2.3)

$$\begin{cases} \dot{x} = \overbrace{(\mu - x^2)x}^{f(x,y)} - 5y \\ \dot{y} = \underbrace{(\mu + 3y^2)y}_{g(x,y)} + \underbrace{5x}_{w} \end{cases} \quad \text{and} \quad \begin{cases} \dot{x} = \overbrace{(\mu - x)x}^{f(x,y)} - (-1)y \\ \dot{y} = \underbrace{(\mu y + 2x^2)}_{g(x,y)} + \underbrace{(-1)x}_{w} \end{cases}$$

a) ans: $\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$

b) ans: $\begin{bmatrix} f_1 \\ g_1 \end{bmatrix} = \{\mu = 0\} = \begin{bmatrix} -x^3 \\ 3y^3 \end{bmatrix}$ $\begin{bmatrix} f_2 \\ g_2 \end{bmatrix} = \{\mu = 0\} = \begin{bmatrix} -x^2 \\ 2x^2 \end{bmatrix}$

c) $f = -x^3, \quad g = 3y^3$

$f'_x = -3x^2$

$f''_{xx} = -6x \quad f''_{xy} = 0 \quad f''_{yy} = 0$

$f'''_{xxx} = -6 \quad f'''_{xyy} = 0$

$g'_y = 9y^2$

$g''_{yy} = 18y \quad g''_{xy} = 0 \quad g''_{xx} = 0$

$g'''_{yyy} = 18 \quad g'''_{xxy} = 0$

$$16a = -6 + 0 + 0 + 18 + \frac{1}{5}\left(0(-6x+0) - 0(0+18y) + 6x \cdot 0 + 0 \cdot 18y\right) =$$

$$\Rightarrow 16a = 12 \Rightarrow a = \frac{12}{16} = \frac{3}{4}$$

$f = -x^2 \quad g = 2x^2$

$f'_x = -2x$

$f''_{xx} = -2 \quad f''_{xy} = 0 \quad f''_{yy} = 0$

$f'''_{xxx} = 0 \quad f'''_{xyy} = 0$

$g'_y = 0$

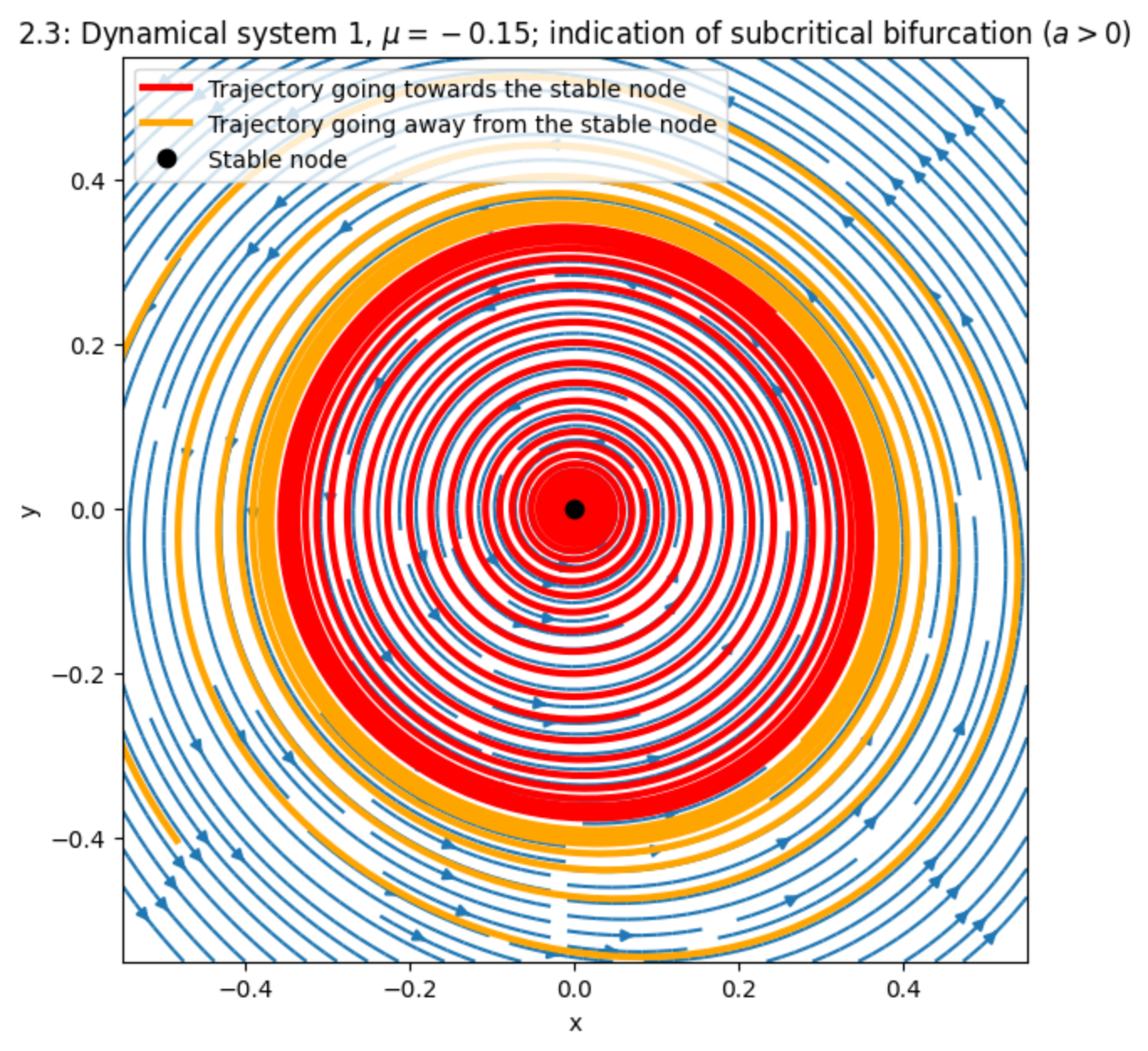
$g''_{yy} = 0 \quad g''_{xy} = 0 \quad g''_{xx} = 4$

