Sparse Planning Graphs for Information Driven Simultaneous Localization and Mapping

1,2,3
The Robotics Institute
Carnegie Mellon University
Pittsburgh, PA 15217
Email: {1,2,3}@cmu.edu

***Abstract*—. . .**

I. Introduction

II. Occupancy Grid Mapping

We model the map as an occupancy grid, and represent the map as a conglomeration of cells: $m = \{m^i\}_{i=1}^N$. The probability that an individual cell is occupied is given by $p\left(m^i \mid x_{1:t}, z_{1:t}\right)$, where $x_{1:t}$ denotes the history of states of the vehicle, and $z_{1:t}$ denotes the history of range observations accumulated by the vehicle. Additionally we assume that cell occupancies are independent of one another: $p\left(m \mid x_{1:t}, z_{1:t}\right) = \prod_i p\left(m^i \mid x_{1:t}, z_{1:t}\right)$. For notational simplicity we write the map conditioned on random variables $x_{1:t}$ and $z_{1:t}$ as $p_t\left(m\right) := p\left(m \mid x_{1:t}, z_{1:t}\right)$.

III. Exploration Cost Functional

The purpose of the planner is to find a dynamically feasible series of control actions from a set of motion primitives, $\mathcal{X}$, over a time interval, $\tau := t{+}1 : t{+}T$, which enable the robot to gather observations that maximize an information metric over its map. We define an *action* as a discrete sequence of states, $x_\tau = [x_{t+1}, \ldots, x_{t+T}]$. While executing an action, the robot will obtain a set of measurements $z_\tau(x_\tau) = [z_{t+1}, \ldots, z_{t+T}]$ by sensing from the states x_τ. Under this notation, the planner must determine x_τ^*, the action that visits locations which allow the robot to obtain the set of measurements, z_τ^*, which, when integrated into the map, maximize an information-theoretic cost function over the map. We choose to maximize Shannon Mutual Information rate between the current map, m, and the measurements z_τ gathered along x_τ.

$$x_\tau^* = \underset{x_\tau \in \mathcal{X}^T}{\operatorname{argmax}} \frac{\mathrm{I_{MI}}\left[m; z_\tau \mid x_\tau\right]}{R\left(T\right)} \tag{1}$$

where $\mathrm{I_{MI}}$ is the Shannon Mutual Information.

$$\mathrm{I_{MI}}\left[m; z_\tau\right] = \mathrm{H}\left[m\right] - \mathrm{H}\left[m \mid z_\tau\right] \tag{2}$$

Since the optimization is performed over x_τ, the map entropy term can be removed.

$$x_\tau^* = \underset{x_\tau \in \mathcal{X}^T}{\operatorname{argmax}} \frac{-\mathrm{H}\left[m \mid z_\tau\right]}{R\left(T\right)} \tag{3}$$

Here we can make several assumptions to simplify the optimization of x_τ^*. Due to cell independence in the occupancy grid formulation, the joint entropy over the map can be expressed as a sum of individual cell entropies. Additionally, let $\mathcal{C}$ to be the set of cells in the map that beams in z_τ pass through. We note that cells $c \notin \mathcal{C}$ are not updated by z_τ and therefore do not contribute to the map's conditional entropy. Using these assumptions, as well as the notation $p_c^\tau \equiv p(c \mid z_\tau)$ and $o(c \mid z_\tau) \equiv p_c^\tau \log p_c^\tau + (1 - p_c^\tau)\log(1 - p_c^\tau)$, we may write the entropy of the map conditioned on z_τ as

$$\begin{aligned}
\mathrm{H}\left[m \mid z_\tau\right] &= \sum_{c \in \mathcal{C}} \mathrm{H}\left[c \mid z_\tau\right] \\
&= \sum_{c \in \mathcal{C}} \underset{c, z_\tau}{\mathbb{E}}\left[-\log p\left(c \mid z_\tau\right)\right] \\
&= -\int_{z_\tau} p\left(z_\tau\right) \sum_{c \in \mathcal{C}} o\left(c \mid z_\tau\right) dz_\tau
\end{aligned} \tag{4}$$

Integration over the space of measurements z_τ is computationally intractable. Instead, we approximate Eq. (4) using N Monte Carlo samples from the distribution $p(z_\tau)$.

