Sparse Planning Graphs for Information Driven Simultaneous Localization and Mapping

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1 Introduction

Exploration is a key capability that enables robotic vehicles to operate in unknown environments. In general, the shortest trajectory over which a robot can greedily explore an environment is the trajectory formed by choosing actions which maximally reduce uncertainty in the environmental belief representation, or map. One strategy to accomplish this is to choose control actions that maximize an information metric between the robot's current map and the robot's map at a future timestep. This solution is one variant of a broad category of exploration strategies known as active Simultaneous Localization and Mapping (SLAM) [3].

State of the art active SLAM implementations generally only compute actions over a one-step planning horizon due to the extremely high computational cost of determining expected information gain over all potential future locations [1], [2]. However, plans over much longer horizons can be generated if expected information gain values can be computed once per planning step, cached, and updated efficiently based on new information. An even more efficient approach would utilize sparse planning graphs to limit expected information gain calculations to a select few feasible future locations in the map. Rapidly Exploring Random Trees (RRT) and lattice graphs are two examples of sparse planning graphs which decompose the reachable space into a sparse set of goal states based on a set of motion primitives for planning purposes.

This project seeks to develop a recursive formulation for efficiently and intelligently updating expected information gain over a finite horizon as the robot moves through an unknown environment and builds a map from sensor observations.

2 Occupancy Grid Mapping

We represent the map as an occupancy grid, which consists of a set of cells: $m = \{m^i\}_{i=1}^{N}$. The probability that an individual cell is occupied is given by $p\left(m^i \mid x_{1:t}, z_{1:t}\right)$, where $x_{1:t}$ denotes the history of states of the vehicle, and $z_{1:t}$ denotes the history of range observations accumulated by the vehicle. We assume that cell occupancy probabilities are independent of one another: $p\left(m \mid x_{1:t}, z_{1:t}\right) = \prod_i p\left(m^i \mid x_{1:t}, z_{1:t}\right)$. For notational simplicity we write the map conditioned on random variables $x_{1:t}$ and $z_{1:t}$ as $p_t\left(m\right) := p\left(m \mid x_{1:t}, z_{1:t}\right)$. Additionally, unobserved grid cells are assigned a uniform prior of being occupied.

We represent the occupancy status of grid cell m^i at time t with a log odds expression

$$l_t := \log \frac{p\left(m^i \mid z_{1:t}\right)}{p\left(\bar{m}^i \mid z_{1:t}\right)} \tag{1}$$

, where $\bar{m}^i$ denotes the probability that m^i is unoccupied. When a new observation z_t is obtained, the log odds update is given by

$$l_t = l_{t-1} + \log \frac{p\left(m^i \mid z_t\right)}{p\left(m^i\right)} - \log \frac{p\left(\bar{m}^i \mid z_t\right)}{p\left(\bar{m}^i\right)} \tag{2}$$

where the last two terms represent the inverse sensor model.

3 Explorative Information Cost Function

The goal of exploration is to find a dynamically feasible sequential set of motions chosen from a library of motion primitives, $\mathcal{X}$, over a time interval, $\tau := t + 1 : t + T$, which enable the robot to explore its environment. We use the criteria that an explorative action is one which allows the robot to position itself in locations that generate observations which are informative to the robot's map. By this criteria, choosing the optimal exploration action will maximize an information metric over the robot's *future map*. In this context, a *future map* is an updated version of the robot's current map, which has used Eq. (**??**) to incorporate all measurements gathered while executing an action. An *action* can be defined as a discrete sequence of states, $x_\tau = [x_{t+1}, \ldots, x_{t+T}]$. While executing an action, the robot will obtain a set of measurements $z_\tau(x_\tau) = [z_{t+1}(x_{t+1}), \ldots, z_{t+T}(x_{t+T})]$ by sensing from the states x_τ. $z_\tau(x_\tau)$ is modeled as a random variable whose distribution is parameterized by a deterministic action, x_τ, generated by a planner. Under this notation, an explorative planner must determine x_τ^*: the action that visits locations which allow the robot to obtain the set of measurements which are most informative to the current map. When integrated with Eq. (**??**), these measurements will maximize an information-theoretic cost function over the map. We choose to maximize Shannon Mutual Information (MI) rate between the current map, m, and the measurements z_τ gathered along x_τ.