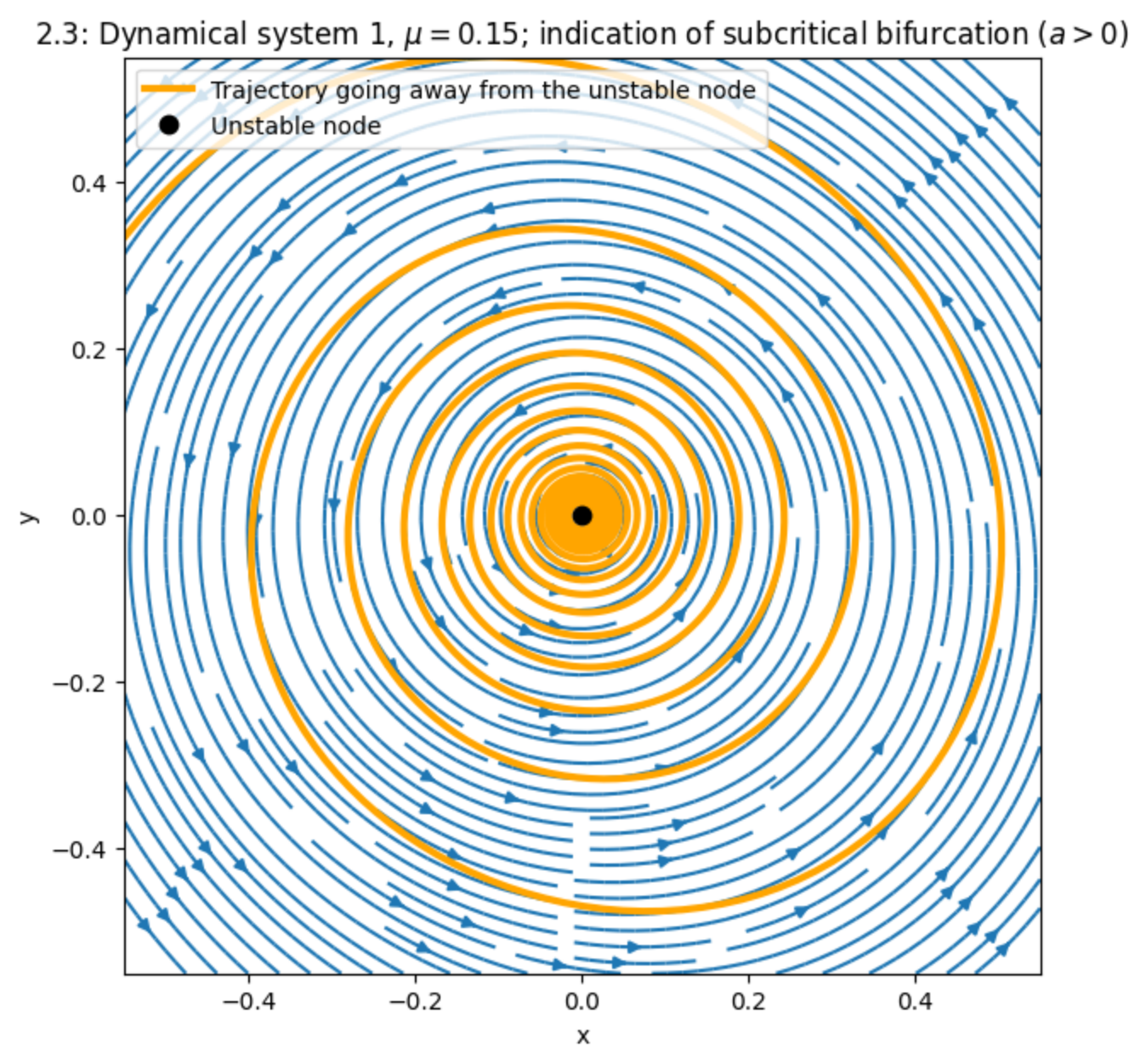
$g'''_{yyy} = 0 \quad g'''_{xxy} = 0$

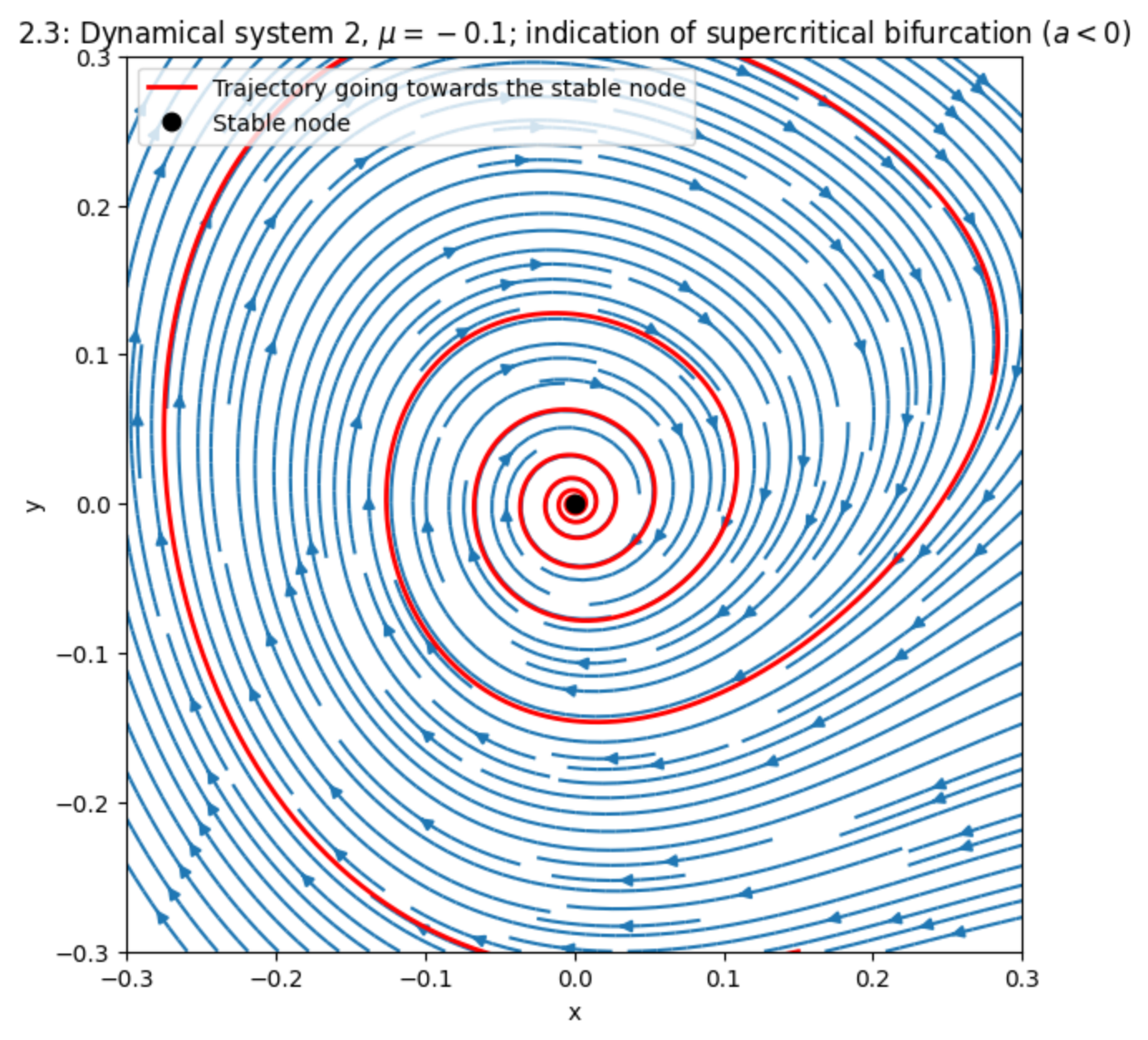
$$16a = 0 + 0 + 0 + 0 + \left(\frac{1}{-1}\right)\left(0(-2+0) - 0(4+0) + 2 \cdot 4 + 0 \cdot 0\right) = -8$$

