

(\*Computation for 1.4 b, e, f.\*)

In[ ]:= Clear[xsol, ysol, x, y, u, v, t]

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{xsol, ysol} = DSolveValue[
  {x'[t] == (p + 1) * x[t] + 3 * y[t],
   y'[t] == -2 * x[t] + (p - 1) * y[t],
   x[0] == u, y[0] == v},
  {x, y}, {t, 0, 20}];
```

xsol;

ysol;

Function[{t},  $\frac{1}{5} e^{pt} \left( v \left( \frac{-1}{2} \right) \cos[\sqrt{5} t] - \sqrt{5} (2u + v) \sin[\sqrt{5} t] \right)$ ];

In[ ]:=  $\left\{ \left\{ x \rightarrow \text{Function}\left[\{t\}, \frac{1}{5} e^{pt} \left( 5u \cos[\sqrt{5} t] + \sqrt{5} u \sin[\sqrt{5} t] + 3\sqrt{5} v \sin[\sqrt{5} t] \right) \right], \right. \right.$   
 $\left. \left. y \rightarrow \text{Function}\left[\{t\}, -\frac{1}{5} e^{pt} \left( -5v \cos[\sqrt{5} t] + 2\sqrt{5} u \sin[\sqrt{5} t] + \sqrt{5} v \sin[\sqrt{5} t] \right) \right] \right\} \right\};$

In[41]:= Clear[u, v, p, r]

u = 1;

v = 1;

p = 0;

$x = v * e^{p * t} * \left( \left( \frac{-1}{2} \right) * \cos[\sqrt{5} t] - \left( \frac{-\sqrt{5}}{2} \right) * \sin[\sqrt{5} t] \right) -$   
 $\frac{(2 * u + v)}{\sqrt{5}} * e^{p * t} * \left( \left( \frac{-\sqrt{5}}{2} \right) * \cos[\sqrt{5} t] + \left( \frac{-1}{2} \right) * \sin[\sqrt{5} t] \right);$

$y = v * e^{p * t} * \cos[\sqrt{5} t] - \frac{(2 * u + v)}{\sqrt{5}} * e^{p * t} * \sin[\sqrt{5} t];$

r = Sqrt[x^2 + y^2] // FullSimplify;

In[48]:= Clear[min, max, ratio]

min = MinValue[r, t] // FullSimplify;

max = MaxValue[r, t] // FullSimplify;

ratio = max / min

Out[51]=

$$\sqrt{\frac{5 + \sqrt{5}}{5 - \sqrt{5}}}$$

In[52]:= tMax = ArgMax[r, t] // FullSimplify;

In[53]:= **Clear**[u, v, p, r]

**u** = 1;

**v** = 1;

**p** = 0;

$$X[t\_]=v * e^{p * t} * \left( \left( \frac{-1}{2} \right) * \cos[\sqrt{5} t] - \left( \frac{-\sqrt{5}}{2} \right) * \sin[\sqrt{5} t] \right) - \frac{(2 * u + v)}{\sqrt{5}} * e^{p * t} * \left( \left( \frac{-\sqrt{5}}{2} \right) * \cos[\sqrt{5} t] + \left( \frac{-1}{2} \right) * \sin[\sqrt{5} t] \right);$$

$$Y[t\_]=v * e^{p * t} * \cos[\sqrt{5} t] - \frac{(2 * u + v)}{\sqrt{5}} * e^{p * t} * \sin[\sqrt{5} t];$$

In[68]:= **Clear**[v]

**v** = {X[tMax], Y[tMax]} // **FullSimplify**

**norm** = **Sqrt**[(X[tMax] - 0)<sup>2</sup> + (Y[tMax] - 0)<sup>2</sup>] // **FullSimplify**

Out[69]=

$$\left\{ \sqrt{\frac{7}{10}} (3 + \sqrt{5}), -\sqrt{\frac{7}{5}} \right\}$$

Out[70]=

$$\sqrt{\frac{7}{10}} (5 + \sqrt{5})$$