

# Bootstrap

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## 1 Théorie

### 1.1 Syntaxe

$e, \sigma, \kappa$  *drulee* :  $\sigma$  *annotated term* *drule*  $x$  *variable* *drule*  $\lambda x \rightarrow e$  *lambda* *drule*  $e' \text{ application}$  *drule*  $\Pi(x : \sigma). \sigma$

where  $e, \sigma, \kappa$  represent general expressions, types and kinds respectively.

### 1.2 Contexte

$\Gamma$	$::=$	$\epsilon$	empty context
		$\Gamma, x : \tau$	adding a variable

$\frac{}{\text{valid}(\epsilon)}$	$\frac{\text{valid}(\Gamma) \quad \Gamma \vdash \tau : \downarrow \star}{\text{valid}(\Gamma, x : \tau)}$
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### 1.3 Evaluation

$\nu, \tau$	$::=$	$n$	neutral term
		$\lambda x \rightarrow \nu$	lambda
		$(\nu, \nu')$	tuple
		$\star$	type of types
		$\Pi(x : \tau). \tau'$	dependent function space
		$\Sigma(x : \tau). \tau'$	dependent pair space

$$\begin{array}{c}
\frac{}{\star \Downarrow \star} \text{ (STAR)} \qquad \frac{}{x \Downarrow x} \text{ (VAR)} \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu} \text{ (ANN)} \\
\\
\frac{e \Downarrow \nu}{\lambda x \rightarrow e \Downarrow \lambda x \rightarrow \nu} \text{ (LAM)} \qquad \frac{e \Downarrow \nu \quad e' \Downarrow \nu'}{(e, e') \Downarrow (\nu, \nu')} \text{ (TUPLE)} \\
\\
\frac{e \Downarrow \lambda x \rightarrow \nu \quad \nu[x \mapsto e'] \Downarrow \nu'}{e \Downarrow e' \Downarrow \nu'} \text{ (APP)} \qquad \frac{e \Downarrow n \quad e' \Downarrow \nu'}{e \Downarrow e' \Downarrow n \Downarrow \nu'} \text{ (NAPP)} \\
\\
\frac{e \Downarrow (\nu, \nu')}{\text{fst } e \Downarrow \nu} \text{ (FST)} \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd } e \Downarrow \nu'} \text{ (SND)} \\
\\
\frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{\Pi(x : \sigma). \sigma' \Downarrow \Pi(x : \tau). \tau'} \text{ (PI)} \qquad \frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma). \sigma' \Downarrow \Sigma(x : \tau). \tau'} \text{ (SIGMA)}
\end{array}$$

## 1.4 Typing

In the following,  $e :_{\uparrow} \tau$  is an expression with inferrable type  $\tau$  while  $e :_{\downarrow} \tau$  is checkable.

$$\begin{array}{c}
\frac{\Gamma \vdash x :_{\uparrow} \tau}{\Gamma \vdash x :_{\downarrow} \tau} \text{ (CHK)} \qquad \frac{\Gamma \vdash \sigma :_{\downarrow} * \quad \sigma \Downarrow \tau \quad \Gamma \vdash e :_{\downarrow} \tau}{\Gamma \vdash (e : \sigma) :_{\uparrow} \tau} \text{ (ANN)} \\
\\
\frac{}{\Gamma \vdash \star :_{\uparrow} \star} \text{ (STAR)} \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x :_{\uparrow} \tau} \text{ (VAR)} \\
\\
\frac{\Gamma, x : \tau \vdash e :_{\downarrow} \tau'}{\Gamma \vdash \lambda x \rightarrow e :_{\downarrow} \Pi(x : \tau). \tau'} \text{ (LAM)} \\
\\
\frac{\Gamma \vdash e :_{\downarrow} \tau \quad \Gamma \vdash e' :_{\downarrow} \tau'}{\Gamma \vdash (e, e') :_{\downarrow} \Sigma((: x). : \tau). \tau'} \text{ (TUPLE)} \\
\\
\frac{\Gamma \vdash e :_{\uparrow} \Pi(x : \tau). \tau' \quad \Gamma \vdash e' :_{\downarrow} \tau \quad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e e' :_{\uparrow} \tau''} \text{ (APP)} \\
\\
\frac{\Gamma \vdash e :_{\uparrow} \Sigma(x : \tau). \tau'}{\Gamma \vdash \text{fst } e :_{\uparrow} \tau} \text{ (FST)} \\
\\
\frac{\Gamma \vdash e :_{\uparrow} \Sigma(x : \tau). \tau' \quad \tau'[x \mapsto \text{fst } e] \Downarrow \tau''}{\Gamma \vdash \text{snd } e :_{\uparrow} \tau''} \text{ (SND)} \\
\\
\frac{\Gamma \vdash \sigma :_{\downarrow} * \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' :_{\downarrow} *}{\Gamma \vdash \Pi(x : \sigma). \sigma' :_{\uparrow} *} \text{ (PI)} \\
\\
\frac{\Gamma \vdash \sigma :_{\downarrow} * \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' :_{\downarrow} *}{\Gamma \vdash \Sigma(x : \sigma). \sigma' :_{\uparrow} *} \text{ (SIGMA)}
\end{array}$$

Une reformulation équivalentes des règles (PI) et (SIGMA), plus adaptée pour l'implémentation :

$$\begin{array}{c}
\frac{\Gamma \vdash \sigma :_{\downarrow} * \quad \sigma \Downarrow \tau \quad \Gamma \vdash \sigma' :_{\downarrow} \Pi(x : \tau). *}{\Gamma \vdash \Pi(x : \sigma). \sigma' :_{\uparrow} *} \text{ (PI)} \\
\\
\frac{\Gamma \vdash \sigma :_{\downarrow} * \quad \sigma \Downarrow \tau \quad \Gamma \vdash \sigma' :_{\downarrow} \Pi(x : \tau). *}{\Gamma \vdash \Sigma(x : \sigma). \sigma' :_{\uparrow} *} \text{ (SIGMA)}
\end{array}$$