Bootstrap

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Contents

1 Minimal calculus

1.1 Syntax

e, σ, κ	$::=$	$e : \sigma$	annotated term
	$\mid$	x	variable
	$\mid$	$\lambda x \mapsto e$	lambda
	$\mid$	$e\ e'$	application
	$\mid$	$\Pi(x : \sigma).\sigma'$	pi type
	$\mid$	(e, e')	tuple
	$\mid$	fst e	fst
	$\mid$	snd e	snd
	$\mid$	$\Sigma(x : \sigma).\sigma'$	sigma type
	$\mid$	$\star$	type of types

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Context

$$\begin{array}{rcll} \Gamma & ::= & \epsilon & \text{empty context} \\ & | & \Gamma, x : \tau & \text{adding a variable} \end{array}$$

$$\frac{}{\text{valid}(\epsilon)} \qquad \frac{\text{valid}(\Gamma) \qquad \Gamma \vdash \tau \Leftarrow \star}{\text{valid}(\Gamma, x : \tau)}$$

1.3 Evaluation

$$\begin{array}{rcll} \nu, \tau & ::= & n & \text{neutral term} \\ & | & \lambda x \mapsto \nu & \text{lambda} \\ & | & \Pi(x : \tau).\tau' & \text{dependent function space} \\ & | & (\nu, \nu') & \text{tuple} \\ & | & \Sigma(x : \tau).\tau' & \text{dependent pair space} \\ & | & \star & \text{type of types} \end{array}$$

$$\begin{array}{rcll} n & ::= & x & \text{variable} \\ & | & n\ \nu & \text{neutral app} \\ & | & \text{fst}\ n & \text{neutral first projection} \\ & | & \text{snd}\ n & \text{neutral second projection} \end{array}$$

$$\frac{}{\star \Downarrow \star}\ (\textsc{Star}) \qquad \frac{}{x \Downarrow x}\ (\textsc{Var}) \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu}\ (\textsc{Ann})$$

$$\frac{e \Downarrow \nu}{\lambda x \mapsto e \Downarrow \lambda x \mapsto \nu}\ (\textsc{Lam}) \qquad \frac{e \Downarrow \nu \qquad e' \Downarrow \nu'}{(e, e')(\Downarrow, \nu)\nu'}\ (\textsc{Tuple})$$

$$\frac{e \Downarrow \lambda x \mapsto \nu \qquad \nu[x \mapsto e'] \Downarrow \nu'}{e\ e' \Downarrow \nu'}\ (\textsc{App}) \qquad \frac{e \Downarrow n \qquad e' \Downarrow \nu'}{e\ e' \Downarrow n\ \nu'}\ (\textsc{NApp})$$

$$\frac{e \Downarrow (\nu, \nu')}{\text{fst}\ e \Downarrow \nu}\ (\textsc{Fst}) \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd}\ e \Downarrow \nu'}\ (\textsc{Snd}) \qquad \frac{e \Downarrow n}{\text{fst}\ e \Downarrow \text{fst}\ n}\ (\textsc{NFst})$$

$$\frac{e \Downarrow n}{\text{snd}\ e \Downarrow \text{snd}\ n}\ (\textsc{NSnd}) \qquad \frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Pi(x : \sigma).\sigma' \Downarrow \Pi(x : \tau).\tau'}\ (\textsc{Pi})$$

$$\frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma).\sigma' \Downarrow \Sigma(x : \tau).\tau'}\ (\textsc{Sigma})$$

1.4 Typing

In the following, $e \Rightarrow \tau$ is an expression whose type synthezises to τ while $e \Leftarrow \tau$ is checkable.

$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash e \Leftarrow \tau}\ (\textsc{Chk}) \qquad \frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash (e : \sigma) \Rightarrow \tau}\ (\textsc{Ann})$$

$$\frac{}{\Gamma \vdash \star \Leftarrow \star}\ (\textsc{Star}) \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau}\ (\textsc{Var})$$

$$\frac{\Gamma, x : \tau \vdash e \Leftarrow \tau'}{\Gamma \vdash \lambda x \mapsto e \Leftarrow \Pi(x : \tau).\tau'}\ (\textsc{Lam})$$

$$\frac{\Gamma \vdash e \Leftarrow \tau \qquad \Gamma \vdash e' \Leftarrow \tau'}{\Gamma \vdash (e, e') \Leftarrow \Sigma(x : \tau).\tau'}\ (\textsc{Tuple})$$

$$\frac{\Gamma \vdash e \Rightarrow \Pi(x : \tau).\tau' \qquad \Gamma \vdash e' \Leftarrow \tau \qquad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e\ e' \Rightarrow \tau''}\ (\textsc{App})$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau).\tau'}{\Gamma \vdash \text{fst}\ e \Rightarrow \tau}\ (\textsc{Fst})$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau).\tau' \qquad \tau'[x \mapsto \text{fst}\ e] \Downarrow \tau''}{\Gamma \vdash \text{snd}\ e \Rightarrow \tau''}\ (\textsc{Snd})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash \Pi(x : \sigma).\sigma' \Leftarrow \star}\ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' \Leftarrow \star}\ (\textsc{Sigma})$$

2 Interlude : booleans

2.1 Syntax

$$
\begin{array}{rcll}
e, \sigma, \kappa & ::= & ... & \\
& | & \text{true} & \\
& | & \text{false} & \\
& | & \text{cond } e\ [x.\sigma]\ e'\ e'' & \text{condition} \\
& | & \text{bool} & \text{type of a bool}
\end{array}
$$

2.2 Evaluation

$$
\begin{array}{rcll}
\nu, \tau & ::= & ... & \\
& | & \text{true} & \\
& | & \text{false} & \\
& | & \text{bool} &
\end{array}
$$

$$
\begin{array}{rcll}
n & ::= & ... & \\
& | & \text{cond } \nu\ [x.\tau]\ \nu'\ \nu'' &
\end{array}
$$

$$\frac{}{\text{true} \Downarrow \text{true}}\ (\textsc{True}) \qquad \frac{}{\text{false} \Downarrow \text{false}}\ (\textsc{False})$$

$$\frac{e \Downarrow \text{true} \qquad e' \Downarrow \nu}{\text{cond } e\ [x.B]\ e'\ e'' \Downarrow \nu}\ (\textsc{CondT}) \qquad \frac{e \Downarrow \text{true} \qquad e'' \Downarrow \nu}{\text{cond } e\ [x.B]\ e'\ e'' \Downarrow \nu}\ (\textsc{CondF})$$

$$\frac{e \Downarrow n \qquad e' \Downarrow \nu \qquad e'' \Downarrow \nu'}{\text{cond } e\ [x.B]\ e'\ e'' \Downarrow \text{cond } n\ [x.\tau]\ \nu\ \nu'}\ (\text{NCond})$$

$$\frac{}{\text{bool} \Downarrow \text{bool}}\ (\textsc{BoolTy})$$

2.3 Typing

$$\frac{}{\text{true} \Leftarrow \text{bool}}\ (\textsc{True}) \qquad \frac{}{\text{false} \Leftarrow \text{bool}}\ (\textsc{False})$$

$$\frac{\Gamma \vdash e \Leftarrow \text{bool} \qquad \Gamma, x : \text{bool} \vdash B \Leftarrow \star \qquad B[x \mapsto e] \Downarrow \tau}{\Gamma \vdash \text{cond } e\ [x.B]\ e'\ e'' \Rightarrow \tau}\ (\textsc{Cond})$$

$$\frac{}{\text{bool} \Leftarrow \star}\ (\textsc{BoolTy})$$