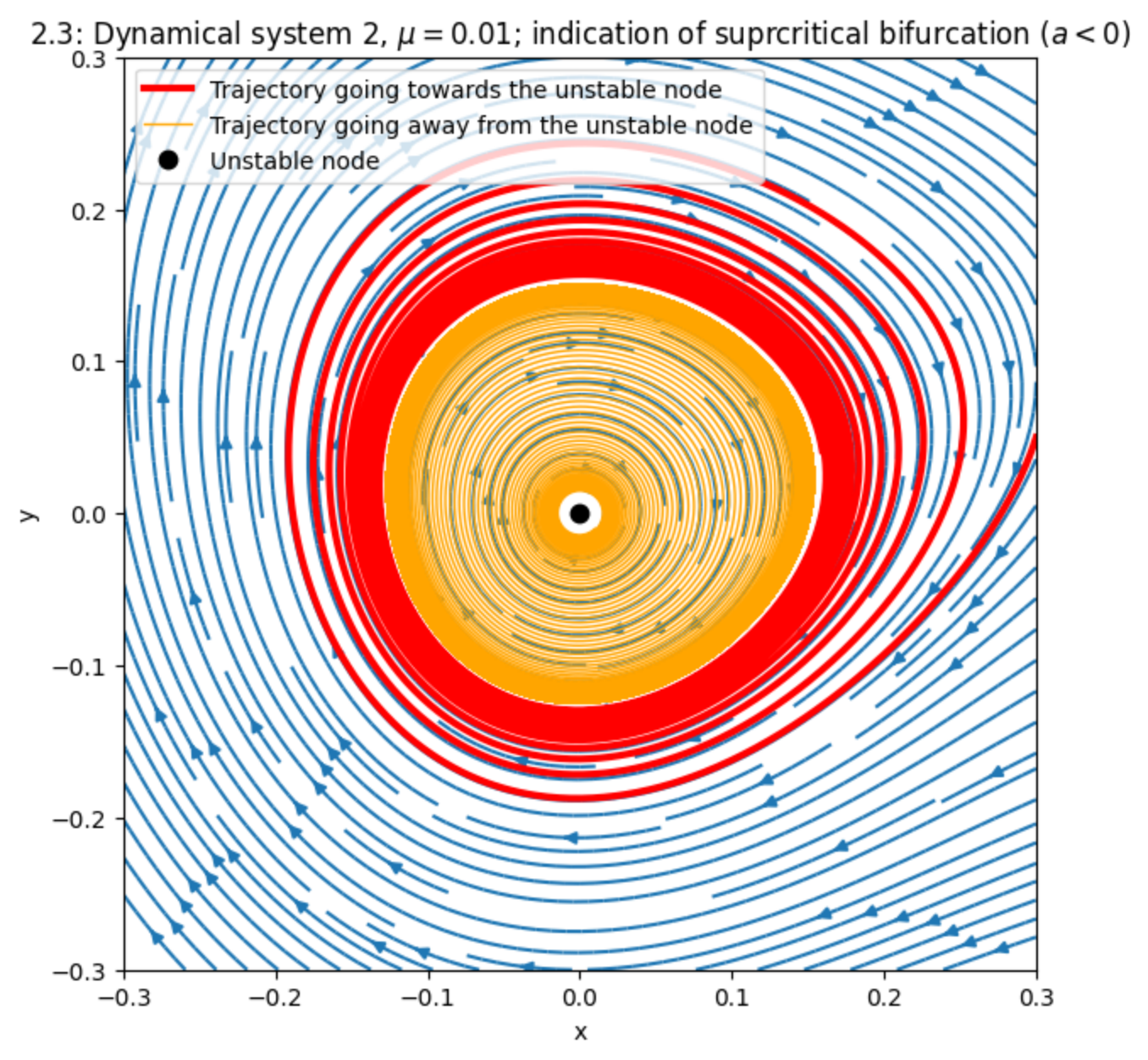
$$\Rightarrow 16a = -8 \Rightarrow a = -\frac{1}{2}$$

ans: $\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3/4 \\ -1/2 \end{bmatrix}$


