Python Lab 4 SSY316

Matthew Newson, Erik Norlin

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Gibbs sampler

Activity 1

Mathematical proofs are in the jupyter notebook as by the instructions.

Figures 1 and 2 show trace plots of how Gibbs sampled μ and σ^2 vary over the number of samples. We can see that both μ and σ^2 converge to the true values where the burn-in period is about 200 samples for μ and about 20 samples for σ^2.

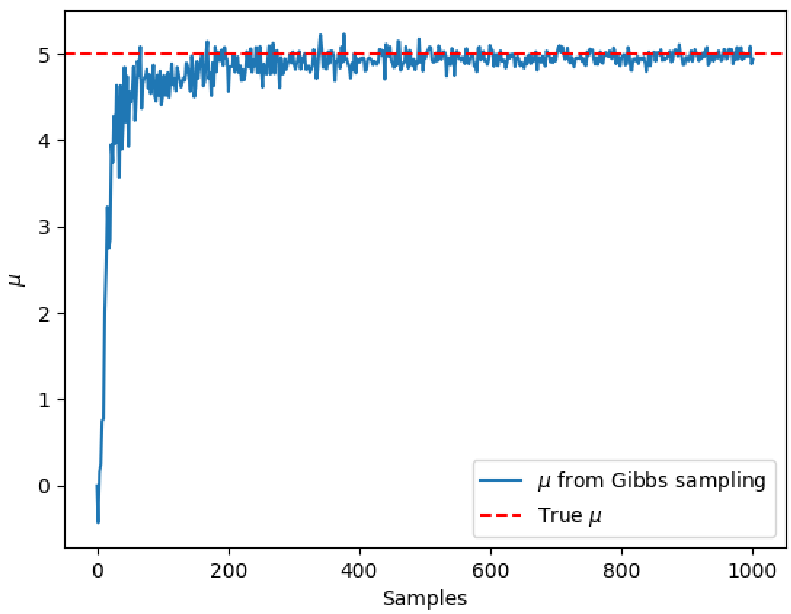

Figure 1: Trace plot of Gibbs sampling for μ varying over 1000 samples.

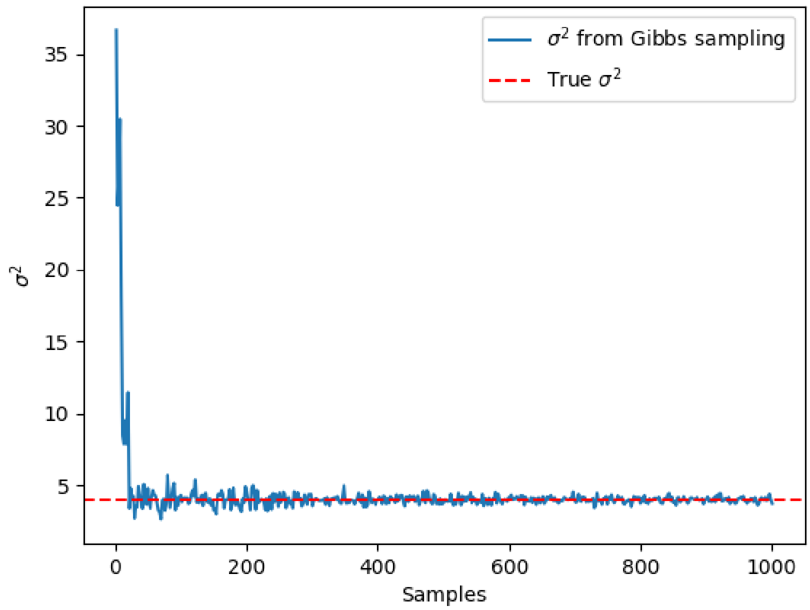

Figure 2: Trace plot of Gibbs sampling for σ^2 varying over 1000 samples.

Activity 2

Mathematical proofs are in the jupyter notebook as by the instructions.

Figure 3 shows trace plot of how Gibbs sampled μ vary over the number of samples. We can see that μ converges to the true value where the burn-in period for is about 100 samples.

