

GlobustVP

Convex Relaxation for Robust Vanishing Point Estimation in Manhattan World

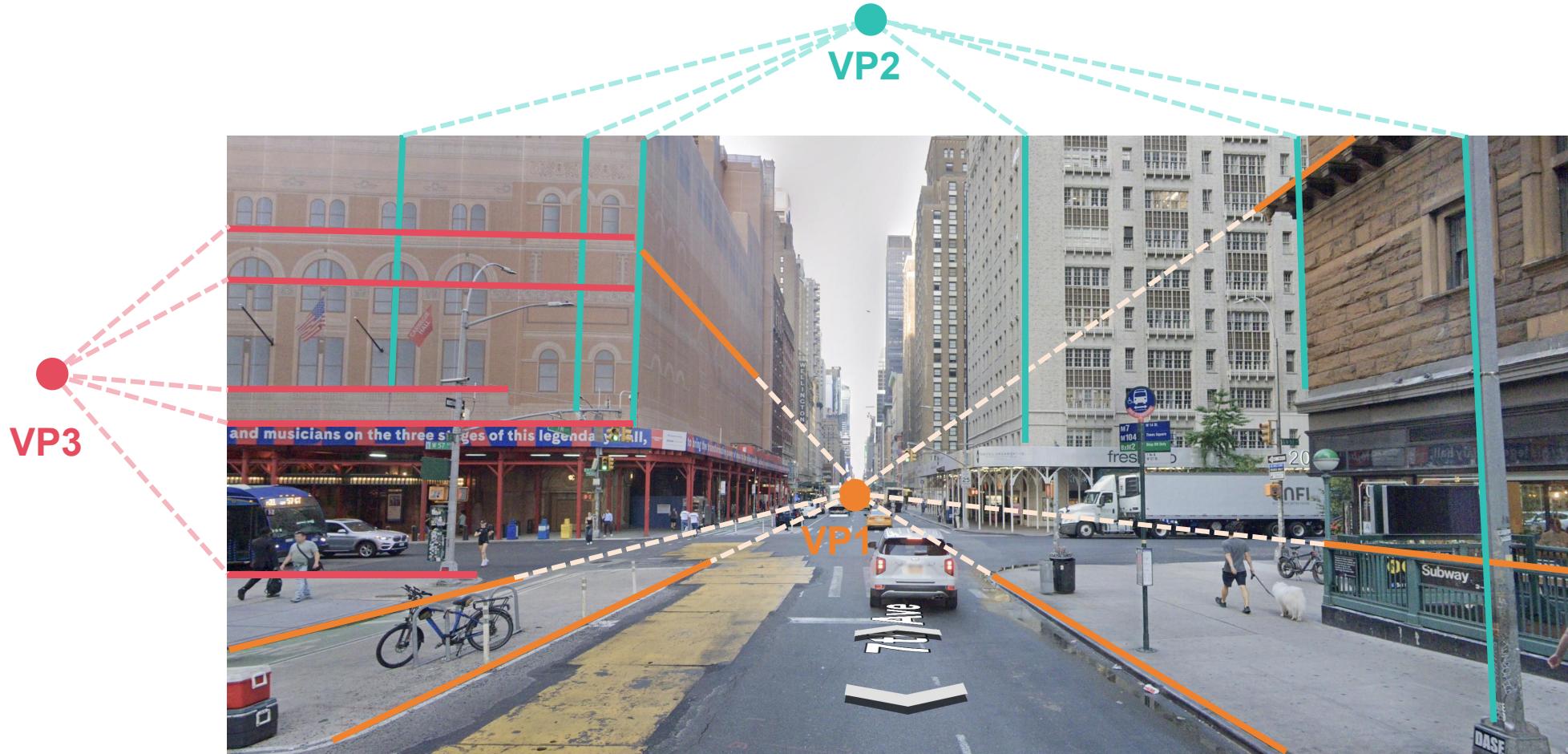
Bangyan Liao^{1,2*}, Zhenjun Zhao^{3*}, Haoang Li⁴, Yi Zhou⁵,
Yingping Zeng⁵, Hao Li¹, Peidong Liu¹

<https://github.com/WU-CVGL/GlobustVP/>



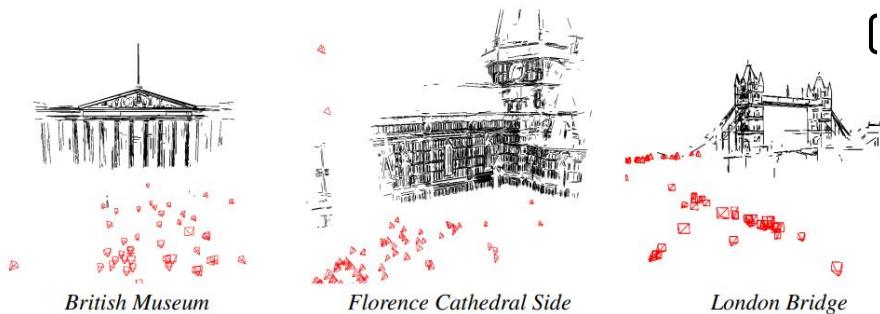
Vanishing Point in Manhattan World

projections of parallel lines intersect at a single point

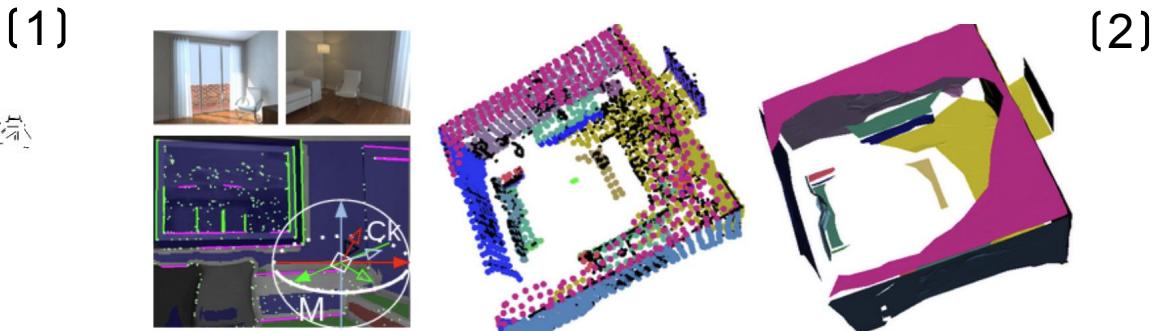


Applications of Vanishing Point

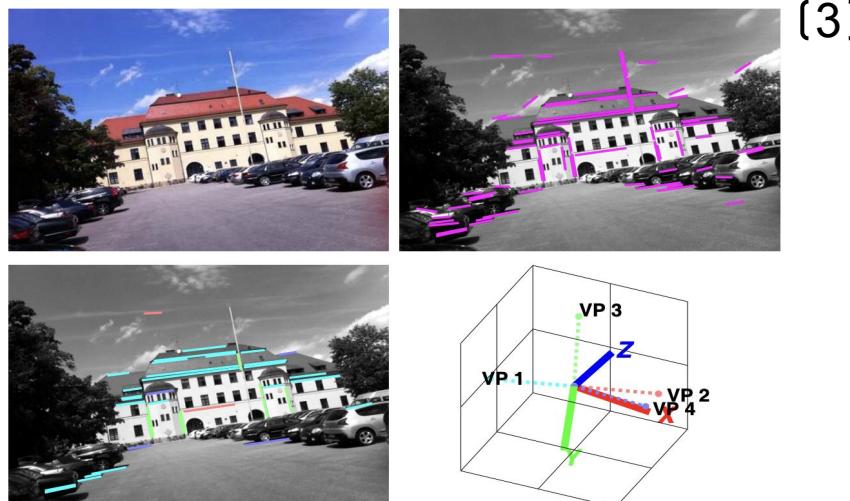
Structure from Motion (SFM)



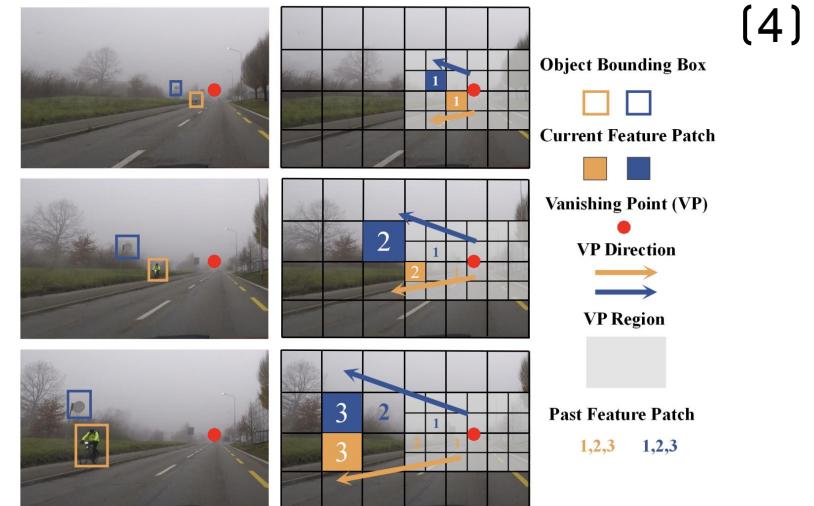
Simultaneous Localization And Mapping (SLAM)



Camera Rotation Estimation



Structural Understanding



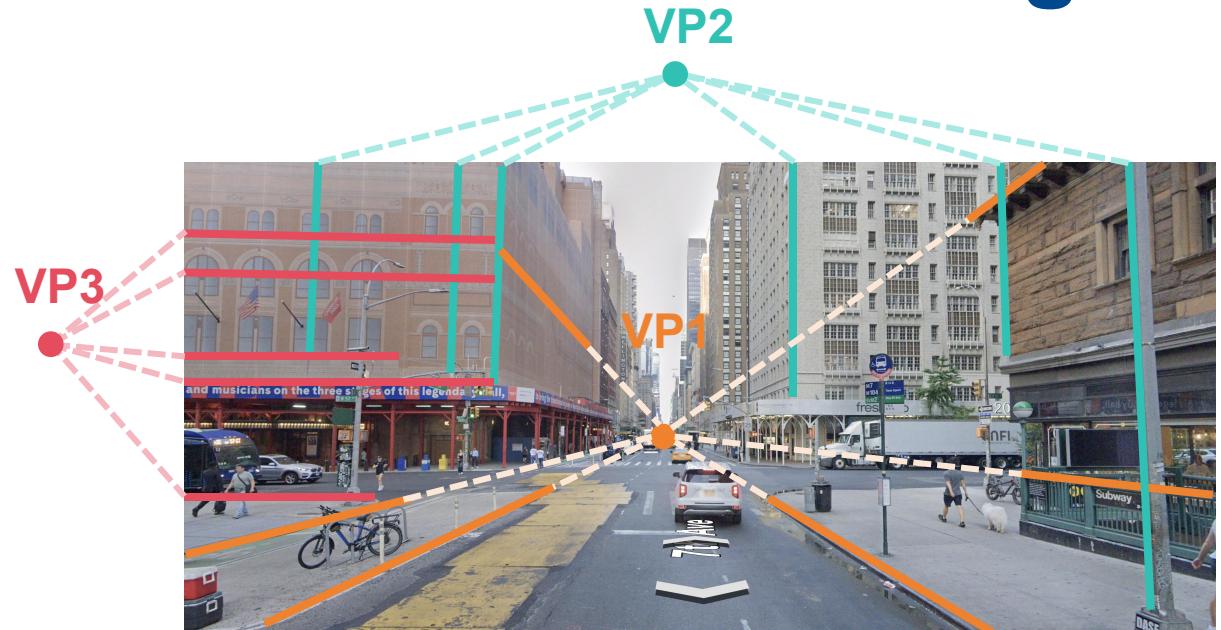
[1] Liu S, Yu Y, Pautrat R, et al. 3d line mapping revisited. CVPR 2023.

