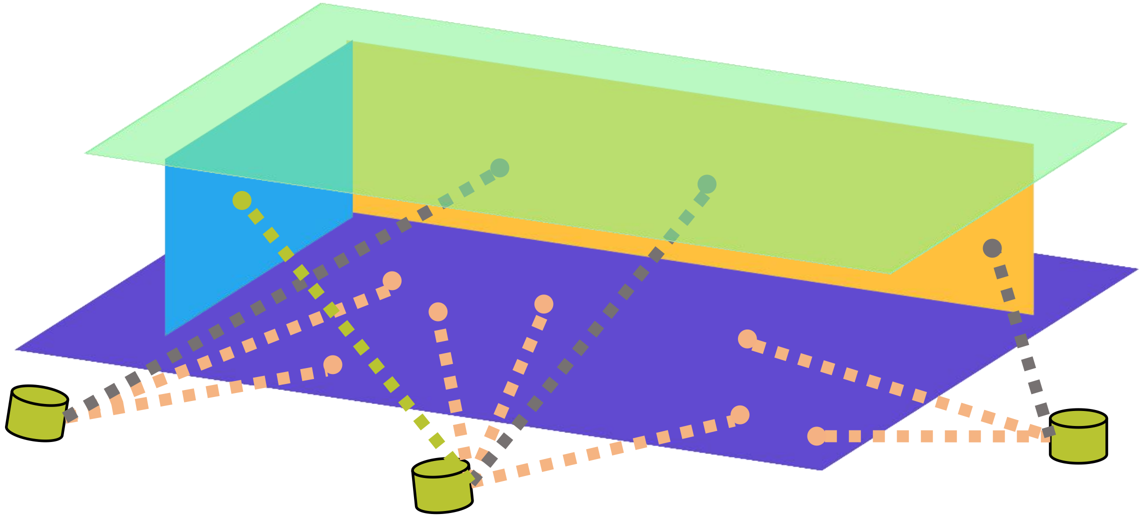
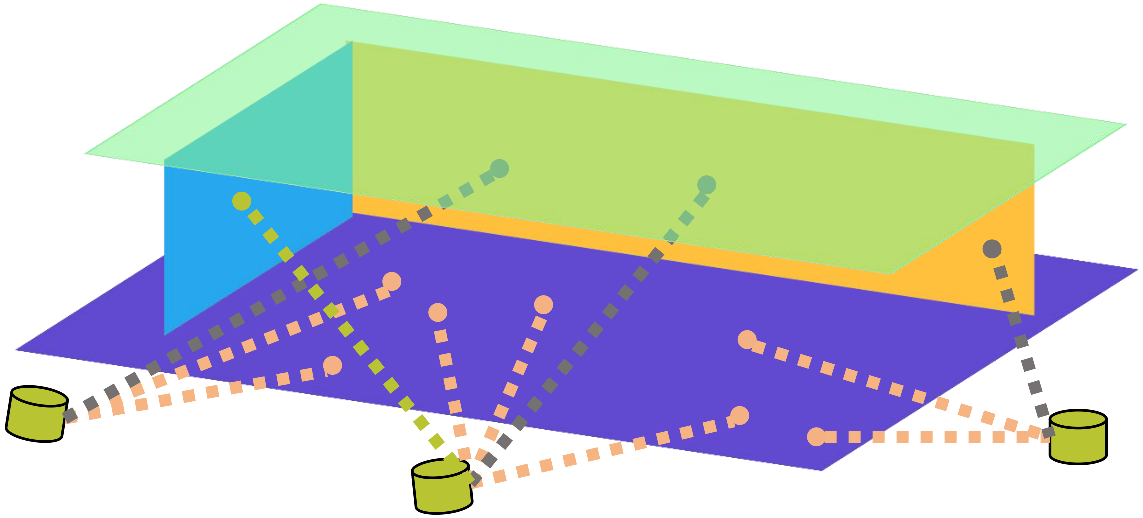
GlobalPointer: *Large-Scale Plane Adjustment with Bi-Convex Relaxation*

