Computer Vision: Lecture 12

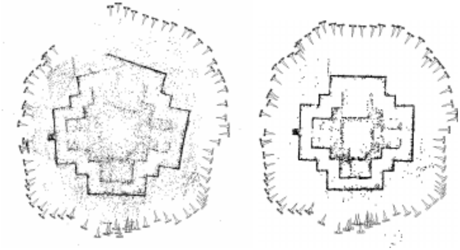
2023-12-07

Today's Lecture

Non-Sequential Methods for SfM

- Last week: rotation averaging and translation registration
- Today: factorization with and without occlusions

Why non-sequential SfM: drift



Minimizing the Reprojection Error

Main goal

For given $\{\mathbf{x}_{ij}\}$ and $\{m_{ij}\}$ find a minimizer

$$\sum_{i,j} m_{ij} \left\| \mathbf{x}_{ij} - \pi(\mathtt{P}_i \mathbf{X}_j) \right\|^2 \to \min_{\{\mathtt{P}_i\},\{\mathbf{X}_j\}}$$

Local optimization needs good starting point

Question

1. Do we always need good initial values for $\{\mathtt{P}_i\}$ and $\{\mathbf{X}_j\}$?
2. Can we (slightly) modify the above objective to obtain e.g. a closed-form solution?

Why is this objective hard to minimize?

- Bilinear, non-convex terms $\mathtt{P}_i \mathbf{X}_j$
- ~~Perspective division $\pi(\mathbf{X}) = \frac{1}{X_3}(X_1, X_2)^\top$~~
- ~~Calibrated SfM: constraints $\mathtt{R}_i \in SO(3)$~~

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Affine Cameras

- Affine cameras use parallel camera rays
- E.g. telephoto lens
- Camera matrix is given by

$$\mathtt{P} = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- $\mathbf{x} = \mathtt{P}\mathbf{X}$ (not only $\mathbf{x} \sim \mathtt{P}\mathbf{X}$)
- We can drop last row of $\mathtt{P}$: $\mathtt{P} \in \mathbb{R}^{2\times 4}$

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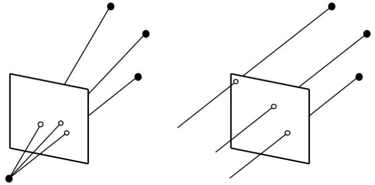
Perspective and affine cameras



Affine Reconstruction

Affine reconstruction: no missing data

For given $\{\mathbf{x}_{ij}\}$ find a minimizer

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where $\mathtt{P}_i \in \mathbb{R}^{2\times 4}$, $\mathbf{X}_j \in \mathbb{R}^4$.

- Let $\mathtt{P}_i = (\mathtt{A}_i \mid \mathbf{t}_i)$

$$\sum_{i,j} \left\|\mathbf{x}_{ij} - \mathtt{A}_i \mathbf{X}_j - \mathbf{t}_i\right\|^2 \to \min_{\{\mathtt{P}_i\},\{\mathbf{X}_j\}}$$

- Closed-form solution for $\mathbf{t}_i$

$$\nabla_{\mathbf{t}_i} = 2 \sum_j (\mathtt{A}_i \mathbf{X}_j + \mathbf{t}_i - \mathbf{x}_{ij}) \stackrel{!}{=} \mathbf{0}$$

$$\Longrightarrow \mathbf{t}_i = \bar{\mathbf{x}}_i - \mathtt{A}_i \bar{\mathbf{X}} \qquad \bar{\mathbf{x}}_i = \frac{1}{M}\sum_j \mathbf{x}_{ij} \qquad \bar{\mathbf{X}} = \frac{1}{M}\sum_j \mathbf{X}_j$$

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Affine Reconstruction

- Centering of the image points

$$\mathbf{x}_{ij}^c = \mathbf{x}_{ij} - \bar{\mathbf{x}}_j \implies \sum_i \mathbf{x}_{ij}^c = \mathbf{0}$$