[2] Li Y, Yunus R, Brasch N, et al. RGB-D SLAM with structural regularities. ICRA 2021.

[3] Lee J K, Yoon K J. Real-time joint estimation of camera orientation and vanishing points. CVPR 2015.

[4] Guo D, Fan D P, Lu T, et al. Vanishing-point-guided video semantic segmentation of driving scenes. CVPR 2024.

Our Problem: VPs Estimation & Line Labeling



Input: Unlabeled Lines
Intrinsic Parameters



Output:

Labeled Lines
Vanishing Points (VP)



*Global
Optimality*



Efficiency



*Outlier
Robustness*



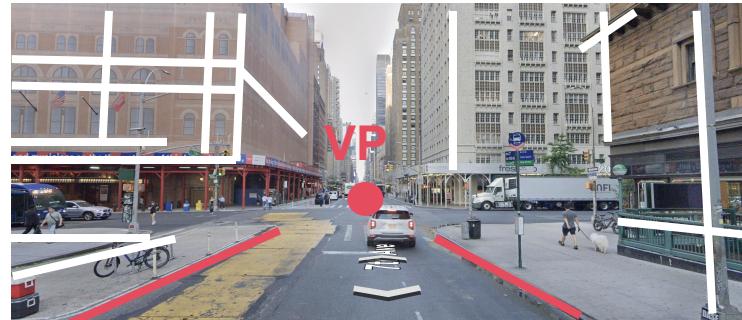
Generalization

Related Works (1/3)

RANdom SAmple Consensus (RANSAC)



Minimal Lines Sampling



VP Hypothesize



Inliers Counting



Global Optimality



Efficiency



Outlier Robustness

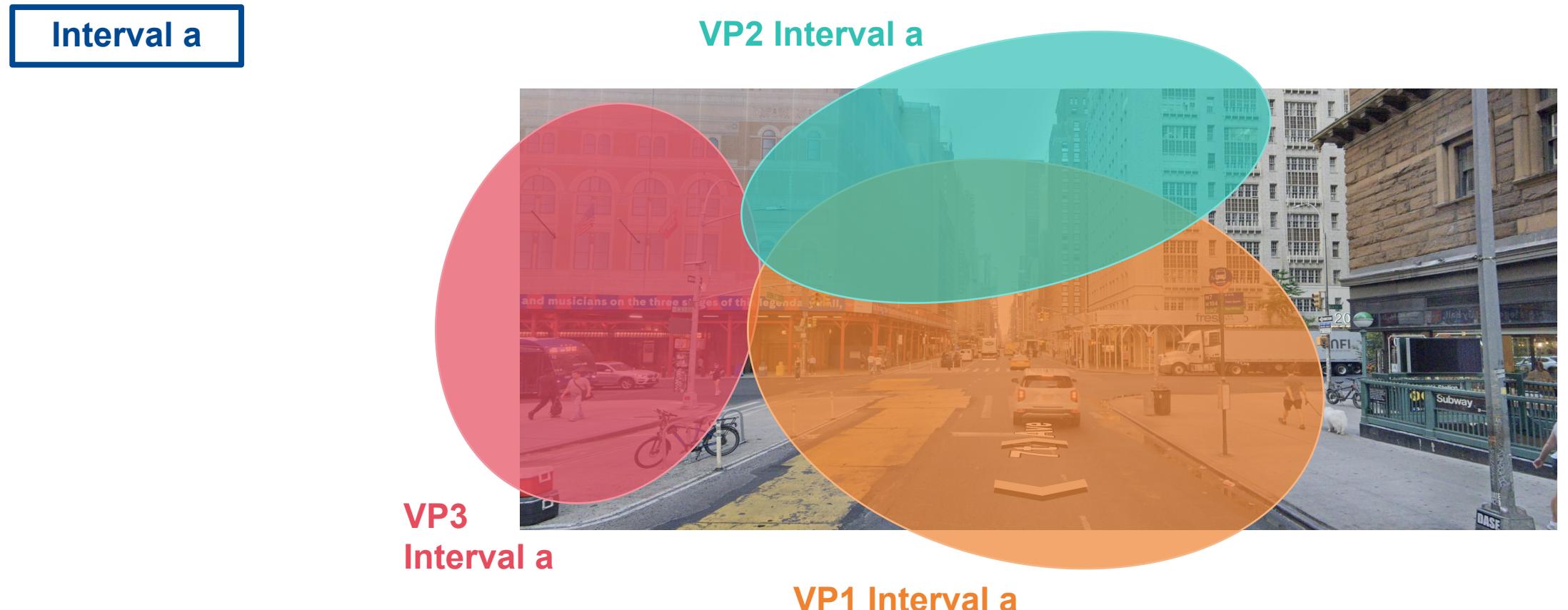


Generalization

- [1] Bazin J C, Pollefeys M. 3-line RANSAC for orthogonal vanishing point detection. IROS 2012.
- [2] Mirzaei F M, Roumeliotis S I. Optimal estimation of vanishing points in a manhattan world. ICCV 2011.
- [3] Sinha S N, Steedly D, Szeliski R, et al. Interactive 3D architectural modeling from unordered photo collections. TOG 2008.
- [4] Tardif J P. Non-iterative approach for fast and accurate vanishing point detection. ICCV 2009.
- [5] Toldo R, Fusillo A. Robust multiple structures estimation with j-linkage. ECCV 2008.
- [6] Zhang L, Lu H, Hu X, et al. Vanishing point estimation and line classification in a manhattan world with a unifying camera model. IJCV 2016.
- [7] Zuliani M, Kenney C S, Manjunath B S. The multiransac algorithm and its application to detect planar homographies. ICIP 2005.

Related Works (2/3)

Branch and Bound (BnB)



[1] Bazin J C, Seo Y, Demonceaux C, et al. Globally optimal line clustering and vanishing point estimation in manhattan world. CVPR 2012.

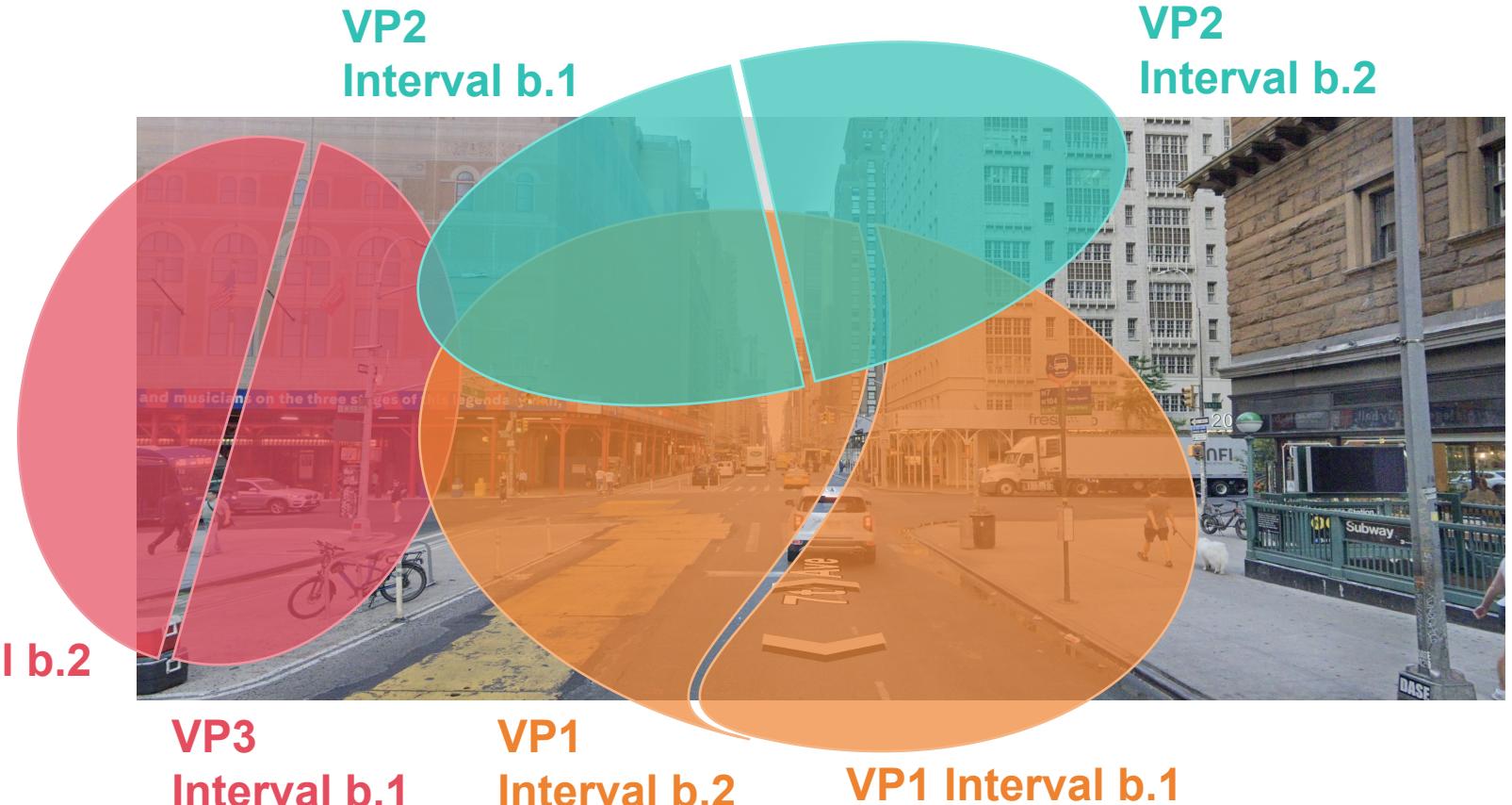
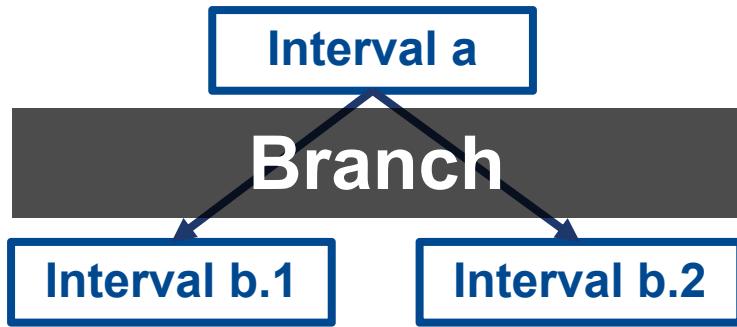
[2] Bazin J C, Seo Y, Pollefeys M. Globally optimal consensus set maximization through rotation search. ACCV 2012.

