Homework 4

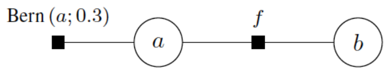
Deadline: November 30, 13:15

Exercise 1

Show that there are $2^{M(M-1)/2}$ distinct undirected graphs over a set of M distinct random variables. Draw the 8 possibilities for the case of M = 3.

Exercise 2

Consider the factor graph below, where a and b can be either true or false. The definition of the factor f is provided in the table, and the prior distribution for a follows the Bernoulli distribution $Bern(0.3)$.



a	b	$f(a, b)$
`true`	`true`	0.9
`true`	`false`	0.1
`false`	`true`	0.2
`false`	`false`	0.8

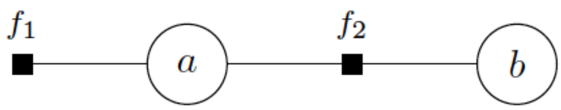
The factor f can be formulated as

$$f(a, b) = 0.9\delta(a)\delta(b) + 0.2\delta(\bar{a})\delta(b) + 0.1\delta(a)\delta(\bar{b}) + 0.8\delta(\bar{a})\delta(\bar{b})$$

Assume that we have observed that b is true. Compute the distribution of a using message passing.

Exercise 3

Consider the factor graph below, where a and b are continuous scalar random variables.



Suppose that $f_1(a) = \mathcal{N}(a; \mu_1, \sigma_1^2)$ and $f_2(a, b) = \mathcal{N}(b; \alpha a, \sigma_2^2)$. Using message passing:

- Compute the marginal distribution of a;
- Compute the marginal distribution of b.

Exercise 4

A company using two different machines produces random amounts of clips and pins each day. Suppose that the production follows a Poisson distribution $P(\lambda)$, where the rate λ depends on the quality of the steel the company is using on a specific day. The Poisson distribution is given by:

$$x \sim P(\lambda) \Leftrightarrow p(x) = P(x; \lambda) = \frac{\lambda^x exp(-\lambda)}{x!}$$

For high-quality steels, $\lambda = 10$, and if the steel is of low quality, $\lambda = 7$. The probability that the company will receive high-quality steel on a specific day is 0.25.
Suppose that at the end of the day the company has produced 10 clips and 8 pins.

- Identify the conditional distributions in the model and draw the Bayesian Network.
- Transform the Bayesian network into a factor graph.
- Using message passing, compute the probability that the company was using high-quality steel.
- Write a program to verify the calculated probability using Monte Carlo simulation.