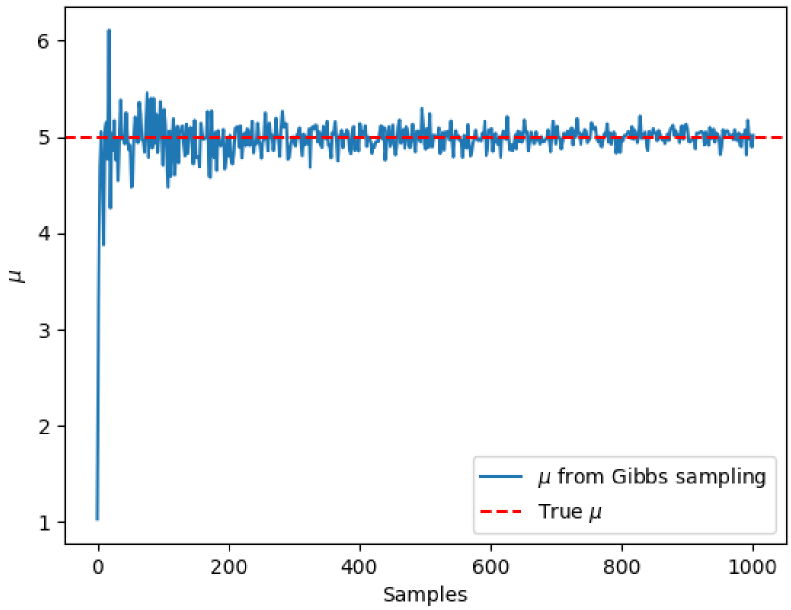

Figure 3: Trace plot of Gibbs sampling for μ varying over 1000 samples.

Mean Field approximation

Activity 3

Table 1 shows the true values, the means of the simulated data, and the means of mean field approximation of μ and τ. The mean field approximation comes close to the true value of μ, however this is not as true for τ. This can have to do with parameter initialization, or perhaps that ELBO converges too soon.

Table 1: True values, means of simulated data, and means of mean field approximation of μ and τ.

	μ	τ
True value	5	2
Simulated data	4.99	2.15
Mean field approximation	4.97	1.38

Activity 4

Table 2 shows the true value, the mean of the simulated data, and the mean of mean field approximation of λ. The mean field approximation comes reasonably close to the true value of λ.

Table 2: True value, mean of simulated data, and mean of mean field approximation of λ.

	λ
True value	1.54
Mean of samples	1.41
Mean field approximation	1.42

Image denosing with Gibbs sampling

Here we are denoising a noisy image (Figure 5) of the the image in Figure 4 using Gibbs sampling. The parameters used for this entire section was $\beta = 2$, $\eta = 1$ with noise $\sigma = 1$ for 15 iterations. During investigation of the effect of different parameter values only one parameter at a time was changed from 0 to 10 while the rest of the parameters remained fixed.

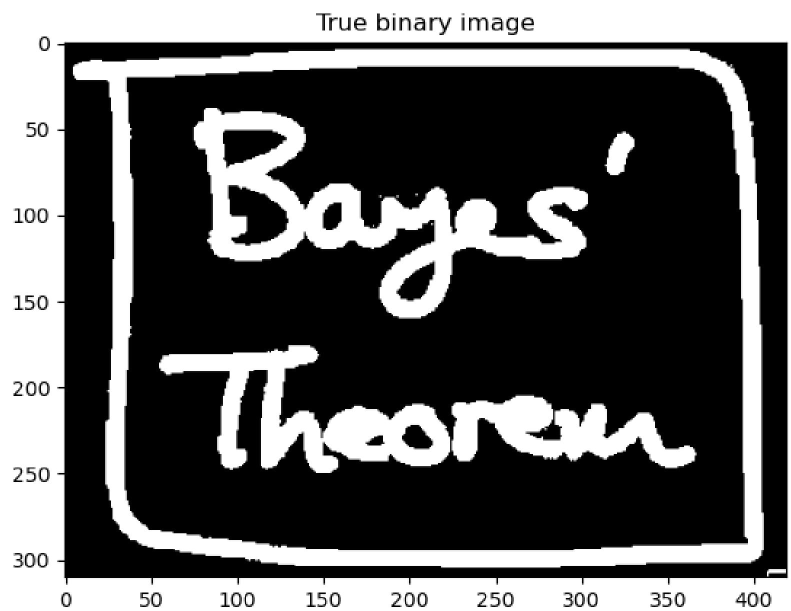

Figure 4: True image.

