Bootstrap

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Contents

1 Théorie

1.1 Syntaxe

$$
\begin{array}{rcll}
e, \sigma, \kappa & ::= & e : \sigma & \text{annotated term} \\
& \mid & x & \text{variable} \\
& \mid & \lambda x \to e & \text{lambda} \\
& \mid & e\ e' & \text{application} \\
& \mid & \Pi(x : \sigma).\sigma' & \text{pi type} \\
& \mid & (e, e') & \text{tuple} \\
& \mid & \text{fst } e & \text{fst} \\
& \mid & \text{snd } e & \text{snd} \\
& \mid & \Sigma(x : \sigma).\sigma' & \text{sigma type} \\
& \mid & \star & \text{type of types}
\end{array}
$$

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Contexte

$$
\begin{array}{rcll}
\Gamma & ::= & \epsilon & \text{empty context} \\
& \mid & \Gamma, x : \tau & \text{adding a variable}
\end{array}
$$

$$\frac{}{\text{valid}(\epsilon)} \qquad \frac{\text{valid}(\Gamma) \qquad \Gamma \vdash \tau :_{\downarrow} \star}{\text{valid}(\Gamma, x : \tau)}$$

1.3 Evaluation

$$\begin{array}{lcll} \nu, \tau & ::= & n & \text{neutral term} \\ & | & \lambda x \rightarrow \nu & \text{lambda} \\ & | & (\nu, \nu') & \text{tuple} \\ & | & \star & \text{type of types} \\ & | & \Pi(x : \tau).\tau' & \text{dependent function space} \\ & | & \Sigma(x : \tau).\tau' & \text{dependent pair space} \end{array}$$

$$\frac{}{\star \Downarrow \star}\ (\textsc{Star}) \qquad \frac{}{x \Downarrow x}\ (\textsc{Var}) \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu}\ (\textsc{Ann})$$

$$\frac{e \Downarrow \nu}{\lambda x \rightarrow e \Downarrow \lambda x \rightarrow \nu}\ (\textsc{Lam}) \qquad \frac{e \Downarrow \nu \qquad e' \Downarrow \nu'}{(e, e') \Downarrow (\nu, \nu')}\ (\textsc{Tuple})$$

$$\frac{e \Downarrow \lambda x \rightarrow \nu \qquad \nu[x \mapsto e'] \Downarrow \nu'}{e\ e' \Downarrow \nu'}\ (\textsc{App}) \qquad \frac{e \Downarrow n \qquad e' \Downarrow \nu'}{e\ e' \Downarrow n\ \nu'}\ (\text{N}\textsc{App})$$

$$\frac{e \Downarrow (\nu, \nu')}{\text{fst } e \Downarrow \nu}\ (\textsc{Fst}) \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd } e \Downarrow \nu'}\ (\textsc{Snd})$$

$$\frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Pi(x : \sigma).\sigma' \Downarrow \Pi(x : \tau).\tau'}\ (\textsc{Pi}) \qquad \frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma).\sigma' \Downarrow \Sigma(x : \tau).\tau'}\ (\textsc{Sigma})$$

1.4 Typing

In the following, $e :_{\uparrow} \tau$ is an expression with inferrable type τ while $e :_{\downarrow} \tau$ is checkable.

$$\frac{\Gamma \vdash x :_{\uparrow} \tau}{\Gamma \vdash x :_{\downarrow} \tau}\ (\textsc{Chk}) \qquad \frac{\Gamma \vdash \sigma :_{\downarrow} * \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash e :_{\downarrow} \tau}{\Gamma \vdash (e : \sigma) :_{\uparrow} \tau}\ (\textsc{Ann})$$

$$\frac{}{\Gamma \vdash \star :_{\uparrow} \star}\ (\textsc{Star}) \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x :_{\uparrow} \tau}\ (\textsc{Var})$$

$$\frac{\Gamma, x : \tau \vdash e :_{\downarrow} \tau'}{\Gamma \vdash \lambda x \to e :_{\downarrow} \Pi(x : \tau).\tau'}\ (\textsc{Lam})$$

$$\frac{\Gamma \vdash e :_{\downarrow} \tau \qquad \Gamma \vdash e' :_{\downarrow} \tau'}{\Gamma \vdash (e, e') :_{\downarrow} \Sigma((: x). : \tau).\tau'}\ (\textsc{Tuple})$$

$$\frac{\Gamma \vdash e :_{\uparrow} \Pi(x : \tau).\tau' \qquad \Gamma \vdash e' :_{\downarrow} \tau \qquad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e\ e' :_{\uparrow} \tau''}\ (\textsc{App})$$

$$\frac{\Gamma \vdash e :_{\uparrow} \Sigma(x : \tau).\tau'}{\Gamma \vdash \mathrm{fst}\ e :_{\uparrow} \tau}\ (\textsc{Fst})$$

$$\frac{\Gamma \vdash e :_{\uparrow} \Sigma(x : \tau).\tau' \qquad \tau'[x \mapsto \mathrm{fst}\ e] \Downarrow \tau''}{\Gamma \vdash \mathrm{snd}\ e :_{\uparrow} \tau''}\ (\textsc{Snd})$$

$$\frac{\Gamma \vdash \sigma :_{\downarrow} \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' :_{\downarrow} \star}{\Gamma \vdash \Pi(x : \sigma).\sigma' :_{\uparrow} \star}\ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \sigma :_{\downarrow} \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' :_{\downarrow} \star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' :_{\uparrow} \star}\ (\textsc{Sigma})$$

Une reformulation équivalentes des règles (Pi) et (Sigma), plus adaptée pour l'implémentation :

$$\frac{\Gamma \vdash \sigma :_{\downarrow} \star \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash \sigma' :_{\downarrow} \Pi(x : \tau).\star}{\Gamma \vdash \Pi(x : \sigma).\sigma' :_{\uparrow} \star}\ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \sigma :_{\downarrow} \star \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash \sigma' :_{\downarrow} \Pi(x : \tau).\star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' :_{\uparrow} \star}\ (\textsc{Sigma})$$