Bangyan Liao*, Zhenjun Zhao*, Lu Chen,
Haoang Li, Daniel Cremers, Peidong Liu



bangyan101.github.io/GlobalPointer

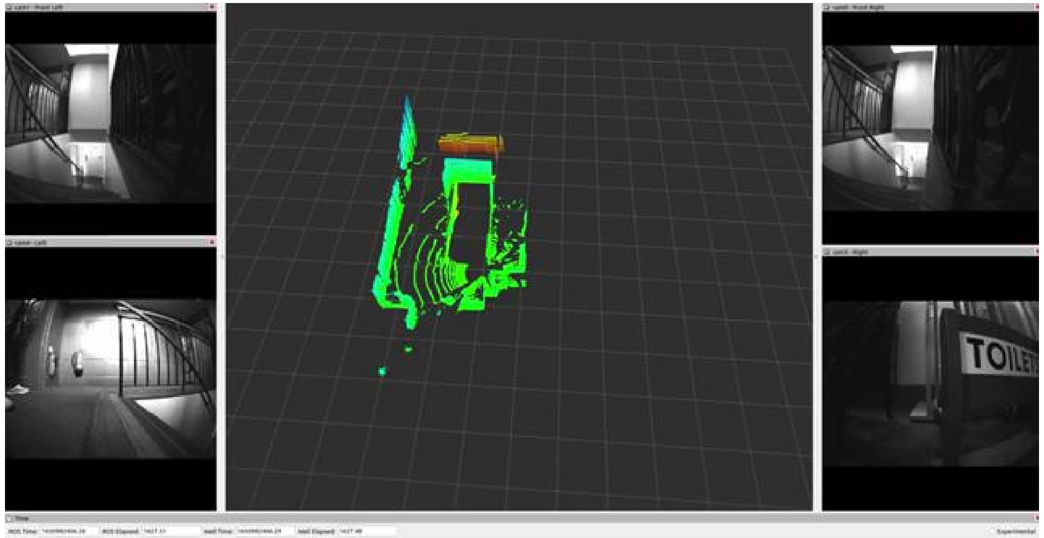


Introduction and Motivation

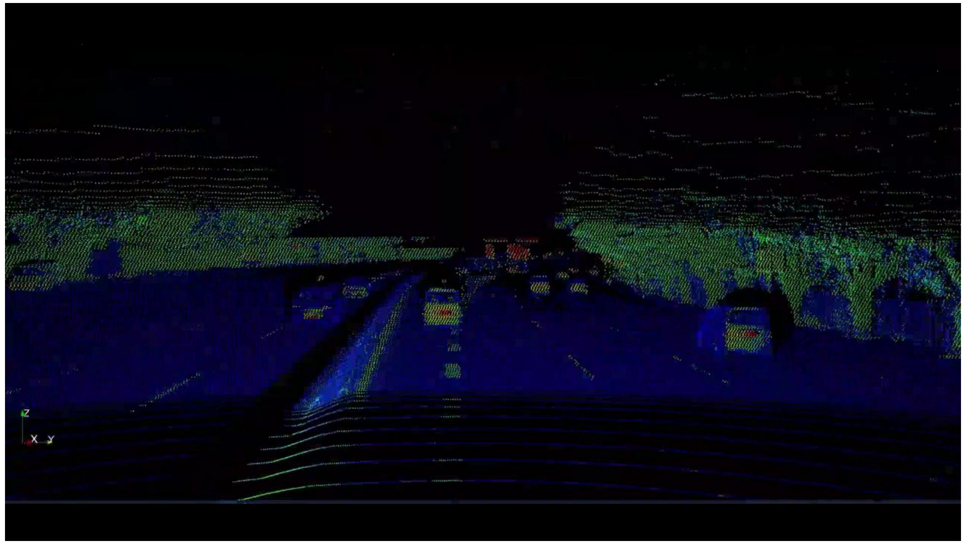
Introduction

Introduction and Motivation

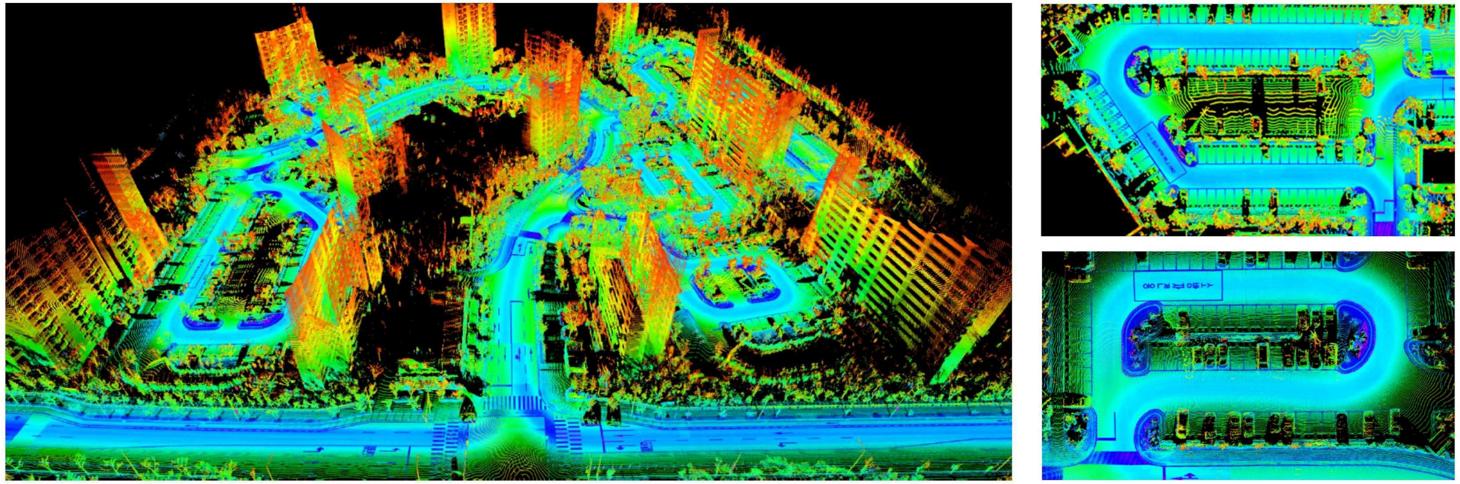
LiDAR SLAM [1][2]



Autonomous Driving



Scene Modeling & Digtal Twin [5]



There is an increasing demand for a more efficient, robust, and accurate multi-frame point cloud registration algorithm.

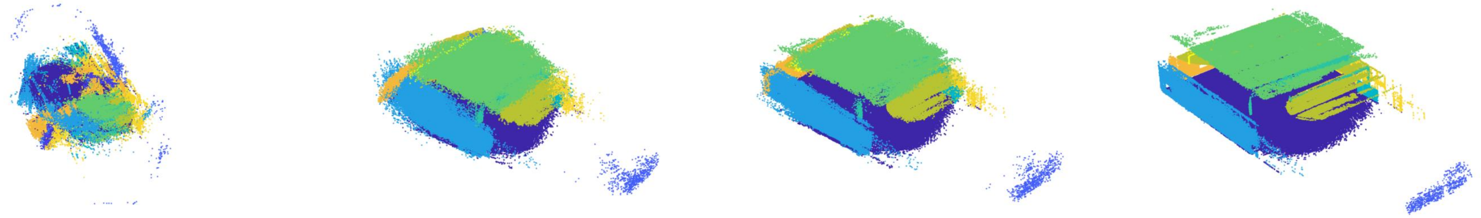
Plane-based Multi-frame Point Cloud Registration

Introduction and Motivation

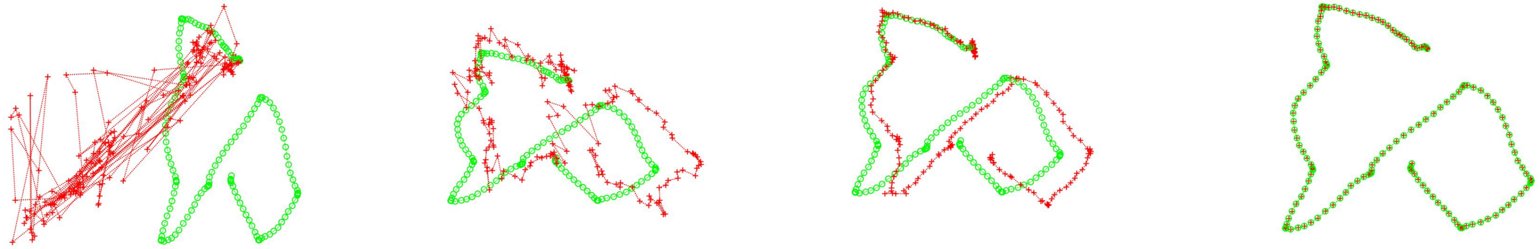
Input: **multiple LiDAR scans with assigned plane labels on each LiDAR point**

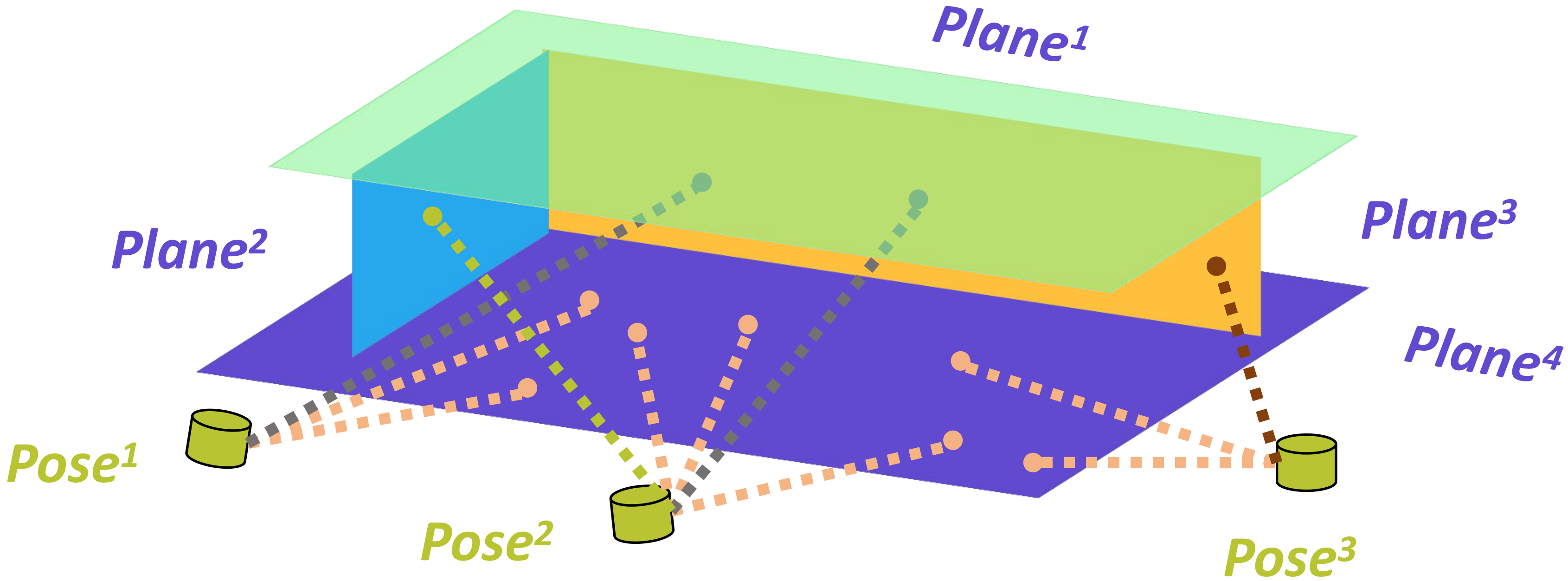
Scan 1 **Scan 50** **Scan 100** **Scan 177**