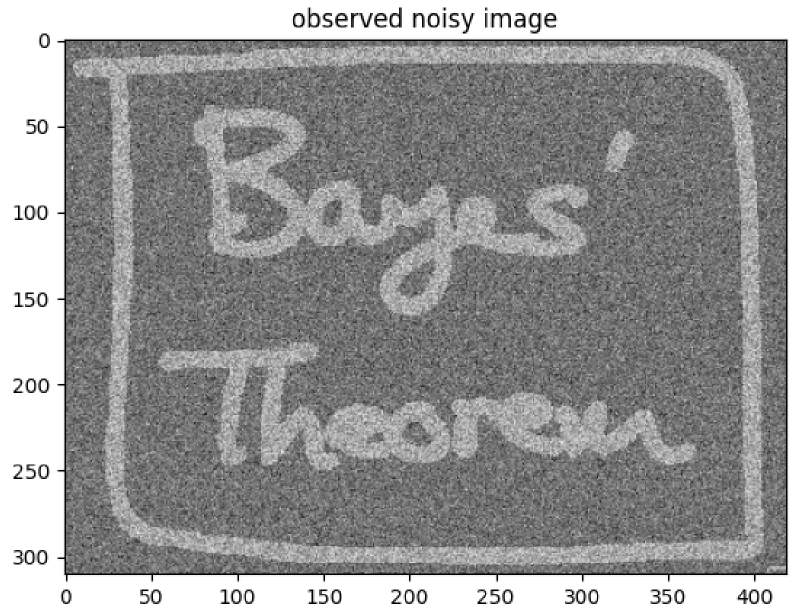

Figure 5: Noisy image with $\sigma = 1$.

Activity 5

Figure 6 shows a denoised image of Figure 5 using Gibbs sampling. We can see that Gibbs sampling does a good job. The denoised image is very similar to the true image.

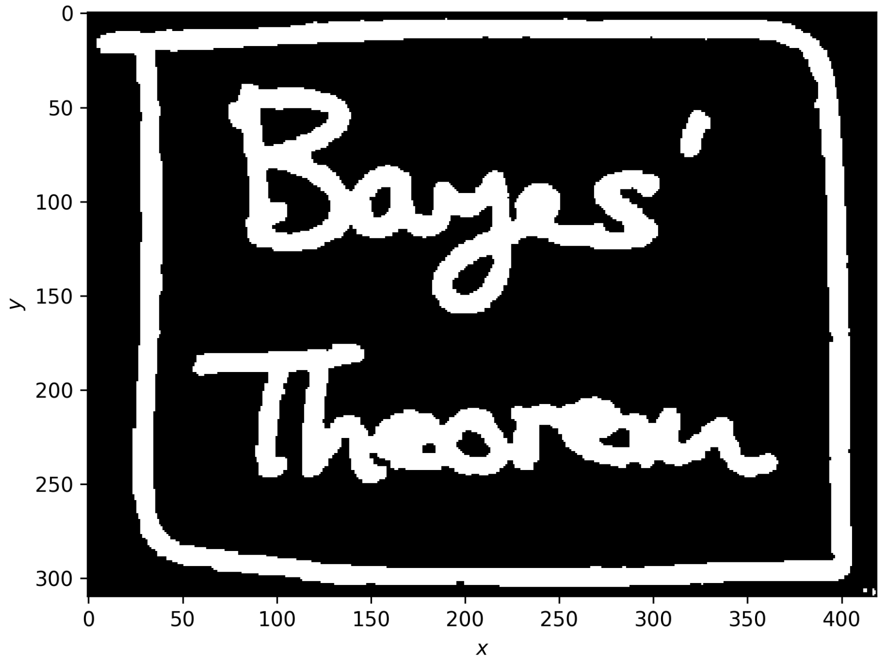

Figure 6: Denoised image of Figure 5 using Gibbs sampling.

Next, the effect that the parameter values has on the NMSE of the denoised image is investigated. Figure 7 shows how the NMSE changes with varying noise level. Low noise obviously gives low NMSE. While the noise level increases, so does the NMSE.

