Fitting the SIR model to data

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5/18/2010

A review of the SIR model

$$\frac{dS}{dt} = -\beta IS$$
$$\frac{dI}{dt} = \beta IS - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

- Describes the movement between susceptible, infected, and recovered pools.
- β and γ are unknown parameters. Can we estimate them from data?

ODEs in matlab

Look in your folders for a file called `sir_ode.m`. This encodes the model for matlab.

```
function dydt = sir_ode(t,y,p)

  beta = p(1);
  gamma = p(2);

  S = y(1);
  I = y(2);
  R = y(3);

  dydt = [-beta*I*S; beta*I*S - gamma*I; gamma*I];
```

Let's load the data

Let's set up some `matlab` variables so we can play with the SIR system.

```
% Input the data and the time window.  Data is the number
% of infected individuals at each time step.
data = [3 6 25 73 222 294 258 237 191 125 69 27 11 4];
time = 1:14;

% Initial conditions: 760 susceptible, 3 infected,
% 0 recovered
y0 = [760 3 0];

% tspan will hold the times for which to solve the system
% of ODEs.  We want to solve it at the times that
% correspond to our data
tspan = time;
```

The matlab ODE solver

Now we can ask matlab to solve the SIR system for a given set of parameters.

```
% Try solving the system at a guess for good parameter value
p0 = [.01 .1];
[t,y] = ode45(@sir_ode,tspan,y0,[],p0);
```

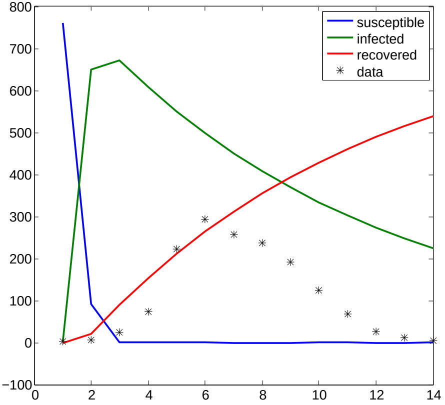
What is the result that matlab passes back (stored in the variables y and t)?

Plotting the solution

Let's plot the data on the same axes as the solution we just obtained for the parameter values $(\beta = .01, \gamma = .1)$.

```
figure(1), clf;
plot(t, y, 'linewidth', 2);
hold on;
plot(time, data, 'k*', 'markersize', 10);
legend('susceptible','infected','recovered','data')
set(gca, 'FontSize', 15)
```

Plotting the solution



Do these parameter values seem to match the data well?

How do we know how well we're doing?

Can you think of a good way to quantify how well the model fits the data, given the parameters you've chosen?

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Your task: Write a function that

- Takes as input `p`, `data`, `tspan`, and `y0`(for some particular value of `p`)
- Solves the SIR system
- Outputs a number that describes the quality of the model fit.

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Here's my function:

```
function disc = sir_discrepancy(p, data, tspan, y0)

[t,y] = ode45(@sir_ode,tspan,y0,[],p);
I = y(:,2);
disc = sum((I-data').^2);
```

Try running your descrepancy (aka loss, regret, etc.) function for different values of `p`. How small can you make it?

Numerical optimization of the parameters

- `matlab` can do optimization using the function `fminsearch`.

```
>> help fminsearch
 FMINSEARCH Multidimensional unconstrained nonlinear
 minimization (Nelder-Mead).
    X = FMINSEARCH(FUN,X0) starts at X0 and attempts to
    find a local minimizer X of the function FUN
```

- Our descrepancy function function takes several inputs, but `fminsearch` only optimized functions with one input.
- Can you figure out a way around this by using the `matlab` help functions?

Numerical optimization of the parameters

My solution:

```
function p_opt = sir_optimize(data, tspan, y0, p0)

p_opt = fminsearch(@sir_disc_nested, p0);
    function disc = sir_disc_nested(p)
      disc = sir_discrepancy(p, data, tspan, y0);
    end
end
```

Now we can run

```
>> p_opt = sir_optimize(data, tspan, y0, p0);
```

Numerical optimization of the parameters

- What were your optimal values of β and γ?
- Now use the ODE solver to solve the SIR system with the optimal parameter values. Plot the results.

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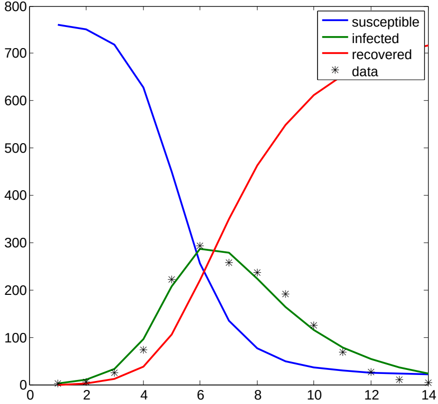

Better than before?