4.1) a) $D_0 = \dfrac{\ln N_{box}(\varepsilon)}{\ln\left(\frac{1}{\varepsilon}\right)} = \dfrac{\ln 4^n}{\ln\left(\frac{1}{\left(\frac{1}{3}\right)^n}\right)} = \dfrac{\ln 4^n}{\ln 3^n} = \dfrac{n \cdot \ln 4}{n \cdot \ln 3} =$

$= \dfrac{\ln 4}{\ln 3}$, Ans: $D_0 = \dfrac{\ln 4}{\ln 3}$

b) $N_{Box}(\varepsilon) = N_A(\varepsilon) + N_B(\varepsilon)$

$N_A(\varepsilon) = N_{Box}\left(\frac{\varepsilon}{\lambda_a}\right)$, $\lambda_a = \frac{1}{4}$

$N_B(\varepsilon) = N_{Box}\left(\frac{\varepsilon}{\lambda_b}\right)$, $\lambda_b = \frac{1}{2}$

$A\varepsilon^{-D_0} = 4A\left(\frac{\varepsilon}{\lambda_a}\right) + A\left(\frac{\varepsilon}{\lambda_b}\right) \Rightarrow \left(\frac{1}{3^n}\right)^{-D_0} = 4\left(\frac{\frac{1}{3^n}}{\left(\frac{1}{4}\right)}\right)^{-D_0} + \left(\frac{\frac{1}{3^n}}{\left(\frac{1}{2}\right)}\right)^{-D_0} =$

$= 4\left(\frac{4}{3^n}\right)^{-D_0} + \left(\frac{2}{3^n}\right)^{-D_0} \Rightarrow \dfrac{3^{nD_0}}{3^{nD_0}} = 4\left(\dfrac{\frac{3^{nD_0}}{4^{D_0}}}{3^{nD_0}}\right) + \left(\dfrac{\frac{3^{nD_0}}{2^{D_0}}}{3^{nD_0}}\right) \Rightarrow$

$\Rightarrow 1 = \frac{4}{4^{D_0}} + \frac{1}{2^{D_0}} \Rightarrow 4^{D_0} = 4 + \frac{4^{D_0}}{2^{D_0}} = 4 + \frac{2^{2D_0}}{2^{D_0}} = 4 + 2^{2D_0 - D_0} =$

$= 4 + 2^{D_0} \Rightarrow 4^{D_0} - 2^{D_0} = 4 = 2^{2D_0} - 2^{D_0} = 4 = \{2^{D_0} = u\} =$

$= u^2 - u = 4 \Rightarrow u^2 - u - 4 = 0 \Rightarrow u = \frac{-(-1)}{2} \pm \sqrt{\frac{1}{4} - (-4)} =$

$= \frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{4 \cdot 4}{1 \cdot 4}} = \frac{1}{2} \pm \sqrt{\frac{17}{4}} = \frac{1 \pm \sqrt{17}}{2} \Rightarrow$

$\Rightarrow 2^{D_0} = \frac{1 \pm \sqrt{17}}{2} \Rightarrow \ln(2^{D_0}) = \ln\left(\frac{1 \pm \sqrt{17}}{2}\right)$, logarithm cannot have negative arguement $\Rightarrow D_0 \ln(2) = \ln\left(\frac{1 + \sqrt{17}}{2}\right) \Rightarrow$

Ans: $D_0 = \dfrac{\ln\left(\frac{1 + \sqrt{17}}{2}\right)}{\ln(2)}$