[3] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and efficient vanishing point estimation in Manhattan world. ICCV 2019.

[4] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and near/true real-time vanishing point estimation in Manhattan world. T-PAMI 2020.

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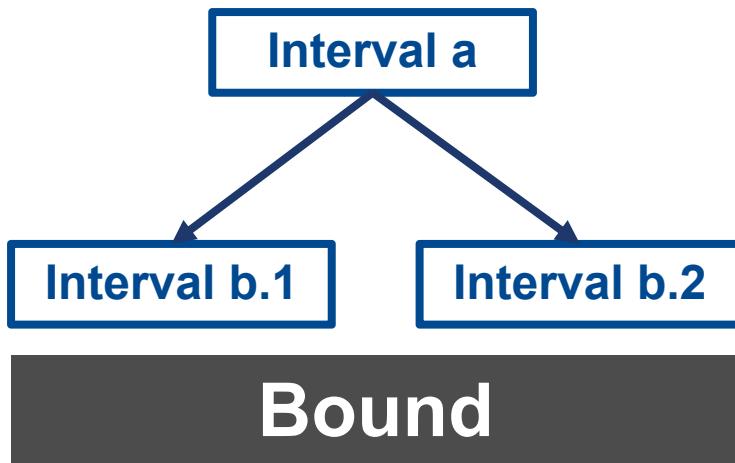
[2] Bazin J C, Seo Y, Pollefeys M. Globally optimal consensus set maximization through rotation search. ACCV 2012.

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Related Works (2/3)

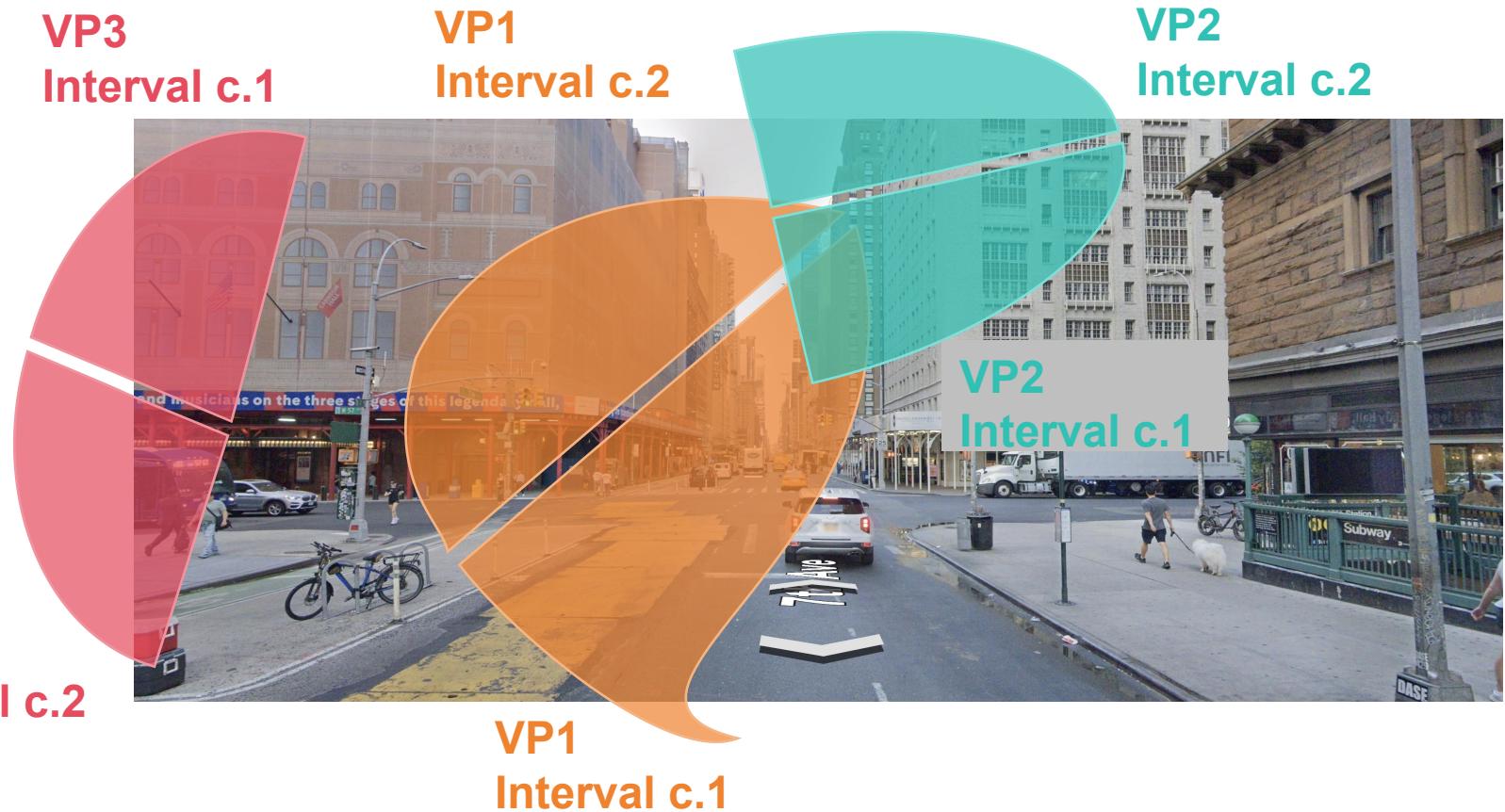
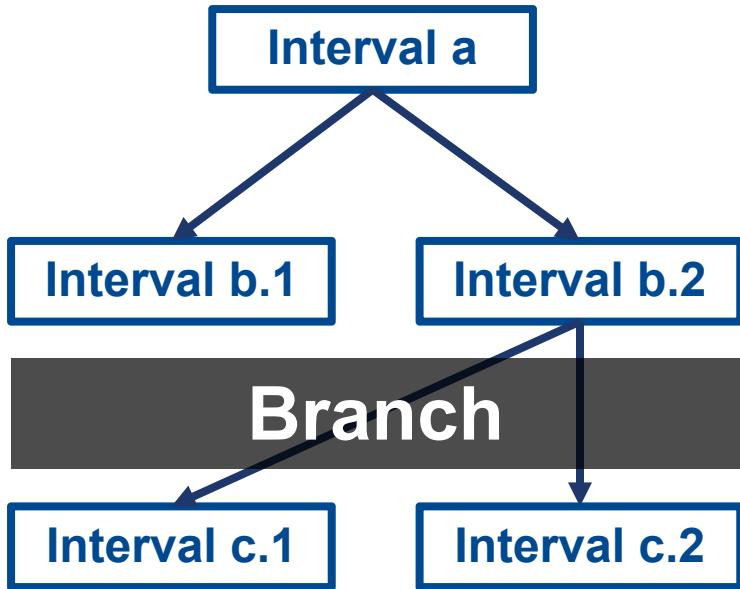
Branch and Bound (BnB)



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- [2] Bazin J C, Seo Y, Pollefeys M. Globally optimal consensus set maximization through rotation search. ACCV 2012.
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Related Works (2/3)

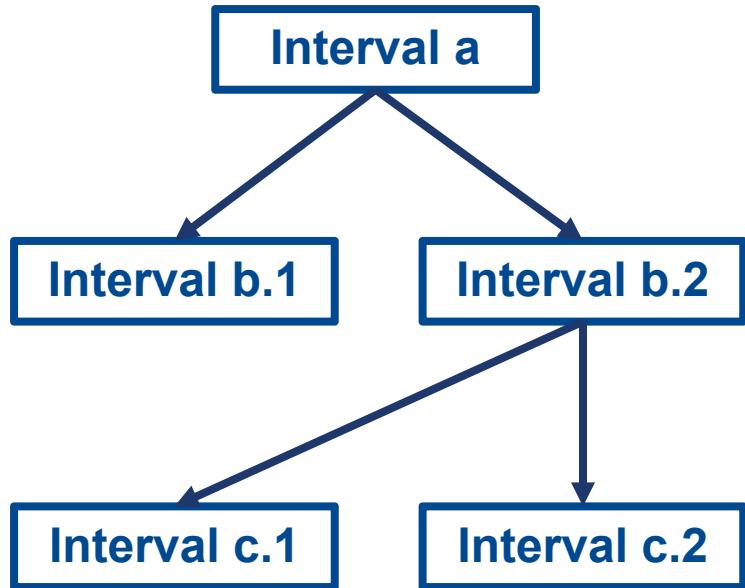
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Related Works (2/3)

Branch and Bound (BnB)



*Global
Optimality*



Efficiency



*Outlier
Robustness*



Generalization

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[4] Li H, Zhao J, Bazin J C, et al. Quasi-globally optimal and near/true real-time vanishing point estimation in Manhattan world. T-PAMI 2020.

Related Works (3/3)

Learning-based



Input: Raw Image
Unlabeled Lines (Optional)



Output: Vanishing Points



Global Optimality



Efficiency



Outlier Robustness



Generalization

- [1] Kluger F, Brachmann E, Ackermann H, et al. Consac: Robust multi-model fitting by conditional sample consensus. CVPR 2020.
- [2] Li H, Chen K, Kim P, et al. Learning icosahedral spherical probability map based on bingham mixture model for vanishing point estimation. ICCV 2021.
- [3] Lin Y, Wiersma R, Pintea S L, et al. Deep vanishing point detection: Geometric priors make dataset variations vanish. CVPR 2022.
- [4] Tong X, Ying X, Shi Y, et al. Transformer based line segment classifier with image context for real-time vanishing point detection in Manhattan world. CVPR 2022.
- [5] Zhai M, Workman S, Jacobs N. Detecting vanishing points using global image context in a non-manhattan world. CVPR 2016.
- [6] Zhou Y, Qi H, Huang J, et al. Neurvps: Neural vanishing point scanning via conic convolution. NeurIPS 2019.
- [7] Kluger F, Rosenhahn B. PARSAc: Accelerating robust multi-model fitting with parallel sample consensus. AAAI 2024.

