Python Lab 3 SSY316

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Bayesian Network

Figure 1 shows the ground truth of the probabilities to be implemented and Figure 2 shows the implemented Bayesian Network and its corresponding probabilities. We can see that they match up as expected. Verifying that the probabilities are correctly implemented we run the command `model.check_model()` which displays `True`.

U=	T	F
P(U)	0.15	0.85

M=	T	F
P(M)	0.05	0.95

B=	T	F
P(B)	0.10	0.90

S=	T	F
P(S|RB)	0.40	0.60
P(S|R!B)	0.05	0.95
P(S|!RB)	0.12	0.88
P(S|!R!B)	0.02	0.98

R=	T	F
P(R|MBU)	0.95	0.05
P(R|MB!U)	0.90	0.10
P(R|M!BU)	0.85	0.15
P(R|M!B!U)	0.76	0.24
P(R|!MBU)	0.18	0.82
P(R|!MB!U)	0.06	0.94
P(R|!M!BU)	0.14	0.86
P(R|!M!B!U)	0.04	0.96

Figure 1: Ground truth probabilities.

U(T)	0.15
U(F)	0.85

M(T)	0.05
M(F)	0.95

B(T)	0.1
B(F)	0.9

R	R(T)	R(T)	R(F)	R(F)
B	B(T)	B(F)	B(T)	B(F)
S(T)	0.4	0.05	0.12	0.02
S(F)	0.6	0.95	0.88	0.98

M	M(T)	M(T)	M(T)	M(T)	M(F)	M(F)	M(F)	M(F)
B	B(T)	B(T)	B(F)	B(F)	B(T)	B(T)	B(F)	B(F)
U	U(T)	U(F)	U(T)	U(F)	U(T)	U(F)	U(T)	U(F)
R(T)	0.95	0.9	0.85	0.76	0.18	0.06	0.14	0.04
R(F)	0.05	0.1	0.15	0.24	0.82	0.94	0.86	0.96

Figure 2: Probabilities of the implemented Bayesian Network.

Table 1 shows the probability that the content should be removed from the platform, i.e., $p(R)$, and the probability that the content should be removed from platform given that the ML model flags it, i.e., $p(R|M)$.

Table 1: Probability distributions for $p(R)$ and $p(R \mid M)$.

$p(R = \texttt{True})$	0.0938
$p(R = \texttt{True} \mid M = \texttt{True})$	0.7869

We verify the probabilities in Table 1 with manual calculations.

$$
\begin{aligned}
p(R=\texttt{True}) = &\sum_{M,U,B} p(R=\texttt{True} \mid M,U,B)p(M)p(U)p(B) \\
&= 0.05\cdot 0.95\cdot 0.15\cdot 0.1 + 0.95\cdot 0.18\cdot 0.15\cdot 0.1 + 0.05\cdot 0.9\cdot 0.85\cdot 0.1 + 0.95\cdot 0.06\cdot 0.85\cdot 0.1 \\
&+ 0.05\cdot 0.85\cdot 0.15\cdot 0.9 + 0.95\cdot 0.14\cdot 0.15\cdot 0.9 + 0.05\cdot 0.76\cdot 0.85\cdot 0.9 + 0.95\cdot 0.04\cdot 0.85\cdot 0.9 \\
&= 0.09378
\end{aligned}
$$

$$
\begin{aligned}
p(R=\texttt{True} \mid M=\texttt{True}) &= \sum_{U,B} p(R=\texttt{True} \mid M=\texttt{True},U,B)p(U)p(B) \\
&= 0.95\cdot 0.15\cdot 0.1 + 0.9\cdot 0.85\cdot 0.1 + 0.85\cdot 0.15\cdot 0.9 + 0.76\cdot 0.85\cdot 0.9 \\
&= 0.7869
\end{aligned}
$$

This verifies that the probabilities stated in Table 1 were computed correctly.

Other probabilities of our interests were the probabilities that a user should be suspended given only one violating observation M, B, and U respectively. Table 2 shows these probabilities as $p(S \mid M)$, $p(S \mid B)$, and $p(S \mid U)$. It shows that there is a quite low probability that a user would be suspended given only one violating observation.

Table 2: Probability distributions for $p(S \mid M)$, $p(S \mid B)$, and $p(S \mid U)$.