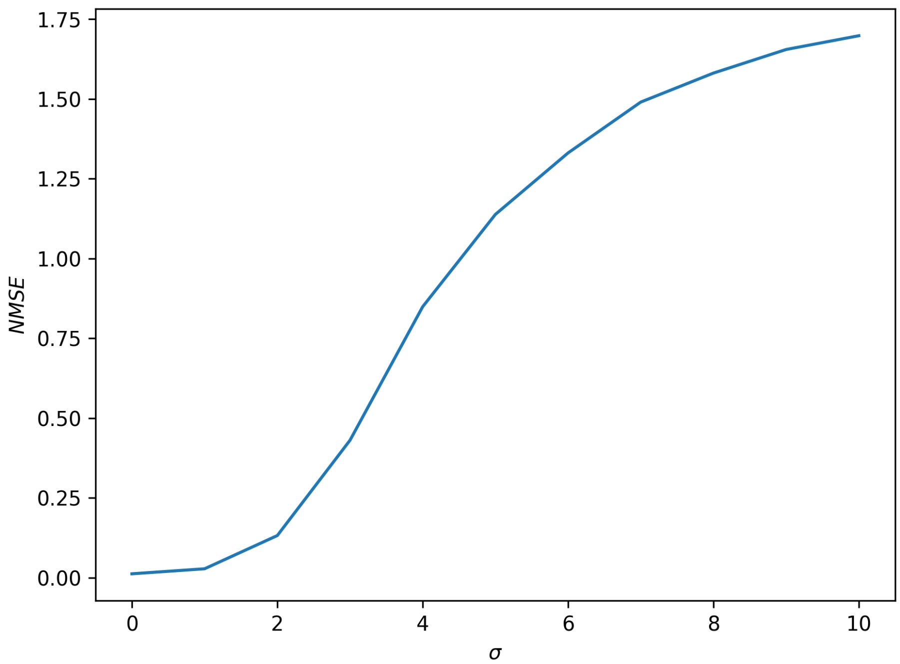

Figure 7: How the NMSE changes with varying noise level.

Figure 8 shows how the NMSE changes with varying β. The NMSE decreases as β increases. However, for $\beta > 1$, the effect of β seems to remain constant.

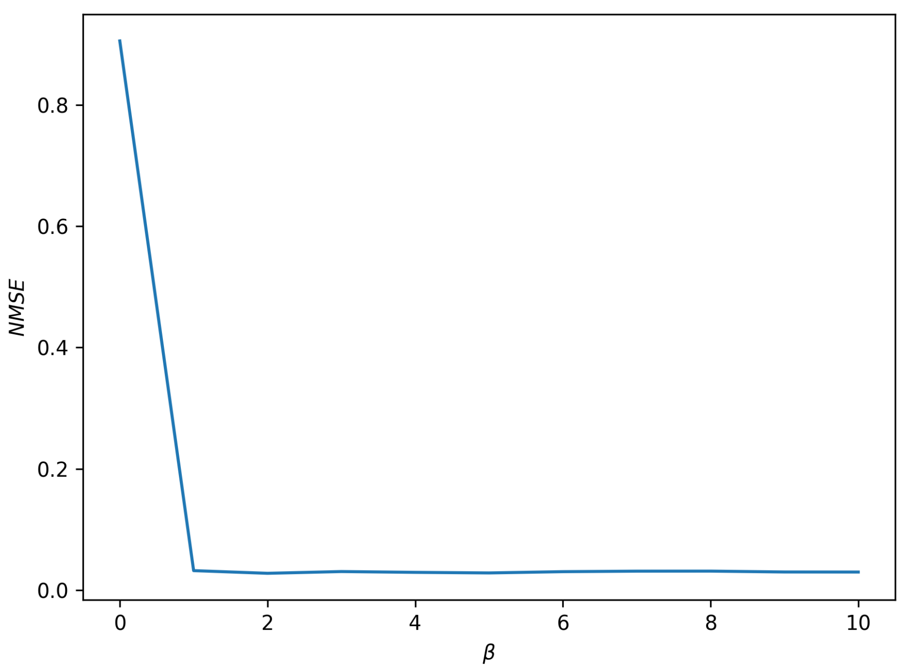

Figure 8: How the NMSE changes with varying β.

Figure 9 shows how the NMSE changes with varying η. The NMSE decreases as η increase to a start but increases as η increases as well.

