

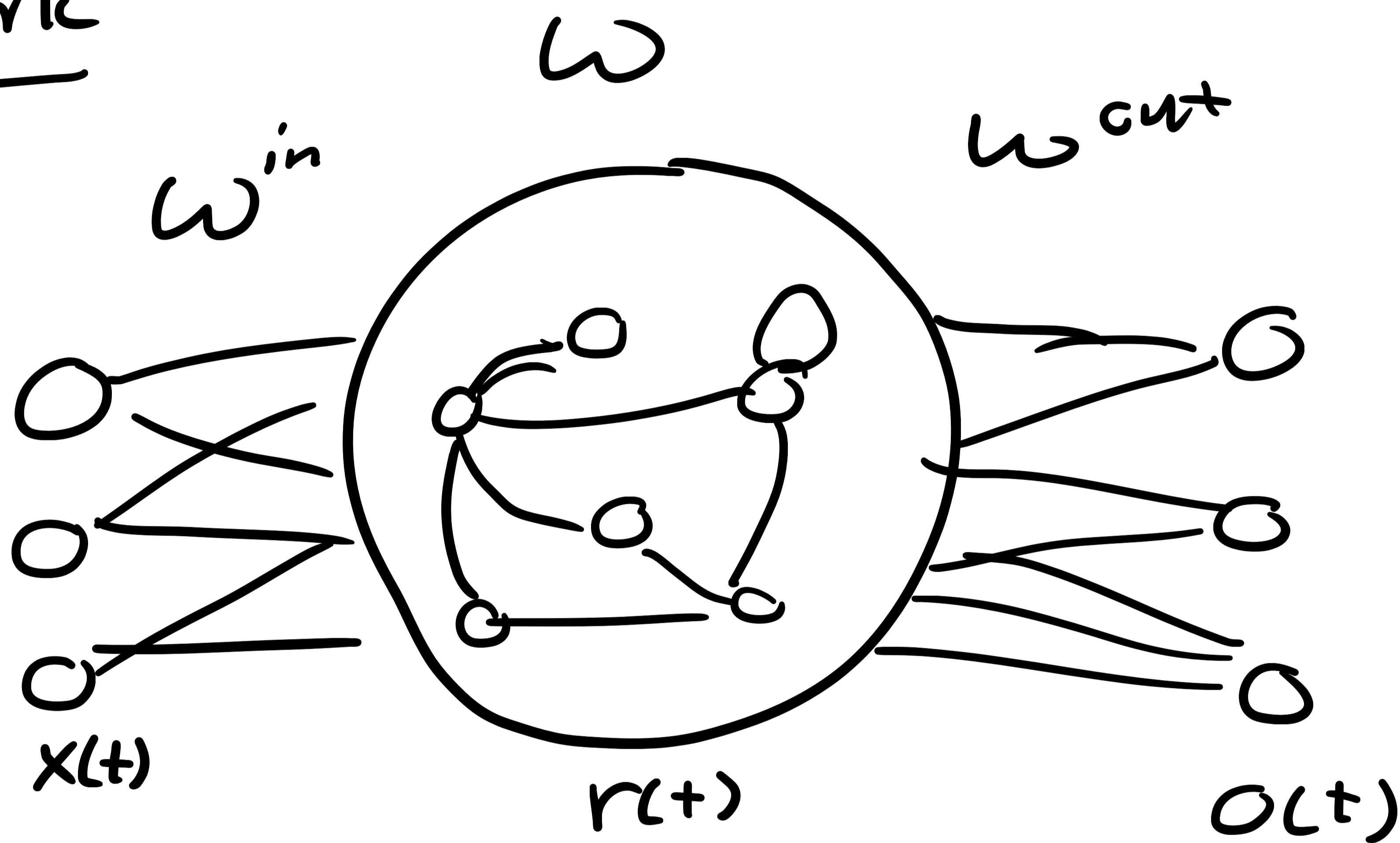
Chaotic time series prediction.

Data
 $X_{\text{train}}, X_{\text{test}}$

$(3, T_{\text{train}})$ $(3, T_{\text{test}})$

No preprocessing needed.

Network



$$w^{in} : (N, 3)$$

$$w : (N, N)$$

$$w^{out} : (3, N)$$

Training dynamics

$$r_i(t+1) = \tanh \left(\sum_j^N w_{ij} r_j(t) + \sum_k^n w_{ik}^{in} x_k(t) \right)$$

$$O_i(t) = \sum_j^N w_{ij}^{out} r_j(t)$$

Prediction dynamics

$$r_i(t+1) = \tanh \left(\sum_j^N w_{ij} r_j(t) + \sum_k^n w_{ik}^{in} O_k(t) \right)$$

Training the reservoir

Initialize: w^{in} , w , $r(0)$.

$$R = \begin{pmatrix} 1 & 1 & & 1 \\ r(0) & r(1) & \dots & r(T_{train}-1) \\ 1 & 1 & & 1 \end{pmatrix}, (N, T_{train})$$

for $t = 0 : (T_{train}-1)$ do:

$$R(:,t) = r(t)$$

$$r_i(t+1) = \tanh\left(\sum_j w_{ij} r_j(t) + \sum_k w_{ik}^{in} x_k(t)\right)$$

end do.

$$x_{train} = \begin{pmatrix} 1 & 1 & & 1 \\ x(0) & x(1) & \dots & x(T_{train}-1) \\ 1 & 1 & & 1 \end{pmatrix}$$