Plane Adjustment (GlobalPointer & GlobalPointer++)



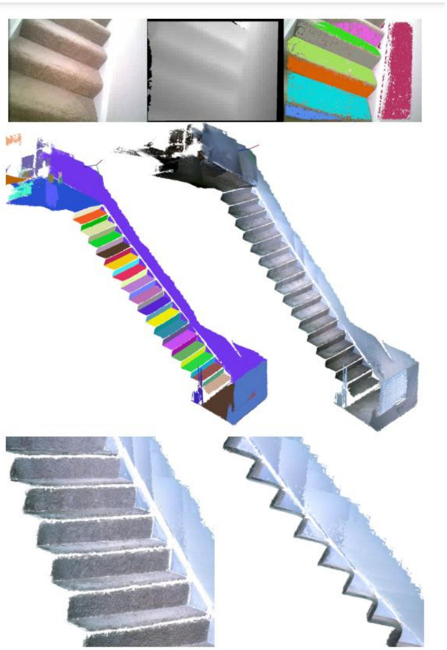
Iteration 1 **Iteration 5** **Iteration 7** **Iteration 25**

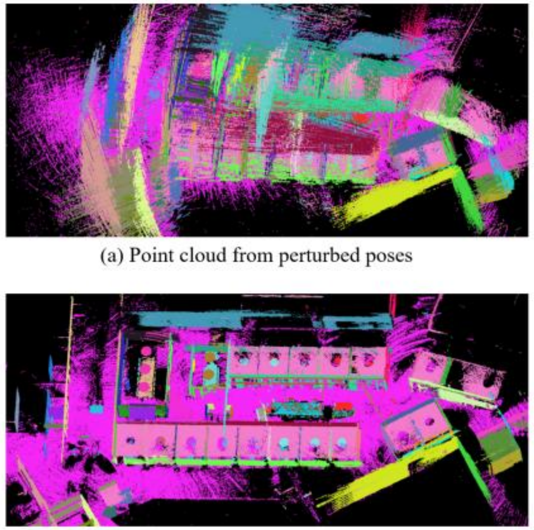




Plane Adjustment

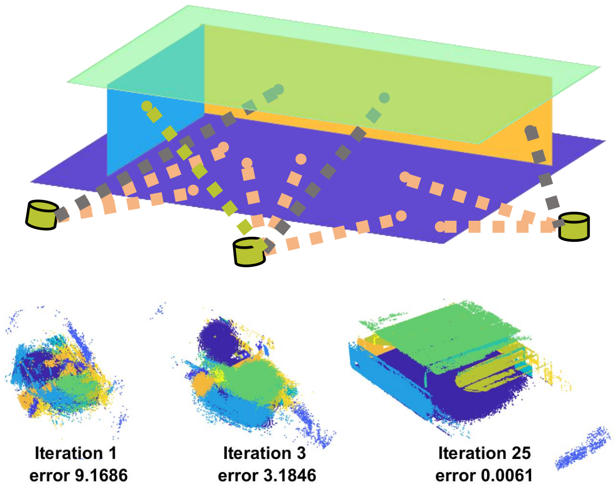
$$\{\mathbf{Plane}^{1,\,2,\,3,\,4*}, \mathbf{Pose}^{1,\,2,\,3*}\} = arg \min_{\mathbf{Plane}^{1,2,3,4}, \mathbf{Pose}^{1,2,3}} \sum_{i=1}^{4} \sum_{j=1}^{3} error(\mathbf{Pose}^{j}, \mathbf{Plane}^{i})$$





(a) Point cloud from perturbed poses

(b) Point cloud from our algorithm



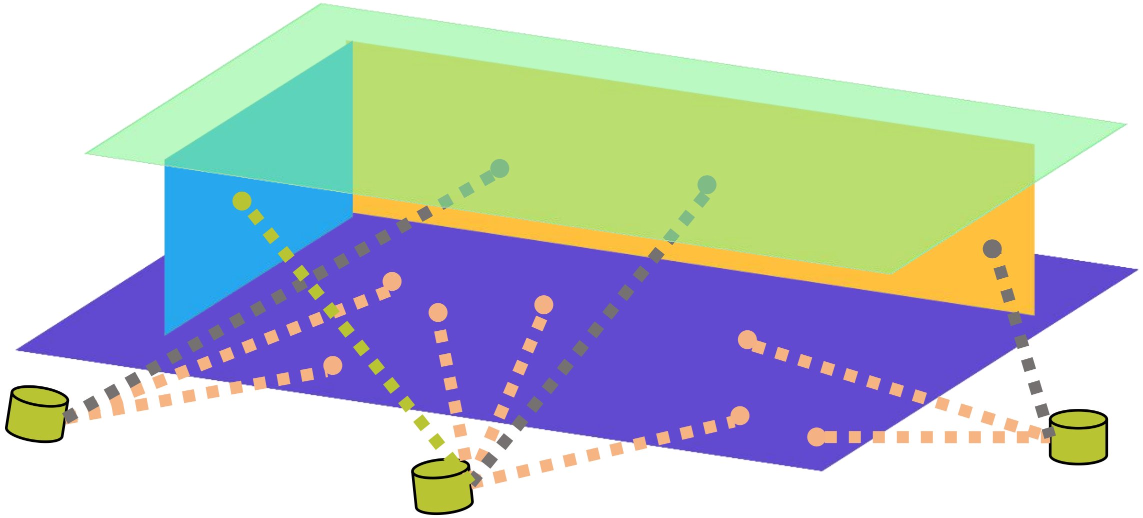
	Nonlinear Least-square based methods[3]	Spectral-based Plane Adjustment[4]	[Ours]
Scalability	✗	✗	✓
Initialization - free	✗	✗	✓

Contribution
Introduction and Motivation

- A novel optimization strategy: ***Bi-Convex Relaxation***
  combines the advantages of both alternating minimization and convex relaxation techniques

- Two algorithmic variants for plane adjustment: ***GlobalPointer*** and ***GlobalPointer++***
  depend on point-to-plane and plane-to-plane errors, respectively

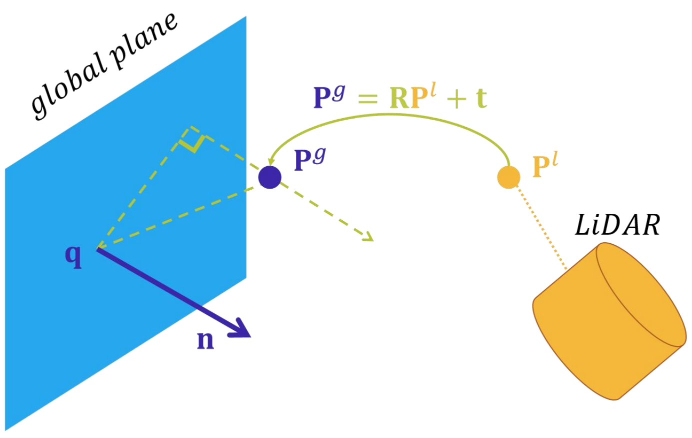
- Extensive synthetic and real experimental evaluations demonstrate
  - **linear time complexity**
  - **robustness to poor initialization**
  - **similar accuracy as prior methods.**