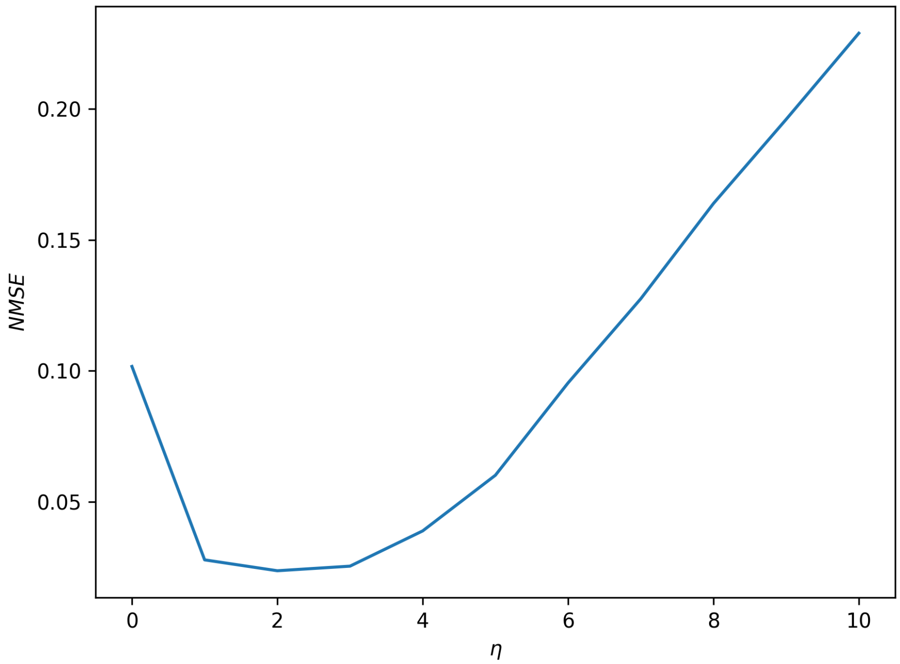

Figure 9: How the NMSE changes with varying η.

Image denosing with Mean-field approximation

Here we are denoising the same noisy image (Figure 5) as before using mean field approximation. The parameters used for in this section was $\beta = 2$, $\eta = 1$ with noise $\sigma = 1$ for 10 iterations in activity 6 and 15 iterations in activity 7. During investigation of the effect of different parameter values only one parameter at a time was changed from 0 to 10 while the rest of the parameters remained fixed.

Activity 6

Figure 10 shows a denoised image of Figure 5 using mean field approximation. We can see that mean field approximation does a good job. The denoised image is very similar to the true image.

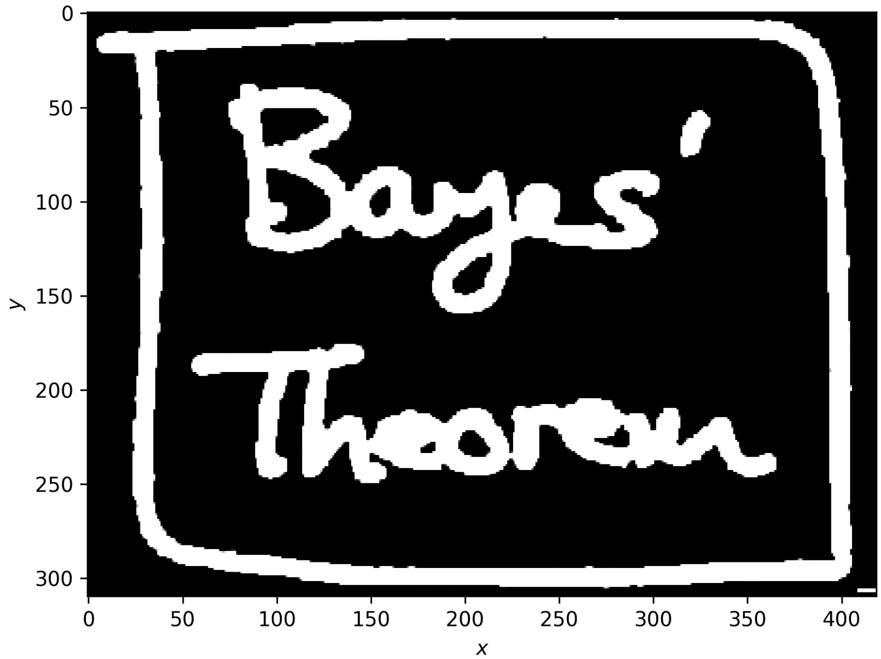

Figure 10: Denoised image of Figure 5 using mean field approximation.

Next, the effect that the parameter values has on the NMSE of the denoised image is investigated. Figure 11 shows how the NMSE changes with varying noise level. Low noise obviously gives low NMSE. While the noise level increases, so does the NMSE.

