Bootstrap

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Contents

1 Minimal calculus

1.1 Syntax

$$
\begin{array}{llll}
e, \sigma, \kappa & ::= & e : \sigma & \text{annotated term} \\
& | & x & \text{variable} \\
& | & \lambda x \mapsto e & \text{lambda} \\
& | & e\ e' & \text{application} \\
& | & \Pi(x : \sigma).\sigma' & \text{pi type} \\
& | & (e, e') & \text{tuple} \\
& | & \mathsf{fst}\ e & \text{fst} \\
& | & \mathsf{snd}\ e & \text{snd} \\
& | & \Sigma(x : \sigma).\sigma' & \text{sigma type} \\
& | & \star & \text{type of types}
\end{array}
$$

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Context

$$
\begin{array}{llll}
\Gamma & ::= & \epsilon & \text{empty context} \\
& | & \Gamma, x : \tau & \text{adding a variable}
\end{array}
$$

$$
\frac{}{\mathrm{valid}(\epsilon)} \qquad \frac{\mathrm{valid}(\Gamma) \qquad \Gamma \vdash \tau \Leftarrow \star}{\mathrm{valid}(\Gamma, x : \tau)}
$$

1.3 Evaluation

$$
\begin{array}{llll}
\nu, \tau & ::= & n & \text{neutral term} \\
& | & \lambda x \mapsto \nu & \text{lambda} \\
& | & \Pi(x : \tau).\tau' & \text{dependent function space} \\
& | & (\nu, \nu') & \text{tuple} \\
& | & \Sigma(x : \tau).\tau' & \text{dependent pair space} \\
& | & \star & \text{type of types}
\end{array}
$$

$$
\begin{array}{llll}
n & ::= & x & \text{variable} \\
& | & n\ \nu & \text{neutral app} \\
& | & \mathsf{fst}\ n & \text{neutral first projection} \\
& | & \mathsf{snd}\ n & \text{neutral second projection}
\end{array}
$$

$$\frac{}{\star \Downarrow \star} \text{ (Star)} \qquad \frac{}{x \Downarrow x} \text{ (Var)} \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu} \text{ (Ann)}$$

$$\frac{e \Downarrow \nu}{\lambda x \mapsto e \Downarrow \lambda x \mapsto \nu} \text{ (Lam)} \qquad \frac{e \Downarrow \nu \quad e' \Downarrow \nu'}{(e, e')(\Downarrow, \nu)\nu'} \text{ (Tuple)}$$

$$\frac{e \Downarrow \lambda x \mapsto \nu \quad \nu[x \mapsto e'] \Downarrow \nu'}{e \ e' \Downarrow \nu'} \text{ (App)} \qquad \frac{e \Downarrow n \quad e' \Downarrow \nu'}{e \ e' \Downarrow n \ \nu'} \text{ (NApp)}$$

$$\frac{e \Downarrow (\nu, \nu')}{\mathsf{fst} \ e \Downarrow \nu} \text{ (Fst)} \qquad \frac{e \Downarrow (\nu, \nu')}{\mathsf{snd} \ e \Downarrow \nu'} \text{ (Snd)} \qquad \frac{e \Downarrow n}{\mathsf{fst} \ e \Downarrow \mathsf{fst} \ n} \text{ (NFst)}$$

$$\frac{e \Downarrow n}{\mathsf{snd} \ e \Downarrow \mathsf{snd} \ n} \text{ (NSnd)} \qquad \frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{\Pi(x : \sigma).\sigma' \Downarrow \Pi(x : \tau).\tau'} \text{ (Pi)}$$

$$\frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma).\sigma' \Downarrow \Sigma(x : \tau).\tau'} \text{ (Sigma)}$$

1.4 Typing

In the following, $e \Rightarrow \tau$ is an expression whose type synthezises to τ while $e \Leftarrow \tau$ is checkable.