$$\mathrm{H}\left[m \mid z_\tau\right] \approx -\frac{1}{N} \sum_{i=1}^{N} \sum_{c \in \mathcal{C}} o\left(c \mid z_\tau^i\right) \tag{5}$$

$$x_\tau^* = \underset{x_\tau \in \mathcal{X}^T}{\operatorname{argmax}} \frac{1}{R\left(T\right)} \sum_{i=1}^{N} \sum_{c \in \mathcal{C}} o\left(c \mid z_\tau^i\right) \tag{6}$$

A. Measurement Model

As explained in Sect. III, each future position x_k will generate a multi-beam range measurement z_m. We express each measurement as a K-tuple random variable, $z_m = \left[z_m^1, \ldots, z_m^K\right]$, with $z_m^k \in [z_{min}, z_{max}]$. A measurement model is required to sample from the distribution $p(z_\tau \mid m)$. Assuming laser beam independence, $p(z_\tau \mid m)$ can be expressed as a product of the measurement probabilities of individual beams at each timestep over the interval $\tau = t + 1 : t + T$.

$$p\left(z_\tau \mid m\right) = \prod_{m=t+1}^{t+T} \prod_{k=1}^{K} p\left(z_m^k \mid m\right) \tag{7}$$

The probability of a single beam measurement can be expressed as a function of the true distance, r, to the obstacle.

$$p\left(z_m^k \mid m\right) = \int_r p\left(z_m^k \mid r, m\right) p\left(r \mid m\right) dr \tag{8}$$

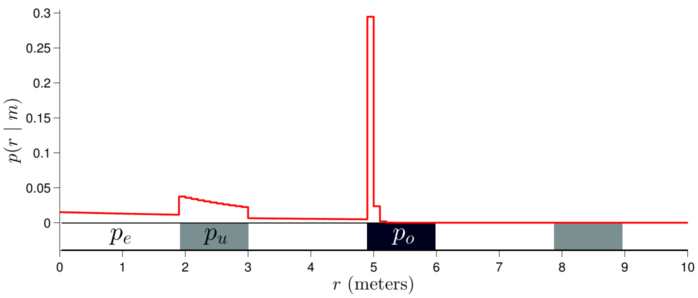

Fig. 1. The distribution $p(r \mid m)$ over a depicted 1-dimensional map with resolution $\Delta r = 0.1$ m, and $p_e = 0.01$, $p_o = 0.92$, $p_u = 0.05$.

$$p\left(z_m^k \mid r, m\right) = \begin{cases} \mathcal{N}(z_m^k - r, \sigma_{hit}^2) & : z_{min} \leq r \leq z_{max} \\ \mathcal{N}(z_{max}, \sigma_{hit}^2) & : z_{max} < r \\ 0 & : r < z_{min} \end{cases} \tag{9}$$

where $\mathcal{N}(x-\mu, \sigma^2)$ is a one-dimensional Gaussian with mean μ and variance σ^2. The Gaussian term in Eq. (9) is an approximation for the true distribution, which is a complex function of both beam angle as well as non-axial distance-dependent noise. The environment-dependent distribution $p(r)$ corresponds to the probability that an obstacle intersects a beam at range r. We model this as a Poisson binomial distribution with probabilities decaying according to the inverse square of the beam range.

On the interval $[z_{min}, z_{\max}]$, $p(z_m^k)$ reduces to a Gaussian convolution with $p(r)$ over r, which can be computed in $O(n \log n)$ operations with a Fast Fourier Transform. This computation can be reduced to $O(n)$ by assuming that σ_{hit} of the range sensor is much smaller than the resolution of the map.

The distribution $p(r)$ represents the probability that the first obstacle along a ray originating at the sensor is found at distance r. $p(r)$ can be expressed as a generalization of the geometric distribution for independent, but not identically distributed (i.n.i.d.) Bernoulli trials.

$$\begin{aligned} p(r = i \mid m) &= p_t(m^i) \prod_{j=1}^{i-1} \left(1 - p_t(m^j)\right) \\ &= p_t(m^i) \left(\frac{1}{p_t(m^{i-1})} - 1\right) p(r = i-1 \mid m) \end{aligned} \tag{10}$$