0^{th} order model for competing microbes

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Background

- Derived from the plankton-rotifer trait-based predator-prey model (`http://dx.doi.org/10.1016/j.ecolmodel.2009.05.005`) by Agostino Merico
- n and m types of microbes A and B; each of the A-type microbes share the same death rate μ_A, but has a stochastic growth rate which provides variety within a given microbe
- In addition, of the $1 - \kappa$ energy available for the microbe, a proportion δ_{A_i} goes to defense and α_{A_i} goes to nutrient allocation
- This model assumes that the microbes compete for nutrients, but otherwise do not interact with each other (hence a 0^{th} order model)
- When microbes die, we assume their biomass is returned into the existing nutrients, N
- Next steps: fit the model to data (***which constants?***), add in intermicrobe interaction, add in immune system interaction

Model

$$\begin{aligned}
\frac{\mathrm{d}A_1}{\mathrm{d}t} &= \left[\gamma_A\left(\frac{N}{N+\frac{K_A}{\alpha_{A_1}}}\right)-\mu_A\right]A_1\\
&\vdots\\
\frac{\mathrm{d}A_n}{\mathrm{d}t} &= \left[\gamma_A\left(\frac{N}{N+\frac{K_A}{\alpha_{A_n}}}\right)-\mu_A\right]A_n\\
\frac{\mathrm{d}B_1}{\mathrm{d}t} &= \left[\gamma_B\left(\frac{N}{N+\frac{K_B}{\alpha_{B_i}}}\right)-\mu_B\right]B_1\\
&\vdots\\
\frac{\mathrm{d}B_m}{\mathrm{d}t} &= \left[\gamma_B\left(\frac{N}{N+\frac{K_B}{\alpha_{B_m}}}\right)-\mu_B\right]B_m\\
\frac{\mathrm{d}N}{\mathrm{d}t} &= -\sum_i\left(\frac{\mathrm{d}A_i}{\mathrm{d}t}+\frac{\mathrm{d}B_i}{\mathrm{d}t}\right)+\beta(N_0-N)
\end{aligned}$$

Fitting Data

- Initially, we try to fit parameters for just one species of microbe

$$\begin{aligned}
\frac{\mathrm{d}A}{\mathrm{d}t} &= \left[\gamma\left(\frac{N}{N+K}\right)-\mu\right]A\\
\frac{\mathrm{d}N}{\mathrm{d}t} &= -\frac{\mathrm{d}A}{\mathrm{d}t}+\beta(N_0-N)
\end{aligned}$$

- From the experimental design, we should know β and N_0
- We need to fit $\gamma,$ K, and μ from data