$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash e \Leftarrow \tau} \ (\text{Chk}) \qquad\qquad \frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash (e : \sigma) \Rightarrow \tau} \ (\text{Ann})$$

$$\frac{}{\Gamma \vdash \star \Leftarrow \star} \ (\text{Star}) \qquad\qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau} \ (\text{Var})$$

$$\frac{\Gamma, x : \tau \vdash e \Leftarrow \tau'}{\Gamma \vdash \lambda x \mapsto e \Leftarrow \Pi(x : \tau).\tau'} \ (\text{Lam})$$

$$\frac{\Gamma \vdash e \Leftarrow \tau \qquad \Gamma \vdash e' \Leftarrow \tau'}{\Gamma \vdash (e, e') \Leftarrow \Sigma(x : \tau).\tau'} \ (\text{Tuple})$$

$$\frac{\Gamma \vdash e \Rightarrow \Pi(x : \tau).\tau' \qquad \Gamma \vdash e' \Leftarrow \tau \qquad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e \ e' \Rightarrow \tau''} \ (\text{App})$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau).\tau'}{\Gamma \vdash \mathsf{fst} \ e \Rightarrow \tau} \ (\text{Fst})$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau).\tau' \qquad \tau'[x \mapsto \mathsf{fst} \ e] \Downarrow \tau''}{\Gamma \vdash \mathsf{snd} \ e \Rightarrow \tau''} \ (\text{Snd})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash \Pi(x : \sigma).\sigma' \Leftarrow \star} \ (\text{Pi})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' \Leftarrow \star} \ (\text{Sigma})$$

2 Interlude : booleans

2.1 Syntax

$$
\begin{array}{llll}
e, \sigma, \kappa & ::= & \dots & \\
& | & \text{true} & \\
& | & \text{false} & \\
& | & \mathsf{cond} \ e \ [x.\sigma] \ e' \ e'' & \text{condition} \\
& | & \text{bool} & \text{type of a bool}
\end{array}
$$

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2.2 Evaluation

$$\nu, \tau \quad ::= \quad \ldots$$
$$| \quad \text{true}$$
$$| \quad \text{false}$$
$$| \quad \text{bool}$$

$$n \quad ::= \quad \ldots$$
$$| \quad \text{cond } \nu \ [x.\tau] \ \nu' \ \nu''$$

$$\frac{}{\text{true} \Downarrow \text{true}} \ (\text{TRUE}) \qquad\qquad \frac{}{\text{false} \Downarrow \text{false}} \ (\text{FALSE})$$

$$\frac{e \Downarrow \text{true} \qquad e' \Downarrow \nu}{\text{cond } e \ [x.B] \ e' \ e'' \Downarrow \nu} \ (\text{CONDT}) \qquad\qquad \frac{e \Downarrow \text{true} \qquad e'' \Downarrow \nu}{\text{cond } e \ [x.B] \ e' \ e'' \Downarrow \nu} \ (\text{CONDF})$$

$$\frac{e \Downarrow n \qquad e' \Downarrow \nu \qquad e'' \Downarrow \nu'}{\text{cond } e \ [x.B] \ e' \ e'' \Downarrow \text{cond } n \ [x.\tau] \ \nu \ \nu'} \ (\text{NCOND})$$

$$\frac{}{\text{bool} \Downarrow \text{bool}} \ (\text{BOOLTY})$$

2.3 Typing

$$\frac{}{\text{true} \Leftarrow \text{bool}} \ (\text{TRUE}) \qquad\qquad \frac{}{\text{false} \Leftarrow \text{bool}} \ (\text{FALSE})$$

$$\frac{\Gamma \vdash e \Leftarrow \text{bool} \qquad \Gamma, x : \text{bool} \vdash B \Leftarrow \star \qquad B[x \mapsto e] \Downarrow \tau}{\Gamma \vdash \text{cond } e \ [x.B] \ e' \ e'' \Rightarrow \tau} \ (\text{COND})$$

