Ebola Dynamics

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Outline

State Diagram

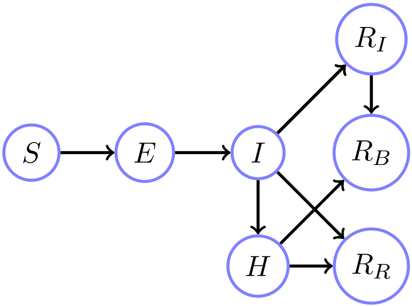


Figure 1: Illustration of mathematical model

Variable Definitions

Parameters:

$\alpha =$ population growth constant[3]

$\beta_1 =$ transmission rate between infected and susceptible

$\beta_2 =$ transmission rate between removed and still infectious and susceptible

$\beta_3 =$ transmission rate between hospitalized and susceptible

$\delta =$ rate at which people move from exposed to infected [4]

$\gamma_1 =$ (average time with disease for unhospitalized individuals)$^{-1}$

$\gamma_2 =$ (average time with disease for hospitalized individuals)$^{-1}$

$\psi =$ (average time that people become hospitalized)$^{-1}$

$\rho_1 = 1.1 \times \rho_2 =$ the proportion of people who die of the disease who are not hospitalized [5]

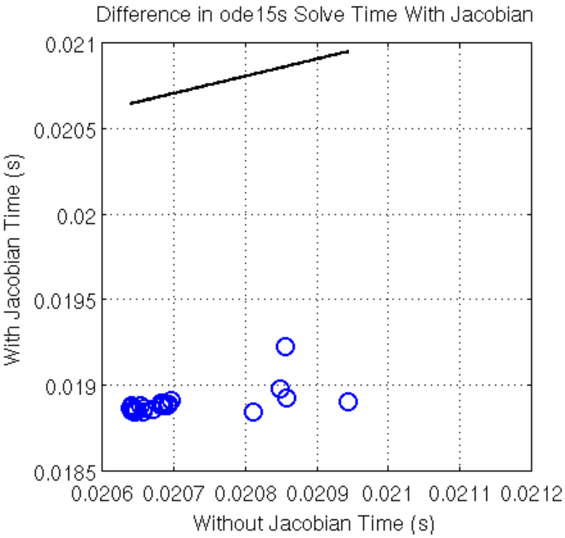
$\rho_2 =$ the proportion of people who die of the disease who are hospitalized [5]

$\omega =$ (time until one is buried)$^{-1}$

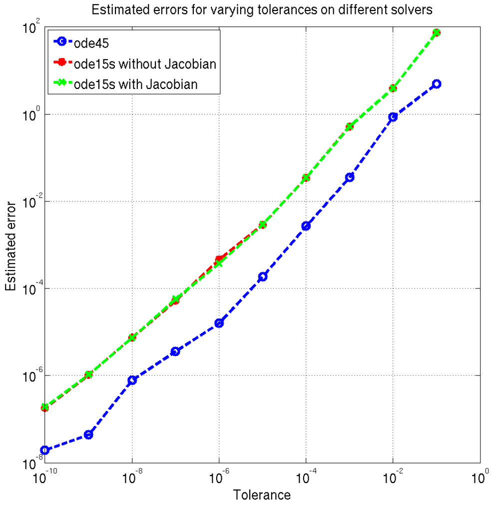
The Model

$$
\begin{aligned}
\frac{dS}{dt} &= \alpha S - \beta_1 SI - \beta_2 SR_I - \beta_3 SH \\
\frac{dE}{dt} &= \beta_1 SI + \beta_2 SR_I + \beta_3 SH - \delta E \\
\frac{dI}{dt} &= \delta E - \gamma_1 I - \psi I \\
\frac{dH}{dt} &= \psi I - \gamma_2 H \\
\frac{dR_I}{dt} &= \rho_1 \gamma_1 I - \omega R_I \\
\frac{dR_B}{dt} &= \omega R_I + \rho_2 \gamma_2 H \\
\frac{dR_R}{dt} &= (1 - \rho_1)\gamma_1 I + (1 - \rho_2)\gamma_2 H
\end{aligned}
$$

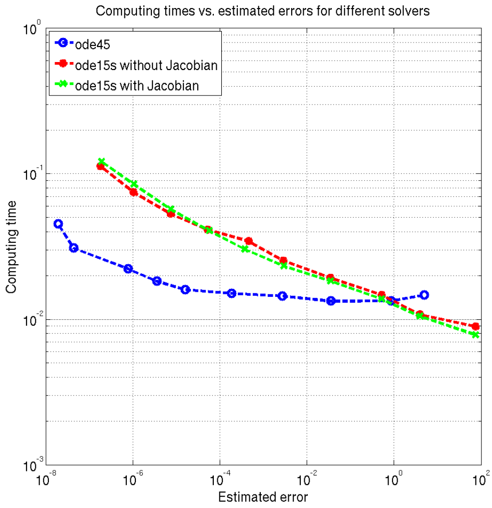
Speed Test


Difference in ode15s Solve Time With Jacobian
With Jacobian Time (s)
0.021
0.0205
0.02
0.0195
0.019
0.0185
0.0206 0.0207 0.0208 0.0209 0.021 0.0211 0.0212
Without Jacobian Time (s)

