Restricted Boltzmann machine

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All equations are taken from the course book *Machine Learning With Neural Networks.*

```
mHiddenNeurons = [1,2,3,4,8];

k = 200;
eta = 0.005;
nTrials = 500;
nMinibatches = 40;
nTrialsOuter = 300;
nTrialsInner = 200;

data =    [-1,-1,-1;-1,1,1;1,-1,1;1,1,-1;1,1,1;-1,1,-1;-1,-1,1;1,-1,-1];
xorData = [-1,-1,-1;-1,1,1;1,-1,1;1,1,-1];

mSetsHiddenNeurons = length(mHiddenNeurons);
nPatterns = height(data);
dataProbs = zeros(nPatterns,mSetsHiddenNeurons);
kullback = zeros(1,mSetsHiddenNeurons);

% CD-k, algorithm 3 in the course book
for mNeurons = 1:mSetsHiddenNeurons

    % Averaging the Kullback-Leibler divergence to get a more precise estimate of the RBM's dis
    nKullbackAvg = 1;
    for iKullbackAvg = 1:nKullbackAvg

        nHiddenNeurons = mHiddenNeurons(mNeurons);
        nVisibleNeurons = 3;

        w_ij = randn(nHiddenNeurons,nVisibleNeurons);
        theta_j = zeros(nVisibleNeurons,1);
        theta_i = zeros(nHiddenNeurons,1);

        h_i = zeros(nHiddenNeurons,1);
        b_i = zeros(nHiddenNeurons,1);
        v_j = zeros(nVisibleNeurons,1);
        b_j = zeros(nVisibleNeurons,1);

        probB_i = zeros(nHiddenNeurons,1);
        probB_j = zeros(nVisibleNeurons,1);

        % Training the Boltzmann machine for a number of trials
        for iTrial = 1:nTrials

            deltaWeight_ij = zeros(nHiddenNeurons,nVisibleNeurons);
            deltaTheta_j = zeros(nVisibleNeurons,1);
            deltaTheta_i = zeros(nHiddenNeurons,1);

            % Training the Boltzmann machine over minibatches
            for iMiniBatch = 1:nMinibatches
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```
        nXorPatterns = height(xorData);
        randPattern = randi(nXorPatterns);
        v_0 = xorData(randPattern,:)';

        % Updating hidden neurons
        for iNeuron = 1:nHiddenNeurons
            b_i(iNeuron) = w_ij(iNeuron,:)*v_0 - theta_i(iNeuron);
            probB_i(iNeuron) = 1/(1 + exp(-2*b_i(iNeuron)));
            r = rand;
            if r < probB_i(iNeuron)
                h_i(iNeuron) = 1;
            else
                h_i(iNeuron) = -1;
            end
        end

        % MCMC
        v_j = v_0;
        b_i0 = b_i;
        for t = 1:k

            % Updating visible neurons
            for jNeuron = 1:nVisibleNeurons
                b_j(jNeuron) = w_ij(:,jNeuron)'*h_i - theta_j(jNeuron);
                probB_j(jNeuron) = 1/(1 + exp(-2*b_j(jNeuron)));
                r = rand;
                if r < probB_j(jNeuron)
                    v_j(jNeuron) = 1;
                else
                    v_j(jNeuron) = -1;
                end
            end

            % Updating hidden neurons
            for iNeuron = 1:nHiddenNeurons
                b_i(iNeuron) = w_ij(iNeuron,:)*v_j - theta_i(iNeuron);
                probB_i(iNeuron) = 1/(1 + exp(-2*b_i(iNeuron)));
                r = rand;
                if r < probB_i(iNeuron)
                    h_i(iNeuron) = 1;
                else
                    h_i(iNeuron) = -1;
                end
            end
        end

        % Computing weight and threshold increments
        for iNeuron = 1:nHiddenNeurons
            deltaWeight_ij(iNeuron,:) = deltaWeight_ij(iNeuron,:) + eta*(tanh(b_i0(iNeu
        end
        deltaTheta_j = deltaTheta_j - eta*(v_0 - v_j);
        deltaTheta_i = deltaTheta_i - eta*(tanh(b_i0) - tanh(b_i));
    end
```

```
        % Updating weights and thresholds
        w_ij = w_ij + deltaWeight_ij;
        theta_j = theta_j + deltaTheta_j;
        theta_i = theta_i + deltaTheta_i;
    end

    % Computing the distribution of the trained Boltzmann machine
    counter = zeros(nPatterns,1);