Methodology

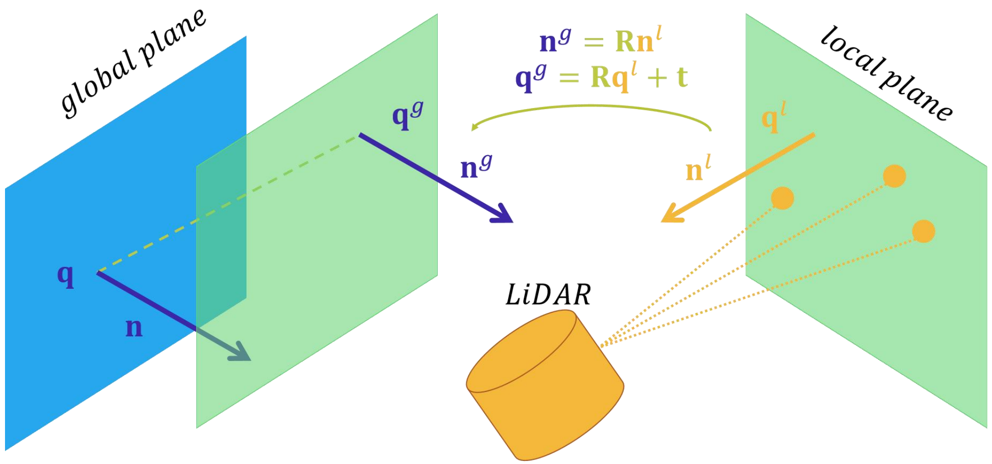
Recall the Point-to-Plane and Plane-to-Plane Error
Methodology

Point-to-Plane Error



$$\mathbf{P}^g = \mathbf{R}\mathbf{P}^l + \mathbf{t}$$
$$e = \left\|\mathbf{n}^{\mathrm{T}}(\mathbf{P}^g - \mathbf{q})\right\|_2^2$$

Plane-to-Plane Error



$$\mathbf{n}^g = \mathbf{R}\mathbf{n}^l, \quad \mathbf{q}^g = \mathbf{R}\mathbf{q}^l + \mathbf{t}$$
$$e = \|\mathbf{n}^g - \mathbf{n}\|_2^2 + \left\|{\mathbf{n}^g}^{\mathrm{T}}\mathbf{q}^g - \mathbf{n}^{\mathrm{T}}\mathbf{q}\right\|_2^2$$

$\mathbf{R}$	rotation matrix
$\mathbf{t}$	translation vector
$\mathbf{n}$	normal vector of plane
$\mathbf{q}$	arbitrary point on plane
$d = -\mathbf{n}^{\mathrm{T}}\mathbf{q}$	auxiliary scalar
$\mathbf{T} = \begin{bmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{bmatrix}$	tranformation matrix
$\mathbf{b} = \begin{bmatrix} \mathbf{n} \\ d \end{bmatrix}$	auxiliary vecor

Plane Adjustment with Point-to-Plane Error

Methodology - *GlobalPointer*

$$\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \sum_{i=1}^{m}\sum_{j=1}^{n} \begin{bmatrix}\mathbf{n}_j \\ d_j\end{bmatrix}^{\mathrm{T}} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}} \begin{bmatrix}\mathbf{n}_j \\ d_j\end{bmatrix}$$

$$s.t. \quad \mathbf{R}_i \in \mathbf{SO}(3), \ \|\mathbf{n}_j\| = 1$$

$$\mathcal{B}(i,j) = \sum_{k \in obs(i,j)} \begin{bmatrix} \mathbf{P}_k^l \mathbf{P}_k^{l\,\mathrm{T}} & \mathbf{P}_k^l \\ \mathbf{P}_k^{l\,\mathrm{T}} & 1 \end{bmatrix}$$

$$i = 1, \dots, m, \quad j = 1, \dots, n.$$

PA (Point-to-Plane Error)

This problem has non-convex objectives and constraints

How to solve?

Bi-convex Relaxation

1. decouple the original complex formulation into two sub-problems
2. reformulate each problem using convex relaxation technique
3. solve each problem alternately until the overall problem converges

Plane-Only SDP

Methodology - *GlobalPointer*

$$
\begin{aligned}
\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} \begin{bmatrix}\mathbf{n}_j\\ d_j\end{bmatrix}^{\mathrm{T}} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}} \begin{bmatrix}\mathbf{n}_j\\ d_j\end{bmatrix} \\
s.t. \quad & \mathbf{R}_i \in \mathbf{SO}(3), \ \|\mathbf{n}_j\| = 1 \\
& \mathcal{B}(i,j) = \sum_{k \in obs(i,j)} \begin{bmatrix} \mathbf{P}_k^l \mathbf{P}_k^{l\,\mathrm{T}} & \mathbf{P}_k^l \\ \mathbf{P}_k^{l\,\mathrm{T}} & 1 \end{bmatrix} \\
& i = 1, \ldots, m, \quad j = 1, \ldots, n.
\end{aligned}
$$

PA (Point-to-Plane Error)

1. fix pose variables $\mathbf{R}, \mathbf{t}$
2. reformulate it as QCQP

$$
\begin{aligned}
\{\mathbf{b}^*\} &= arg \min_{\mathbf{b}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} \mathbf{b}_j^{\mathrm{T}} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}} \mathbf{b}_j \\
&= arg \min_{\mathbf{b}} \quad \sum_{j=1}^{n} \mathbf{b}_j^{\mathrm{T}} \left(\sum_{i=1}^{m} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}} \right) \mathbf{b}_j \\
& s.t. \quad \|\mathbf{n}_j\| = 1 \\
& i = 1, \ldots, m, \quad j = 1, \ldots, n.
\end{aligned}
$$

Plane-Only QCQP

1. replace variables $\mathbf{b}_j \mathbf{b}_j^{\mathrm{T}}$ with matrix $\mathbf{Y}_j$
2. relax the constraint to get the SDP

Plane-Only SDP

$$
\begin{aligned}
\{\mathbf{Y}^*\} = arg \min_{\mathbf{Y}} \quad & \sum_{j=1}^{n} trace\left(\mathcal{D}(j) \mathbf{Y}_j\right) \\
s.t. \quad & \mathcal{D}(j) = \sum_{i=1}^{m} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}}, \\
& \mathbf{Y}_j \succcurlyeq 0, \quad \|\mathbf{n}_j\| = 1, \\
& i = 1, \ldots, m, \quad j = 1, \ldots, n.
\end{aligned}
$$

This problem is now convex and can be solved with off-the-shelf SDP solver.

