Bootstrap

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Contents

1 Théorie

1.1 Syntaxe

$$
\begin{array}{rcll}
e, \rho, \kappa & ::= & e : \rho & \text{annotated term} \\
& | & x & \text{variable} \\
& | & \lambda x \to e & \text{lambda} \\
& | & e\ e' & \text{application} \\
& | & \Pi(x : \rho).\rho' & \text{pi type} \\
& | & (e, e') & \text{tuple} \\
& | & \text{fst}\ e & \text{fst} \\
& | & \text{snd}\ e & \text{snd} \\
& | & \Sigma(x : \rho).\rho' & \text{sigma type} \\
& | & * & \text{type of types}
\end{array}
$$

where e, ρ, κ represent general expressions, types and kinds respectively.

1.2 Contexte

$$
\begin{array}{rcll}
\Gamma & ::= & \epsilon & \text{empty context} \\
& | & \Gamma, x : \tau & \text{adding a variable}
\end{array}
$$

$$\frac{}{\mathrm{valid}(\epsilon)} \qquad \frac{\mathrm{valid}(\Gamma) \qquad \Gamma \vdash \tau :_{\downarrow} \star}{\mathrm{valid}(\Gamma, x : \tau)}$$

1.3 Evaluation

$$\begin{array}{rcll}
\nu, \tau & ::= & n & \text{neutral term} \\
& | & \lambda x \to \nu & \text{lambda} \\
& | & (\nu, \nu') & \text{tuple} \\
& | & \star & \text{type of types} \\
& | & \Pi(x : \tau).\tau' & \text{dependent function space} \\
& | & \Sigma(x : \tau).\tau' & \text{dependent pair space}
\end{array}$$

$$\frac{}{\star \Downarrow *}\ (\textsc{Star}) \qquad \frac{}{x \Downarrow x}\ (\textsc{Var}) \qquad \frac{e \Downarrow \nu}{e : \rho \Downarrow \nu}\ (\textsc{Ann})$$

$$\frac{e \Downarrow \nu}{\lambda x \to e \Downarrow \lambda x \to \nu}\ (\textsc{Lam}) \qquad \frac{e \Downarrow \nu \qquad e' \Downarrow \nu'}{(e, e') \Downarrow (\nu, \nu')}\ (\textsc{Tuple})$$

$$\frac{e \Downarrow \lambda x \to \nu \qquad \nu[x \mapsto e'] \Downarrow \nu'}{e\ e' \Downarrow \nu'}\ (\textsc{App}) \qquad \frac{e \Downarrow n \qquad e' \Downarrow \nu'}{e\ e' \Downarrow n\ \nu'}\ (\textsc{NApp})$$

$$\frac{e \Downarrow (\nu, \nu')}{\mathrm{fst}\ e \Downarrow \nu}\ (\textsc{Fst}) \qquad \frac{e \Downarrow (\nu, \nu')}{\mathrm{snd}\ e \Downarrow \nu'}\ (\textsc{Snd})$$

$$\frac{\rho \Downarrow \tau \qquad \rho' \Downarrow \tau'}{\Pi(x : \rho).\rho' \Downarrow \Pi(x : \tau).\tau'}\ (\textsc{Pi}) \qquad \frac{\rho \Downarrow \tau \qquad \rho' \Downarrow \tau'}{\Sigma(x : \rho).\rho' \Downarrow \Sigma(x : \tau).\tau'}\ (\textsc{Sigma})$$

1.4 Typing

In the following, $e :_{\uparrow} \tau$ is an expression with inferrable type τ while $e :_{\downarrow} \tau$ is checkable.

$$\frac{\Gamma \vdash x :_{\uparrow} \tau}{\Gamma \vdash x :_{\downarrow} \tau} \ (\textsc{Chk}) \qquad \frac{\Gamma \vdash \rho :_{\downarrow} * \qquad \rho \Downarrow \tau \qquad \Gamma \vdash e :_{\downarrow} \tau}{\Gamma \vdash (e : \rho) :_{\uparrow} \tau} \ (\textsc{Ann})$$

$$\frac{}{\Gamma \vdash * :_{\uparrow} *} \ (\textsc{Star}) \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash e :_{\uparrow} \tau} \ (\textsc{Var})$$

$$\frac{\Gamma, x : \tau \vdash e :_{\downarrow} \tau'}{\Gamma \vdash \lambda x \to e :_{\downarrow} \Pi(x : \tau)).\tau'} \ (\textsc{Lam})$$

$$\frac{\Gamma \vdash e :_{\downarrow} \tau \qquad \Gamma \vdash e' :_{\downarrow} \tau'}{\Gamma \vdash (e, e') :_{\downarrow} \Sigma(x : \tau).\tau'} \ (\textsc{Tuple})$$

$$\frac{\Gamma \vdash e :_{\uparrow} \Pi(x : \tau).\tau' \qquad \Gamma \vdash e' :_{\downarrow} \tau \qquad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e\ e' :_{\uparrow} \tau''} \ (\textsc{App})$$

$$\frac{\Gamma \vdash e :_{\uparrow} \Sigma(x : \tau).\tau'}{\Gamma \vdash \text{fst } e :_{\uparrow} \tau} \ (\textsc{Fst})$$

$$\frac{\Gamma \vdash e :_{\uparrow} \Sigma(x : \tau).\tau' \qquad \tau'[x \mapsto \text{fst } e] \Downarrow \tau''}{\Gamma \vdash \text{snd } e :_{\uparrow} \tau''} \ (\textsc{Snd})$$

$$\frac{\Gamma \vdash \rho :_{\downarrow} * \qquad \rho \Downarrow \tau \qquad \Gamma, x : \tau \vdash \rho' :_{\downarrow} *}{\Gamma \vdash \Pi(x : \rho).\rho' :_{\uparrow} *} \ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \rho :_{\downarrow} * \qquad \rho \Downarrow \tau \qquad \Gamma, x : \tau \vdash \rho' :_{\downarrow} *}{\Gamma \vdash \Sigma(x : \rho).\rho' :_{\uparrow} *} \ (\textsc{Sigma})$$

Une reformulation équivalentes des règles (Pi) et (Sigma), plus adaptée pour l'implémentation :

$$\frac{\Gamma \vdash \rho :_{\downarrow} * \qquad \rho \Downarrow \tau \qquad \Gamma \vdash \rho' :_{\downarrow} \Pi(x : \tau).*}{\Gamma \vdash \Pi(x : \rho).\rho' :_{\uparrow} *} \ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \rho :_{\downarrow} * \qquad \rho \Downarrow \tau \qquad \Gamma \vdash \rho' :_{\downarrow} \Pi(x : \tau).*}{\Gamma \vdash \Sigma(x : \rho).\rho' :_{\uparrow} *} \ (\textsc{Sigma})$$