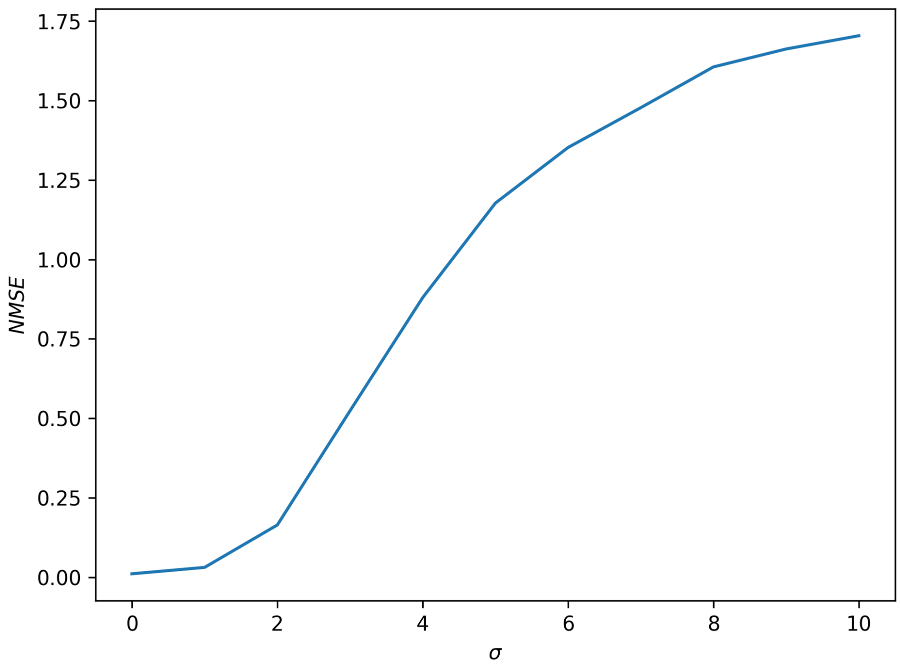

Figure 11: How the NMSE changes with varying noise level.

Figure 12 shows how the NMSE changes with varying β. The NMSE decreases as β increases. However, for $\beta > 1$, the effect of β seems to remain constant.

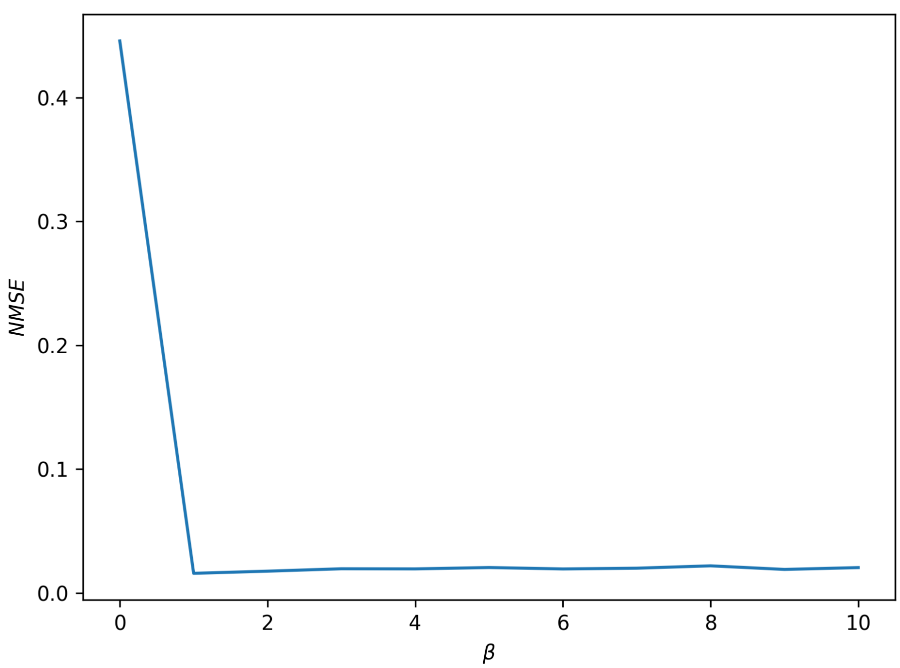

Figure 12: How the NMSE changes with varying β.

Figure 13 shows how the NMSE changes with varying η. The NMSE decreases as η increase to a start but increases as η increases as well.

