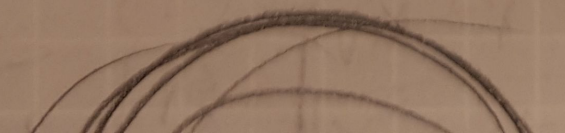
ar 0

2.1 b) $\begin{cases} \dot{r} = h(r) \sim ar, & h(r) \sim ar + O(r^2) \\ \dot{\theta} = 0 \end{cases}$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases} \Rightarrow \begin{cases} \dot{x} = -r\dot{\theta}\sin\theta + \dot{r}\cos\theta \\ \dot{y} = r\dot{\theta}\cos\theta + \dot{r}\sin\theta \end{cases}$$

$$\Rightarrow \begin{cases} \dot{x} = \dot{r}\cos\theta \sim ar\cos\theta = a\sqrt{x^2+y^2}\cos\left(\arctan\left(\frac{y}{x}\right)\right) \\ \dot{y} = \dot{r}\sin\theta \sim ar\sin\theta = a\sqrt{x^2+y^2}\sin\left(\arctan\left(\frac{y}{x}\right)\right) \end{cases}$$

$n = -2$



2.1 d)

$n = 2$: $I = 2$.

$n = -2$: $I = -2$

$n = 4$: $I = 4$

$n = -4$: $I = -4$

$n = 6$: $I = 6$

$n = -6$: $I = -6$

$\Rightarrow$ Ans: $I = n$

In[1]:=
```
Clear["Global`*"]
minx = -2;
miny = -2;
maxx = 2;
maxy = 2;
tmin = -100;
tmax = 10;
dist = 0.1;


(*
f = {y-x,x^2};
sol[x0_,y0_] := NDSolve[
  {x'[t]==y[t]-x[t],
   y'[t]==x[t]^2,
   x[0]==x0,y[0]==y0},
  {x,y},{t,tmin,tmax}]
*)


(*
f = {y^3,x};
sol[x0_,y0_] := NDSolve[
  {x'[t]==y[t]^3,
   y'[t]==x[t],
   x[0]==x0,y[0]==y0},
  {x,y},{t,tmin,tmax}]
*)


n = 2;
f = {(x^2 + y^2)^(Abs[n] / 2) * Cos[n * ArcTan[y / x]],
    (x^2 + y^2)^(Abs[n] / 2) * Sin[n * ArcTan[y / x]]};
sol[x0_, y0_] := NDSolve[
  {x'[t] == (x[t]^2 + y[t]^2)^(Abs[n] / 2) * Cos[n * ArcTan[y[t] / x[t]]],
   y'[t] == (x[t]^2 + y[t]^2)^(Abs[n] / 2) * Sin[n * ArcTan[y[t] / x[t]]],
   x[0] == x0, y[0] == y0},
  {x, y}, {t, tmin, tmax}]



initialCond = Join[
   Table[{x , miny}, {x , minx , maxx, dist}],
   Table[{x, maxy}, {x, minx , maxx, dist}],
   Table[{minx , y } , {y, miny , maxy, dist}],
   Table[{maxx, y} , {y, miny , maxy, dist}]
  ];

(*ParametricPlot[
 Evaluate[{x[t],y[t]}/. sol[initialCond[[50,1]],initialCond[[50,2]]],
```

```
 {t,tmin,tmax}, PlotRange→{{minx,maxx},{miny,maxy}}]*)

p0 = Show[
  Table[
   ParametricPlot[
    Evaluate[{x[t], y[t]} /. sol[initialCond[[i, 1]], initialCond[[i, 2
        ]]]], {t, tmin, tmax}, PlotRange → {{minx, maxx}, {miny, maxy}}],
   {i, 1, Length[initialCond]}],
  StreamPlot[f, {x, minx, maxx}, {y, miny, maxy}],
  PlotRange → {{minx, maxx}, {miny, maxy}}
 ]
```

Power: Infinite expression $\frac{1}{0.}$ encountered.

Power: Infinite expression $\frac{1}{0.}$ encountered.

NDSolve: Encountered non-numerical value for a derivative at t == 0.`.

ReplaceAll: {NDSolve[{x′[t] == Cos[2 ArcTan[≪1≫]] (x[≪1≫]^2 + y[≪1≫]^2), y′[t] == Sin[2 ArcTan[≪1≫]] (x[≪1≫]^2 + y[≪1≫]^2), x[0] = cannot be used for replacing.