Ours Method: GlobustVP

$$\begin{array}{ll} \min_{\mathbf{D}, \mathbf{Q}} & \langle (\mathbf{DN}; \mathbf{c}\mathbf{1}_m^\top)^2, \mathbf{Q} \rangle \\ \text{s.t.} & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{3,*}[\mathbf{D}]_{3,*}^\top = 1, [\mathbf{D}]_{1,*}[\mathbf{D}]_{2,*}^\top = 0, \\ & [\mathbf{D}]_{2,*}[\mathbf{D}]_{3,*}^\top = 0, [\mathbf{D}]_{1,*}[\mathbf{D}]_{3,*}^\top = 0, \\ & \quad (\text{Primal Problem}) \end{array}$$

1) Primal Problem

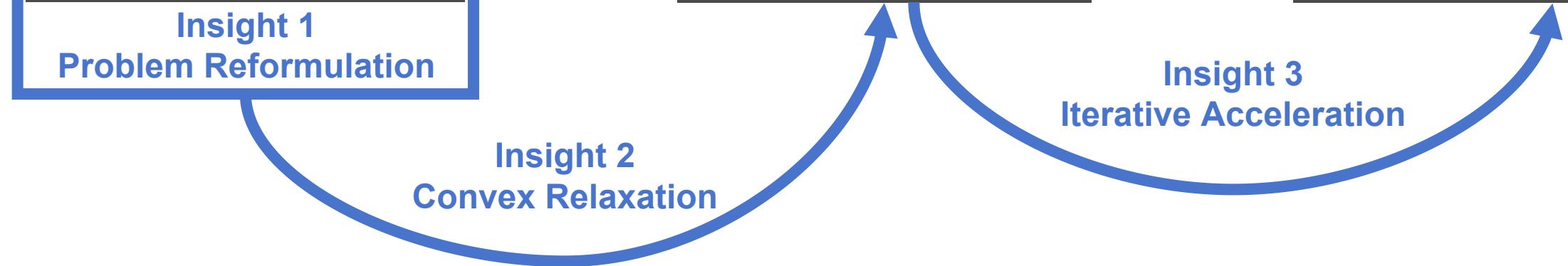
Insight 1
Problem Reformulation

$$\begin{array}{ll} \min_{\mathbf{W}} & \text{trace}(\mathbf{CW}) \\ \text{s.t.} & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,k} = \mathbf{W}_{k,0}^\top, \quad k = 1, \dots, 4 \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq 0, \quad (\text{Full SDP Problem}) \end{array}$$

2) SDP Problem

$$\begin{array}{ll} \min_{\mathbf{W}} & \text{trace}(\mathbf{CW}) \\ \text{s.t.} & \mathbf{W}_{0,0,1} = \sum_{i=1}^2 \mathbf{W}_{0,i,i}, \quad \forall i \in \{1, 2\}, \quad j = 1, \dots, m, \\ & \text{trace}(\mathbf{W}_{0,0,1}) = 1, \quad \mathbf{W}_{0,0,1} = \mathbf{W}_{0,0,2}, \\ & \mathbf{W}_{*,*,i} \succeq 0, \quad \forall i \in \{1, 2\}. \quad (\text{Single Block SDP Problem}) \end{array}$$

3) Iterative SDP Problem



*Global
Optimality*



Efficiency



*Outlier
Robustness*



Generalizable

Insight 1: Primal Problem Reformulation

$$\min_{\substack{\text{VP1}, \text{VP2}, \text{VP3}, \\ \text{Permutation}}} \text{Loss}(\text{Lines}, \text{VP1}, \text{VP2}, \text{VP3}, \text{Permutation})$$

How to represent discrete labels?

-> continuous constraint

$$q = q^2 \iff q = 1 \text{ or } q = 0$$

$$q_{\text{VP1}} + q_{\text{VP2}} + q_{\text{VP3}} + q_{\text{Outlier}} = 1$$

-> scales to all lines

	VP1	VP2	VP3	Outlier	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6
VP1	0	1	0	0	0	0	0	0	0	0
VP2	1	0	0	0	0	1	0	0	0	0
VP3	0	0	0	0	0	0	0	0	0	1
Outlier	0	0	1	1	0	0	0	0	0	0

Permutation Matrix

Column Sum

	VP1	VP2	VP3	Outlier	
VP1	0	1	0	0	0
VP2	1	0	0	0	1
VP3	0	0	0	0	0
Outlier	0	0	1	1	0

Line 1 Line 2 Line 3 Line 4 Line 5 Line 6

Column Sum

	VP1	VP2	VP3	Outlier	
VP1	0	1	0	0	0
VP2	1	0	0	0	1
VP3	0	0	0	0	0
Outlier	0	0	1	1	0

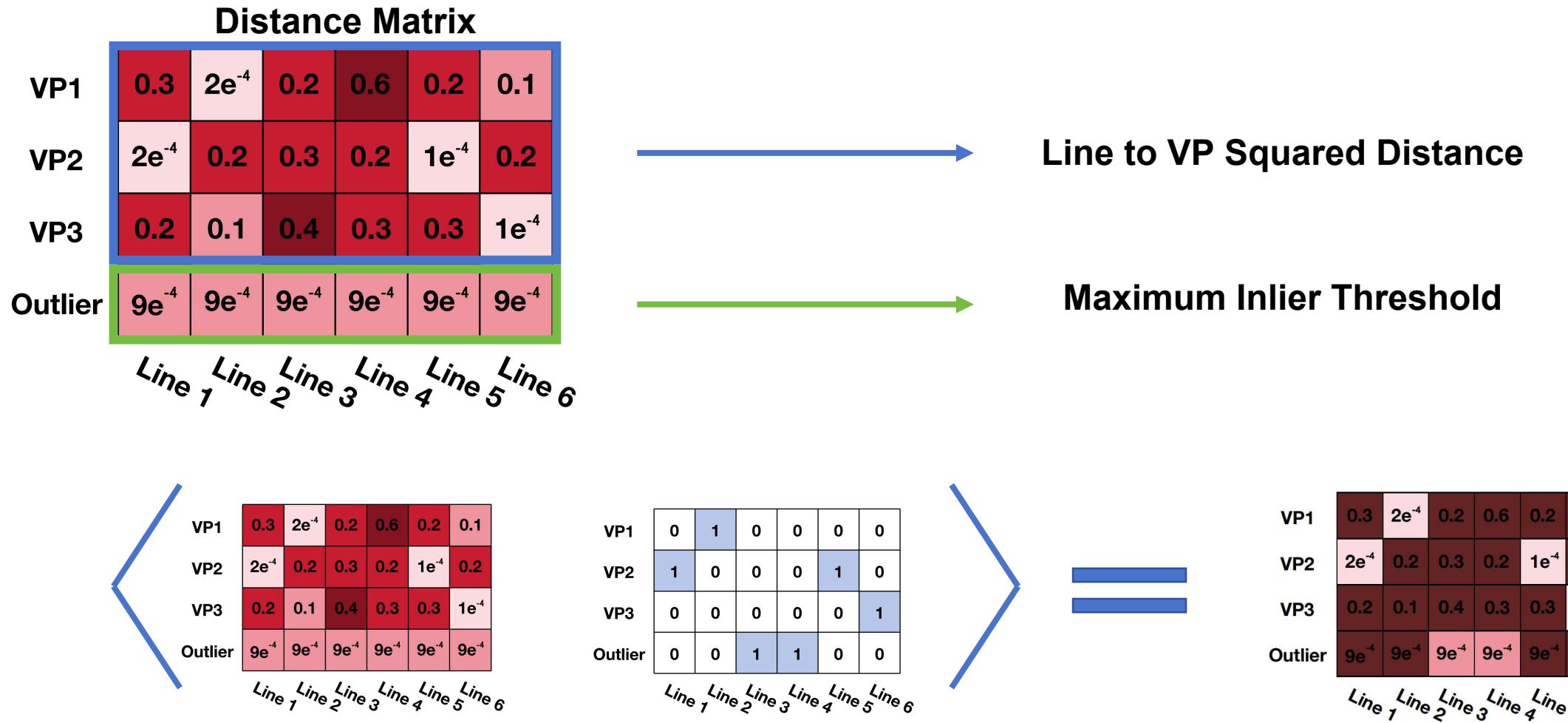
	VP1	VP2	VP3	Outlier	
VP1	0	1	0	0	0
VP2	1	0	0	0	1
VP3	0	0	0	0	0
Outlier	0	0	1	1	0

	VP1	VP2	VP3	Outlier	
VP1	0	1	0	0	0
VP2	1	0	0	0	1
VP3	0	0	0	0	0
Outlier	0	0	1	1	0

	VP1	VP2	VP3	Outlier	
VP1	0	1	0	0	0
VP2	1	0	0	0	1
VP3	0	0	0	0	0
Outlier	0	0	1	1	0

	VP1	VP2	VP3	Outlier	
VP1	0	1	0	0	0
VP2	1	0	0	0	1
VP3	0	0	0	0	0
Outlier	0	0	1	1	0

Insight 1: Primal Problem Reformulation



Insight 1: Primal Problem Reformulation

Primal Problem

$$\min_{\mathbf{D}, \mathbf{Q}} \langle ([\mathbf{DN}; c\mathbf{1}_m^\top]^2, \mathbf{Q} \rangle$$

s.t.

$$\mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q},$$

$$\begin{aligned} [\mathbf{D}]_{1,*}[\mathbf{D}]_{1,*}^\top &= 1, [\mathbf{D}]_{2,*}[\mathbf{D}]_{2,*}^\top = 1, \\ [\mathbf{D}]_{3,*}[\mathbf{D}]_{3,*}^\top &= 1, [\mathbf{D}]_{1,*}[\mathbf{D}]_{2,*}^\top = 0, \\ [\mathbf{D}]_{2,*}[\mathbf{D}]_{3,*}^\top &= 0, [\mathbf{D}]_{1,*}[\mathbf{D}]_{3,*}^\top = 0, \end{aligned}$$

Manhattan Constraint

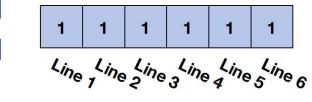
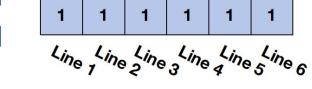
$$\min_{\text{VP1, VP2, VP3}} \text{Permutation Matrix}$$

	VP1	VP2	VP3	Outlier		
Line 1	0.3	$2e^{-4}$	0.2	0.6	0.2	0.1
Line 2	$2e^{-4}$	0.2	0.3	0.2	$1e^{-4}$	0.2
Line 3	0.2	0.1	0.4	0.3	0.3	$1e^{-4}$
Line 4	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$	$9e^{-4}$
Line 5						
Line 6						

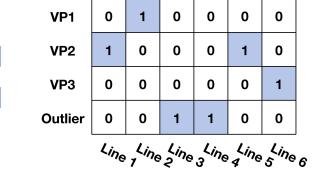
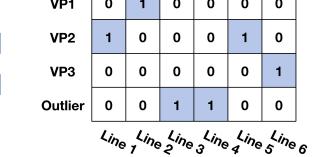
	VP1	VP2	VP3	Outlier		
Line 1	0	1	0	0	0	0
Line 2	1	0	0	0	1	0
Line 3	0	0	0	0	0	1
Line 4	0	0	1	1	0	0
Line 5						
Line 6						

Column Sum

	VP1	VP2	VP3	Outlier		
Line 1	0	1	0	0	0	0
Line 2	1	0	0	0	1	0
Line 3	0	0	0	0	0	1
Line 4	0	0	1	1	0	0
Line 5						
Line 6						

=	
=	

	VP1	VP2	VP3	Outlier		
Line 1	0	1	0	0	0	0
Line 2	1	0	0	0	1	0
Line 3	0	0	0	0	0	1
Line 4	0	0	1	1	0	0
Line 5						
Line 6						

=	
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Our Method: GlobustVP

$$\min_{\mathbf{D}, \mathbf{Q}} \quad <([\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top]^2, \mathbf{Q}>$$

$$\text{s.t.} \quad \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, \quad (\mathbf{Q})^2 = \mathbf{Q},$$