Pose-Only SDP

Methodology - *GlobalPointer*

$$\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R,t,n,q}} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} \begin{bmatrix}\mathbf{n}_j \\ d_j\end{bmatrix}^{\mathrm{T}} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}} \begin{bmatrix}\mathbf{n}_j \\ d_j\end{bmatrix}$$

$$s.t. \quad \mathbf{R}_i \in \mathbf{SO}(3), \ \|\mathbf{n}_j\| = 1$$

$$\mathcal{B}(i,j) = \sum_{k \in obs(i,j)} \begin{bmatrix} \mathbf{P}_k^l \mathbf{P}_k^{l\,\mathrm{T}} & \mathbf{P}_k^l \\ \mathbf{P}_k^{l\,\mathrm{T}} & 1 \end{bmatrix}$$

$$i = 1, \ldots, m, \quad j = 1, \ldots, n.$$

PA (Point-to-Plane Error)

1. fix plane variables $\mathbf{n}, d$
2. reformulate it as QCQP

$$\{vec(\mathbf{T})^*\} = arg \min_{vec(\mathbf{T})} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} \mathbf{b}_j^{\mathrm{T}} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i^{\mathrm{T}} \mathbf{b}_j$$

$$= arg \min_{vec(\mathbf{T})} \quad \sum_{i=1}^{m} vec(\mathbf{T}_i)^{\mathrm{T}} \mathcal{C}(i) vec(\mathbf{T}_i)$$

$$s.t. \quad \mathcal{C}(i) = \sum_{j=1}^{n} \left(\mathbf{b}_j \mathbf{b}_j^{\mathrm{T}}\right) \otimes \mathcal{B}(i,j)$$

$$\mathbf{R}_i \in \mathbf{SO}(3)$$

$$i = 1, \ldots, m, \quad j = 1, \ldots, n.$$

Pose-Only QCQP

1. replace $vec(\mathbf{T}_i) vec(\mathbf{T}_i)^{\mathrm{T}}$ with matrix $\mathbf{X}_j$
2. relax the constraint to get the SDP

Pose-Only SDP

$$\{\mathbf{X}^*\} = arg \min_{\mathbf{X}} \quad \sum_{i=1}^{m} trace(\mathcal{C}(i) \mathbf{X}_i)$$

$$s.t. \quad \mathcal{C}(i) = \sum_{j=1}^{n} \left(\mathbf{b}_j \mathbf{b}_j^{\mathrm{T}}\right) \otimes \mathcal{B}(i,j),$$

$$\mathbf{X}_i \succcurlyeq 0, \quad \{\text{redundant rotation constraints}\},$$

$$i = 1, \ldots, m, \quad j = 1, \ldots, n.$$

This problem is now convex and can be solved with off-the-shelf SDP solver.

GlobalPointer

Plane-Only SDP

$$
\begin{aligned}
\{\mathbf{Y}^*\} = arg \min_{\mathbf{Y}} \quad & \sum_{j=1}^{n} trace\big(\mathcal{D}(j)\mathbf{Y}_j\big) \\
s.t. \quad & \mathcal{D}(j) = \sum_{i=1}^{m} \mathbf{T}_i \mathcal{B}(i,j) \mathbf{T}_i{}^{\mathrm{T}}, \\
& \mathbf{Y}_j \succcurlyeq 0, \quad \left\|\mathbf{n}_j\right\| = 1, \\
& i = 1, \dots, m, \quad j = 1, \dots, n.
\end{aligned}
$$

Rounding $\{\mathbf{Y}^*\}$ to get plane variables $\mathbf{b}$

Pose-Only SDP

$$
\begin{aligned}
\{\mathbf{X}^*\} = arg \min_{\mathbf{X}} \quad & \sum_{i=1}^{m} trace(\mathcal{C}(i)\mathbf{X}_i) \\
s.t. \quad & \mathcal{C}(i) = \sum_{j=1}^{n} \left(\mathbf{b}_j \mathbf{b}_j^{\mathrm{T}}\right) \otimes \mathcal{B}(i,j), \\
& \mathbf{X}_i \succcurlyeq 0, \quad \{\text{redundant rotation constraints}\}, \\
& i = 1, \dots, m, \quad j = 1, \dots, n.
\end{aligned}
$$

Rounding $\{\mathbf{X}^*\}$ to get pose variables $\mathbf{R}, \mathbf{t}$

Plane Adjustment with Plane-to-Plane Error

Methodology - *GlobalPointer++*

$$
\begin{aligned}
\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \quad & \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i \mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|{\mathbf{n}_{ij}^{l*}}^{\mathrm{T}} {\mathbf{R}_i}^{\mathrm{T}} \mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2 \\
s.t. \quad & \mathbf{R}_i \in \mathbf{SO}(3),\ \left\|\mathbf{n}_j\right\| = 1 \\
& i = 1, \ldots, m, \quad j = 1, \ldots, n.
\end{aligned}
$$

$$
\left(\mathbf{n}_{ij}^{l*}, d_{ij}^{l*}\right) = arg \min_{\mathbf{n}_{ij}^{l}, d_{ij}^{l}} \begin{bmatrix} \mathbf{n}_{ij}^{l} \\ d_{ij}^{l} \end{bmatrix}^{\mathrm{T}} \mathcal{B}(i,j) \begin{bmatrix} \mathbf{n}_{ij}^{l} \\ d_{ij}^{l} \end{bmatrix}
$$

PA (Plane-to-Plane Error)

Plane-Only Closed-Form Solver

Methodology - *GlobalPointer++*

$$\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|{\mathbf{n}_{ij}^{l*}}^{\mathrm{T}}{\mathbf{R}_i}^{\mathrm{T}}\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$

$$s.t. \quad \mathbf{R}_i \in \mathbf{SO}(3), \ \left\|\mathbf{n}_j\right\| = 1$$

$$i = 1, \ldots, m, \quad j = 1, \ldots, n.$$

$$\left(\mathbf{n}_{ij}^{l*}, d_{ij}^{l*}\right) = arg \min_{\mathbf{n}_{ij}^{l}, d_{ij}^{l}} \begin{bmatrix}\mathbf{n}_{ij}^{l}\\ d_{ij}^{l}\end{bmatrix}^{\mathrm{T}} \mathcal{B}(i,j) \begin{bmatrix}\mathbf{n}_{ij}^{l}\\ d_{ij}^{l}\end{bmatrix}$$

PA (Plane-to-Plane Error)

1. fix pose variables $\mathbf{R}, \mathbf{t}$
2. optimize plane variables $\mathbf{n}, d$ with closed-form solver

$$\{\mathbf{n}^*, d^*\} = arg \min_{\mathbf{n},d} \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|{\mathbf{n}_{ij}^{l*}}^{\mathrm{T}}{\mathbf{R}_i}^{\mathrm{T}}\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$

$$s.t. \quad \left\|\mathbf{n}_j\right\| = 1, \quad j = 1, \ldots, n.$$

Plane-Only Closed-Form Solver

Pose-Only Closed-Form Solver

Methodology - *GlobalPointer++*

$$\{\mathbf{R}^*, \mathbf{t}^*, \mathbf{n}^*, \mathbf{q}^*\} = arg \min_{\mathbf{R},\mathbf{t},\mathbf{n},\mathbf{q}} \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i \mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|{\mathbf{n}_{ij}^{l*}}^{\mathrm{T}} {\mathbf{R}_i}^{\mathrm{T}} \mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$

$$s.t. \quad \mathbf{R}_i \in \mathbf{SO}(3),\ \left\|\mathbf{n}_j\right\| = 1$$

$$i = 1, \ldots, m, \quad j = 1, \ldots, n.$$

$$\left(\mathbf{n}_{ij}^{l*}, d_{ij}^{l*}\right) = arg \min_{\mathbf{n}_{ij}^{l}, d_{ij}^{l}} \begin{bmatrix} \mathbf{n}_{ij}^{l} \\ d_{ij}^{l} \end{bmatrix}^{\mathrm{T}} \mathcal{B}(i,j) \begin{bmatrix} \mathbf{n}_{ij}^{l} \\ d_{ij}^{l} \end{bmatrix}$$

PA (Plane-to-Plane Error)