$$x_\tau^* = \underset{x_\tau \in \mathcal{X}^T}{\operatorname{argmax}} \frac{\mathrm{I_{MI}}\left[m; z_\tau(x_\tau)\right]}{R\left(x_\tau\right)} = \underset{x_\tau \in \mathcal{X}^T}{\operatorname{argmax}} \frac{\mathrm{H}\left[m\right] - \mathrm{H}\left[m \mid z_\tau(x_\tau)\right]}{R\left(x_\tau\right)} \tag{3}$$

where I_{MI} is the Shannon Mutual Information, and $R : \mathcal{X}^T \to \mathbb{R}^+$ is a function which returns the time required to execute an action. In contrast to MI, MI rate is used so that actions with different execution times can be compared with a common metric.

TALK ABOUT WHY $\mathrm{H}[m]$ IS EASY.

Several assumptions can be made to simplify the optimization of x^*_τ. Due to cell independence in the occupancy grid formulation, the joint conditional entropy over the map can be expressed as a sum of individual cell conditional entropies. Additionally, let $\mathcal{C}$ to be the set of cells in the map that beams in z_τ pass through. Given the measurement model (described in Sect. 3.1), cells $c \notin \mathcal{C}$ are not updated by z_τ and therefore do not contribute to the map's conditional entropy. Using these assumptions, we may write the entropy of the map conditioned on z_τ as

$$\begin{aligned} \mathrm{H}[m \mid z_\tau] &= \sum_{c \in \mathcal{C}} \mathrm{H}[c \mid z_\tau] \\ &= \sum_{c \in \mathcal{C}} \mathop{\mathbb{E}}_{c, z_\tau} \left[-\log p\left(c \mid z_\tau\right)\right] \\ &= -\int_{z_\tau} p\left(z_\tau\right) \sum_{c \in \mathcal{C}} o\left(c \mid z_\tau\right) dz_\tau \end{aligned} \tag{4}$$

where

$$o(c \mid z_\tau) = p(c \mid z_\tau) \log p(c \mid z_\tau) + (1 - p(c \mid z_\tau)) \log\left(1 - p(c \mid z_\tau)\right) \tag{5}$$

Computing this term requires integration over the space of measurements, z_τ, which is intractable for an online planner. Instead. Instead, we approximate Eq. (4) with N Monte Carlo samples, $\{z^i_\tau\}^N_{i=1}$, drawn from the distribution $p(z_\tau)$.

$$\mathrm{H}[m \mid z_\tau] \approx -\frac{1}{N} \sum_{i=1}^{N} \sum_{c \in \mathcal{C}} o\left(c \mid z^i_\tau\right) \tag{6}$$

Sampling

$$x^*_\tau = \operatorname*{argmax}_{x_\tau \in \mathcal{X}^T} \frac{1}{R\left(x_\tau\right)} \left(\mathrm{H}[m] + \sum_{i=1}^{N} \sum_{c \in \mathcal{C}} o\left(c \mid z^i_\tau(x_\tau)\right)\right) \tag{7}$$

3.1 Measurement Model

As explained in Sect. 3, each future position x_j will generate a multi-beam range measurement z_j. We express each measurement as a K-tuple random variable, $z_j = [z_{j,1}, \ldots, z_{j,K}]$, with $z_{j,k} \in [z_{min}, z_{max}]$. A measurement model is required to sample from the distribution $p(z_\tau \mid m)$. Assuming laser beam independence,

$p(z_\tau \mid m)$ can be expressed as a product of the measurement probabilities of individual beams at each timestep over the interval $\tau = t+1 : t+T$.

$$p\left(z_\tau \mid m\right) = \prod_{j=t+1}^{t+T} \prod_{k=1}^{K} p\left(z_{j,k} \mid m\right) \tag{8}$$

The probability of a single beam measurement can be expressed as a function of the true distance, r, to the obstacle.

