Sparse Planning Graphs for Information Driven Exploration

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***Abstract*—...**

I. Introduction

Exploration is a key capability that enables robotic vehicles to operate in unknown environments. In this project we develop an active perception policy for robotic exploration. Active perception exploration formulations choose control actions which optimize an information-theoretic objective function such as Shannon's mutual information or entropy [] over the robot's map, given a sensor measurement model. Other common exploration techniques, such as Frontier exploration [], use geometric reasoning to infer explorative paths. While these strategies work well in practice, they operate on a maximum likelihood estimate of the map, and apply heuristics to determine the most uncertain locations in the environment. In contrast, active perception strategies do not utilize geometric or maximum likelihood assumptions, and instead interpret the map as a binary random variable, choosing actions which directly minimize the random variable's uncertainty. Julian et al. prove that maximizing mutual information between a robot's map and expected future map naturally yields explorative behaviors [].

Active perception formulations seek to optimize information-theoretic objectives. While this optimization is real-time for short planning horizons, these metrics are often expensive to compute online, requiring double integration over possible future robot states and measurements, or Monte Carlo sampling from the distribution of measurements. In this project, we aim to develop an efficient active perception exploration strategy which evaluates the information-theoretic objective in a sparse set of poses across the configuration space. This strategy allows one to evaluate the objective function a limited number of times, while still generating paths that explore the space.

To achieve real-time active perception exploration, we use a Rapidly-Exploring Random Tree (RRT) to generate sets of dynamically feasible actions over a finite planning horizon []. RRT planners trade trajectory optimality for efficiency, allowing for evaluation of many potential future locations in the configuration space during a single planning step. In addition, RRT planners are anytime, allowing one to generate potential trajectories for a prespecified amount of time before evaluating the most optimal sampled trajectory. Our strategy evaluates each RRT leaf-node using the information-theoretic objective function, and stores the resulting reward in the tree. After planning for a specified amount of time, the maximum reward leaf-node is chosen as the next location to visit, and the RRT is traversed to generate a dynamically feasible trajectory to that location.

A recent work by Charrow et al. [] has proposed the Cauchy-Schwarz Quadratic Mutual Information (CSQMI) as an efficient information-theoretic objective function. CSQMI is theoretically well-motivated: it is derived from Renyi's Quadratic Entropy, a generalization of Shannon's entropy. However, in contrast to Shannon's mutual information (which is derived directly from Shannon's entropy), CSQMI is shown to have superior computational efficiency.

II. State Estimation

State estimation addresses the problem of determining the robot's pose in the environment given noisy sensor observations. This state estimation pipeline's pose output is a superior alternative to directly feeding in exteroceptive sensor observations to the controller and planner. In this section, we present an Unscented Kalman Filter for fusing inertial measurement unit (IMU) and localization observations for a 2D robot.

A. Process Model

The vehicle state $\mathbf{x}$ consists of the global position $\mathbf{p} = [p_x\ p_y]^T$, global heading angle θ, global velocity $\mathbf{v} = [v_x\ v_y]^T$, IMU angular velocity bias in the z-direction b_ω and IMU linear acceleration biases in the x- and y-directions $\mathbf{b}_a = [b_{ax}\ b_{ay}]^T$.

$$\dot{\mathbf{p}} = \mathbf{v} \tag{1}$$
$$\dot{\theta} = \omega - b_\omega - n_\omega \tag{2}$$
$$\dot{\mathbf{v}} = \mathbf{C}(\theta)\left(\mathbf{a} - \mathbf{b}_a - \mathbf{n}_a\right) \tag{3}$$
$$\dot{b}_\omega = n_{b\omega} \tag{4}$$
$$\dot{\mathbf{b}}_a = \mathbf{n}_{ba} \tag{5}$$

The IMU measurements are comprised of the z-direction angular velocity ω as well as the body frame x- and y-direction linear accelerations $\mathbf{a} = [a_x \ a_y]^T$. Both measurements are modelled as being corrupted by additive Gaussian white noise and a random walk bias driven by Gaussian white noise (4), (5).

$$\mathbf{n} = \begin{bmatrix} \mathbf{n}_a \\ n_\omega \\ \mathbf{n}_{ba} \\ n_{b\omega} \end{bmatrix} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}) \tag{6}$$

$$\mathbf{Q} = \text{diag}\{\sigma_a^2, \sigma_a^2, \sigma_\omega^2, \sigma_{ba}^2, \sigma_{ba}^2, \sigma_{b\omega}^2\} \tag{7}$$

Since the IMU measurements are in the body frame, we use a 2×2 rotation matrix $\mathbf{C}(\theta)$ to rotate them into the global reference frame (3). Each component of the noise vector in (6) is assumed to be independent, and their corresponding covariance sigma values are chosen by offline sensor characterization tests (7).

B. Correction Model

The laser scans are used to construct a map of the environment, and a grid-based localization algorithm is used to compute the global pose (position and heading) of the vehicle with respect to the map. The sensor model for the localization algorithm is given by

$$\mathbf{z} = \begin{bmatrix} \mathbf{p} \\ \theta \end{bmatrix} + \begin{bmatrix} \mathbf{C}(\theta) & \mathbf{0} \\ \mathbf{0}^T & 1 \end{bmatrix} \mathbf{w} \tag{8}$$

$$\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \mathbf{R}) \tag{9}$$

The observation noise covariance matrix $\mathbf{R}$ (9) is computed by fitting a multivariate Gaussian to the posterior probability grid of the localization algorithm. Because this covariance is expressed in the scanner frame (assumed to be coincident with the body frame), we rotate its associated noise vector $\mathbf{w}$ into the world frame in (8).

C. Unscented Kalman Filter

The nonlinearities introduced by the rotation in the process and correction models motivated the choice of the Unscented Kalman Filter in this project. For brevity, we omit the complete presentation of all the steps involved in the UKF (we refer the reader to [1]). We apply the process model and correction model equations when the relevant measurement is received by the state estimator. Outlier exteroceptive observations are rejected by a chi-squared innovation gate.

REFERENCES

[1] S. J. Julier and J. K. Uhlmann. A new extension of the kalman filter to nonlinear systems. In *Proc. SPIE*, volume 3068, pages 182–193, July 1997.