$p(S=\texttt{True} \mid M=\texttt{True})$	0.0763
$p(S=\texttt{True} \mid B=\texttt{True})$	0.1535
$p(S=\texttt{True} \mid U=\texttt{True})$	0.0409

Markov Random Fields

While comparing both the original image (Figure 3) and the de-noised image (Figure 4) we can observe that both images are very similar. However we can differentiate the de-noised image quite easily because there are still little imperfections with white grains spread around the image. Nevertheless, the image is extremely similar to the original considered it had some noise on it previously.

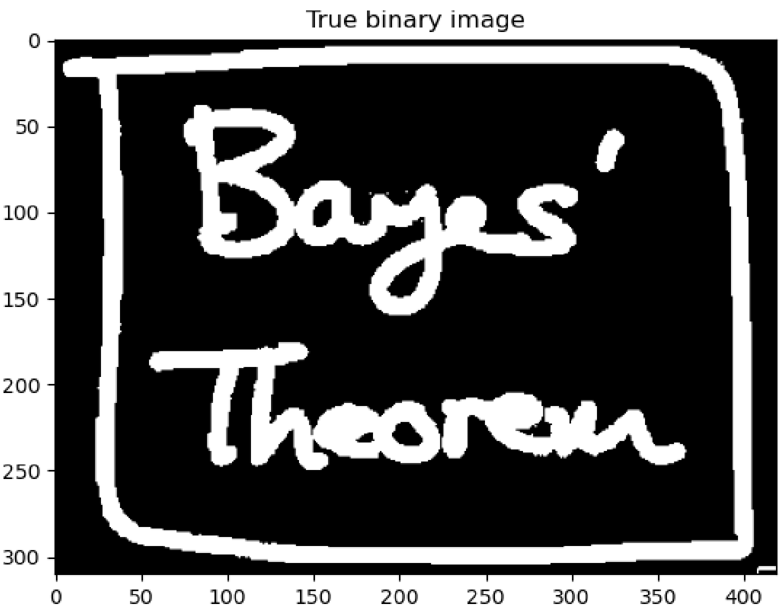

Figure 3: Original image.

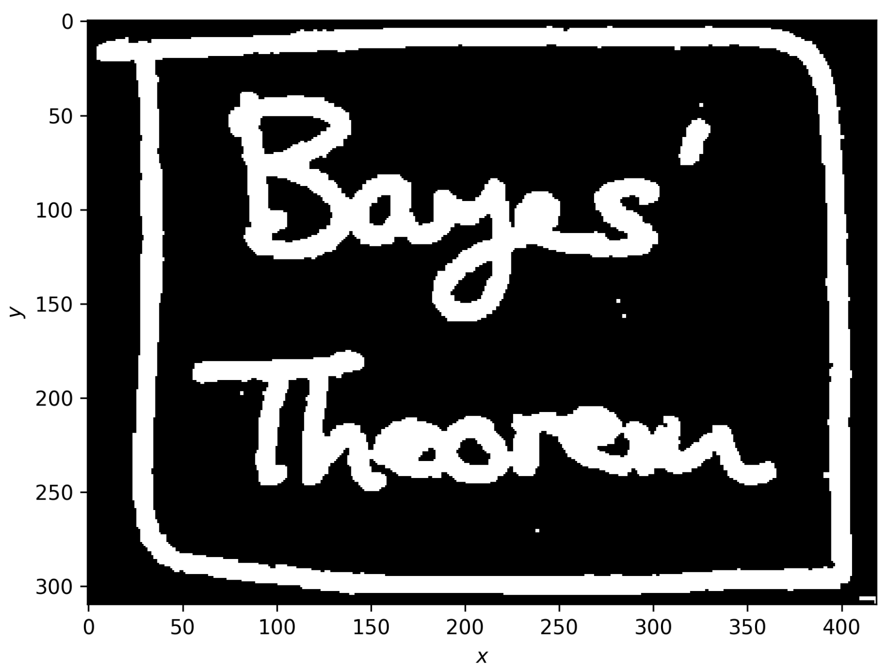

Figure 4: De-noised image for 5 iterations where the noise level was $\sigma = 1$ with $\beta = 2$, $\eta = 1$, and $h = 0$.

