Bootstrap

Erik Helmers

March 16, 2023

Contents

1 Minimal calculus

1.1 Syntax

$$
\begin{array}{llll}
e, \sigma, \kappa & ::= & e : \sigma & \text{annotated term} \\
& | & x & \text{variable} \\
& | & \lambda x \mapsto e & \text{lambda} \\
& | & e\ e' & \text{application} \\
& | & (x : \sigma) \to \sigma' & \text{pi type} \\
& | & (e, e') & \text{tuple} \\
& | & \text{fst } e & \text{fst} \\
& | & \text{snd } e & \text{snd} \\
& | & (x : \sigma) \times \sigma' & \text{sigma type} \\
& | & \star & \text{type of types}
\end{array}
$$

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Context

$$
\begin{array}{llll}
\Gamma & ::= & \epsilon & \text{empty context} \\
& | & \Gamma, x : \tau & \text{adding a variable}
\end{array}
$$

$$
\frac{}{\text{valid}(\epsilon)} \qquad \frac{\text{valid}(\Gamma) \qquad \Gamma \vdash \tau \Leftarrow \star}{\text{valid}(\Gamma, x : \tau)}
$$

1.3 Evaluation

$$
\begin{array}{llll}
\nu, \tau & ::= & n & \text{neutral term} \\
& | & \lambda x \mapsto \nu & \text{lambda} \\
& | & (x : \tau) \to \tau' & \text{dependent function space} \\
& | & (\nu, \nu') & \text{tuple} \\
& | & (x : \tau) \times \tau' & \text{dependent pair space} \\
& | & \star & \text{type of types}
\end{array}
$$

$$
\begin{array}{llll}
n & ::= & x & \text{variable} \\
& | & n\ \nu & \text{neutral app} \\
& | & \text{fst } n & \text{neutral first projection} \\
& | & \text{snd } n & \text{neutral second projection}
\end{array}
$$

$$\frac{}{\star \Downarrow \star} \text{ (STAR)} \qquad \frac{}{x \Downarrow x} \text{ (VAR)} \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu} \text{ (ANN)}$$

$$\frac{e \Downarrow \nu}{\lambda x \mapsto e \Downarrow \lambda x \mapsto \nu} \text{ (LAM)} \qquad \frac{e \Downarrow \nu \quad e' \Downarrow \nu'}{(e, e')(\Downarrow, \nu)\nu'} \text{ (TUPLE)}$$

$$\frac{e \Downarrow \lambda x \mapsto \nu \quad \nu[x \mapsto e'] \Downarrow \nu'}{e \; e' \Downarrow \nu'} \text{ (APP)} \qquad \frac{e \Downarrow n \quad e' \Downarrow \nu'}{e \; e' \Downarrow n \; \nu'} \text{ (NAPP)}$$

$$\frac{e \Downarrow (\nu, \nu')}{\text{fst } e \Downarrow \nu} \text{ (FST)} \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd } e \Downarrow \nu'} \text{ (SND)} \qquad \frac{e \Downarrow n}{\text{fst } e \Downarrow \text{fst } n} \text{ (NFST)}$$

$$\frac{e \Downarrow n}{\text{snd } e \Downarrow \text{snd } n} \text{ (NSND)} \qquad \frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{(x : \sigma) \to \sigma' \Downarrow (x : \tau) \to \tau'} \text{ (PI)}$$

$$\frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{(x : \sigma) \times \sigma' \Downarrow (x : \tau) \times \tau'} \text{ (SIGMA)}$$

1.4 Typing

In the following, $e \Rightarrow \tau$ is an expression whose type synthezises to τ while $e \Leftarrow \tau$ is checkable.