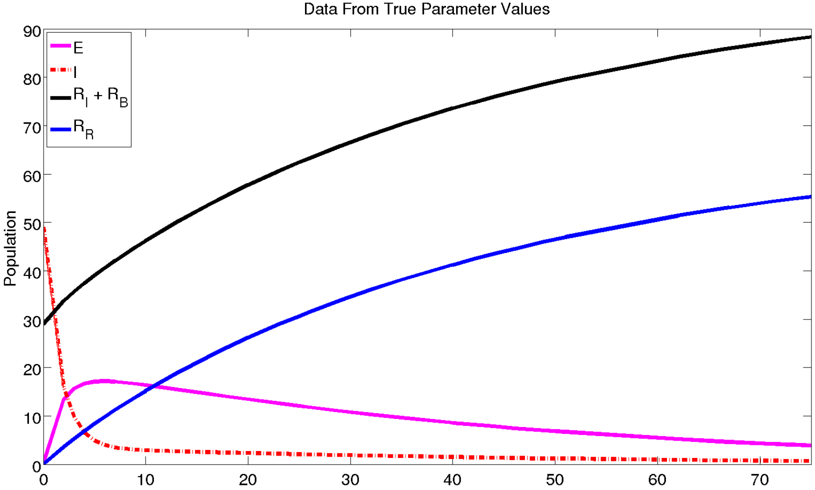
Errors for varying tolerances



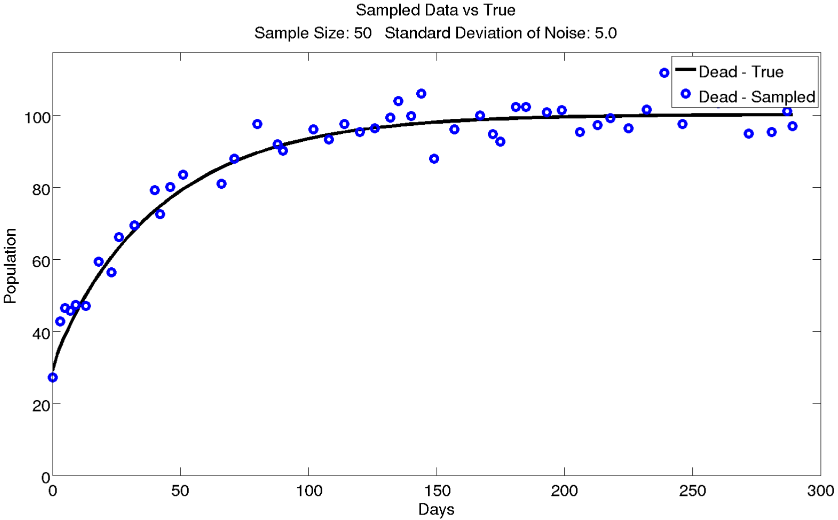
Times to compute and resulting errors



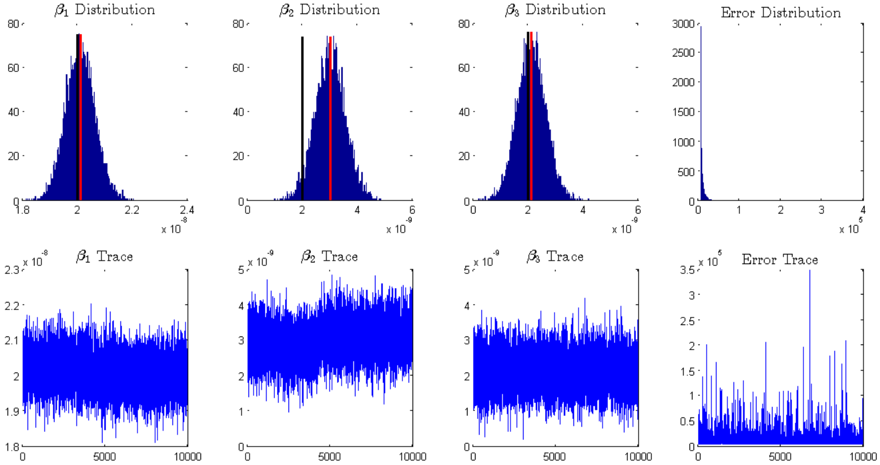
Model Simulation



Sensitivity in the Inverse Problem



MCMC


β_1 Distribution
β_2 Distribution
β_3 Distribution
Error Distribution
β_1 Trace
β_2 Trace
β_3 Trace
Error Trace
x 10^-8
x 10^-9
x 10^5

Results

Sample Size	Noise $\sim N(0, s^2)$	$\bar{\beta_1}$	$\bar{\beta_2}$	$\bar{\beta_3}$	$SD(\hat{\beta_1})$	$SD(\hat{\beta_2})$	$SD(\hat{\beta_3})$
25	0	1.83e-8	5.49e-9	2.16e-9	4.98e-10	5.04e-10	4.97e-10
50	0	1.99e-8	2.01e-9	2.00e-9	5.17e-10	5.60e-10	4.99e-10
25	5	2.00e-8	7.02e-9	1.82e-9	5.28e-10	6.16e-10	5.00e-10
50	5	1.55e-8	4.40e-9	2.59e-9	5.01e-10	5.04e-10	4.96e-10
25	10	na	na	na	na	na	na
50	10	2.01e-8	2.99e-9	2.11e-9	5.077e-10	5.23e-10	5.01e-10

Table 1: Results of MCMC for different sample sizes and different random noise

Questions?

References

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