

# Bootstrap

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# 1 Minimal calculus

## 1.1 Syntax

|                         |                                    |                |
|-------------------------|------------------------------------|----------------|
| $e, \sigma, \kappa ::=$ | $e : \sigma$                       | annotated term |
|                         | $x$                                | variable       |
|                         | $\lambda x \mapsto e$              | lambda         |
|                         | $e \ e'$                           | application    |
|                         | $(x : \sigma) \rightarrow \sigma'$ | pi type        |
|                         | $(e, e')$                          | tuple          |
|                         | $\text{fst } e$                    | fst            |
|                         | $\text{snd } e$                    | snd            |
|                         | $(x : \sigma) \times \sigma'$      | sigma type     |
|                         | $\star$                            | type of types  |

where  $e, \sigma, \kappa$  represent general expressions, types and kinds respectively.

## 1.2 Context

|              |                    |                   |
|--------------|--------------------|-------------------|
| $\Gamma ::=$ | $\epsilon$         | empty context     |
|              | $\Gamma, x : \tau$ | adding a variable |

$$\frac{}{\text{valid}(\epsilon)} \quad \frac{\text{valid}(\Gamma) \quad \Gamma \vdash \tau \Leftarrow \star}{\text{valid}(\Gamma, x : \tau)}$$

## 1.3 Evaluation

|                 |                                |                           |
|-----------------|--------------------------------|---------------------------|
| $\nu, \tau ::=$ | $n$                            | neutral term              |
|                 | $\lambda x \mapsto \nu$        | lambda                    |
|                 | $(x : \tau) \rightarrow \tau'$ | dependent function space  |
|                 | $(\nu, \nu')$                  | tuple                     |
|                 | $(x : \tau) \times \tau'$      | dependent pair space      |
|                 | $\star$                        | type of types             |
| $n ::=$         | $x$                            | variable                  |
|                 | $n \ \nu$                      | neutral app               |
|                 | $\text{fst } n$                | neutral first projection  |
|                 | $\text{snd } n$                | neutral second projection |

$$\begin{array}{c}
\frac{}{\star \Downarrow \star} \text{ (STAR)} \qquad \frac{}{x \Downarrow x} \text{ (VAR)} \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu} \text{ (ANN)} \\
\\
\frac{e \Downarrow \nu}{\lambda x \mapsto e \Downarrow \lambda x \mapsto \nu} \text{ (LAM)} \qquad \frac{e \Downarrow \nu \quad e' \Downarrow \nu'}{(e, e')(\Downarrow, \nu)\nu'} \text{ (TUPLE)} \\
\\
\frac{e \Downarrow \lambda x \mapsto \nu \quad \nu[x \mapsto e'] \Downarrow \nu'}{e \ e' \Downarrow \nu'} \text{ (APP)} \qquad \frac{e \Downarrow n \quad e' \Downarrow \nu'}{e \ e' \Downarrow n \ \nu'} \text{ (NAPP)} \\
\\
\frac{e \Downarrow (\nu, \nu')}{\text{fst } e \Downarrow \nu} \text{ (FST)} \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd } e \Downarrow \nu'} \text{ (SND)} \qquad \frac{e \Downarrow n}{\text{fst } e \Downarrow \text{fst } n} \text{ (NFST)} \\
\\
\frac{e \Downarrow n}{\text{snd } e \Downarrow \text{snd } n} \text{ (NSND)} \qquad \frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{(x : \sigma) \rightarrow \sigma' \Downarrow (x : \tau) \rightarrow \tau'} \text{ (PI)} \\
\\
\frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{(x : \sigma) \times \sigma' \Downarrow (x : \tau) \times \tau'} \text{ (SIGMA)}
\end{array}$$

## 1.4 Typing

In the following,  $e \Rightarrow \tau$  is an expression whose type synthesises to  $\tau$  while  $e \Leftarrow \tau$  is checkable.