$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash e \Leftarrow \tau} \ (\text{CHK})
\qquad
\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash (e : \sigma) \Rightarrow \tau} \ (\text{ANN})$$

$$\frac{}{\Gamma \vdash \star \Leftarrow \star} \ (\text{STAR})
\qquad
\frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau} \ (\text{VAR})$$

$$\frac{\Gamma, x : \tau \vdash e \Leftarrow \tau'}{\Gamma \vdash \lambda x \mapsto e \Leftarrow (x : \tau) \to \tau'} \ (\text{LAM})$$

$$\frac{\Gamma \vdash e \Leftarrow \tau \quad \Gamma \vdash e' \Leftarrow \tau'}{\Gamma \vdash (e, e') \Leftarrow (x : \tau) \times \tau'} \ (\text{TUPLE})$$

$$\frac{\Gamma \vdash e \Rightarrow (x : \tau) \to \tau' \quad \Gamma \vdash e' \Leftarrow \tau \quad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e \ e' \Rightarrow \tau''} \ (\text{APP})$$

$$\frac{\Gamma \vdash e \Rightarrow (x : \tau) \times \tau'}{\Gamma \vdash \text{fst } e \Rightarrow \tau} \ (\text{FST})$$

$$\frac{\Gamma \vdash e \Rightarrow (x : \tau) \times \tau' \quad \tau'[x \mapsto \text{fst } e] \Downarrow \tau''}{\Gamma \vdash \text{snd } e \Rightarrow \tau''} \ (\text{SND})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash (x : \sigma) \to \sigma' \Leftarrow \star} \ (\text{PI})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash (x : \sigma) \times \sigma' \Leftarrow \star} \ (\text{SIGMA})$$

2 Interlude : booleans

2.1 Syntax

$$
\begin{array}{lll}
e, \sigma, \kappa \quad ::= & ... \\
& \mid & \text{true} \\
& \mid & \text{false} \\
& \mid & \text{cond } e \ [x.\sigma] \ e' \ e'' \quad \text{condition} \\
& \mid & \text{boolty} \quad\quad\quad\quad\quad \text{type of a bool}
\end{array}
$$

2.2 Evaluation

$$
\begin{aligned}
\nu, \tau \quad ::= \quad & \ldots \\
| \quad & \text{true} \\
| \quad & \text{false} \\
| \quad & \text{bool}
\end{aligned}
$$

$$
\begin{aligned}
n \quad ::= \quad & \ldots \\
| \quad & \text{cond } \nu \; [x.\tau] \; \nu' \; \nu''
\end{aligned}
$$

$$
\frac{}{\text{true} \Downarrow \text{true}} \; (\textsc{True})
\qquad\qquad
\frac{}{\text{false} \Downarrow \text{false}} \; (\textsc{False})
$$

$$
\frac{e \Downarrow \text{true} \qquad e' \Downarrow \nu}{\text{cond } e \; [x.B] \; e' \; e'' \Downarrow \nu} \; (\textsc{CondT})
\qquad\qquad
\frac{e \Downarrow \text{true} \qquad e'' \Downarrow \nu}{\text{cond } e \; [x.B] \; e' \; e'' \Downarrow \nu} \; (\textsc{CondF})
$$

$$
\frac{e \Downarrow n \qquad e' \Downarrow \nu \qquad e'' \Downarrow \nu'}{\text{cond } e \; [x.B] \; e' \; e'' \Downarrow \text{cond } n \; [x.\tau] \; \nu \; \nu'} \; (\textsc{NCond})
$$

$$
\frac{}{\text{boolty} \Downarrow \text{bool}} \; (\textsc{BoolTy})
$$

2.3 Typing

$$
\frac{}{\text{true} \Leftarrow \text{bool}} \; (\textsc{True})
\qquad\qquad
\frac{}{\text{false} \Leftarrow \text{bool}} \; (\textsc{False})
$$

$$
\frac{\Gamma \vdash e \Leftarrow \text{bool} \qquad \Gamma, x : \text{bool} \vdash B \Leftarrow \star \qquad B[x \mapsto e] \Downarrow \tau}{\Gamma \vdash \text{cond } e \; [x.B] \; e' \; e'' \Rightarrow \tau} \; (\textsc{Cond})
$$

$$
\frac{}{\text{boolty} \Leftarrow \star} \; (\textsc{BoolTy})
$$

3 Enumerations

3.1 Syntax

$$
\begin{array}{lll}
e, \sigma, \kappa & ::= & \ldots \\
& | & \text{nil} \\
& | & \text{unit} \\
& | & 't \qquad \text{label} \\
& | & \text{label} \qquad \text{label type} \\
& | & \text{nil} \\
& | & [t\ l] \\
& | & \text{labels} \qquad \text{labels type}
\end{array}
$$