$$p\left(z_{j,k} \mid m\right) = \int_r p\left(z_{j,k} \mid r\right) p\left(r \mid m\right) dr \tag{9}$$

Note that in the inverse sensor model $p(z_{j,k} \mid r)$, $z_{j,k}$ is conditionally independent of the map given the true range, so m can safely be omitted as a conditioning variable.

$$p\left(z_{j,k} \mid r\right) = \begin{cases} \mathcal{N}(z_{j,k} - r, \sigma_{hit}^2) & : z_{min} \leq r \leq z_{max} \\ \mathcal{N}(z_{max}, \sigma_{hit}^2) & : z_{max} < r \\ 0 & : r < z_{min} \end{cases} \tag{10}$$

where $\mathcal{N}(x-\mu, \sigma^2)$ is a one-dimensional Gaussian with mean μ and variance σ^2. The Gaussian term in Eq. (10) is an approximation for the true distribution, which is a complex function of both beam angle as well as non-axial distance-dependent noise. The environment-dependent distribution $p(r)$ corresponds to the probability that an obstacle intersects a beam at range r. We model this as a Poisson binomial distribution with probabilities decaying according to the inverse square of the beam range.

On the interval $[z_{min}, z_{\max}]$, $p(z_{j,k})$ reduces to a Gaussian convolution with $p(r)$ over r, which can be computed in $O(n \log n)$ operations with a Fast Fourier Transform. This computation can be reduced to $O(n)$ by assuming that σ_{hit} of the range sensor is much smaller than the resolution of the map.

The distribution $p(r)$ represents the probability that the first obstacle along a ray originating at the sensor is found at distance r. $p(r)$ can be expressed as a generalization of the geometric distribution for independent, but not identically distributed (i.n.i.d.) Bernoulli trials.

$$\begin{aligned} p(r = i \mid m) &= p_t(m^i) \prod_{j=1}^{i-1} \left(1 - p_t(m^j)\right) \\ &= p_t(m^i) \left(\frac{1}{p_t(m^{i-1})} - 1\right) p(r = i-1 \mid m) \end{aligned} \tag{11}$$

References

[1] Frederic Bourgault, Alexei A Makarenko, Stefan B Williams, Ben Grocholsky, and Hugh F Durrant-Whyte. Information based adaptive robotic exploration. In *Intelligent Robots and Systems, 2002. IEEE/RSJ International Conference on*, volume 1, pages 540–545. IEEE, 2002.

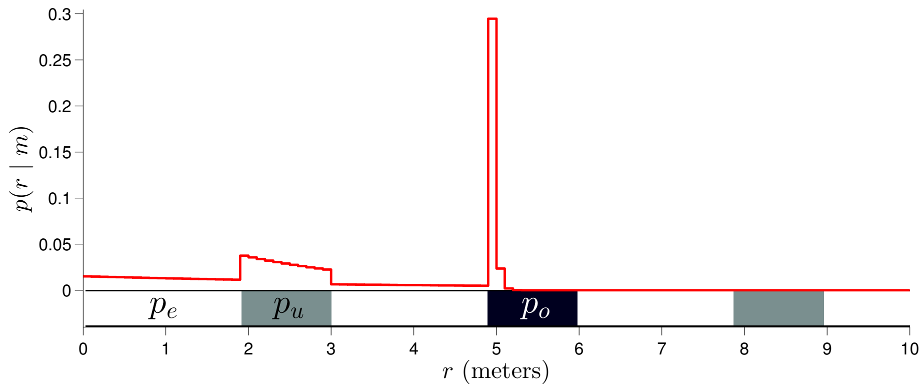

Figure 1: The distribution $p(r \mid m)$ over a depicted 1-dimensional map with resolution $\Delta r = 0.1$ m, and $p_e = 0.01$, $p_o = 0.92$, $p_u = 0.05$.

[2] Cyrill Stachniss, Giorgio Grisetti, and Wolfram Burgard. Information gain-based exploration using rao-blackwellized particle filters. In *Robotics: Science and Systems*, volume 2, 2005.

[3] Sebastian Thrun, Wolfram Burgard, and Dieter Fox. *Probabilistic robotics*. MIT press, 2005.