$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash e \Leftarrow \tau} \text{ (CHK)} \qquad \frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash (e : \sigma) \Rightarrow \tau} \text{ (ANN)}$$

$$\frac{}{\Gamma \vdash \star \Leftarrow \star} \text{ (STAR)} \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau} \text{ (VAR)}$$

$$\frac{\Gamma, x : \tau \vdash e \Leftarrow \tau'}{\Gamma \vdash \lambda x \mapsto e \Leftarrow (x : \tau) \rightarrow \tau'} \text{ (LAM)}$$

$$\frac{\Gamma \vdash e \Leftarrow \tau \quad \Gamma \vdash e' \Leftarrow \tau'}{\Gamma \vdash (e, e') \Leftarrow (x : \tau) \times \tau'} \text{ (TUPLE)}$$

$$\frac{\Gamma \vdash e \Rightarrow (x : \tau) \rightarrow \tau' \quad \Gamma \vdash e' \Leftarrow \tau \quad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e e' \Rightarrow \tau''} \text{ (APP)}$$

$$\frac{\Gamma \vdash e \Rightarrow (x : \tau) \times \tau'}{\Gamma \vdash \text{fst } e \Rightarrow \tau} \text{ (FST)}$$

$$\frac{\Gamma \vdash e \Rightarrow (x : \tau) \times \tau' \quad \tau'[x \mapsto \text{fst } e] \Downarrow \tau''}{\Gamma \vdash \text{snd } e \Rightarrow \tau''} \text{ (SND)}$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash (x : \sigma) \rightarrow \sigma' \Leftarrow \star} \text{ (PI)}$$

$$\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' \Leftarrow \star}{\Gamma \vdash (x : \sigma) \times \sigma' \Leftarrow \star} \text{ (SIGMA)}$$

## 2 Interlude : booleans

### 2.1 Syntax

$$\begin{array}{ll} e, \sigma, \kappa & ::= \dots \\ & | \text{ true} \\ & | \text{ false} \\ & | \text{ cond } e \text{ } [x.\sigma] \text{ } e' \text{ } e'' & \text{ condition} \\ & | \text{ bool} & \text{ type of a bool} \end{array}$$

## 2.2 Evaluation

$$\begin{array}{lcl} \nu, \tau & ::= & \dots \\ & | & \text{true} \\ & | & \text{false} \\ & | & \text{bool} \end{array}$$

$$\begin{array}{lcl} n & ::= & \dots \\ & | & \text{cond } \nu \ [x.\tau] \ \nu' \ \nu'' \end{array}$$

$$\frac{}{\text{true} \Downarrow \text{true}} \text{ (TRUE)}$$

$$\frac{}{\text{false} \Downarrow \text{false}} \text{ (FALSE)}$$

$$\frac{e \Downarrow \text{true} \quad e' \Downarrow \nu}{\text{cond } e \ [x.B] \ e' \ e'' \Downarrow \nu} \text{ (COND T)}$$

$$\frac{e \Downarrow \text{true} \quad e'' \Downarrow \nu}{\text{cond } e \ [x.B] \ e' \ e'' \Downarrow \nu} \text{ (COND F)}$$

$$\frac{e \Downarrow n \quad e' \Downarrow \nu \quad e'' \Downarrow \nu'}{\text{cond } e \ [x.B] \ e' \ e'' \Downarrow \text{cond } n \ [x.\tau] \ \nu \ \nu'} \text{ (NCOND)}$$

$$\frac{}{\text{bool} \Downarrow \text{bool}} \text{ (BOOLT Y)}$$

## 2.3 Typing

$$\frac{}{\text{true} \Leftarrow \text{bool}} \text{ (TRUE)}$$

$$\frac{}{\text{false} \Leftarrow \text{bool}} \text{ (FALSE)}$$

$$\frac{\Gamma \vdash e \Leftarrow \text{bool} \quad \Gamma, x : \text{bool} \vdash B \Leftarrow \star \quad B[x \mapsto e] \Downarrow \tau}{\Gamma \vdash \text{cond } e \ [x.B] \ e' \ e'' \Rightarrow \tau} \text{ (COND)}$$

$$\frac{}{\text{bool} \Leftarrow \star} \text{ (BOOLT Y)}$$

### 3 Enumerations

#### 3.1 Syntax

|                     |       |                          |
|---------------------|-------|--------------------------|
| $e, \sigma, \kappa$ | $::=$ | ...                      |
|                     |       | nil                      |
|                     |       | unit                     |
|                     |       | 't            label      |
|                     |       | label       label type   |
|                     |       | nil                      |
|                     |       | [t l]                    |
|                     |       | labels       labels type |