3.2 Evaluation

$$
\frac{l \Downarrow \text{nil}}{\text{record } l \text{ as } x \text{ return } B \Downarrow \text{unit}} \ (\text{RecordNil})
$$

$$
\frac{l \Downarrow [t\ l'] \qquad B[x \mapsto 0] \Downarrow \tau \qquad \text{record } l' \text{ as } x \text{ return } B[x \mapsto 1+\ x] \Downarrow \tau'}{\text{record } l \text{ as } x \text{ return } B \Downarrow (\_ : \tau) \times \tau'} \ (\text{RecordCons})
$$

$$
\frac{l \Downarrow n}{\text{record } l \text{ as } x \text{ return } B \Downarrow \text{record } n \text{ as } x \text{ return } B} \ (\text{NRecord})
$$

$$
\frac{e \Downarrow 0 \qquad \text{fst cs} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \nu} \ (\text{CaseZe})
$$

$$
\frac{\text{snd cs} \Downarrow \text{cs'} \qquad e \Downarrow 1+\ e' \qquad \text{case } e' \text{ as } x \text{ return } B[x \mapsto 1+\ x] \text{ with cs'} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \nu} \ (\text{CaseSuc})
$$

$$
\frac{e \Downarrow n \qquad \text{cs} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \text{case } n \text{ as } x \text{ return } B \text{ with } \nu} \ (\text{CaseZe})
$$

3.3 Typing

$$\frac{}{\text{nil} \Leftarrow \text{unit}} \; (\text{Nil}) \qquad\qquad \frac{}{\text{unit} \Leftarrow \star} \; (\text{Unit})$$

$$\frac{}{{}'t \; \Leftarrow \text{label}} \; (\text{Label}) \qquad\qquad \frac{}{\text{label} \Leftarrow \star} \; (\text{LabelTy})$$

$$\frac{}{\text{nil} \Leftarrow \text{labels}} \; (\text{NilL}) \qquad \frac{\Gamma \vdash t \Leftarrow \text{label} \quad \Gamma \vdash l \Leftarrow \text{labels}}{[t\ l] \Leftarrow \text{labels}} \; (\text{ConsL})$$

$$\frac{}{\text{labels} \Leftarrow \star} \; (\text{LabelsTy})$$

$$\frac{\Gamma \vdash t \Leftarrow \text{label} \quad \Gamma \vdash l \Leftarrow \text{labels}}{\Gamma \vdash 0 \Leftarrow \text{enum} \; [t\ l]} \; (\text{Zero})$$

$$\frac{\Gamma \vdash t \Leftarrow \text{label} \quad \Gamma \vdash l \Leftarrow \text{labels} \quad \Gamma \vdash n \Leftarrow \text{enum} \; l}{\Gamma \vdash 1+ n \Leftarrow \text{enum} \; [t\ l]} \; (\text{Suc})$$

$$\frac{\Gamma \vdash l \Leftarrow \text{labels}}{\Gamma \vdash \text{enum} \; l \Leftarrow \star} \; (\text{Enum})$$

$$\frac{\Gamma \vdash l \Leftarrow \text{labels} \quad \Gamma, x : \text{enum} \; l \vdash B \Leftarrow \star}{\Gamma \vdash \text{record} \; l \; \text{as} \; x \; \text{return} \; B \Rightarrow \star} \; (\text{Record})$$

$$\frac{\begin{array}{c} \Gamma \vdash e \Leftarrow \text{enum} \; l \quad \Gamma, x : \text{enum} \; l \vdash B \Leftarrow \star \\ B[x \mapsto e] \Downarrow \tau \quad \Gamma \vdash \text{cs} \Leftarrow \text{record} \; l \; \text{as} \; x \; \text{return} \; B \end{array}}{\Gamma \vdash \text{case} \; e \; \text{as} \; x \; \text{return} \; B \; \text{with} \; \text{cs} \Rightarrow \tau} \; (\text{Case})$$