Supervision report 21st June 2021

Erik Hörmann

Mathematical institute
University of Oxford

UNIVERSITY OF OXFORD
Mathematical Institute

Online, 21st June 2021

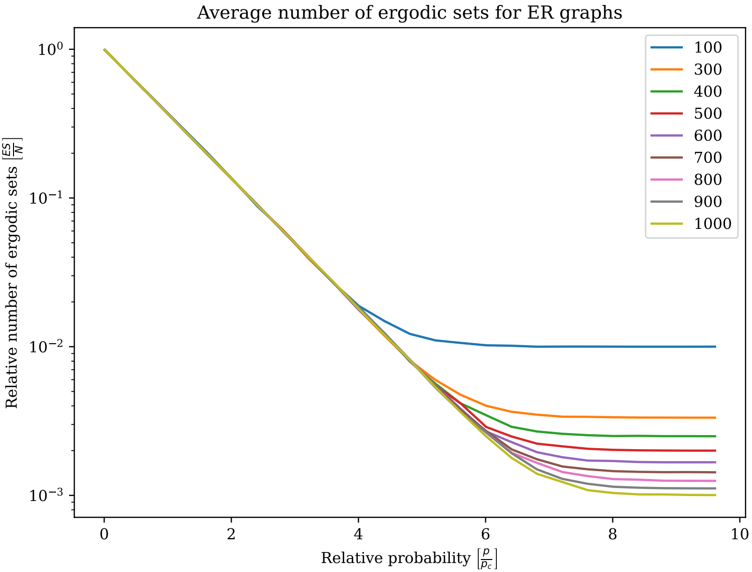
Outline

Quantities of interest

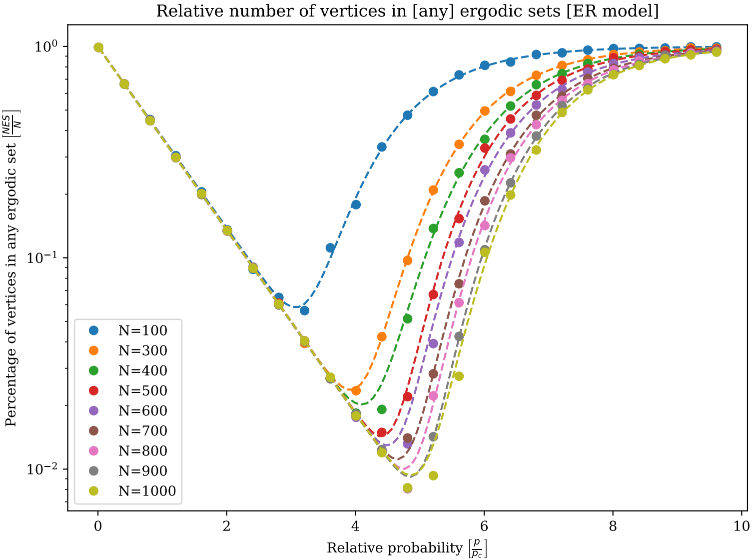
We simulated an Erdös-Renyi Model $\mathcal{G}(N, p)$ and we study the following quantities as functions of both N, p:

- number of ergodic sets [ES]
- number of vertices in ANY ergodic set [NES]
- number of vertices in THE LARGEST ergodic set [GSCC]

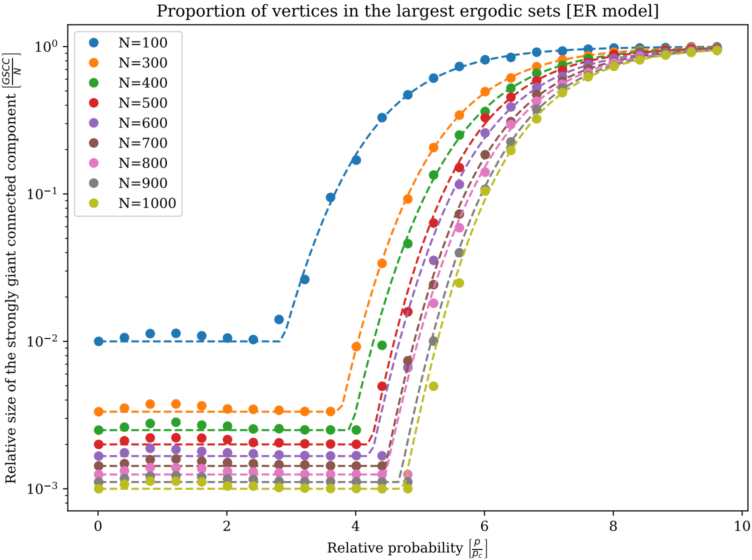
Number of ergodic sets [ER-ES]