- W.l.o.g. we can assume $\bar{\mathbf{X}} = \mathbf{0}$
  - Affine ambiguity

Affine reconstruction: no missing data

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- Using matrix notation

$$\mathtt{W} = \underbrace{\begin{pmatrix} \mathbf{x}^c_{11} & \dots & \mathbf{x}^c_{1M} \\ \vdots & \vdots & \vdots \\ \mathbf{x}^c_{N1} & \dots & \mathbf{x}^c_{NM} \end{pmatrix}}_{\in \mathbb{R}^{2N \times M}} \qquad \mathtt{B} = \underbrace{\begin{pmatrix} \mathtt{A}_1 \\ \vdots \\ \mathtt{A}_N \end{pmatrix}}_{\in \mathbb{R}^{2N \times 3}} \qquad \mathtt{C} = \underbrace{\begin{pmatrix} \mathbf{X}_1^\top \\ \vdots \\ \mathbf{X}_M^\top \end{pmatrix}}_{\in \mathbb{R}^{M \times 3}}$$

- Matrix factorization problem

$$\min_{\mathtt{B},\mathtt{C}} \left\| \mathtt{W} - \mathtt{B}\mathtt{C}^\top \right\|_F^2$$

Affine Reconstruction

Matrix factorization problem

$$\min_{\mathtt{B},\mathtt{C}} \left\| \mathtt{W} - \mathtt{B}\mathtt{C}^\top \right\|_F^2$$

- $\mathtt{B} \in \mathbb{R}^{2N \times 3}$, $\mathtt{C} \in \mathbb{R}^{M \times 3}$ $\implies$ $\mathtt{B}\mathtt{C}^\top$ has rank ≤ 3

Equivalent problem:

$$\min_{\hat{\mathtt{W}}} \left\| \mathtt{W} - \hat{\mathtt{W}} \right\|_F^2 \qquad \text{s.t. } \hat{\mathtt{W}} \text{ has rank } 3$$

Solve via SVD: $\mathtt{W} = \mathtt{U}\Sigma\mathtt{V}^\top$

$$\hat{\mathtt{W}} = \mathtt{U}_{:,1:3}\,\Sigma_{1:3,1:3}\,\mathtt{V}_{:,1:3}^\top \qquad \mathtt{B} = \mathtt{U}_{:,1:3}\,\Sigma_{1:3,1:3} \qquad \mathtt{C} = \mathtt{V}_{:,1:3}$$

- Q: what would the rank of $\hat{\mathtt{W}}$ be without the centering assumption?

Affine Reconstruction

Matrix factorization problem

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Affine Reconstruction

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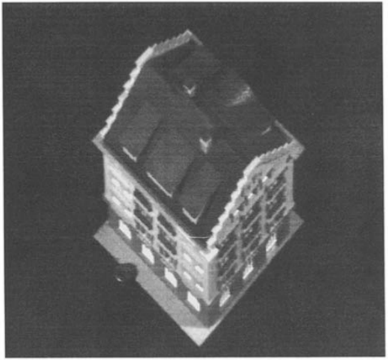
- Affine ambiguity

$$\hat{\mathtt{W}} = \mathtt{B}\mathtt{C}^\top = \mathtt{B}\mathtt{Q}\mathtt{Q}^{-1}\mathtt{C}^\top \qquad \text{for any invertible } \mathtt{Q} \in \mathbb{R}^{3\times 3}$$

  Square pixels: find $\mathtt{Q}$ such that $\mathtt{A}_i\mathtt{Q}$ is orthonormal camera, $\mathtt{A}_i\mathtt{Q}\mathtt{Q}^\top\mathtt{A}_i^\top = \mathtt{I}$

