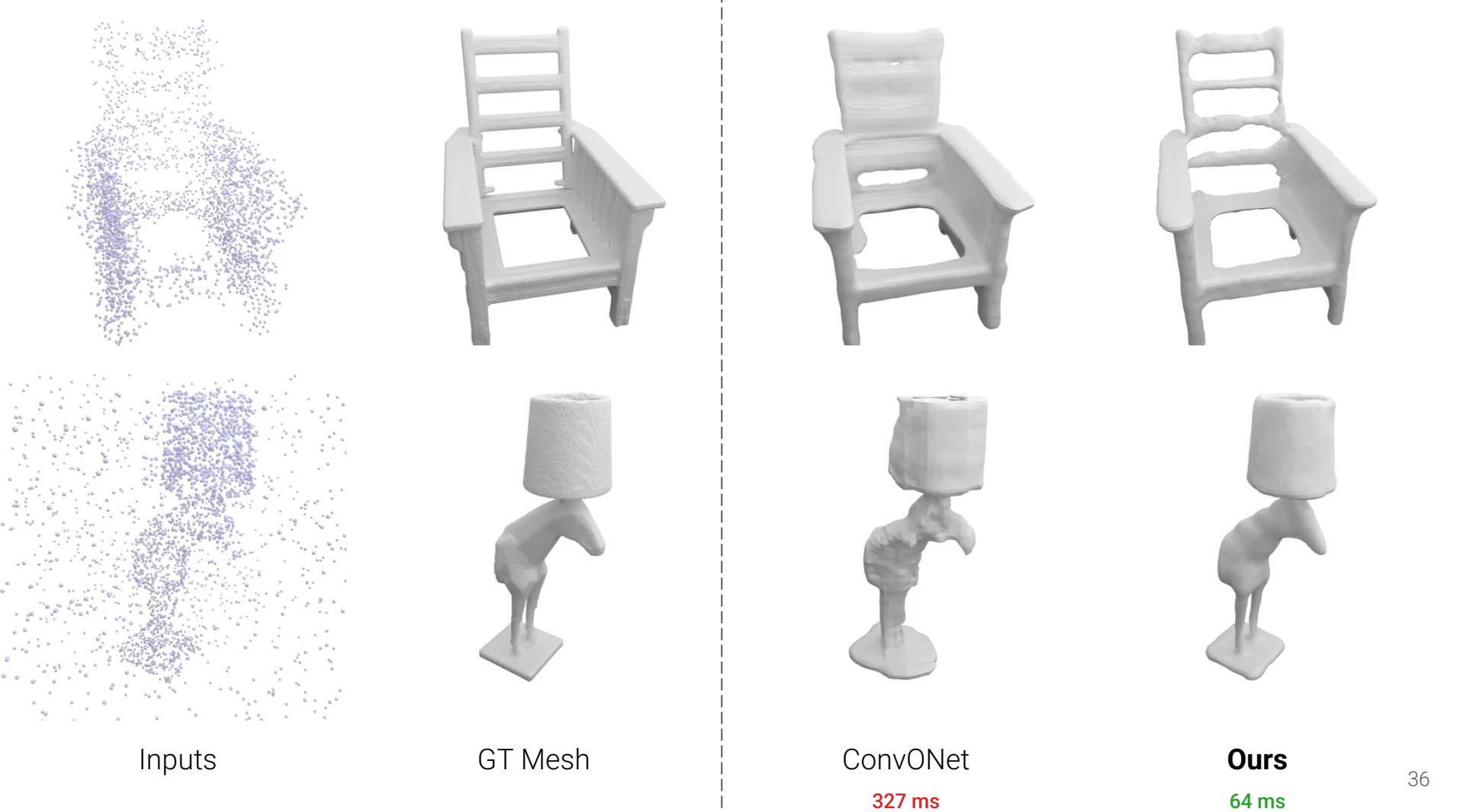


GT Mesh

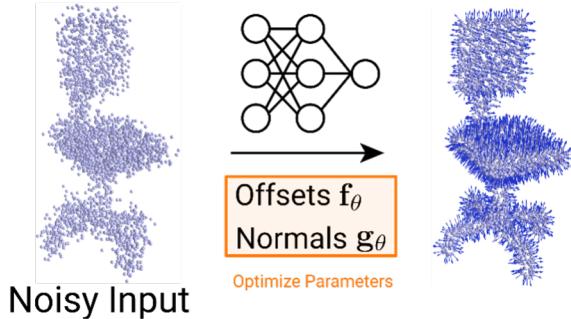


ConvONet

327 ms



Benefit of Geometric Initialization



Chamfer distance over the training process

Iterations	10K	50K	100K	200K	Best
ConvONet	0.082	0.058	0.055	0.050	0.044
Ours	0.041	0.036	0.035	0.034	0.034

SAP converges much faster!

Conclusions

- SAP is **interpretable, lightweight** and guarantees HQ watertight meshes
- SAP is also **topology agnostic**, enables fast inference
- Our Poisson solver is **differentiable** and GPU-accelerated

Limitation: Cubic memory requirements limits SAP for small scenes

Thank You!

<https://pengsongyou.github.io/sap>