2.3: Dynamical system 1, $\mu = -0.15$; indication of subcritical bifurcation ($a > 0$)
Trajectory going towards the stable node
Trajectory going away from the stable node
Stable node
0.4
0.2
0.0
−0.2
−0.4
y
−0.4
−0.2
0.0
0.2
0.4
x


2.3: Dynamical system 1, $\mu = 0.15$; indication of subcritical bifurcation ($a > 0$)
Trajectory going away from the unstable node
Unstable node
0.4
0.2
0.0
−0.2
−0.4
y
−0.4
−0.2
0.0
0.2
0.4
x


2.3: Dynamical system 2, $\mu = -0.1$; indication of supercritical bifurcation ($a < 0$)
Trajectory going towards the stable node
Stable node
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
y
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
x


2.3: Dynamical system 2, $\mu = 0.01$; indication of suprcritical bifurcation ($a < 0$)
Trajectory going towards the unstable node
Trajectory going away from the unstable node
Unstable node
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
y
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
x

```python
import numpy as np
import matplotlib.pyplot as plt
from scipy.integrate import odeint
import sys

xmin = -0.55
ymin = -0.55
xmax = 0.55
ymax = 0.55

# Streamplot
no_points = 1000
x_points = np.linspace(xmin, xmax, no_points)
y_points = np.linspace(ymin, ymax, no_points)
X, Y = np.meshgrid(x_points, y_points)

mu = -0.15
dxdt_streamplot = mu*X - 5*Y - X**2
dydt_streamplot = 5*X + mu*Y + 3*Y**3
# dxdt_streamplot = mu*X + Y - X**2
# dydt_streamplot = -X + mu*Y + 2*X**2

# Numerical integration
T = 100
t = np.linspace(0,T,T*100)
x = np.zeros(T)
y = x.copy()
fp = np.array([[0,0], [0,0]])
x[0] = fp[0,0] + 0.24
y[0] = fp[0,1] + 0.24
T2 = 21
t2 = np.linspace(0,T2,T2*100)
x2 = np.zeros(T2)
y2 = x2.copy()
x2[0] = fp[0,0] + 0.01
y2[0] = fp[0,1] - 0.01

def dynamical_system(xy, t):
    mu = 0.15
    x = xy[0]
    y = xy[1]
    dxdt_integration = mu*x - 5*y - x**2
    dydt_integration = 5*x + mu*y + 3*y**3
    # dxdt_integration = mu*x + y - x**2
    # dydt_integration = -x + mu*y + 2*x**2
    return [dxdt_integration, dydt_integration]

x0y0 = [x[0],y[0]]
xy = odeint(dynamical_system, x0y0, t)
x = xy[:,0]
y = xy[:,1]
x0y02 = [x2[0],y2[0]]
xy2 = odeint(dynamical_system, x0y02, t2)
x2 = xy2[:,0]
y2 = xy2[:,1]

fig, ax = plt.subplots(figsize=(7,7))
ax.streamplot(X, Y, dxdt_streamplot, dydt_streamplot, density = 2)
```

```
# ax.plot(x, y, '-', color='red', linewidth=3, label='Trajectory going towards the stable node')
ax.plot(x2, y2, '-', color='orange', linewidth=3, label='Trajectory going away from the stable node')
ax.plot(fp[0,0],fp[0,1], '.', color='black', markersize=15, label='Stable node')
# ax.plot(fp[1,0],fp[1,1], '.', color='magenta', markersize=15, label='Unstable spiral')
ax.set_title('2.3: Dynamical system 1, $\mu = 0.15$; indication of subcritical bifurcation ($a>0$)')
ax.set_xlabel('x')
ax.set_ylabel('y')
ax.set_xlim(xmin,xmax)
ax.set_ylim(xmin,xmax)
ax.set_box_aspect(1)

plt.legend(loc="upper left")
plt.savefig('23_d1mu0.15.png', bbox_inches='tight')
plt.show()

```