#### 3.2 Evaluation

$$\frac{l \Downarrow \text{nil}}{\text{record } l \text{ as } x \text{ return } B \Downarrow \text{unit}} \text{ (RECORDNIL)}$$

$$\frac{B[x \mapsto 0] \Downarrow \tau \quad \text{record } l' \text{ as } x \text{ return } B[x \mapsto 1 + x] \Downarrow \tau'}{\text{record } l \text{ as } x \text{ return } B \Downarrow \tau \times \tau'} \text{ (RECORDCONS)}$$

$$\frac{l \Downarrow n}{\text{record } l \text{ as } x \text{ return } B \Downarrow \text{record } n \text{ as } x \text{ return } B} \text{ (NRECORD)}$$

$$\frac{e \Downarrow 0 \quad \text{fst cs} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \nu} \text{ (CASEZE)}$$

$$\frac{\text{snd cs} \Downarrow \text{cs}' \quad \text{case } e' \text{ as } x \text{ return } B[x \mapsto 1 + x] \text{ with cs}' \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \nu} \text{ (CASESUC)}$$

$$\frac{e \Downarrow n \quad \text{cs} \Downarrow \nu}{\text{case } e \text{ as } x \text{ return } B \text{ with cs} \Downarrow \text{case } n \text{ as } x \text{ return } B \text{ with } \nu} \text{ (CASEZE)}$$

### 3.3 Typing

$$\begin{array}{c}
\frac{}{\text{nil} \Leftarrow \text{unit}} \text{ (NIL)} \qquad \frac{}{\text{unit} \Leftarrow \star} \text{ (UNIT)} \\
\\
\frac{}{t \Leftarrow \text{label}} \text{ (LABEL)} \qquad \frac{}{\text{label} \Leftarrow \star} \text{ (LABELTY)} \\
\\
\frac{}{\text{nil} \Leftarrow \text{labels}} \text{ (NILL)} \qquad \frac{\Gamma \vdash t \Leftarrow \text{label} \quad \Gamma \vdash l \Leftarrow \text{labels}}{[t \ l] \Leftarrow \text{labels}} \text{ (CONSL)} \\
\\
\frac{}{\text{labels} \Leftarrow \star} \text{ (LABELSTY)} \\
\\
\frac{\Gamma \vdash t \Leftarrow \text{label} \quad \Gamma \vdash l \Leftarrow \text{labels}}{\Gamma \vdash 0 \Leftarrow \text{enum } [t \ l]} \text{ (ZERO)} \\
\\
\frac{\Gamma \vdash t \Leftarrow \text{label} \quad \Gamma \vdash l \Leftarrow \text{labels} \quad \Gamma \vdash n \Leftarrow \text{enum } l}{\Gamma \vdash 1 + n \Leftarrow \text{enum } [t \ l]} \text{ (SUC)} \\
\\
\frac{\Gamma \vdash l \Leftarrow \text{labels}}{\Gamma \vdash \text{enum } l \Leftarrow \star} \text{ (ENUM)} \\
\\
\frac{\Gamma \vdash l \Leftarrow \text{labels} \quad \Gamma, x : \text{enum } l \vdash B \Leftarrow \star}{\Gamma \vdash \text{record } l \text{ as } x \text{ return } B \Rightarrow \star} \text{ (RECORD)} \\
\\
\frac{\Gamma \vdash e \Leftarrow \text{enum } l \quad \Gamma, x : \text{enum } l \vdash B \Leftarrow \star \quad B[x \mapsto e] \Downarrow \tau \quad \Gamma \vdash \text{cs} \Leftarrow \text{record } l \text{ as } x \text{ return } B}{\Gamma \vdash \text{case } e \text{ as } x \text{ return } B \text{ with cs} \Rightarrow \tau} \text{ (CASE)}
\end{array}$$