- Extends to non-rigid objects by allowing rank > 3: guest lecture on Dec. 11

Affine Reconstruction



Projective Reconstruction

- Assume projective depths λ_{ij} are known: $\lambda_{ij}\mathbf{x}_{ij} \approx \mathtt{P}_i \mathbf{X}_j$

$$\Lambda \odot \mathtt{W} = \underbrace{\begin{pmatrix} \lambda_{11}\mathbf{x}_{11} & \dots & \lambda_{1M}\mathbf{x}_{1M} \\ \vdots & \vdots & \vdots \\ \lambda_{N1}\mathbf{x}_{N1} & \dots & \lambda_{MN}\mathbf{x}_{NM} \end{pmatrix}}_{\in \mathbb{R}^{3N\times M}} \qquad \mathtt{B} = \underbrace{\begin{pmatrix} \mathtt{P}_1 \\ \vdots \\ \mathtt{P}_N \end{pmatrix}}_{\in \mathbb{R}^{3N\times 4}} \qquad \mathtt{C} = \underbrace{\begin{pmatrix} \mathbf{X}_1^\top \\ \vdots \\ \mathbf{X}_M^\top \end{pmatrix}}_{\in \mathbb{R}^{M\times 4}}$$

$\mathtt{P}_i \in \mathbb{R}^{3\times 4}$, $\mathbf{X}_j \in \mathbb{R}^4$

Projective factorization

$$\min_{\mathtt{B},\mathtt{C}^\top} \left\| \Lambda \odot \mathtt{W} - \mathtt{B}\mathtt{C}^\top \right\|_F^2 \iff \min_{\hat{\mathtt{W}}} \left\| \Lambda \odot \mathtt{W} - \hat{\mathtt{W}} \right\|_F^2 \qquad \text{s.t. } \hat{\mathtt{W}} \text{ has rank 4}$$

Projective Reconstruction

Iterative projective factorization (Sturm-Triggs method)

1. Initialize $\lambda_{ij} = 1$
2. Factorize $\mathtt{\Lambda} \odot \mathtt{W}$ to obtain $\mathtt{B}$, $\mathtt{C}$ (SVD as in the affine case)

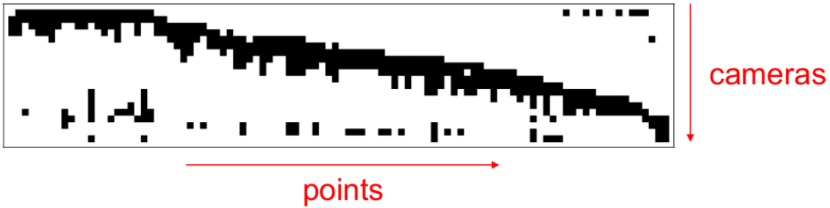
$$\min_{\mathtt{B},\mathtt{C}^\top} \left\| \mathtt{\Lambda} \odot \mathtt{W} - \mathtt{B}\mathtt{C}^\top \right\|_F^2$$

3. Re-estimate all λ_{ij} (triangulation)
4. If $\mathtt{\Lambda} \odot \mathtt{W}$ is almost rank 4, then stop. Otherwise goto 2.

- Caveat: global optimum is $\mathtt{\Lambda} = 0$, $\mathtt{B} = 0$ or $\mathtt{C} = 0$
    - Some modifications to avoid degenerate solution
- No guarantees on convergence
- Projective ambiguity
    - Auto-calibration, self-calibration

Factorization with Missing Data

- What about affine/projective factorization with missing data?



Factorization with Missing Data

- What about affine/projective factorization with missing data?

$$\mathtt{M} = \begin{bmatrix} \mathbf{x}_{11} & \mathbf{x}_{12} & \mathbf{x}_{13} & ? & \cdots & \mathbf{x}_{1M} \\ \mathbf{x}_{21} & ? & \mathbf{x}_{23} & ? & \cdots & ? \\ \vdots & & & & \ddots & \vdots \\ \mathbf{x}_{N1} & ? & ? & \mathbf{x}_{N3} & \cdots & ? \end{bmatrix} = \begin{bmatrix} \mathtt{P}_1 \\ \vdots \\ \mathtt{P}_N \end{bmatrix} \begin{bmatrix} \mathbf{X}_1 & \cdots & \mathbf{X}_M \end{bmatrix}^\top$$

$$\mathbf{x}_{ij} \in \mathbb{R}^2$$

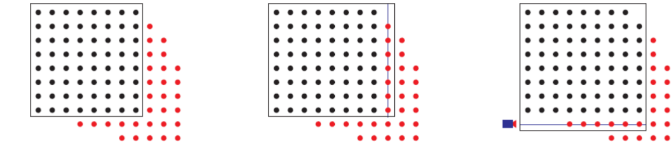
- SVD is not applicable
  - We cannot set missing entries in $\mathtt{W}$ e.g. to 0
- Factorization with missing data

$$\min_{\mathtt{B},\mathtt{C}} \left\| \mathtt{M} \odot \left(\mathtt{W} - \mathtt{B}\mathtt{C}^\top \right) \right\|_F^2$$