Optimize: $\underline{x}(+) \approx o(+)$ = $w^{out} r(+)$

$$x_{train} \approx w^{out} R$$

Linear regression: $x_{train} R^+ \approx w^{out}$

Ridge regression:

$$w^{out} = x_{train} R^T (R R^T + k I)^{-1}$$

ridge parameter

Prediction

Feed x_{test} using

$$r_i(t+1) = \tanh \left(\sum_j w_{ij} r_j(t) + \sum_k w_{ik}^{in} x_k(t) \right)$$

Then, predict by replacing $x(t)$ with $O(t)$,

$$x(t) \approx O(t)$$

$$\sum_t^{\bar{T}_{\text{train}}} (x(t) - O(t))^2$$

\Downarrow

$$(x(t) - \underbrace{w^{\text{out}} r(t)}_{})^2$$

$$r(t+1) > \tanh (w_r(t) + w^{in} x(t))$$

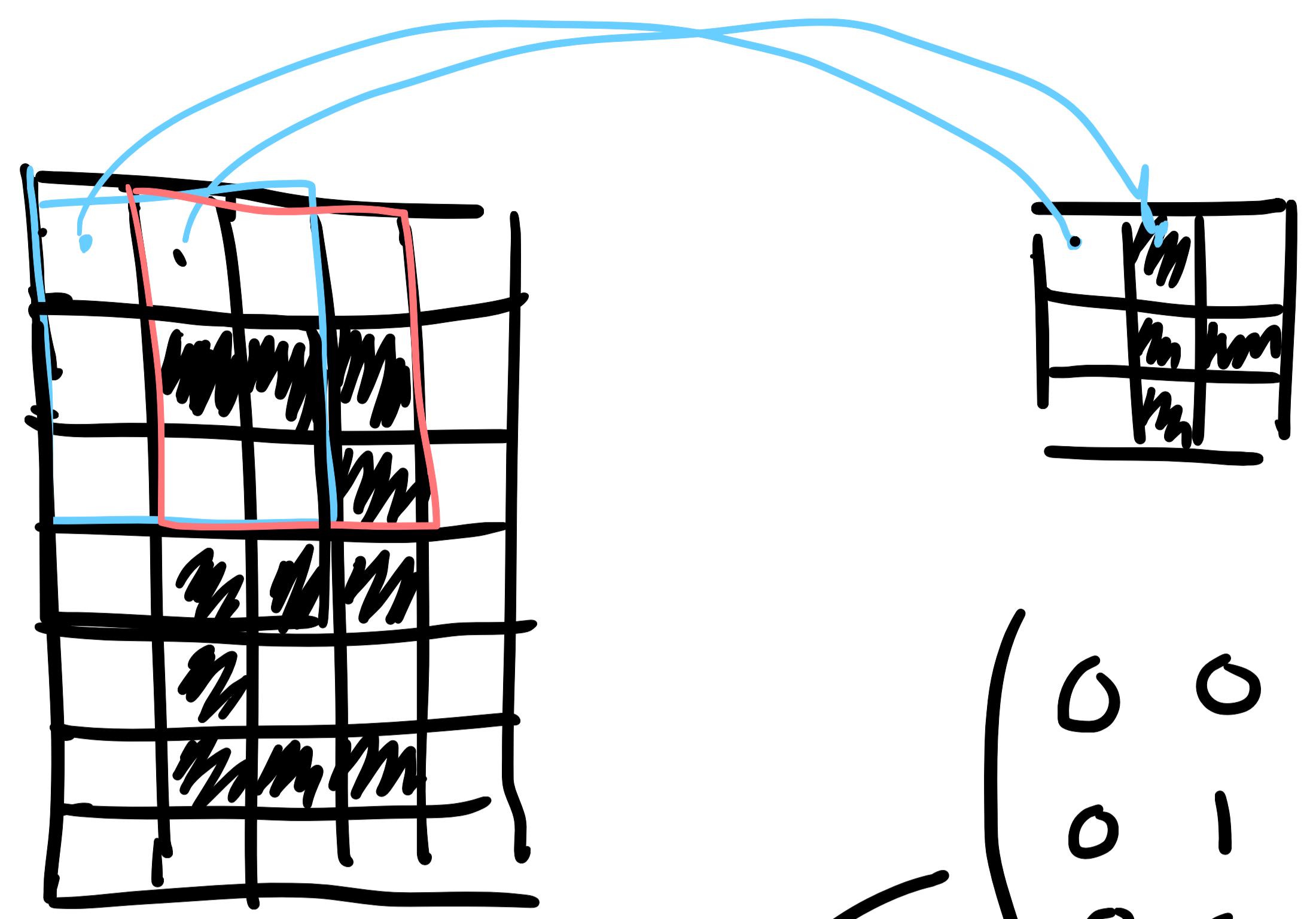
$$\uparrow$$

Final: $x(\bar{T}_{\text{test}} - 1)$

$$\downarrow$$

$r(\bar{T}_{\text{test}})$

$$r(\bar{T}_{\text{test}} + 1)$$



$\blacksquare - 1$
 $\square - 0$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$V^{(2)} = \boxed{\begin{array}{ccc} 2 & 2 & 2 \\ 2 & 3 & 3 \\ 3 & 2 & 2 \\ 3 & 2 & 2 \\ 3 & 2 & 1 \end{array}} \quad 5 \times 3$$

$$M^{(2)} = \begin{pmatrix} 3 & & \\ 3 & & \\ 3 & & \end{pmatrix}$$