Number of vertices in ANY ergodic set [ER-NES]



Size of the giant strongly connected component [ER-GSCC]



Fitting functions

The following fitting function have been used:

- for the number of ergodic sets [ES], no fit was performed
  (just an exponential decay with different plateau at 1/N)
- for [NES], the total number of vertices in ANY ergodic set:

$$f_{NES}(p,[\alpha_1,\beta,\gamma,\delta]) = e^{-\alpha_1 p} + \left[\frac{1}{(1+e^{-\beta\,(x-\gamma)})}\right]^{\delta} \tag{1}$$

- for [GSCC], the size of the giant strongly connected component [GSCC]:

$$f_{GSCC}(p,[\alpha_2,\beta,\gamma,\delta]) = \mathbb{1}_{p<\alpha_2}\frac{1}{N} + \mathbb{1}_{p>\alpha_2}\left[\frac{1}{(1+e^{-\beta\,(x-\gamma)})}\right]^{\delta} \tag{2}$$

Fitting parameters for the Erdös-Renyi model

The exponential decay factor α_1 stays constant for all the sizes considered:

$$N = [100, 200, 300, 400, 500, 600, 700, 800, 900, 1000]$$

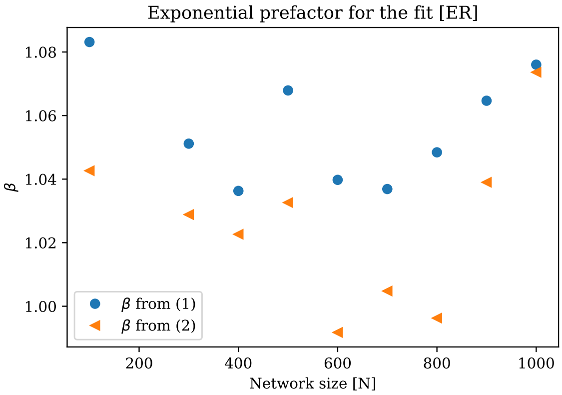
and the reported value is

$$\alpha_1 = 0.998 \pm 0.004 \tag{3}$$

The parameter α_2 depends on the granularity of the dataset and thus is not reported, as the probability step of 0.4 is not fine enough to provide an accuarate estimate.

Fitting parameters for the Erdös-Renyi model

The exponential prefactor β has no clear distribution, but the results are consistent between the two methods. The following figure shows the agreement.



Fitting parameters for the Erdös-Renyi model

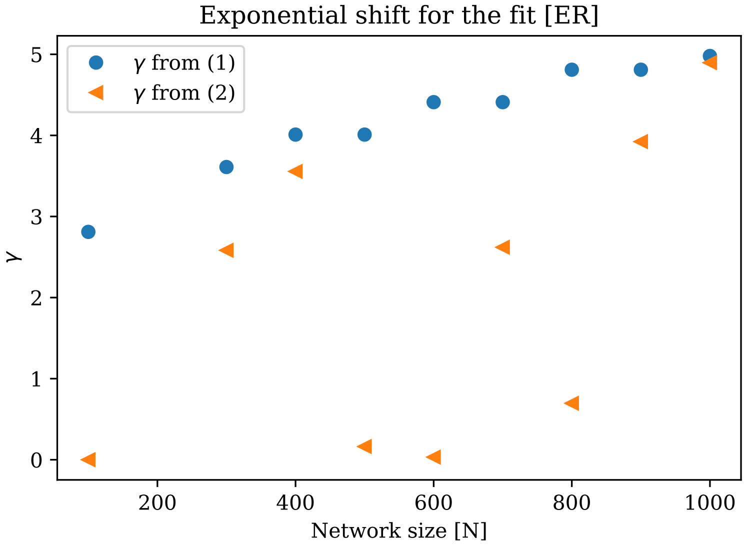
The exponential prefactor β is slightly dependent on the method, and calling β_1 the estimate from the fitting of equation (1) and β_2 the estimate from the fitting of equation (2)