- Visibility matrix: $\mathtt{M} \in \{0,1\}^{2N \times M}$

Factorization with Missing Data

- Option 1: "bottom-up SfM"
  1. Decompose problem into fully visible sub-blocks
  2. Apply affine/projective factorization for each block
  3. Merge the results
- Option 2: incremental/sequential SfM
  1. Apply factorization on an initial fully visible sub-block
  2. Triangulate new 3D points and resection new cameras



- Option 3: directly solve

$$\min_{\mathtt{B},\mathtt{C}} \left\| \mathtt{M} \odot \left(\mathtt{W} - \mathtt{B}\mathtt{C}^\top \right) \right\|_F^2$$

Factorization with Missing Data

Affine factorization with missing data

$$\min_{\mathtt{B},\mathtt{C}} \left\| \mathtt{M} \odot (\mathtt{W} - \mathtt{B}\mathtt{C}^\top) \right\|_F^2$$

- Surprising observation
  - The right optimization algorithm has $\approx 90\%$ chance of finding the optimal solution
  - From random starting points
- Algorithm outline: variable projection (VarPro) method, Wiberg algorithm
  - Eliminate e.g. C from the objective in closed form

$$\mathtt{C}^*(\mathtt{B}) = \arg\min_{\mathtt{C}} \left\| \mathtt{M} \odot (\mathtt{W} - \mathtt{B}\mathtt{C}^\top) \right\|_F^2$$

  - Solve cost in the remaining unknowns using 2nd order minimization method

$$\min_{\mathtt{B}} \left\| \mathtt{M} \odot (\mathtt{W} - \mathtt{B}\mathtt{C}^*(\mathtt{B})^\top) \right\|_F^2$$

  - Fewer unknowns, more difficult objective

Factorization with Missing Data

Affine factorization with missing data

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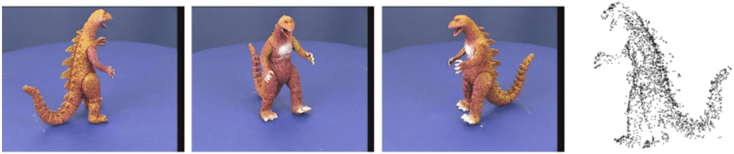
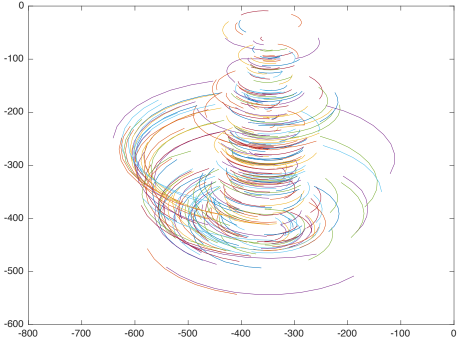
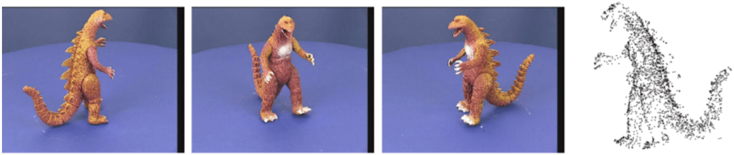
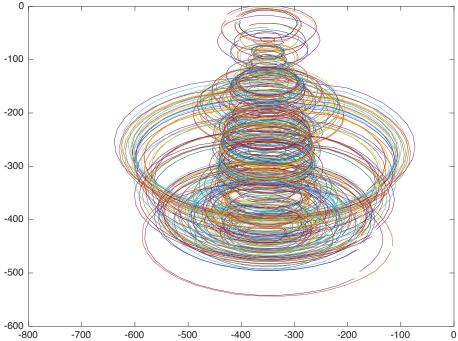