1. fix plane variables $\mathbf{n}, d$
2. optimize pose variables $\mathbf{R}, \mathbf{t}$ with closed-form solver

$$\{\mathbf{R}^*, \mathbf{t}^*\} = arg \min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i \mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|{\mathbf{n}_{ij}^{l*}}^{\mathrm{T}} {\mathbf{R}_i}^{\mathrm{T}} \mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$

$$= arg \min_{\mathbf{q},\mathbf{t}} \sum_{i=1}^{m}\sum_{j=1}^{n} {\mathbf{q}_i}^{\mathrm{T}} \mathbf{M}_{ij} \mathbf{q}_i + \left\|{\mathbf{n}_{ij}^{l*}}^{\mathrm{T}} {\mathbf{R}_i}^{\mathrm{T}} \mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$

$$s.t. \quad \left\|\mathbf{q}_j\right\| = 1, \quad i = 1, \ldots, m.$$

Pose-Only Closed-Form Solver

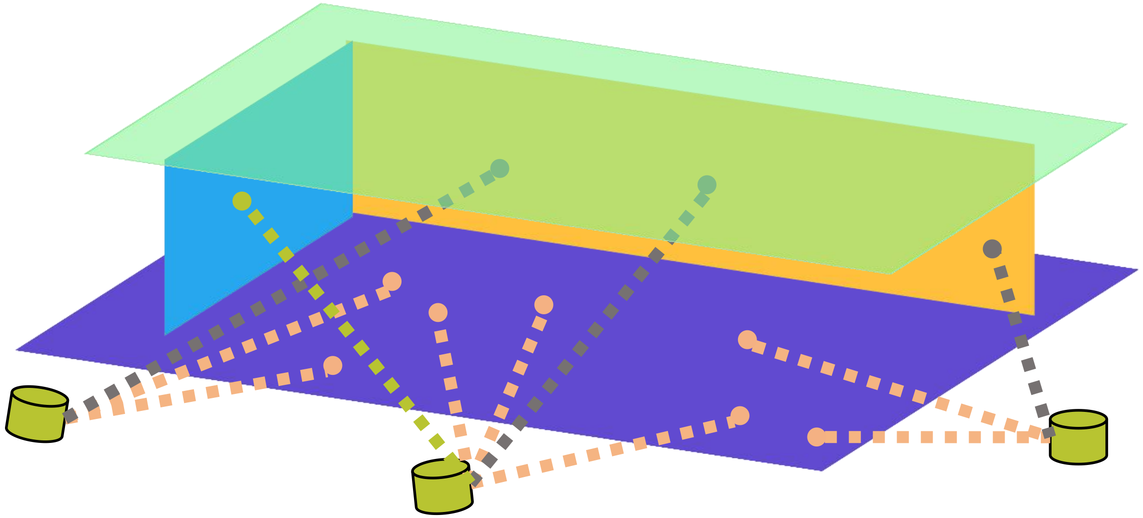
GlobalPointer++

$$\{\mathbf{n}^*, d^*\} = arg \min_{\mathbf{n},d} \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|\mathbf{n}_{ij}^{l*\mathrm{T}}\mathbf{R}_i^{\mathrm{T}}\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$s.t. \quad \left\|\mathbf{n}_j\right\| = 1, \quad j = 1,\ldots,n.$$

Plane-Only Closed-Form Solver

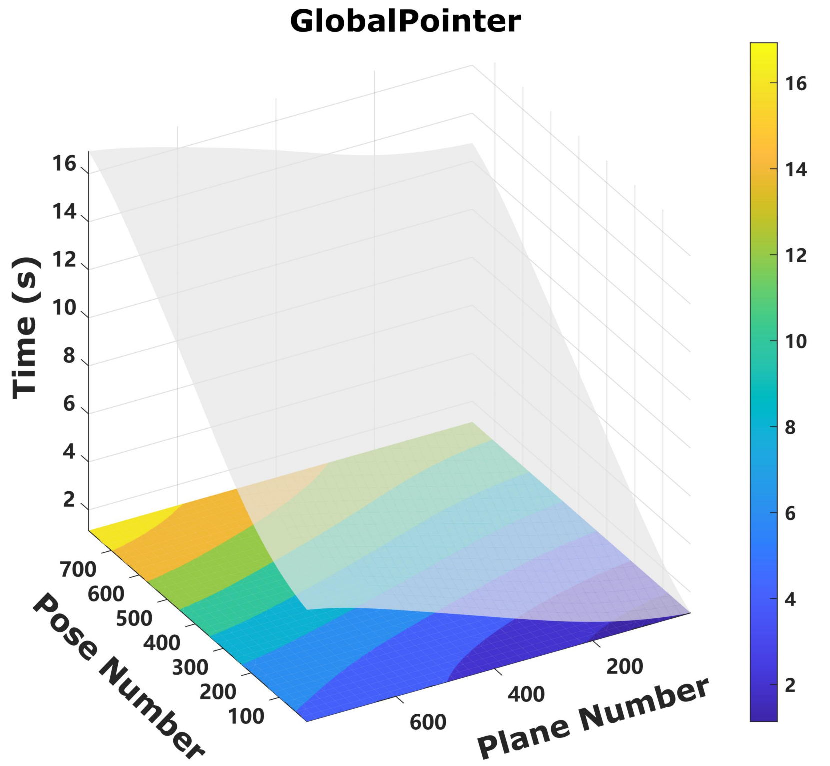
$$\{\mathbf{R}^*, \mathbf{t}^*\} = arg \min_{\mathbf{R},\mathbf{t}} \sum_{i=1}^{m}\sum_{j=1}^{n} \left\|\mathbf{n}_j - \mathbf{R}_i\mathbf{n}_{ij}^{l*}\right\|_2^2 + \left\|\mathbf{n}_{ij}^{l*\mathrm{T}}\mathbf{R}_i^{\mathrm{T}}\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$= arg \min_{\mathbf{q},\mathbf{t}} \sum_{i=1}^{m}\sum_{j=1}^{n} \mathbf{q}_i^{\mathrm{T}}\mathbf{M}_{ij}\mathbf{q}_i + \left\|\mathbf{n}_{ij}^{l*\mathrm{T}}\mathbf{R}_i^{\mathrm{T}}\mathbf{t}_i + d_j - d_{ij}^{l*}\right\|_2^2$$
$$s.t. \quad \left\|\mathbf{q}_j\right\| = 1, \quad i = 1,\ldots,m.$$