1) Primal Problem
 $[\mathbf{D}]_{3,*}[\mathbf{D}]_{3,*}^\top = 1, [\mathbf{D}]_{1,*}[\mathbf{D}]_{1,*}^\top = 1,$
 $[\mathbf{D}]_{2,*}[\mathbf{D}]_{3,*}^\top = 0, [\mathbf{D}]_{1,*}[\mathbf{D}]_{3,*}^\top = 0,$
(Primal Problem)

Insight 1

Problem Reformulation



Generalizable

Insight 2: QCQP \rightarrow Convex Relaxation \rightarrow SDP

$$\begin{aligned} \min_{\mathbf{D}, \mathbf{Q}} \quad & <([\mathbf{D}\mathbf{N}; c\mathbf{1}_m^\top]^2, \mathbf{Q} > \\ \text{s.t.} \quad & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]^{1,*}[\mathbf{D}]^\top = 1, [\mathbf{D}]^{2,*}[\mathbf{D}]^\top = 1 \\ & [\mathbf{D}]_{3,*}[\mathbf{D}]^\top = 1, [\mathbf{D}]_{1,*}[\mathbf{D}]^\top = 0, \\ & [\mathbf{D}]_{2,*}[\mathbf{D}]^\top = 0, [\mathbf{D}]_{1,*}[\mathbf{D}]^\top = 0, \\ & \qquad \qquad \qquad \text{(Primal Problem)} \end{aligned}$$

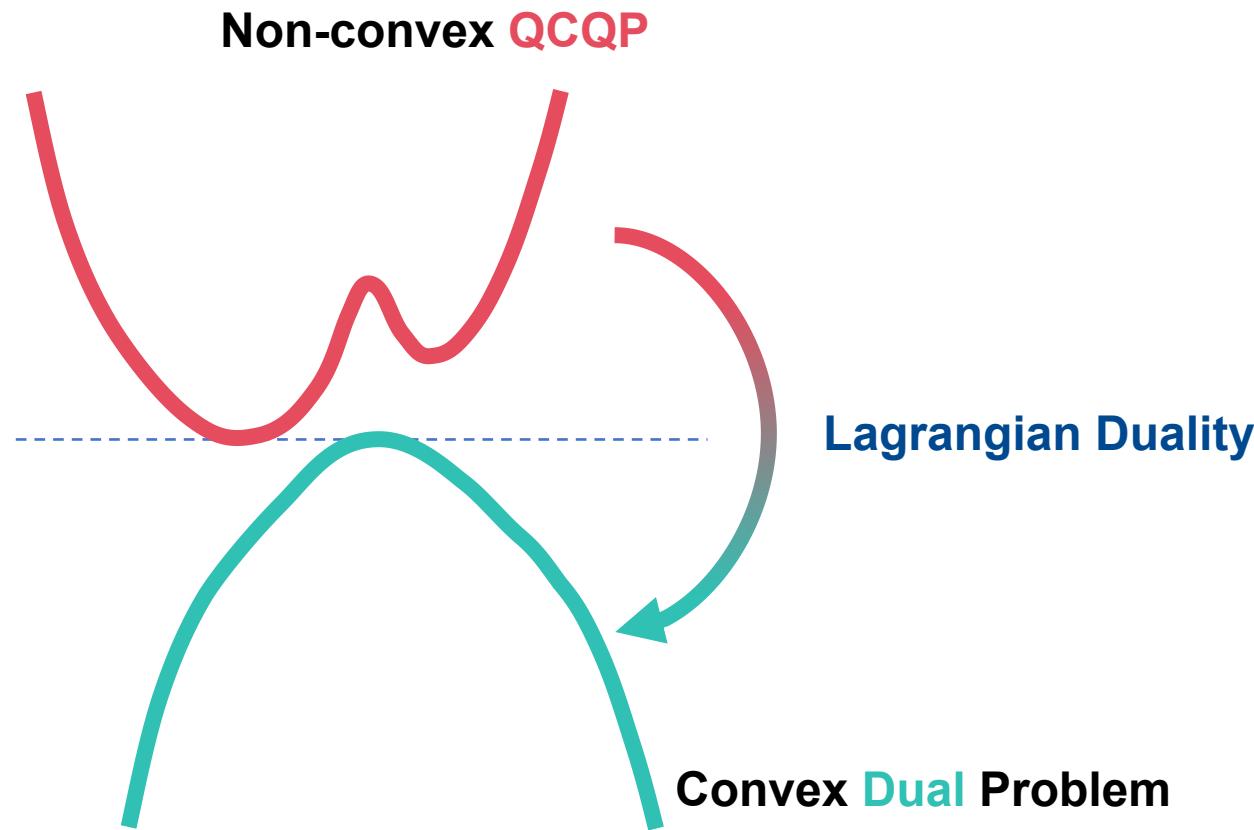
$$\begin{aligned} \min_{\omega} \quad & \omega^\top \mathbf{C} \omega \\ \text{s.t.} \quad & \omega_0 \omega_0^\top = \sum_{i=1}^4 \omega_0 \omega_{i,j}^\top, \quad j = 1, \dots, m, \\ & \{\omega_0\}_1^\top \{\omega_0\}_1 = 1, \quad \{\omega_0\}_2^\top \{\omega_0\}_2 = 1, \\ & \{\omega_0\}_3^\top \{\omega_0\}_3 = 1, \quad \{\omega_0\}_1^\top \{\omega_0\}_2 = 0, \\ & \{\omega_0\}_1^\top \{\omega_0\}_3 = 0, \quad \{\omega_0\}_2^\top \{\omega_0\}_3 = 0. \\ & \qquad \qquad \qquad \text{(Full QCQP Problem)} \end{aligned}$$

Equivalent to Higher Dimension QCQP

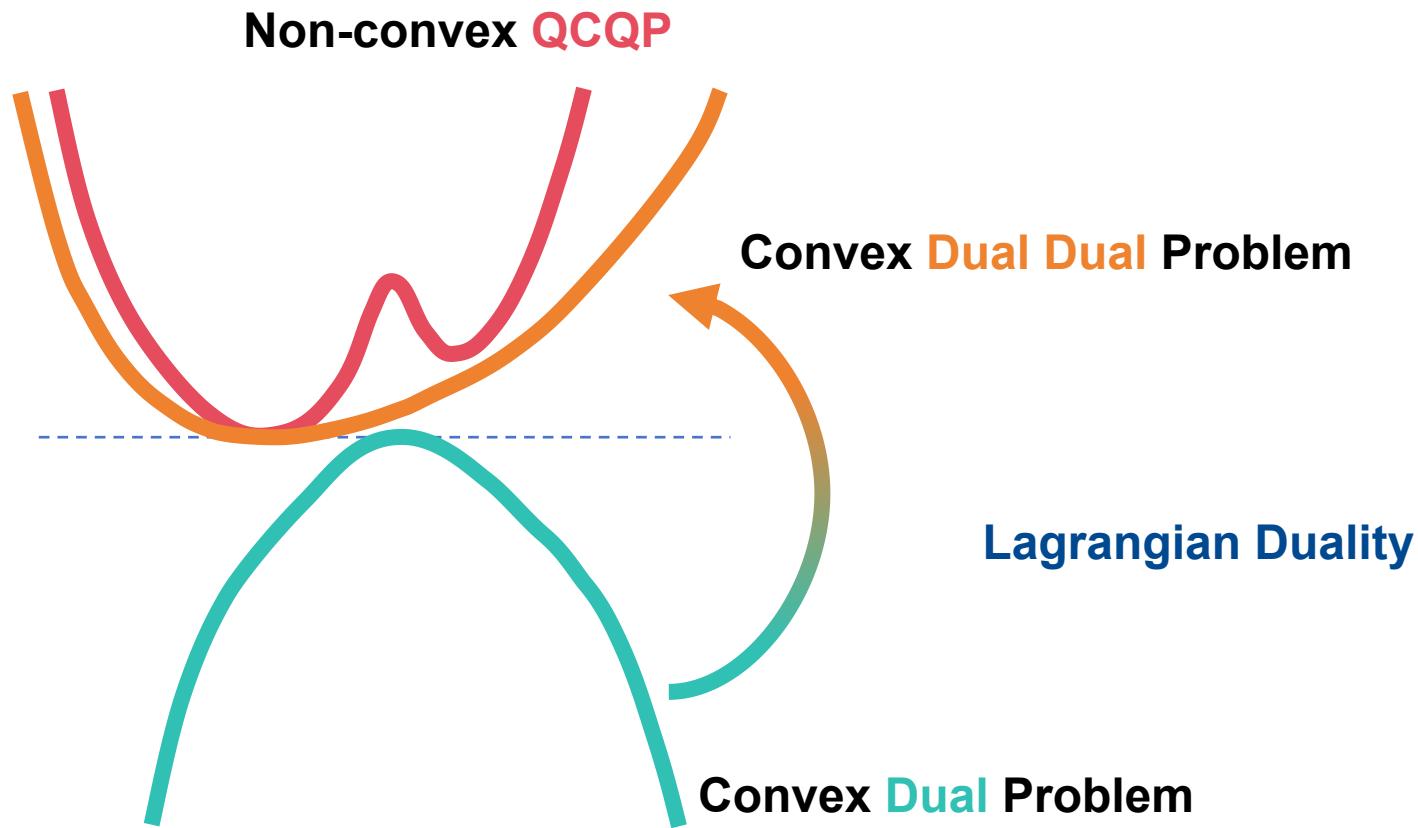
Insight 2: QCQP \rightarrow Convex Relaxation \rightarrow SDP



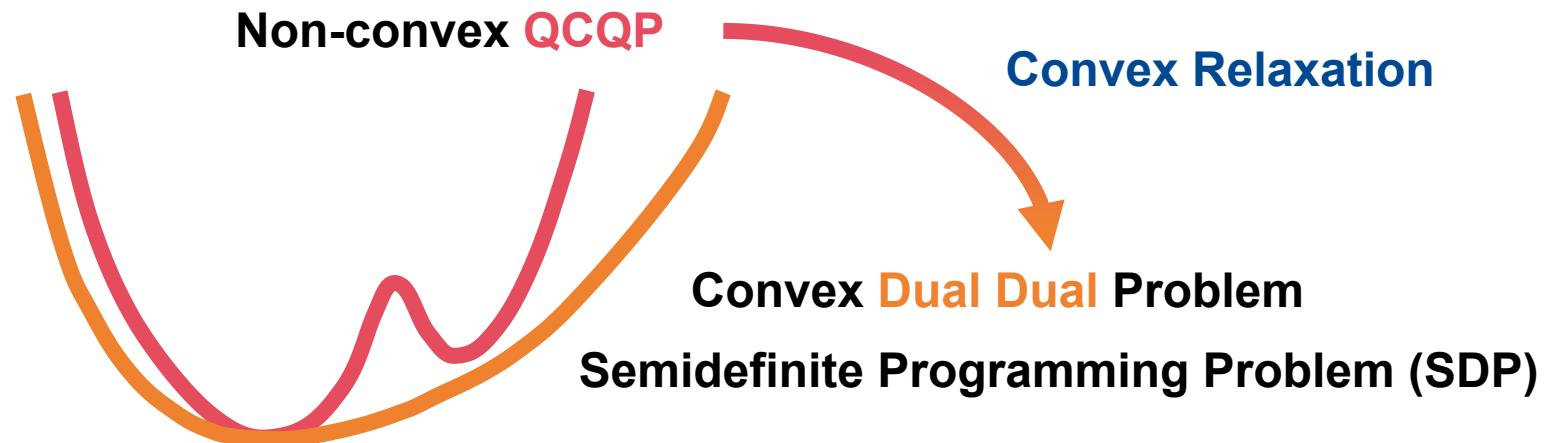
Insight 2: QCQP \rightarrow Convex Relaxation \rightarrow SDP