    for iTrialOuter = 1:nTrialsOuter

        randPattern = randi(nPatterns);
        v_j = data(randPattern,:)';

        % Updating hidden neurons
        for iNeuron = 1:nHiddenNeurons
            b_i(iNeuron) = w_ij(iNeuron,:)*v_j - theta_i(iNeuron);
            probB_i(iNeuron) = 1/(1 + exp(-2*b_i(iNeuron)));
            r = rand;
            if r < probB_i(iNeuron)
                h_i(iNeuron) = 1;
            else
                h_i(iNeuron) = -1;
            end
        end

        for iTrialInner = 1:nTrialsInner

            % Updating visible neurons
            for jNeuron = 1:nVisibleNeurons
                b_j(jNeuron) = w_ij(:,jNeuron)'*h_i - theta_j(jNeuron);
                probB_j(jNeuron) = 1/(1 + exp(-2*b_j(jNeuron)));
                r = rand;
                if r < probB_j(jNeuron)
                    v_j(jNeuron) = 1;
                else
                    v_j(jNeuron) = -1;
                end
            end

            % Updating hidden neurons
            for iNeuron = 1:nHiddenNeurons
                b_i(iNeuron) = w_ij(iNeuron,:)*v_j - theta_i(iNeuron);
                probB_i(iNeuron) = 1/(1 + exp(-2*b_i(iNeuron)));
                r = rand;
                if r < probB_i(iNeuron)
                    h_i(iNeuron) = 1;
                else
                    h_i(iNeuron) = -1;
                end
            end

            for muPattern = 1:nPatterns
```

```
                    target = data(muPattern,:)';
                    if isequal(v_j,target)
                        counter(muPattern) = counter(muPattern) + 1;
                        break
                    end
                end
            end
        end

        % Avergaing the distribution due to inner and outer trial loops
        for muPattern = 1:nPatterns
            dataProbs(muPattern,mNeurons) = counter(muPattern)/(nTrialsOuter*nTrialsInner);
        end

        % Computing the Kullback-Leibler divergence with the numerical results of the distribut
        pB = dataProbs(:,mNeurons);
        logPb = zeros(nPatterns,1);

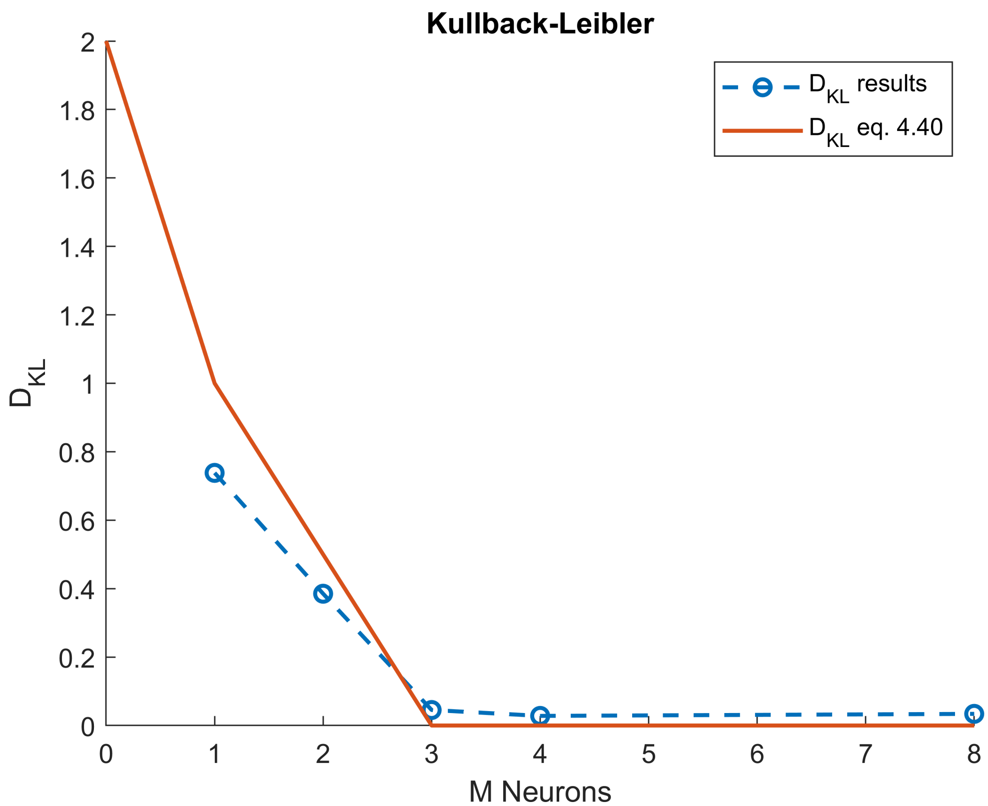
        pData = 1/nXorPatterns;
        logPdata = log(pData);

        kullbackTemp = 0;
        for muPattern = 1:4
            if pB(muPattern) == 0
                logPb(muPattern) = 0;
            else
                logPb(muPattern) = log(pB(muPattern));
            end
            if muPattern > nXorPatterns
                pData = 0;
                logPdata = 0;
            end
            kullback(mNeurons) = kullback(mNeurons) + pData*(logPdata - logPb(muPattern));
        end
    end
    % Avergaing the Kullback-Leibler divergence
    kullback(mNeurons) = kullback(mNeurons) / nKullbackAvg;
end

% Computing the Kullback-Leibler divergence theory
kullbackTheoryX = 0:1:8;
kullbackTheoryY = kullbackTheoryX;
kullbackTheoryY = nVisibleNeurons - floor(log2(kullbackTheoryY' + 1)) - (kullbackTheoryY' + 1)
kullbackTheoryY(4:9) = 0;

% Plotting the Kullback-Leibler divergence theory and numerical results as a function of M hidd
figure
hold on
plot(mHiddenNeurons,kullback,"--o","Linewidth",1.5)
plot(kullbackTheoryX,kullbackTheoryY,"-","Linewidth",1.5)
title("Kullback-Leibler")
xlabel("M Neurons")
ylabel("D_{KL}")
legend("D_{KL} results","D_{KL} eq. 4.40")
```

```
hold off
```




```
dataProbs
```

```
dataProbs = 8×5
    0.1297    0.1033    0.1975    0.1952    0.2681
    0.1038    0.1231    0.2719    0.2586    0.2994
    0.0800    0.2920    0.2925    0.3136    0.2388
    0.1892    0.2254    0.2074    0.2205    0.1778
    0.0916    0.0075    0.0044    0.0024    0.0054
    0.1688    0.1365    0.0072    0.0030    0.0040
    0.0905    0.1041    0.0102    0.0037    0.0036
    0.1464    0.0081    0.0089    0.0029    0.0028
```