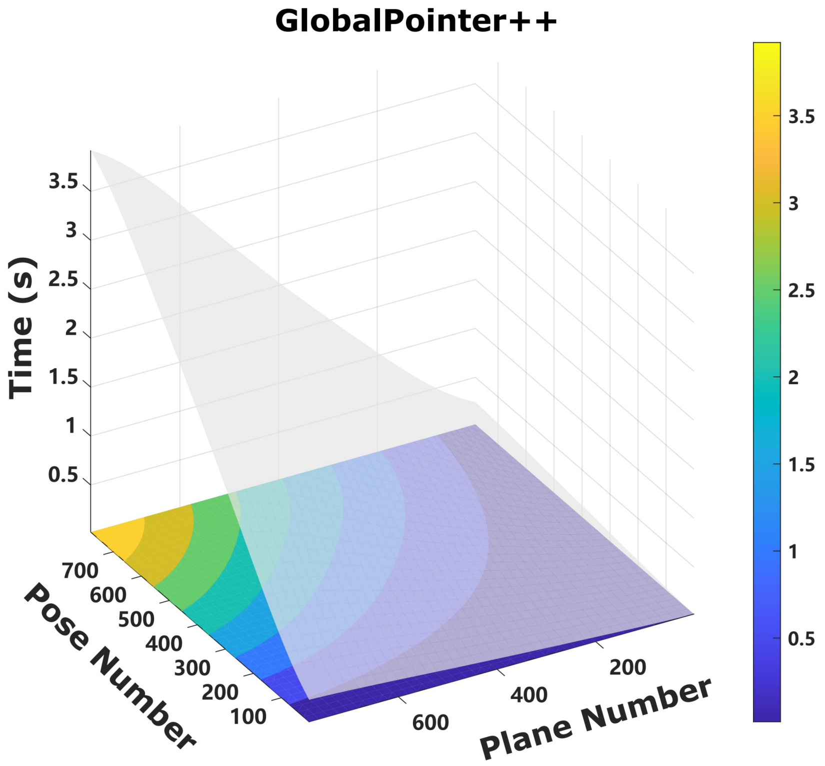
Pose-Only Closed-Form Solver



Experiments

Runtime
Experiments - Synthetic Data

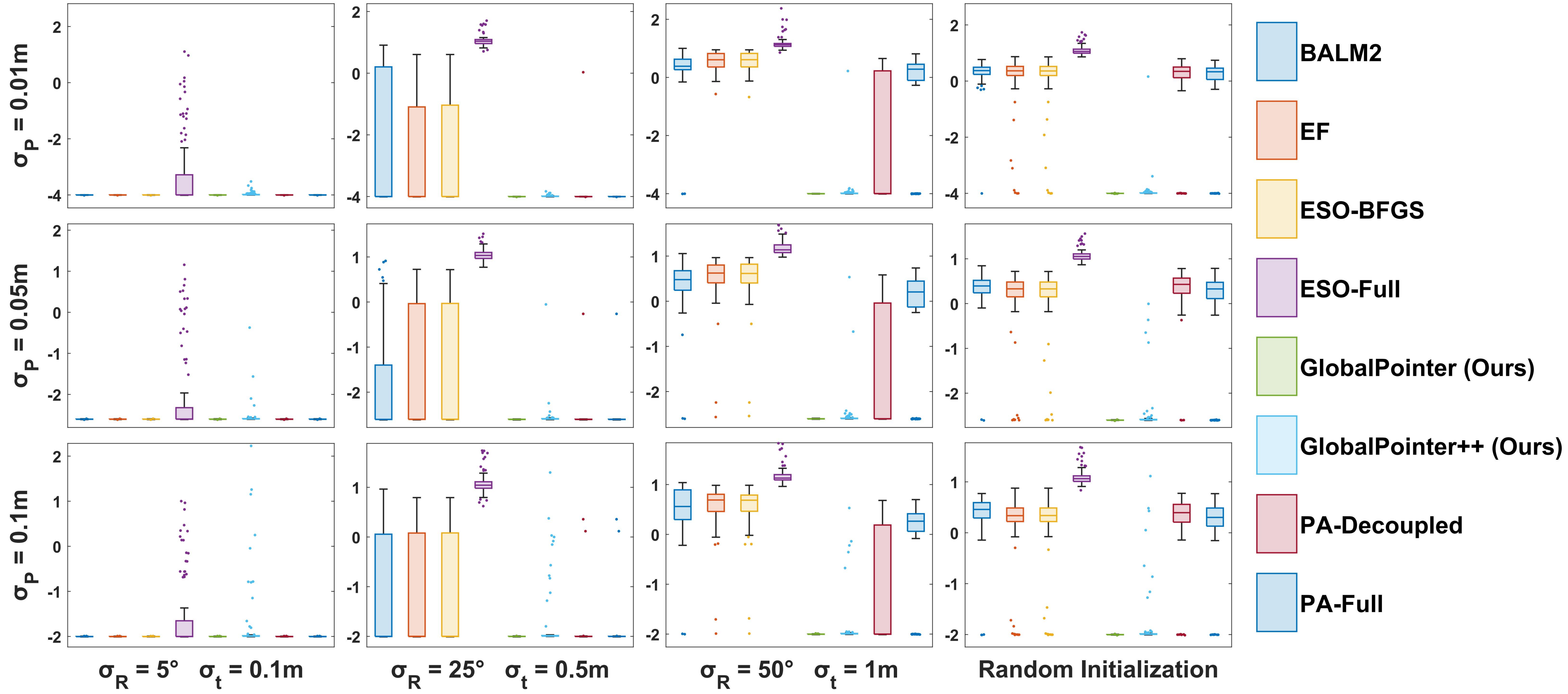




Total optimization time analysis with increasing poses and planes.

Accuracy

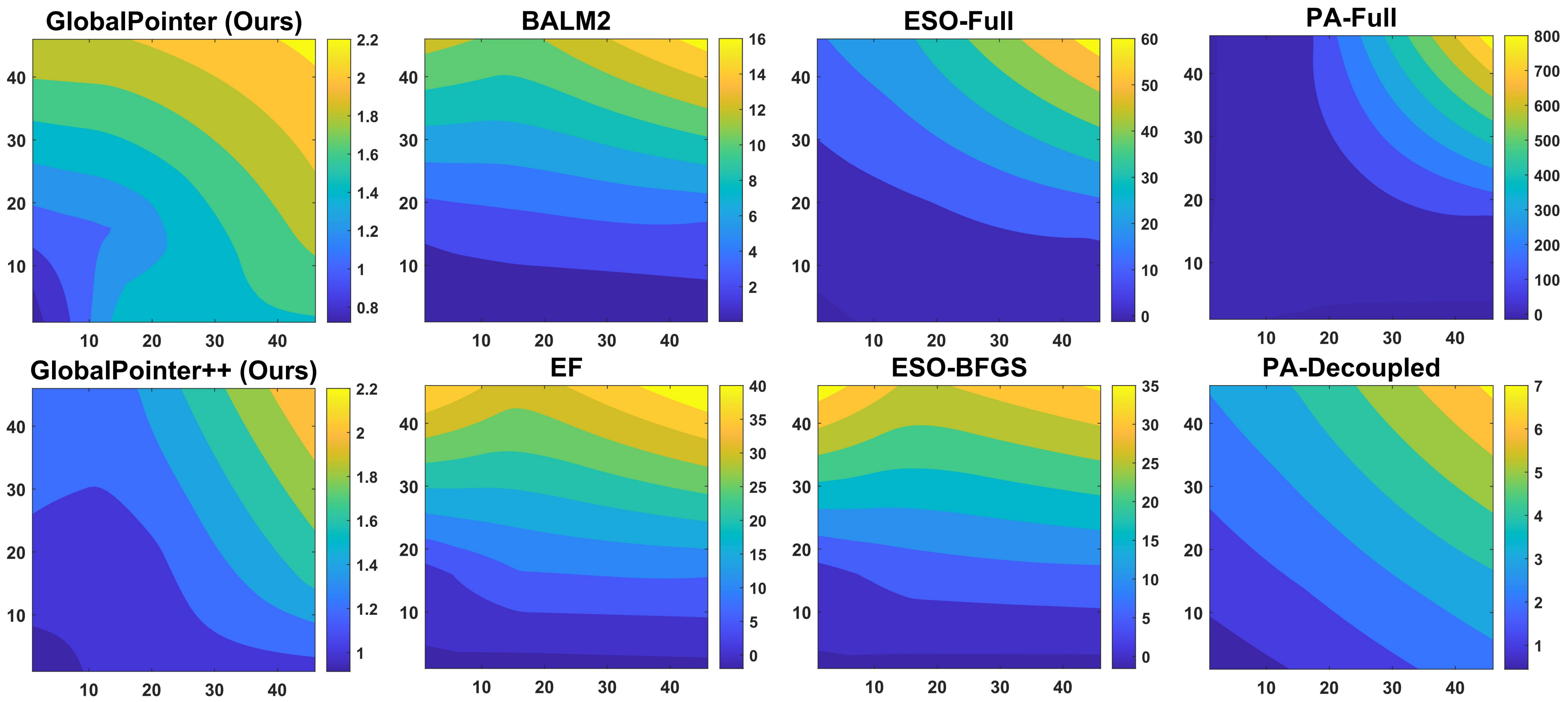
Experiments - Synthetic Data



Accuracy comparisons on the synthetic dataset under varying point cloud noise levels and pose initialization noise levels. The y axis represents the total point-to-plane error in the log10 scale.