Insight 2: QCQP \rightarrow Convex Relaxation \rightarrow SDP



Insight 2: QCQP \rightarrow Convex Relaxation \rightarrow SDP



$$\begin{aligned} \min_{\omega} \quad & \omega^\top C \omega \\ \text{s.t.} \quad & \omega_0 \omega_0^\top = \sum_{i=1}^4 \omega_0 \omega_{i,j}^\top, \quad j = 1, \dots, m, \\ & \omega_0 \omega_{i,j}^\top = \omega_{i,j}, \quad i = 1, \dots, 4 \\ & \{\omega_0\}_1^\top \{\omega_0\}_1 = 1, \quad \{\omega_0\}_2^\top \{\omega_0\}_2 = 1, \\ & \{\omega_0\}_3^\top \{\omega_0\}_3 = 1, \quad \{\omega_0\}_1^\top \{\omega_0\}_2 = 0, \\ & \{\omega_0\}_1^\top \{\omega_0\}_3 = 0, \quad \{\omega_0\}_2^\top \{\omega_0\}_3 = 0. \end{aligned}$$

(Full QCQP Problem)

equivalent under
=====
tight relaxation

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,k} = \mathbf{0} \quad k = 1, \dots, 4 \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq 0, \end{aligned}$$

(Full SDP Problem)

Ours Method: GlobustVP

$$\begin{array}{ll} \min_{\mathbf{D}, \mathbf{Q}} & \langle ([\mathbf{D}\mathbf{N}; \mathbf{c}\mathbf{1}_m^\top]^2, \mathbf{Q}) \rangle \\ \text{s.t.} & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{3,*}[\mathbf{D}]_{3,*}^\top = 1, [\mathbf{D}]_{1,*}[\mathbf{D}]_{2,*}^\top = 0, \\ & [\mathbf{D}]_{2,*}[\mathbf{D}]_{3,*}^\top = 0, [\mathbf{D}]_{1,*}[\mathbf{D}]_{3,*}^\top = 0, \\ & \quad (\text{Primal Problem}) \end{array}$$

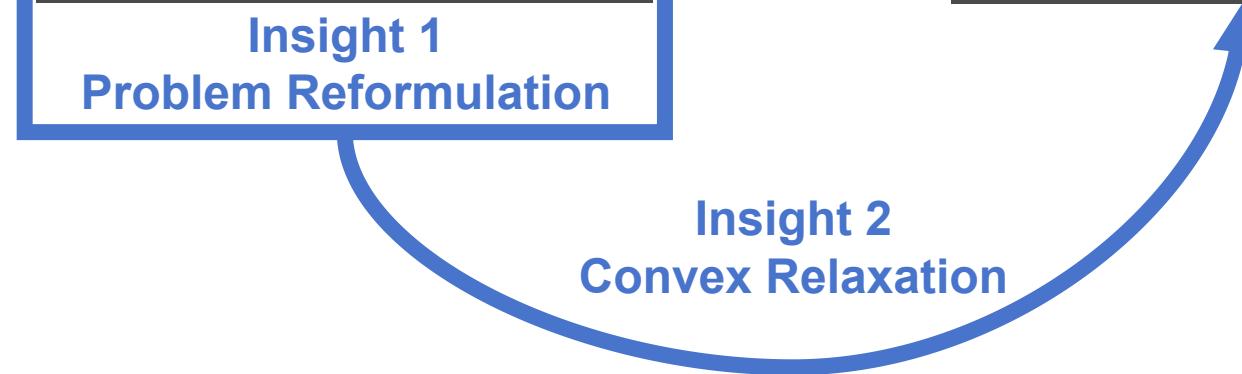
1) Primal Problem

Insight 1
Problem Reformulation

$$\begin{array}{ll} \min_{\mathbf{W}} & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,k} = \mathbf{W}_{k,0}^\top, \quad k = 1, \dots, 4 \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq 0, \end{array}$$

2) SDP Problem

(Full SDP Problem)



*Global
Optimality*



*Outlier
Robustness*



Generalizable

Insight 3: Iterative SDP

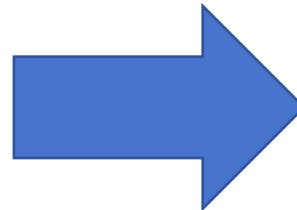
$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{CW}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{*,k} = \mathbf{0}_{(m-1) \times 1}, \quad k = 1, 2, 3, \dots, n, \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq 0, \end{aligned}$$

(Full SDP Problem)

$$\begin{aligned} \min_{\mathbf{W}} \quad & \text{trace}(\mathbf{CW}) \\ \text{s.t.} \quad & \mathbf{W}_{0,0,1} = \sum_{i=1}^2 \mathbf{W}_{0,j,i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,0,1} = \mathbf{W}_{0,0,2}, \\ & \mathbf{W}_{*,*,i} \succeq 0, \quad \forall i \in \{1, 2\}. \end{aligned}$$

(Single Block SDP Problem)

Large SDP problem is not efficient enough



Find VPs one by one!

VP1	0.3	2e ⁻⁴	0.2	0.6	0.2	0.1
VP2	2e ⁻⁴	0.2	0.3	0.2	1e ⁻⁴	0.2
VP3	0.2	0.1	0.4	0.3	0.3	1e ⁻⁴
Outlier	9e ⁻⁴					

Line 1 Line 2 Line 3 Line 4 Line 5 Line 6

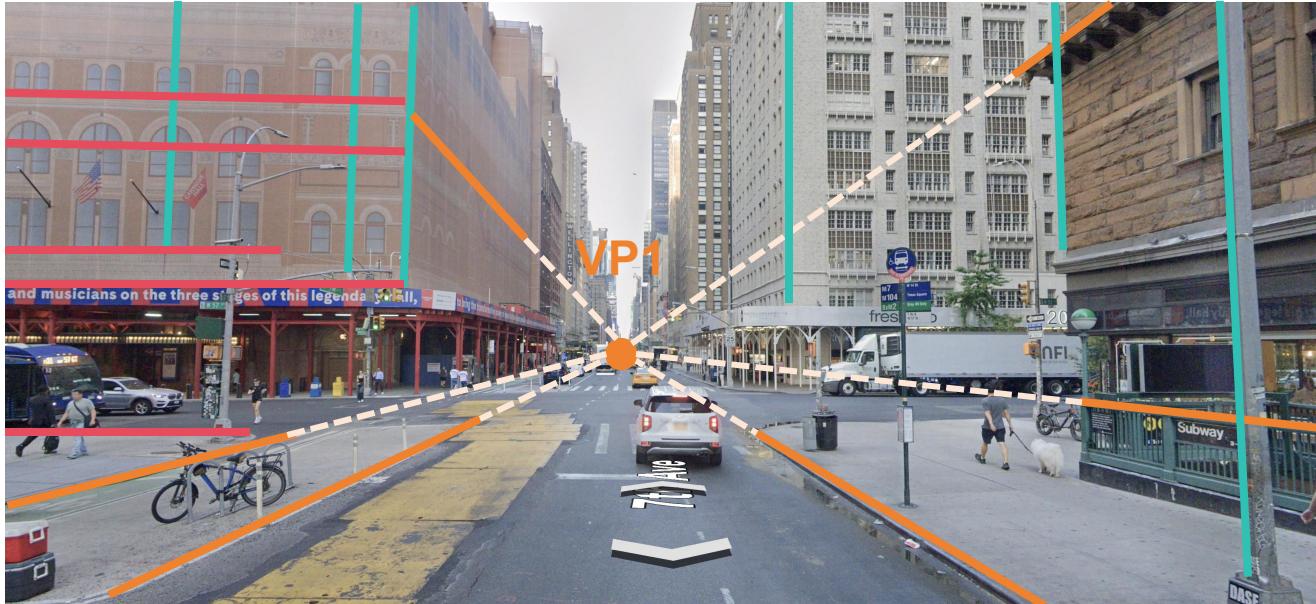
VP1	0	1	0	0	0	0
VP2	1	0	0	0	1	0
VP3	0	0	0	0	0	1
Outlier	0	0	1	1	0	0

Line 1 Line 2 Line 3 Line 4 Line 5 Line 6

VP	0.2	2e ⁻⁴	0.4	1e ⁻⁴	0.3	1e ⁻⁴
Outlier	9e ⁻⁴					
	<i>Line 1</i>	<i>Line 2</i>	<i>Line 3</i>	<i>Line 4</i>	<i>Line 5</i>	<i>Line 6</i>

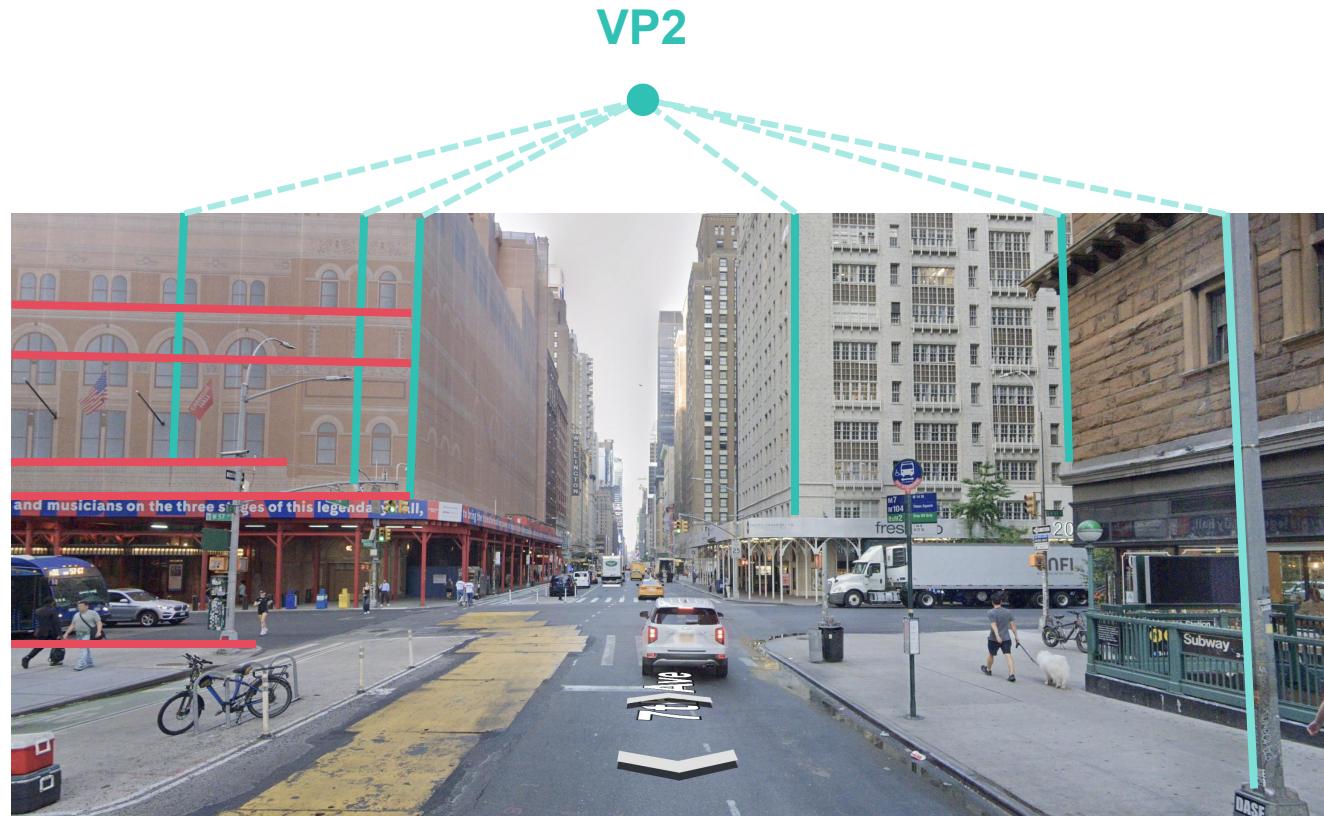
VP	0	1	0	1	0	1
Outlier	1	0	1	0	1	0
	<i>Line 1</i>	<i>Line 2</i>	<i>Line 3</i>	<i>Line 4</i>	<i>Line 5</i>	<i>Line 6</i>