$$\beta_1 = 1.056 \pm 0.017 \tag{4}$$

$$\beta_2 = 1.026 \pm 0.024 \tag{5}$$

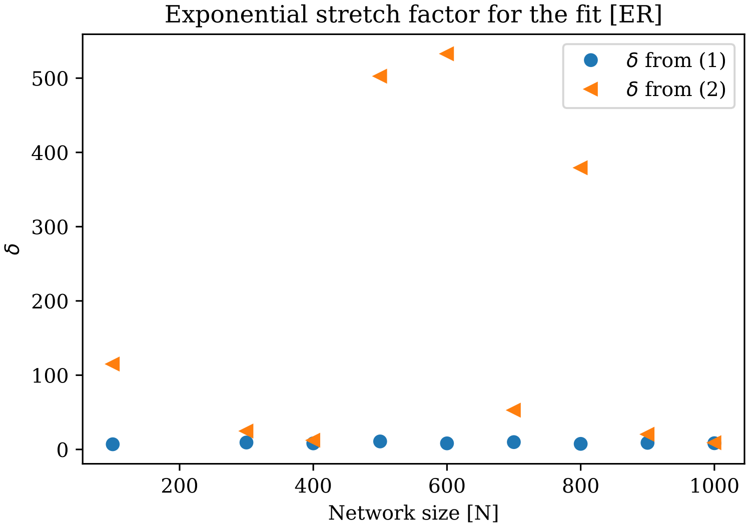
Fitting parameters for the Erdös-Renyi model

The exponential shift parameter γ has no clear distribution.

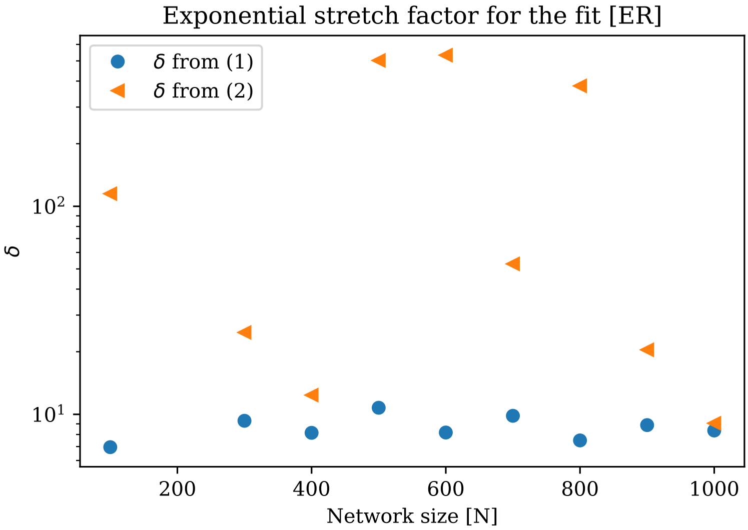


Fitting parameters for the Erdös-Renyi model

The exponential stretch parameter δ has no clear distribution.



Fitting parameters for the Erdös-Renyi model



Fitting parameters recap

Exponential decay:

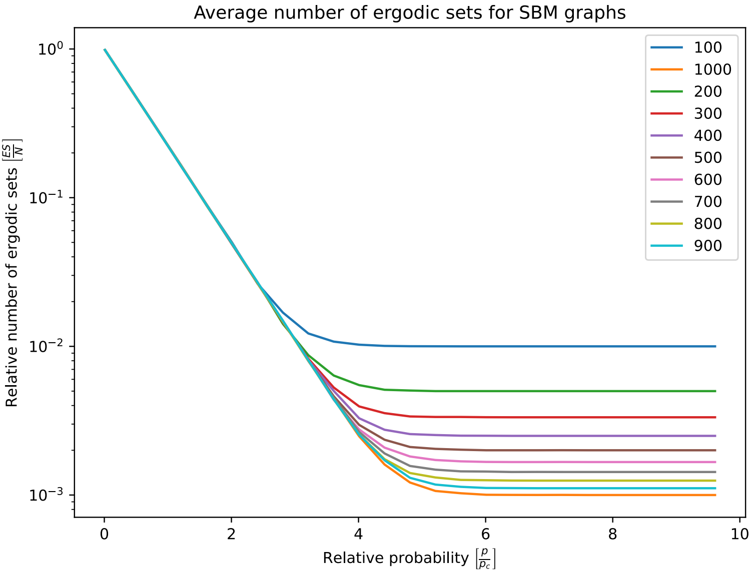
$$\alpha_1 = 0.998 \pm 0.004 \tag{6}$$

Exponential prefactor:

