

Bootstrap

Erik Helmers

March 7, 2023

Contents

1	Théorie	1
1.1	Syntaxe	1
1.2	Contexte	1
1.3	Evaluation	2
1.4	Typing	2

1 Théorie

1.1 Syntaxe

e, σ, κ	$::=$	$e : \sigma$	annotated term
		x	variable
		$\lambda x \mapsto e$	lambda
		$e \ e'$	application
		$\Pi(x : \sigma).\sigma'$	pi type
		(e, e')	tuple
		$\text{fst } e$	fst
		$\text{snd } e$	snd
		$\Sigma(x : \sigma).\sigma'$	sigma type
		\star	type of types

where e, σ, κ represent general expressions, types and kinds respectively.

1.2 Contexte

Γ	$::=$	ϵ	empty context
		$\Gamma, x : \tau$	adding a variable

$$\frac{}{\text{valid}(\epsilon)} \quad \frac{\text{valid}(\Gamma) \quad \Gamma \vdash \tau \Leftarrow \star}{\text{valid}(\Gamma, x : \tau)}$$

1.3 Evaluation

$\nu, \tau ::= n$	neutral term
$\lambda x \mapsto \nu$	lambda
(ν, ν')	tuple
\star	type of types
$\Pi(x : \tau). \tau'$	dependent function space
$\Sigma(x : \tau). \tau'$	dependent pair space
$n ::= x$	variable
$n \nu$	neutral app
$\text{fst } n$	neutral first projection
$\text{snd } n$	neutral second projection

$$\begin{array}{c} \frac{}{\star \Downarrow \star} \text{ (STAR)} \qquad \frac{}{x \Downarrow x} \text{ (VAR)} \qquad \frac{e \Downarrow \nu}{e : \sigma \Downarrow \nu} \text{ (ANN)} \\[10pt] \frac{e \Downarrow \nu}{\lambda x \mapsto e \Downarrow \lambda x \mapsto \nu} \text{ (LAM)} \qquad \frac{e \Downarrow \nu \quad e' \Downarrow \nu'}{(e, e') \Downarrow (\nu, \nu')} \text{ (TUPLE)} \\[10pt] \frac{e \Downarrow \lambda x \mapsto \nu \quad \nu[x \mapsto e'] \Downarrow \nu'}{e \Downarrow e' \Downarrow \nu'} \text{ (APP)} \qquad \frac{e \Downarrow n \quad e' \Downarrow \nu'}{e \Downarrow e' \Downarrow n \Downarrow \nu'} \text{ (NAPP)} \\[10pt] \frac{e \Downarrow (\nu, \nu')}{\text{fst } e \Downarrow \nu} \text{ (FST)} \qquad \frac{e \Downarrow (\nu, \nu')}{\text{snd } e \Downarrow \nu'} \text{ (SND)} \qquad \frac{e \Downarrow n}{\text{fst } e \Downarrow \text{fst } n} \text{ (NFST)} \\[10pt] \frac{e \Downarrow n}{\text{snd } e \Downarrow \text{snd } n} \text{ (NSND)} \qquad \frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{\Pi(x : \sigma). \sigma' \Downarrow \Pi(x : \tau). \tau'} \text{ (PI)} \\[10pt] \frac{\sigma \Downarrow \tau \quad \sigma' \Downarrow \tau'}{\Sigma(x : \sigma). \sigma' \Downarrow \Sigma(x : \tau). \tau'} \text{ (SIGMA)} \end{array}$$

1.4 Typing

In the following, $e \Rightarrow \tau$ is an expression whose type synthesises to τ while $e \Leftarrow \tau$ is checkable.

$$\frac{\Gamma \vdash e \Rightarrow \tau}{\Gamma \vdash e \Leftarrow \tau} \text{ (CHK)} \qquad \frac{\Gamma \vdash \sigma \Leftarrow * \quad \sigma \Downarrow \tau \quad \Gamma \vdash e \Leftarrow \tau}{\Gamma \vdash (e : \sigma) \Rightarrow \tau} \text{ (ANN)}$$

$$\frac{}{\Gamma \vdash \star \Leftarrow \star} \text{ (STAR)} \qquad \frac{\Gamma(x) = \tau}{\Gamma \vdash x \Rightarrow \tau} \text{ (VAR)}$$

$$\frac{\Gamma, x : \tau \vdash e \Leftarrow \tau'}{\Gamma \vdash \lambda x \mapsto e \Leftarrow \Pi(x : \tau). \tau'} \text{ (LAM)}$$

$$\frac{\Gamma \vdash e \Leftarrow \tau \quad \Gamma \vdash e' \Leftarrow \tau'}{\Gamma \vdash (e, e') \Leftarrow \Sigma(x : \tau). \tau'} \text{ (TUPLE)}$$

$$\frac{\Gamma \vdash e \Rightarrow \Pi(x : \tau). \tau' \quad \Gamma \vdash e' \Leftarrow \tau \quad \tau'[x \mapsto e'] \Downarrow \tau''}{\Gamma \vdash e e' \Rightarrow \tau''} \text{ (APP)}$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau). \tau'}{\Gamma \vdash \text{fst } e \Rightarrow \tau} \text{ (FST)}$$

$$\frac{\Gamma \vdash e \Rightarrow \Sigma(x : \tau). \tau' \quad \tau'[x \mapsto \text{fst } e] \Downarrow \tau''}{\Gamma \vdash \text{snd } e \Rightarrow \tau''} \text{ (SND)}$$

$$\frac{\Gamma \vdash \sigma \Leftarrow * \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' \Leftarrow *}{\Gamma \vdash \Pi(x : \sigma). \sigma' \Leftarrow *} \text{ (PI)}$$

$$\frac{\Gamma \vdash \sigma \Leftarrow * \quad \sigma \Downarrow \tau \quad \Gamma, x : \tau \vdash \sigma' \Leftarrow *}{\Gamma \vdash \Sigma(x : \sigma). \sigma' \Leftarrow *} \text{ (SIGMA)}$$

Une reformulation équivalentes des règles (PI) et (SIGMA), plus adaptée pour l'implémentation :

$$\frac{\Gamma \vdash \sigma \Leftarrow * \quad \sigma \Downarrow \tau \quad \Gamma \vdash \sigma' \Leftarrow \Pi(x : \tau). \star}{\Gamma \vdash \Pi(x : \sigma). \sigma' \Leftarrow *} \text{ (PI)}$$

$$\frac{\Gamma \vdash \sigma \Leftarrow * \quad \sigma \Downarrow \tau \quad \Gamma \vdash \sigma' \Leftarrow \Pi(x : \tau). \star}{\Gamma \vdash \Sigma(x : \sigma). \sigma' \Leftarrow *} \text{ (SIGMA)}$$