The Figures 5, 6, 7, and 8 show how the normalised mean square error (NMSE) performs against the parameters

σ, h, β and η respectively. β, and η were increased from zero since they are constrained to be larger than zero, whereas h is not. Unsurprisingly, it shows that the NMSE increases as the noise level increases. Interestingly enough, varying h shows a funny looking curve where the NMSE seems to be constant for $h < -2.5$ and $h > 3.5$ with a minimum at $h \approx 0$. For both β and η the NMSE decreases drastically as both parameters are increased from zero. However, as the NMSE decreases with increasing β, it slowly starts to increase again as η increases. The minimum NMSE for varying β looks like to never be obtained, but choosing $\beta \geq 2$ seems to give promising results. Varying η, the minimum NMSE looks like to be obtained when $\eta \approx 1$. Thus, the optimal parameter values for the Ising model from this analysis seem to be approximately $\beta \geq 2$, $\eta = 1$, and $h = 0$. These are the exact parameter values used to obtain the de-noised image in Figure 4, which shows almost a perfectly de-noised image.

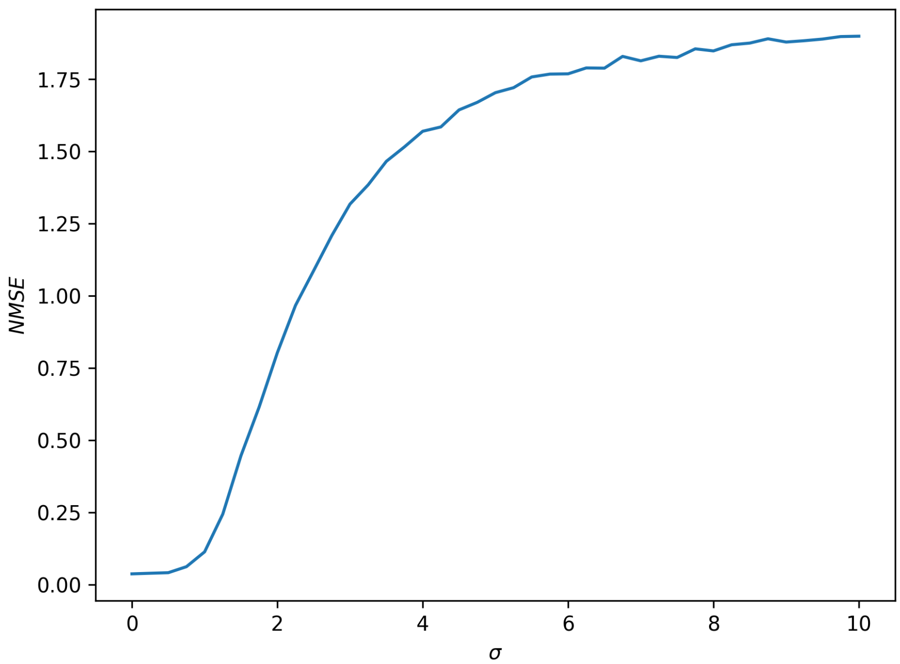

Figure 5: NMSE vs noise σ, with $\beta = 1$, $h = 1$ and $\eta = 1$.

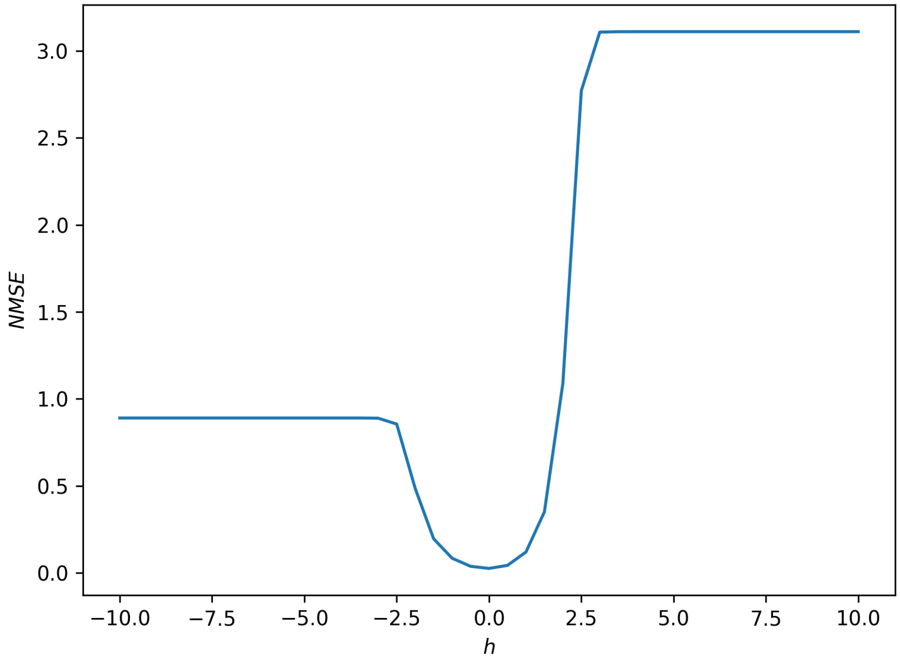

Figure 6: NMSE vs h, with $\beta = 1$, $\eta = 1$ and $\sigma = 1$.