Time Complexity

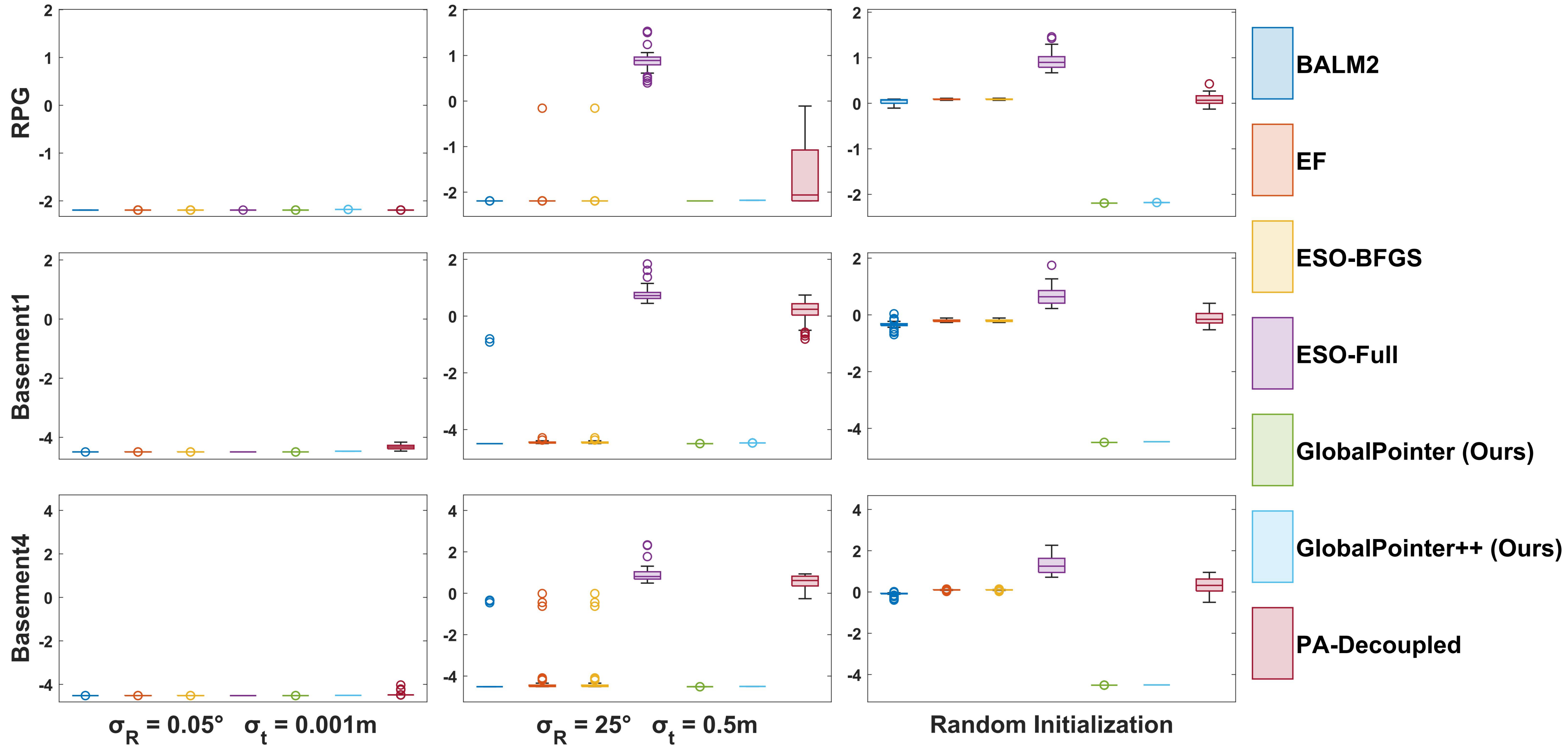
Experiments - Synthetic Data



Relative time complexity comparisons. The x axis represents the number of planes and the y axis represents the number of poses. The color bar on the right side of each subplot indicates the mapping relationship between the multiplier of runtime growth and the colors.

Accuracy

Experiments - Real Data



Accuracy comparisons on the real dataset under varying pose initialization noise levels. The y axis represents the total point-to-plane error in the log10 scale.

Reference

[1] Zhang L, Helmberger M, Fu L F T, et al. Hilti-oxford dataset: A millimeter-accurate benchmark for simultaneous localization and mapping[J]. IEEE Robotics and Automation Letters, 2022, 8(1): 408-415.
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[3] Kaess M. Simultaneous localization and mapping with infinite planes[C]//2015 IEEE International Conference on Robotics and Automation (ICRA). IEEE, 2015: 4605-4611.
[4] Zhou L. Efficient second-order plane adjustment[C]//Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition. 2023: 13113-13121.
[5] Lv J, Hu K, Xu J, et al. CLINS: Continuous-time trajectory estimation for LiDAR-inertial system[C]//2021 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS). IEEE, 2021: 6657-6663.

***GlobalPointer**: Large-Scale Plane Adjustment with Bi-Convex Relaxation*

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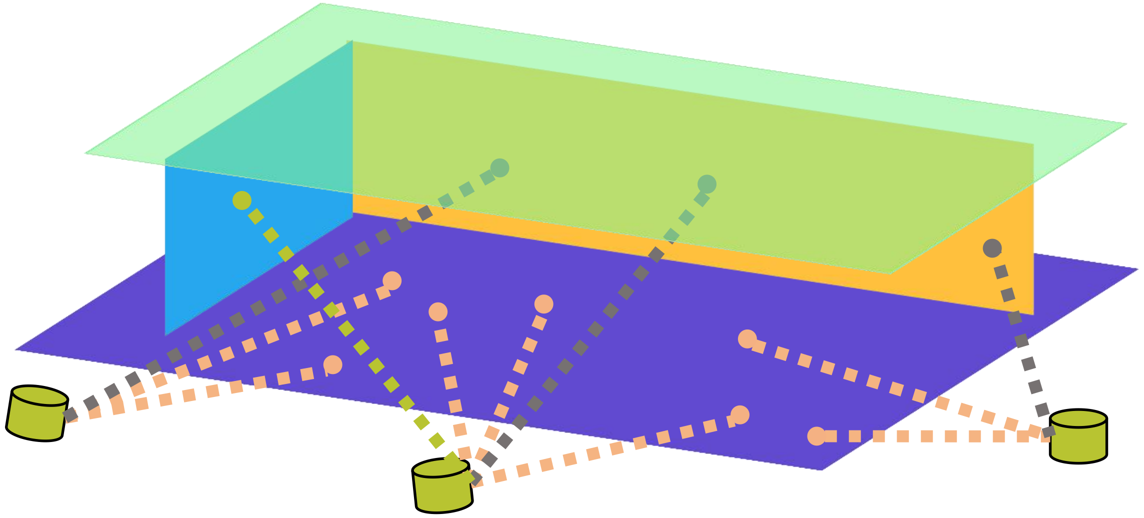

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