$$\frac{}{\text{bool} \Leftarrow \star} \ (\text{BOOLTY})$$

3 Enumerations

3.1 Syntax

$$e, \sigma, \kappa \quad ::= \quad ...$$

	nil	
	unit	
	$'$t	label
	label	label type
	nil	
	$[t\ l]$	
	labels	labels type

3.2 Evaluation

$$\frac{l \Downarrow \text{nil}}{\text{record } l \text{ as } x \text{ return } B \Downarrow \text{unit}} \text{ (\textsc{RecordNil})}$$

$$\frac{l \Downarrow [t\ l'] \qquad \qquad}{B[x \mapsto 0] \Downarrow \tau \qquad \text{record } l' \text{ as } x \text{ return } B[x \mapsto 1+\ x] \Downarrow \tau'}{\text{record } l \text{ as } x \text{ return } B \Downarrow \Sigma(\_ : \tau).\tau'} \text{ (\textsc{RecordCons})}$$

$$\frac{l \Downarrow n}{\text{record } l \text{ as } x \text{ return } B \Downarrow \text{record } n \text{ as } x \text{ return } B} \text{ (\textsc{NRecord})}$$

$$\frac{e \Downarrow 0 \qquad \text{fst cs} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \nu} \text{ (\textsc{CaseZe})}$$

$$\frac{e \Downarrow 1+\ e'}{\text{snd cs} \Downarrow \text{cs'} \qquad \text{case } e' \text{ as } x \text{ return } B[x \mapsto 1+\ x] \text{ with cs'} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \nu} \text{ (\textsc{CaseSuc})}$$

$$\frac{e \Downarrow n \qquad \text{cs} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \text{case } n \text{ as } x \text{ return } B \text{ with } \nu} \text{ (\textsc{CaseZe})}$$

3.3 Typing

$$\frac{}{\text{nil} \Leftarrow \text{unit}} \text{ (NIL)} \qquad\qquad \frac{}{\text{unit} \Leftarrow \star} \text{ (UNIT)}$$

$$\frac{}{'t \ \Leftarrow \text{label}} \text{ (LABEL)} \qquad\qquad \frac{}{\text{label} \Leftarrow \star} \text{ (LABELTY)}$$

$$\frac{}{\text{nil} \Leftarrow \text{labels}} \text{ (NILL)} \qquad \frac{\Gamma \vdash t \Leftarrow \text{label} \qquad \Gamma \vdash l \Leftarrow \text{labels}}{[t\ l] \Leftarrow \text{labels}} \text{ (CONSL)}$$

$$\frac{}{\text{labels} \Leftarrow \star} \text{ (LABELSTY)}$$

$$\frac{\Gamma \vdash t \Leftarrow \text{label} \qquad \Gamma \vdash l \Leftarrow \text{labels}}{\Gamma \vdash 0 \Leftarrow \text{enum } [t\ l]} \text{ (ZERO)}$$

$$\frac{\Gamma \vdash t \Leftarrow \text{label} \qquad \Gamma \vdash l \Leftarrow \text{labels} \qquad \Gamma \vdash n \Leftarrow \text{enum } l}{\Gamma \vdash 1+ n \Leftarrow \text{enum } [t\ l]} \text{ (SUC)}$$

$$\frac{\Gamma \vdash l \Leftarrow \text{labels}}{\Gamma \vdash \text{enum } l \Leftarrow \star} \text{ (ENUM)}$$

$$\frac{\Gamma \vdash l \Leftarrow \text{labels} \qquad \Gamma, x : \text{enum } l \vdash B \Leftarrow \star}{\Gamma \vdash \text{record } l \text{ as } x \text{ return } B \Rightarrow \star} \text{ (RECORD)}$$

$$\frac{\Gamma \vdash e \Leftarrow \text{enum } l \qquad \Gamma, x : \text{enum } l \vdash B \Leftarrow \star \qquad B[x \mapsto e] \Downarrow \tau \qquad \Gamma \vdash \text{cs} \Leftarrow \text{record } l \text{ as } x \text{ return } B}{\Gamma \vdash \text{case } e \text{ as } x \text{ return } B \text{ with cs} \Rightarrow \tau} \text{ (CASE)}$$