

```

1 import numpy as np
2 import matplotlib.pyplot as plt
3 from scipy.integrate import odeint
4 import sys
5 import os
6
7 # Numerical integration
8 omega = 1
9 nu = 1
10 mu = 1/10
11 T = 2*np.pi/(omega+nu*mu)
12 timesteps = int(T*10)
13 t_array = np.linspace(0,T,timesteps)
14 dt = T/len(t_array)
15 xmin = 0
16 xmax = T
17
18 # J11 = 1/10-X2**2-2*X10[0]*X20[0]-3*X10[0]
19 # J12 = -3*X20**2-2*X10[0]*X20[0]-X10[0]**2-1
20 # J21 = 1+X20**2+3*X10[0]**2-2*X10[0]*X20[0]
21 # J22 = 1/10+2*X10[0]*X20[0]-3*X20[0]**2-X10[0]**2
22
23 # J = np.array([[J11, J12],\
24 #               [J21, J22]])
25
26 def dynamical_system(IC, t):
27     X1 = IC[0]
28     X2 = IC[1]
29     M11 = IC[2]
30     M12 = IC[3]
31     M21 = IC[4]
32     M22 = IC[5]
33
34     dX1 = (1/10)*X1-X2**3-X1*X2**2-X1**2*X2-X2-X1**3
35     dX2 = X1+(1/10)*X2+X1*X2**2+X1**3-X2**3-X1**2*X2
36
37     J11 = 1/10-X2**2-2*X1*X2-3*X1**2
38     J12 = -3*X2**2-2*X1*X2-X1**2-1
39     J21 = 1+X2**2+3*X1**2-2*X1*X2
40     J22 = 1/10+2*X1*X2-3*X2**2-X1**2
41
42     dM11 = J11*M11+J12*M21
43     dM12 = J11*M12+J12*M22
44     dM21 = J21*M11+J22*M21
45     dM22 = J21*M12+J22*M22
46     return [dX1, dX2, dM11, dM12, dM21, dM22]
47
48 X1 = mu**0.5
49 X2 = 0
50 M11 = 1
51 M12 = 0
52 M21 = 0
53 M22 = 1
54 IC = [X1,X2,M11,M12,M21,M22]
55 eqs = odeint(dynamical_system, IC, t_array)
56
57 X1 = eqs[:,0]
58 X2 = eqs[:,1]
59
60 M11 = eqs[:,2]
61 M12 = eqs[:,3]
62 M21 = eqs[:,4]
63 M22 = eqs[:,5]
64
65 M11_last = M11[-1]
66 M12_last = M12[-1]
67 M21_last = M21[-1]
68 M22_last = M22[-1]
69
70
71 print('M11: ', np.round(M11_last,4))
72 print('M12: ', np.round(M12_last,4))
73 print('M12: ', M12_last)
74 print('M21: ', np.round(M21_last,4))
75 print('M22: ', np.round(M22_last,4))
76
77 M = np.array([[M11_last,M12_last],[M21_last,M22_last]])
78 eig_values, eig_vectors = np.linalg.eig(M)

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```

79 print(eig_values)
80 stab_exps = np.log(eig_values)/T
81 print('Stability exponents: ', np.round(stab_exps,4))
82
83
84 # for t in (t_array-1):
85
86 #     t = int(t)
87 # J11 = 1/10-X2[t]**2-2*X1[t]*X2[t]-3*X1[-1]
88 # J12 = -3*X2[t]**2-2*X1[t]*X2[t]-X1[t]**2-1
89 # J21 = 1+X2[t]**2+3*X1[t]**2-2*X1[-1]*X2[-1]
90 # J22 = 1/10+2*X1[t]*X2[t]-3*X2[t]**2-X1[-1]**2
91
92 #     J = np.array([[J11, J12],\
93 #                   [J21, J22]])
94
95 #     M = M + np.matmul(J,M)*dt
96
97 #     M11[t+1] = M[0,0]
98 #     M12[t+1] = M[0,1]
99 #     M21[t+1] = M[1,0]
100 #     M22[t+1] = M[1,1]
101
102 fig, ax = plt.subplots(figsize=(7,7))
103 ax.plot(t_array, X1, '-', linewidth=2, label='X1')
104 ax.plot(t_array, X2, '-', linewidth=2, label='X2')
105 ax.plot(t_array, M11, '-', linewidth=2, label='M11')
106 ax.plot(t_array, M12, '-', linewidth=2, label='M12')
107 ax.plot(t_array, M21, '-', linewidth=2, label='M21')
108 ax.plot(t_array, M22, '-', linewidth=2, label='M22')
109
110 ax.set_title('$3.2d$')
111 ax.set_xlabel('t')
112 ax.set_ylabel('X')
113 ax.set_xlim(xmin,xmax)
114 ax.set_box_aspect(1)
115
116 plt.legend(loc="lower right", prop={'size': 8})
117 plt.savefig('Dynamical systems/DS HW3/3.2/3.2d.png', bbox_inches='tight')
118 # plt.show()

```