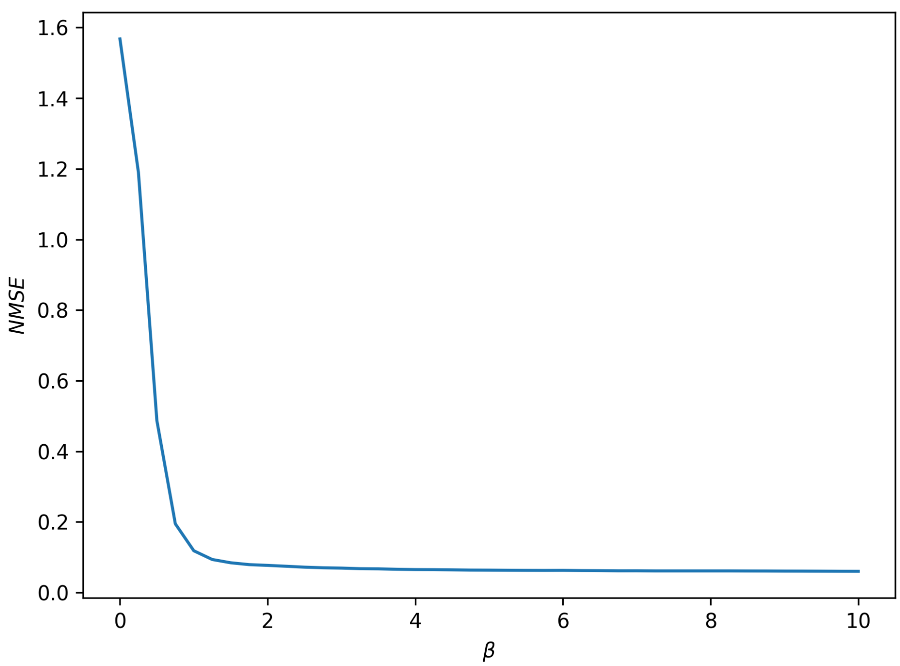

Figure 7: NMSE vs β, with $h = 1$, $\eta = 1$ and $\sigma = 1$.

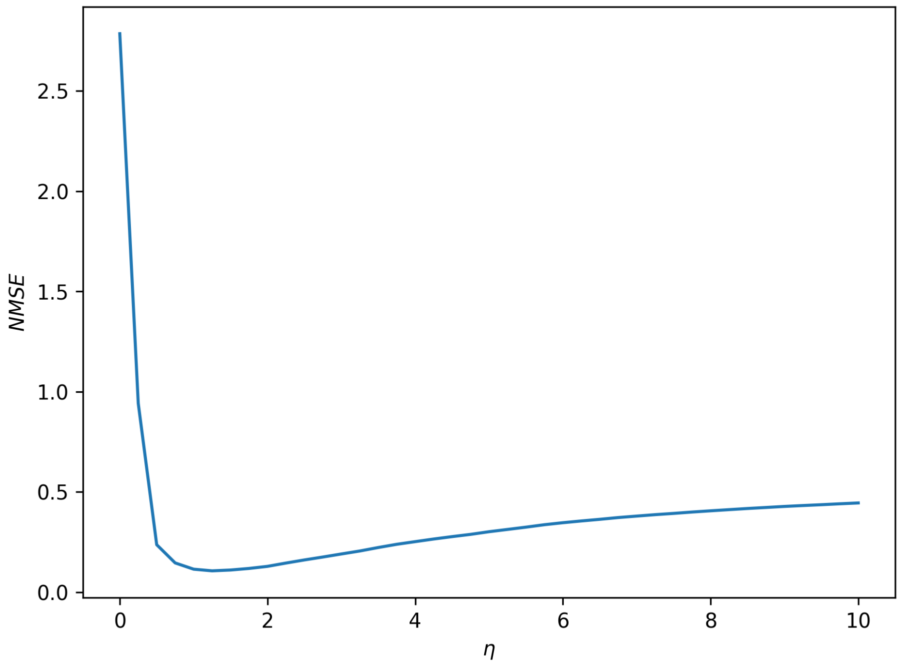

Figure 8: NMSE vs η, with $\beta = 1$, $h = 1$ and $\sigma = 1$.