Insight 3: Iterative SDP



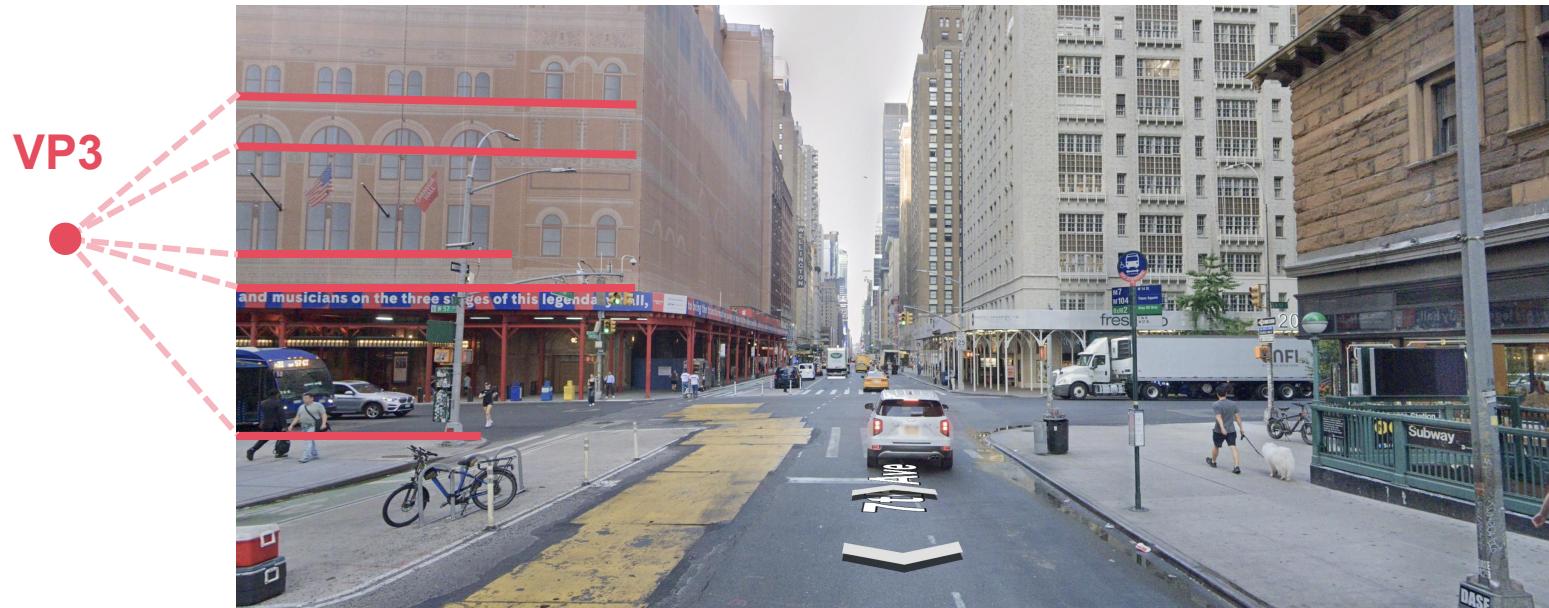
Treat **Line** and **Line** as Outliers, find **VP1**

Insight 3: Iterative SDP



Treat **Line** as Outliers, find **VP2**

Insight 3: Iterative SDP



Find VP3

Ours Method: GlobustVP

$$\begin{array}{ll} \min_{\mathbf{D}, \mathbf{Q}} & \langle ([\mathbf{D}\mathbf{N}; \mathbf{c}\mathbf{1}_m^\top]^2, \mathbf{Q}) \rangle \\ \text{s.t.} & \mathbf{Q}^\top \mathbf{1}_4 = \mathbf{1}_m, (\mathbf{Q})^2 = \mathbf{Q}, \\ & [\mathbf{D}]_{1,*}[\mathbf{D}]_{1,*}^\top = 1, [\mathbf{D}]_{2,*}[\mathbf{D}]_{2,*}^\top = 1, \\ & [\mathbf{D}]_{3,*}[\mathbf{D}]_{3,*}^\top = 1, [\mathbf{D}]_{1,*}[\mathbf{D}]_{2,*}^\top = 0, \\ & [\mathbf{D}]_{2,*}[\mathbf{D}]_{3,*}^\top = 0, [\mathbf{D}]_{1,*}[\mathbf{D}]_{3,*}^\top = 0, \\ & \quad (\text{Primal Problem}) \end{array}$$

1) Primal Problem

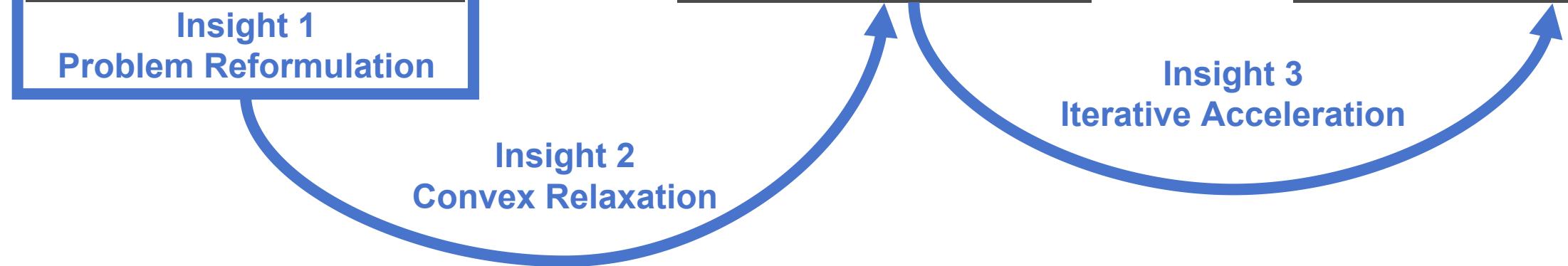
Insight 1
Problem Reformulation

$$\begin{array}{ll} \min_{\mathbf{W}} & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} & \mathbf{W}_{0,0} = \sum_{i=1}^4 \mathbf{W}_{0,4(j-1)+i}, \quad j = 1, \dots, m, \\ & \mathbf{W}_{0,k} = \mathbf{W}_{k,0}^\top, \quad k = 1, \dots, 4 \\ & \text{trace}(\{\mathbf{W}_{0,0}\}_{i,j}) = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad \forall i, j \in \{1, 2, 3\} \\ & \mathbf{W} \succeq 0, \quad (\text{Full SDP Problem}) \end{array}$$

2) SDP Problem

$$\begin{array}{ll} \min_{\mathbf{W}} & \text{trace}(\mathbf{C}\mathbf{W}) \\ \text{s.t.} & \mathbf{W}_{0,0,1} = \sum_{i=1}^2 \mathbf{W}_{0,i,i}, \quad \forall i \in \{1, 2\}, \quad j = 1, \dots, m, \\ & \text{trace}(\mathbf{W}_{0,0,1}) = 1, \quad \mathbf{W}_{0,0,1} = \mathbf{W}_{0,0,2}, \\ & \mathbf{W}_{*,*,i} \succeq 0, \quad \forall i \in \{1, 2\}. \quad (\text{Single Block SDP Problem}) \end{array}$$