NDSolve: −99.9978 cannot be used as a variable.

ReplaceAll: {NDSolve[{x′[−99.9978] == Cos[2 ArcTan[≪1≫]] (x[≪1≫]^2 + y[≪1≫]^2), y′[−99.9978] == Sin[2 ArcTan[≪1≫]] (x[≪1≫]^2 + y[≪1≫]^2), x[0] == 0., y[0] == −2}, {x, y}, {−99.9978, −100, 10}]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

NDSolve: −99.9978 cannot be used as a variable.

ReplaceAll: {NDSolve[{x′[−99.9978] == Cos[2. ArcTan[≪1≫]] (x[≪1≫]^2 + y[≪1≫]^2), y′[−99.9978] == Sin[2. ArcTan[≪1≫]] (x[≪1≫]^2 + y[≪1≫]^2), x[0.] == 0., y[0.] == −2.}, {x, y}, {−99.9978, −100., 10.}]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

NDSolve: −97.7529 cannot be used as a variable.

General: Further output of NDSolve::dsvar will be suppressed during this calculation.

Power: Infinite expression $\frac{1}{0.}$ encountered.

General: Further output of Power::infy will be suppressed during this calculation.

NDSolve: Encountered non-numerical value for a derivative at t == 0.`.

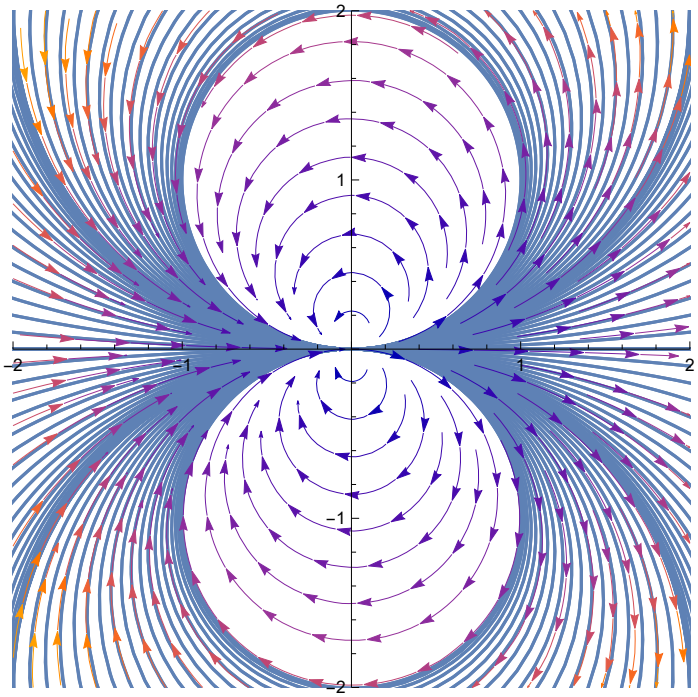
NDSolve: At t == −0.5, step size is effectively zero; singularity or stiff system suspected.

InterpolatingFunction: Input value {−99.9978} lies outside the range of data in the interpolating function. Extrapolation will be used.

InterpolatingFunction: Input value {−99.9978} lies outside the range of data in the interpolating function. Extrapolation will be used.

NDSolve: At t == 0.4999998785463731`, step size is effectively zero; singularity or stiff system suspected.

Out[13]=




```
Clear[x, y, f1, f2, f3, f4, min, max]
min = -1;
max = 1;

(*
n =1;
f1 = ArcTan[Tan[n*ArcTan[y]]];
f2 = ArcTan[Tan[n*ArcTan[1/x]]];
f3 = ArcTan[Tan[n*ArcTan[-y]]];
f4 = ArcTan[Tan[n*ArcTan[-1/x]]];
*)
f1 = ArcTan[1/y^3];
f2 = ArcTan[x];
f3 = ArcTan[-1/y^3];
f4 = ArcTan[-x];

(*ansD = 1/(2*π)*(Integrate[f1,{y,min,max}]+Integrate[f2,{x,max,min}])+
    Integrate[f3,{y,max,min}]+Integrate[f4,{x,min,max}]])//FullSimplify*)
ansD =
  1/(2 * π) * (Integrate[f1, y] + Integrate[f2, x] + Integrate[f3, y] + Integrate[f4, x]) //
   FullSimplify;
```

Out[ ]=

```
0
```