Observed parts of `W`

Factorization with Missing Data

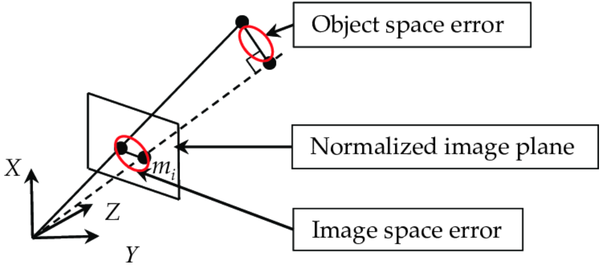




Completed $\mathtt{W}$

Factorization with Missing Data

- Can we use VarPro in the projective setting?
- Affine camera model is not approximating a pinhole camera well
- $\pi(\cdot)$ is too non-linear
- Object-space error: point line distance



Factorization with Missing Data

- Objective based on object-space error

$$\ell_{\mathsf{OSE}} := \sum\nolimits_{i,j} m_{ij} \left\| \mathtt{P}_{i,1:2}\mathbf{X}_j - (\mathtt{P}_{i,3}\mathbf{X}_j)\mathbf{x}_{ij} \right\|^2$$

Degenerate optimal solution $\mathtt{P}_i = 0$ or $\mathbf{X}_j = \mathbf{0}$

- Affine factorization cost: no such degeneracy

$$\ell_{\mathsf{Affine}} := \sum\nolimits_{i,j} m_{ij} \left\| \mathtt{P}_{i,1:2}\mathbf{X}_j - \mathbf{x}_{ij} \right\|^2$$

- Use convex combination of affine and OSE costs

$$\ell_{\mathsf{pOSE}} := (1-\eta)\ell_{\mathsf{OSE}} + \eta\ell_{\mathsf{Affine}}$$

- Benefits
  - Better approximation of pinhole cameras
  - We can still apply the VarPro algorithm

Factorization with Missing Data

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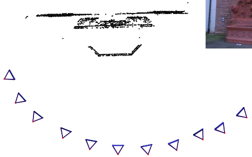
Factorization with Missing Data

- "Pseudo object-space error": $\ell_{\mathsf{pOSE}} := (1-\eta)\ell_{\mathsf{OSE}} + \eta\ell_{\mathsf{Affine}}$



"ground truth"

$\eta = 0.5$

Factorization with Missing Data

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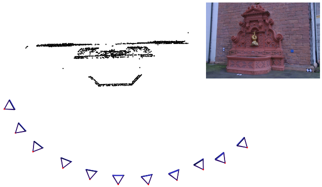

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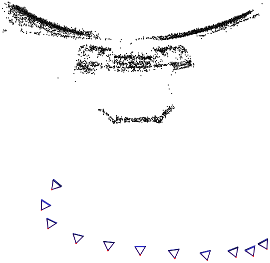
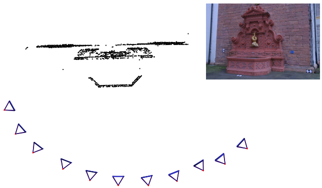


$\eta = 0.1$

Factorization with Missing Data

- "Pseudo object-space error": $\ell_{\mathsf{pOSE}} := (1-\eta)\ell_{\mathsf{OSE}} + \eta\ell_{\mathsf{Affine}}$



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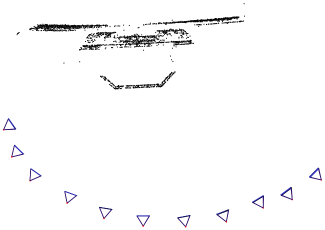

$\eta = 0.01$

Factorization with Missing Data

Video

Factorization with Missing Data

Limitations

- VarPro has $\approx 75 - 90\%$ success rate: may need to run algorithm multiple times
- All factorization methods are non-robust
- Return 3D models up to projective ambiguity

Upcoming

Monday, Dec 11: guest lecture

IDE-D2505-7, MT12, MT14 & SB-D042