## 4 Descriptions

### 4.1 Syntax

$$\begin{array}{lcl}
 e, \sigma & ::= & \dots \\
 & | & \text{'unit} \\
 & | & \text{'var} \quad \text{identity functor} \\
 & | & \text{'}\Sigma \sigma e \\
 & | & \text{'}\Pi \sigma e \\
 & | & \llbracket e \rrbracket \sigma \\
 & | & \text{desc} \quad \text{descriptor type} \\
 & | & \mu e \\
 & | & \text{ctor } e
 \end{array}$$

### 4.2 Evaluation

$$\begin{array}{lcl}
 \nu, \tau & ::= & \dots \\
 & | & \text{'unit} \\
 & | & \text{'var} \quad \text{identity functor} \\
 & | & \text{'}\Sigma \tau D \\
 & | & \text{'}\Pi \tau D \\
 & | & \text{desc} \quad \text{descriptor type} \\
 & | & \mu n \quad \text{fixpoint} \\
 & | & \text{ctor } n \quad \text{constructor}
 \end{array}$$

$$\begin{array}{lcl}
 n & ::= & \dots \\
 & | & \llbracket n \rrbracket \tau
 \end{array}$$

$$\frac{D \Downarrow \text{'unit}}{\llbracket D \rrbracket \sigma \Downarrow \text{unit}} \text{ (DECODENIL)} \qquad \frac{D \Downarrow \text{'var} \quad \sigma \Downarrow \tau}{\llbracket D \rrbracket \sigma \Downarrow \tau} \text{ (DECODEVAR)}$$

$$\frac{D \Downarrow \text{'}\Sigma \tau D' \quad (e : \tau) \times \llbracket D e \rrbracket \sigma \Downarrow \tau'}{\llbracket D \rrbracket \sigma \Downarrow \tau'} \text{ (DECODESIGMA)}$$

$$\frac{D \Downarrow \text{'}\Pi \tau D' \quad (e : \tau) \rightarrow \llbracket D' e \rrbracket \sigma \Downarrow \tau'}{\llbracket D \rrbracket \sigma \Downarrow \tau'} \text{ (DECODEPI)}$$



### 4.3 Typing

$$\begin{array}{c}
\overline{\Gamma \vdash \text{'unit'} \Leftarrow \text{desc}} \qquad \overline{\Gamma \vdash \text{'var'} \Leftarrow \text{desc}} \qquad \overline{\Gamma \vdash \text{desc} \Leftarrow \star} \\
\\
\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma \vdash D \Leftarrow \tau \rightarrow \text{desc}}{\Gamma \vdash \text{'}\Sigma \sigma D \Leftarrow \text{desc}} \text{ (DSIGMA)} \\
\\
\frac{\Gamma \vdash \sigma \Leftarrow \star \quad \sigma \Downarrow \tau \quad \Gamma \vdash D \Leftarrow \tau \rightarrow \text{desc}}{\Gamma \vdash \text{'}\Pi \sigma D \Leftarrow \text{desc}} \text{ (DP1)} \\
\\
\frac{\Gamma \vdash D \Leftarrow \text{desc} \quad \Gamma \vdash \sigma \Leftarrow \star}{\Gamma \vdash \llbracket D \rrbracket \sigma \Rightarrow \star} \text{ (DECODE)} \qquad \frac{\Gamma \vdash D \Leftarrow \text{desc}}{\Gamma \vdash \mu D \Leftarrow \star} \text{ (MU)} \\
\\
\frac{\Gamma \vdash \sigma \Leftarrow \llbracket D \rrbracket (\mu D)}{\Gamma \vdash \text{ctor } \sigma \Leftarrow \mu D}
\end{array}$$