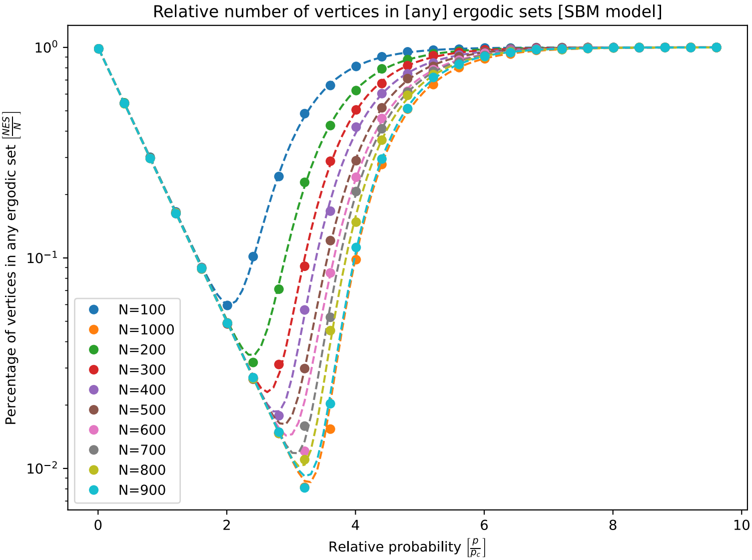
$$\beta_1 = 1.056 \pm 0.017 \tag{7}$$

$$\beta_2 = 1.026 \pm 0.024 \tag{8}$$

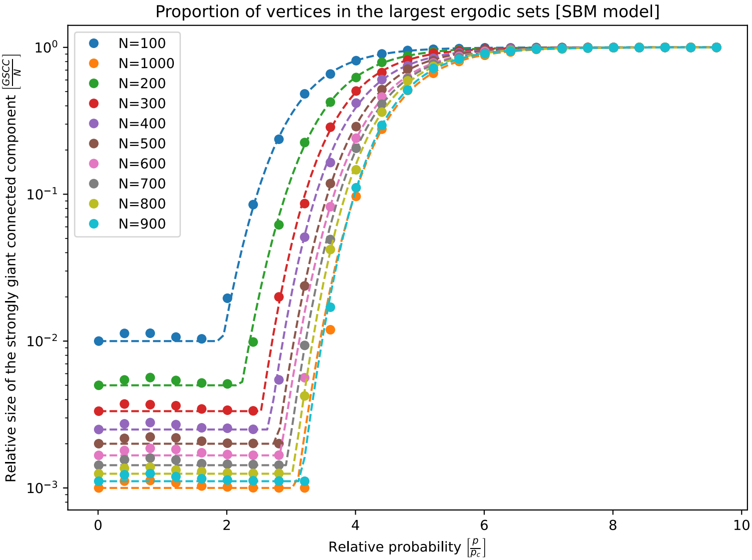
Number of ergodic sets [SBM-ES]



Number of vertices in ANY ergodic set [SBM-NES]



Size of the giant strongly connected component [SBM-GSCC]



Fitting functions

We used exactly the same functions as with the Erdös-Renyi model:

- for the number of ergodic sets [ES], no fit was performed (just an exponential decay with different plateau at 1/N)
- for [NES], the total number of vertices in ANY ergodic set:

$$f_{NES}(p, [\alpha, \beta, \gamma, \delta]) = e^{-\alpha\, p} + \left[\frac{1}{\left(1 + e^{-\beta\,(x-\gamma)}\right)}\right]^{\delta} \tag{9}$$

- for [GSCC], the size of the giant strongly connected component [GSCC]:

$$f_{GSCC}(p, [\alpha, \beta, \gamma, \delta]) = \mathbb{1}_{p<\alpha}\frac{1}{N} + \mathbb{1}_{p>\alpha}\left[\frac{1}{\left(1 + e^{-\beta\,(x-\gamma)}\right)}\right]^{\delta} \tag{10}$$

Thank you!

erik.hormann@maths.ox.ac.uk