Bootstrap

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Contents

1 Théorie

1.1 Syntaxe

$$
\begin{array}{rcll}
e, \sigma, \kappa & ::= & e : \sigma & \text{annotated term} \\
& | & x & \text{variable} \\
& | & \lambda x \mapsto e & \text{lambda} \\
& | & e\ e' & \text{application} \\
& | & \Pi(x : \sigma).\sigma' & \text{pi type} \\
& | & (e, e') & \text{tuple} \\
& | & \text{fst } e & \text{fst} \\
& | & \text{snd } e & \text{snd} \\
& | & \Sigma(x : \sigma).\sigma' & \text{sigma type} \\
& | & \star & \text{type of types}
\end{array}
$$

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Contexte

$$
\begin{array}{rcll}
\Gamma & ::= & \epsilon & \text{empty context} \\
& | & \Gamma, x : \tau & \text{adding a variable}
\end{array}
$$

$$\frac{}{\text{valid}(\epsilon)} \qquad \frac{\text{valid}(\Gamma) \qquad \Gamma \vdash \tau \Leftarrow \star}{\text{valid}(\Gamma, x : \tau)}$$

1.3 Evaluation

$$\begin{array}{rcll}
\nu, \tau & ::= & n & \text{neutral term} \\
& \mid & \lambda x \mapsto \nu & \text{lambda} \\
& \mid & (\nu, \nu') & \text{tuple} \\
& \mid & \star & \text{type of types} \\
& \mid & \Pi(x : \tau).\tau' & \text{dependent function space} \\
& \mid & \Sigma(x : \tau).\tau' & \text{dependent pair space}
\end{array}$$

$$\begin{array}{rcll}
n & ::= & x & \text{variable} \\
& \mid & n\ \nu & \text{neutral app} \\
& \mid & \text{fst } n & \text{neutral first projection} \\
& \mid & \text{snd } n & \text{neutral second projection}
\end{array}$$

$$\frac{}{\star \Downarrow \star}\ (\textsc{Star}) \qquad \frac{}{x \Downarrow x}\ (\textsc{Var}) \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu}\ (\textsc{Ann})$$

$$\frac{e \Downarrow \nu}{\lambda x \mapsto e \Downarrow \lambda x \mapsto \nu}\ (\textsc{Lam}) \qquad \frac{e \Downarrow \nu \qquad e' \Downarrow \nu'}{(e, e') \Downarrow (\nu, \nu')}\ (\textsc{Tuple})$$

$$\frac{e \Downarrow \lambda x \mapsto \nu \qquad \nu[x \mapsto e'] \Downarrow \nu'}{e\ e' \Downarrow \nu'}\ (\textsc{App}) \qquad \frac{e \Downarrow n \qquad e' \Downarrow \nu'}{e\ e' \Downarrow n\ \nu'}\ (\textsc{NApp})$$

$$\frac{e \Downarrow (\nu, \nu')}{\text{fst } e \Downarrow \nu}\ (\textsc{Fst}) \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd } e \Downarrow \nu'}\ (\textsc{Snd}) \qquad \frac{e \Downarrow n}{\text{fst } e \Downarrow \text{fst } n}\ (\textsc{NFst})$$

$$\frac{e \Downarrow n}{\text{snd } e \Downarrow \text{snd } n}\ (\textsc{NSnd}) \qquad \frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Pi(x : \sigma).\sigma' \Downarrow \Pi(x : \tau).\tau'}\ (\textsc{Pi})$$

$$\frac{\sigma \Downarrow \tau \qquad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma).\sigma' \Downarrow \Sigma(x : \tau).\tau'}\ (\textsc{Sigma})$$

1.4 Typing

In the following, $e \Rightarrow \tau$ is an expression whose type synthezises to τ while $e \Leftarrow \tau$ is checkable.

$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash e \Leftarrow \tau}\ (\textsc{Chk}) \qquad \frac{\Gamma \vdash \sigma \Leftarrow * \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash (e : \sigma) \Rightarrow \tau}\ (\textsc{Ann})$$

$$\frac{}{\Gamma \vdash \star \Leftarrow \star}\ (\textsc{Star}) \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau}\ (\textsc{Var})$$

$$\frac{\Gamma, x : \tau \vdash e \Leftarrow \tau'}{\Gamma \vdash \lambda x \mapsto e \Leftarrow \Pi(x : \tau).\tau'}\ (\textsc{Lam})$$

$$\frac{\Gamma \vdash e \Leftarrow \tau \qquad \Gamma \vdash e' \Leftarrow \tau'}{\Gamma \vdash (e, e') \Leftarrow \Sigma(x : \tau).\tau'}\ (\textsc{Tuple})$$

$$\frac{\Gamma \vdash e \Rightarrow \Pi(x : \tau).\tau' \qquad \Gamma \vdash e' \Leftarrow \tau \qquad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e\ e' \Rightarrow \tau''}\ (\textsc{App})$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau).\tau'}{\Gamma \vdash \text{fst}\ e \Rightarrow \tau}\ (\textsc{Fst})$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau).\tau' \qquad \tau'[x \mapsto \text{fst}\ e] \Downarrow \tau''}{\Gamma \vdash \text{snd}\ e \Rightarrow \tau''}\ (\textsc{Snd})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash \Pi(x : \sigma).\sigma' \Leftarrow \star}\ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' \Leftarrow \star}\ (\textsc{Sigma})$$

Une reformulation équivalentes des règles (Pi) et (Sigma), plus adaptée pour l'implémentation :

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash \sigma' \Leftarrow \Pi(x : \tau).\star}{\Gamma \vdash \Pi(x : \sigma).\sigma' \Leftarrow \star}\ (\textsc{Pi})$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \qquad \sigma \Downarrow \tau \qquad \Gamma \vdash \sigma' \Leftarrow \Pi(x : \tau).\star}{\Gamma \vdash \Sigma(x : \sigma).\sigma' \Leftarrow \star}\ (\textsc{Sigma})$$