Bootstrap

Erik Helmers

February 22, 2023

Contents

1 Théorie

1.1 Syntaxe

$e, \sigma, \kappa \, drule \, e : \sigma \, annotated \, term \, drule \, x \, variable \, drule \, \lambda x \to e \, lambda \, drule \, e \, e' \, application \, drule \, \Pi(x : \sigma).\sigma$

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Contexte

$$
\begin{array}{rcll}
\Gamma & ::= & \epsilon & \text{empty context} \\
& | & \Gamma, x : \tau & \text{adding a variable}
\end{array}
$$

$$
\frac{}{\text{valid}(\epsilon)} \qquad \frac{\text{valid}(\Gamma) \qquad \Gamma \vdash \tau :_\downarrow \star}{\text{valid}(\Gamma, x : \tau)}
$$

1.3 Evaluation

$$
\begin{array}{rcll}
\nu, \tau & ::= & n & \text{neutral term} \\
& | & \lambda x \to \nu & \text{lambda} \\
& | & (\nu, \nu') & \text{tuple} \\
& | & \star & \text{type of types} \\
& | & \Pi(x : \tau).\tau' & \text{dependent function space} \\
& | & \Sigma(x : \tau).\tau' & \text{dependent pair space}
\end{array}
$$

$$
\begin{array}{lll}
n & ::= & x & \text{variable} \\
  & | & n\ \nu & \text{neutral app} \\
  & | & \text{fst}\ n & \text{neutral first projection} \\
  & | & \text{snd}\ n & \text{neutral second projection}
\end{array}
$$

$$
\frac{}{\star \Downarrow \star}\ (\textsc{Star})
\qquad
\frac{}{x \Downarrow x}\ (\textsc{Var})
\qquad
\frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu}\ (\textsc{Ann})
$$

$$
\frac{e \Downarrow \nu}{\lambda x \to e \Downarrow \lambda x \to \nu}\ (\textsc{Lam})
\qquad
\frac{e \Downarrow \nu \qquad e' \Downarrow \nu'}{(e, e') \Downarrow (\nu, \nu')}\ (\textsc{Tuple})
$$

$$
\frac{e \Downarrow \lambda x \to \nu \qquad \nu[x \mapsto e'] \Downarrow \nu'}{e\ e' \Downarrow \nu'}\ (\textsc{App})
\qquad
\frac{e \Downarrow n \qquad e' \Downarrow \nu'}{e\ e' \Downarrow n\ \nu'}\ (\textsc{NApp})
$$

$$
\frac{e \Downarrow (\nu, \nu')}{\text{fst}\ e \Downarrow \nu}\ (\textsc{Fst})
\qquad
\frac{e \Downarrow (\nu, \nu')}{\text{snd}\ e \Downarrow \nu'}\ (\textsc{Snd})
\qquad
\frac{e \Downarrow n}{\text{fst}\ e \Downarrow \text{fst}\ n}\ (\textsc{NFst})
$$

$$
\frac{e \Downarrow n}{\text{snd}\ e \Downarrow \text{snd}\ n}\ (\textsc{NSnd})
\qquad
\frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Pi(x : \sigma).\sigma' \Downarrow \Pi(x : \tau).\tau'}\ (\textsc{Pi})
$$

$$
\frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma).\sigma' \Downarrow \Sigma(x : \tau).\tau'}\ (\textsc{Sigma})
$$

1.4 Typing

In the following, $e :_\uparrow \tau$ is an expression with inferrable type τ while $e :_\downarrow \tau$ is checkable.

$$\frac{\Gamma \vdash x :_\uparrow \tau}{\Gamma \vdash x :_\downarrow \tau} \text{ (CHK)} \qquad\qquad \frac{\Gamma \vdash \sigma :_\downarrow * \quad \sigma \Downarrow \tau \quad \Gamma \vdash e :_\downarrow \tau}{\Gamma \vdash (e : \sigma) :_\uparrow \tau} \text{ (ANN)}$$

$$\frac{}{\Gamma \vdash \star :_\uparrow \star} \text{ (STAR)} \qquad\qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x :_\uparrow \tau} \text{ (VAR)}$$

$$\frac{\Gamma, x : \tau \vdash e :_\downarrow \tau'}{\Gamma \vdash \lambda x \to e :_\downarrow \Pi(x : \tau).\tau'} \text{ (LAM)}$$

$$\frac{\Gamma \vdash e :_\downarrow \tau \quad \Gamma \vdash e' :_\downarrow \tau'}{\Gamma \vdash (e, e') :_\downarrow \Sigma((: x).: \tau).\tau'} \text{ (TUPLE)}$$

$$\frac{\Gamma \vdash e :_\uparrow \Pi(x : \tau).\tau' \quad \Gamma \vdash e' :_\downarrow \tau \quad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e\ e' :_\uparrow \tau''} \text{ (APP)}$$

$$\frac{\Gamma \vdash e :_\uparrow \Sigma(x : \tau).\tau'}{\Gamma \vdash \text{fst } e :_\uparrow \tau} \text{ (FST)}$$

$$\frac{\Gamma \vdash e :_\uparrow \Sigma(x : \tau).\tau' \quad \tau'[x \mapsto \text{fst } e] \Downarrow \tau''}{\Gamma \vdash \text{snd } e :_\uparrow \tau''} \text{ (SND)}$$

$$\frac{\Gamma \vdash \sigma :_\downarrow \star \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' :_\downarrow \star}{\Gamma \vdash \Pi(x : \sigma).\sigma' :_\uparrow \star} \text{ (PI)}$$

$$\frac{\Gamma \vdash \sigma :_\downarrow \star \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' :_\downarrow \star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' :_\uparrow \star} \text{ (SIGMA)}$$

Une reformulation équivalentes des règles (PI) et (SIGMA), plus adaptée pour l'implémentation :

$$\frac{\Gamma \vdash \sigma :_\downarrow \star \quad \sigma \Downarrow \tau \quad \Gamma \vdash \sigma' :_\downarrow \Pi(x : \tau).\star}{\Gamma \vdash \Pi(x : \sigma).\sigma' :_\uparrow \star} \text{ (PI)}$$

$$\frac{\Gamma \vdash \sigma :_\downarrow \star \quad \sigma \Downarrow \tau \quad \Gamma \vdash \sigma' :_\downarrow \Pi(x : \tau).\star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' :_\uparrow \star} \text{ (SIGMA)}$$