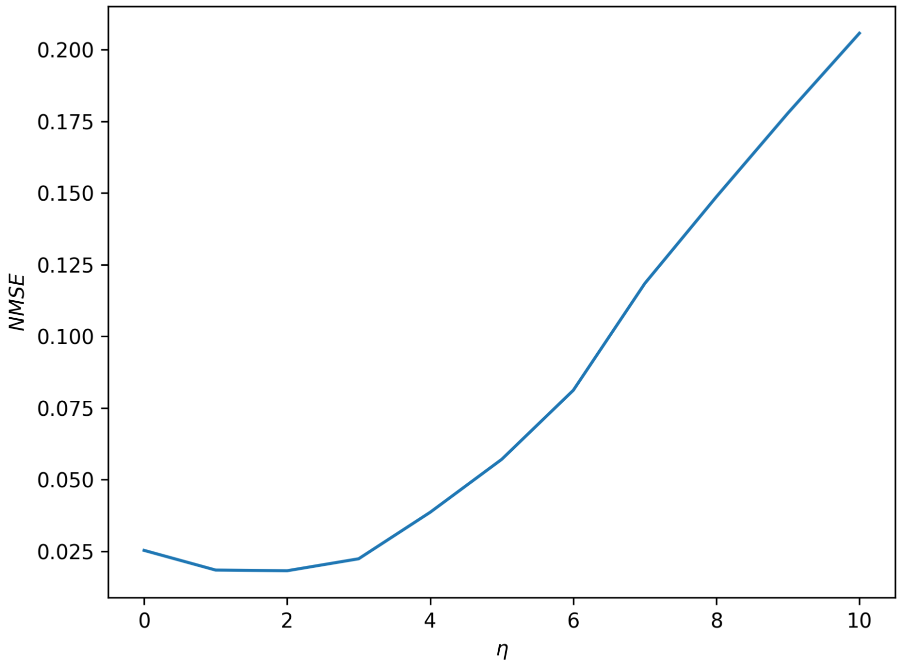

Figure 13: How the NMSE changes with varying η.

Overall, the effect that the parameter values has on the NMSE for mean field approximation follows a similar pattern as Gibbs sampling.

Activity 7

Here we investigate how Markov random fields, Gibbs sampling and mean field approximation compare when denoising the same image (Figure 5) by looking at the NMSE with varying parameter values.

Figures 11, 12 and 13 show how the NMSE changes with varying parameter values for all three methods with the fixed parameters $\sigma = 1$, $\beta = 2$ and $\eta = 1$ ($h = 0$ for Markov random fields). All simulations of every method were run for 15 iterations. They all exhibit strongly similar patterns in how the NMSE changes with respect to the parameters. We can see from the figures that mean field approximation always perform better than the other methods. Gibbs sampling and Markov random fields follow each other very closely. Between the two, for some parameter values Markov random fields seems to be the winner, whereas for other parameter values Gibbs sampling seems to be the winner. Either way, for $\beta > 1$ and $1 < \eta < 3$ all three methods perform very similar in denoising the image.

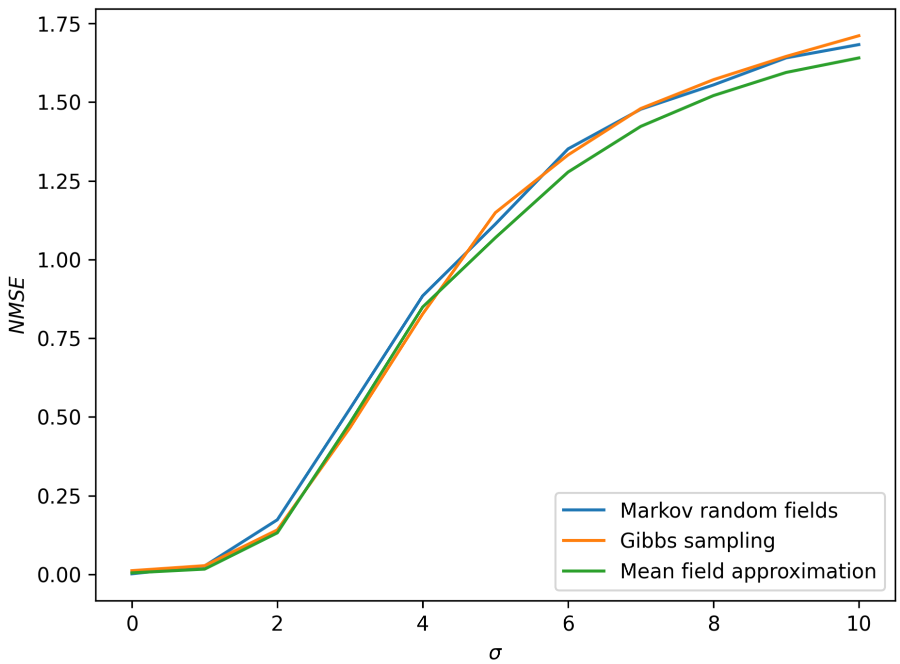

Figure 14: How the NMSE changes with varying noise level.