3) Iterative SDP Problem



*Global
Optimality*



Efficiency

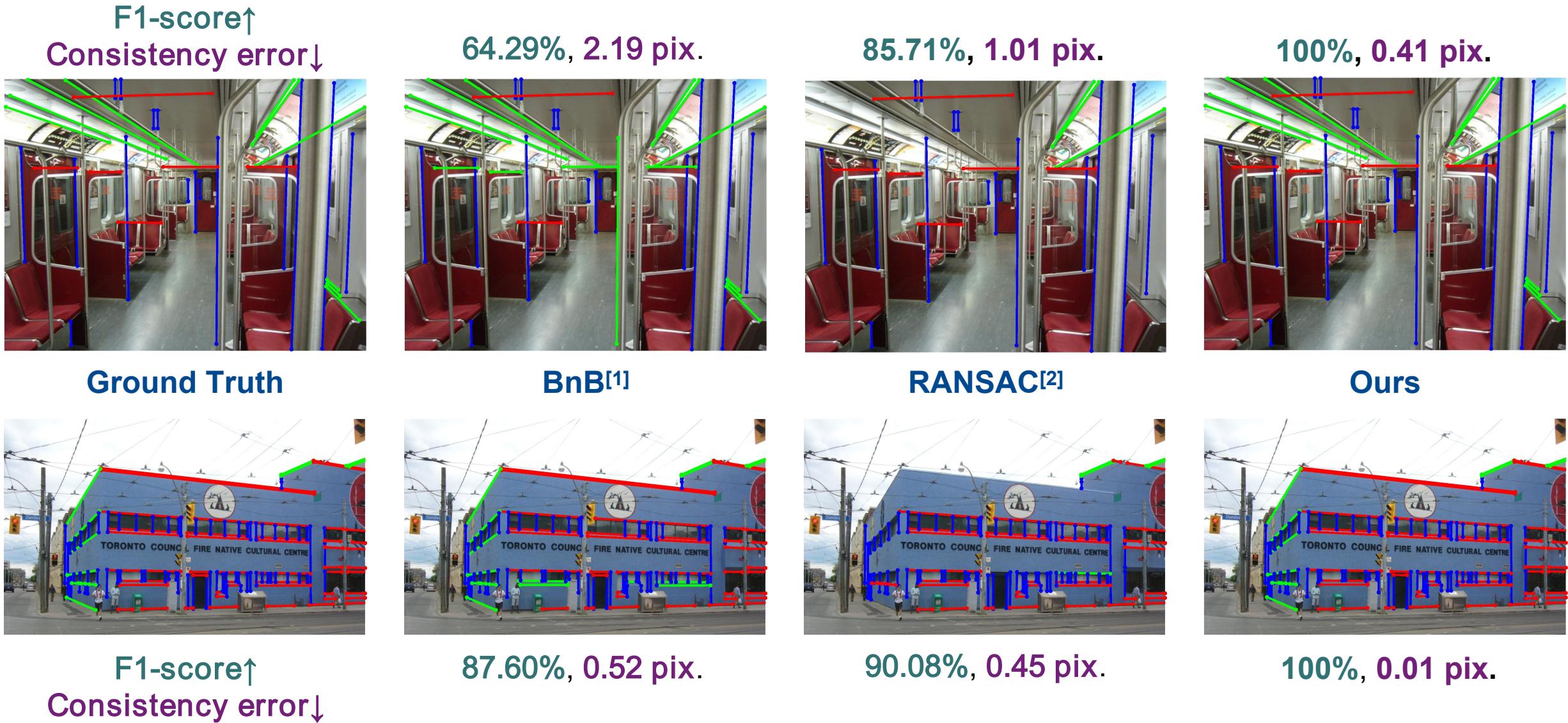


*Outlier
Robustness*



Generalizable

Some Qualitative Results

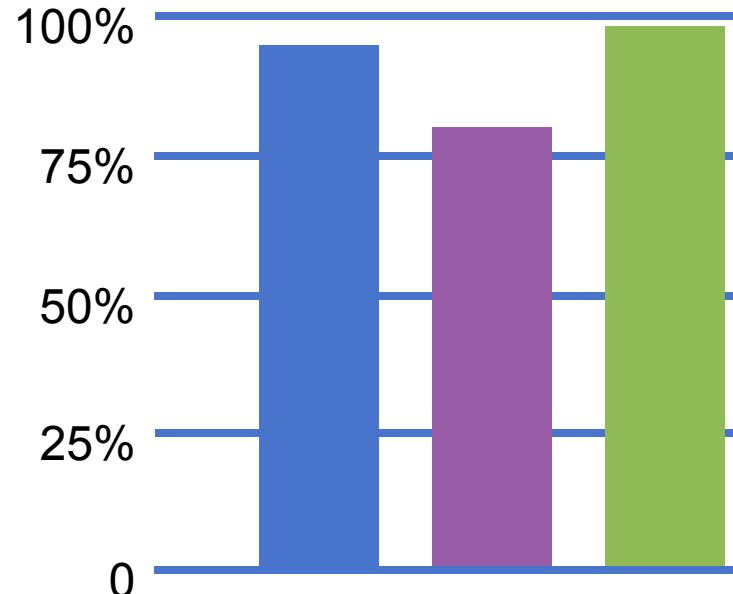


[1] Bazin J C, Seo Y, Demonceaux C, et al. Globally optimal line clustering and vanishing point estimation in manhattan world. CVPR 2012.

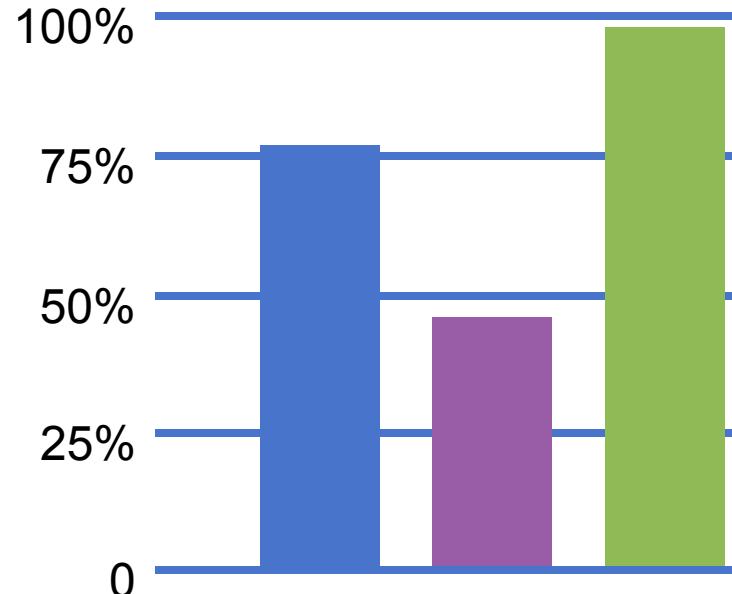
[2] Zhang L, Lu H, Hu X, et al. Vanishing point estimation and line classification in a manhattan world with a unifying camera model. IJCV 2016.

Synthetic Comparison

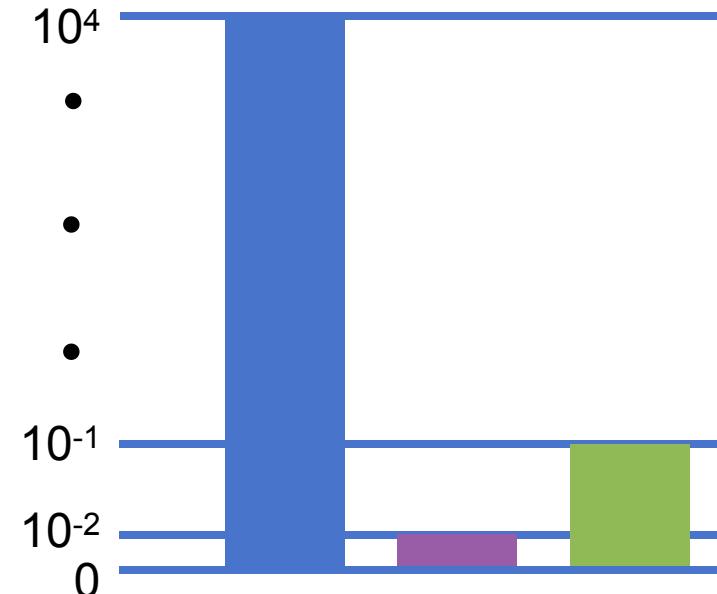
F1 Score with 30% outlier



F1 Score with 70% outlier



Time (s) with 20% outlier



BnB^[1]

RANSAC^[2]

Ours

[1] Bazin J C, Seo Y, Demonceaux C, et al. Globally optimal line clustering and vanishing point estimation in manhattan world. CVPR 2012.

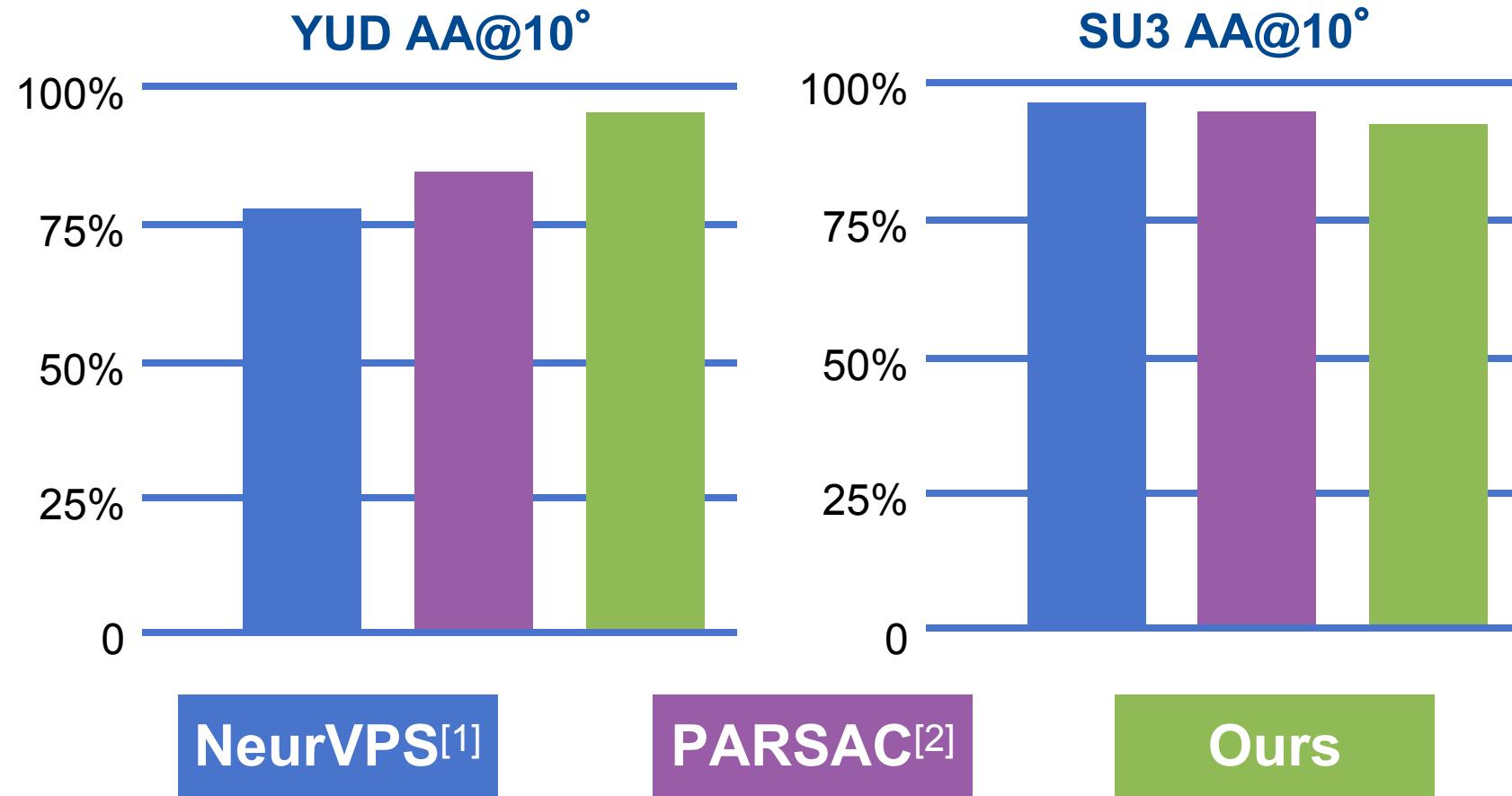
[2] Zhang L, Lu H, Hu X, et al. Vanishing point estimation and line classification in a manhattan world with a unifying camera model. IJCV 2016.

Comparison against Learning-based Method

YUD Dataset



SU3 Dataset



[1] Zhou Y, Qi H, Huang J, et al. Neurvps: Neural vanishing point scanning via conic convolution. NeurIPS 2019.

[2] Kluger F, Rosenhahn B. PARSAC: Accelerating robust multi-model fitting with parallel sample consensus. AAAI 2024.

GlobustVP = Globally Optimal Outlier-Robust Vanishing Point Solver



*Global
Optimality*



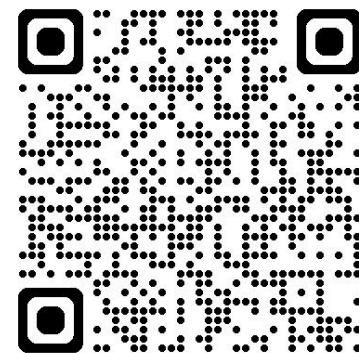
Efficiency



*Outlier
Robustness*



Generalizable



<https://github.com/WU-CVGL/GlobustVP/>



See you in Poster Session 4 #102