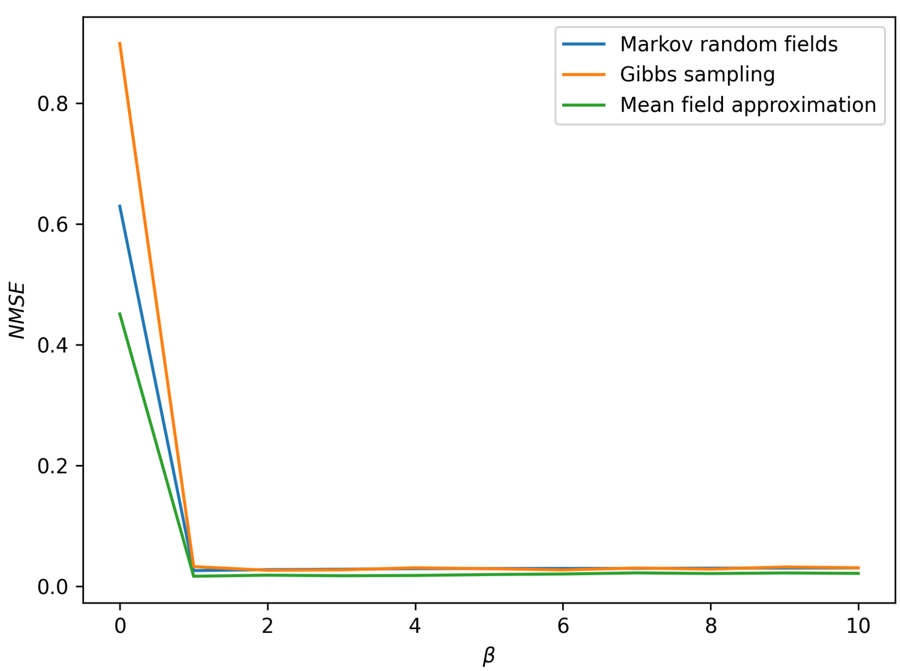

Figure 15: How the NMSE changes with varying β.

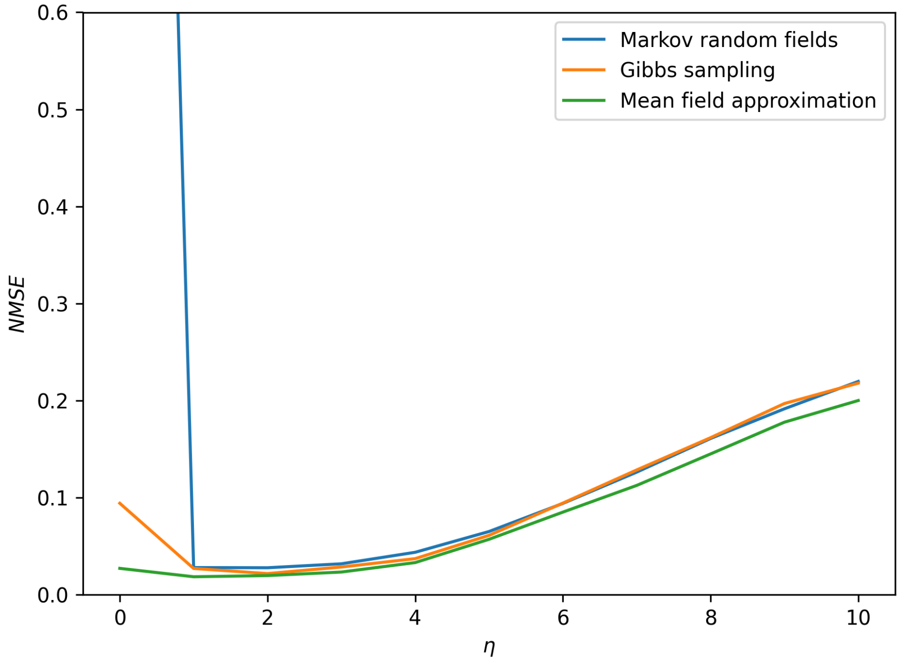

Figure 16: How the NMSE changes with varying η.