2.2)

a-d)

$$\ddot{\theta} = -\frac{g}{l}\sin\theta - \frac{\gamma}{m}\dot{\theta}$$

$$\begin{cases} \dot{\theta} = \omega \\ \dot{\omega} = -\frac{g}{l}\sin\theta - \frac{\gamma}{m}\omega \end{cases}$$

Substitute with: $\begin{cases} \theta = \theta_0 x \\ \omega = \omega_0 y \\ t = t_0 t' \end{cases}$

$$\Rightarrow \begin{cases} \frac{d\theta_0 x}{dt_0 t'} = \omega_0 y & (1) \\ \frac{d\omega_0 y}{dt_0 t'} = -\frac{g}{l}\sin(\theta_0 x) - \frac{\gamma}{m}\omega_0 y & (2) \end{cases}$$

$$\Rightarrow \begin{cases} \frac{dx}{dt'} = \frac{t_0 \omega_0 y}{\theta_0} \\ \frac{dy}{dt'} = -\frac{t_0 g}{\omega_0 l}\sin(\theta_0 x) - \underbrace{\frac{t_0 \gamma}{m}}_{\sigma} y \end{cases}$$

We choose $\theta_0 = 1$

(1): $\frac{\omega_0 t_0}{\theta_0} = 1 \Rightarrow \omega_0 = \frac{1}{t_0}$ (3)

(2): $\frac{t_0 g}{\omega_0 l} = 1 \Rightarrow t_0 = \frac{\omega_0 l}{g} = \frac{l}{t_0 g} \Rightarrow t_0 = \sqrt{\frac{l}{g}}$

(3): $\omega_0 = \frac{1}{\sqrt{\frac{l}{g}}} = \sqrt{\frac{g}{l}}$

$\sigma = \frac{\gamma}{m}\sqrt{\frac{l}{g}}$

Ans: $\theta_0 = 1$

$\omega_0 = \sqrt{\frac{g}{l}}$

$t_0 = \sqrt{\frac{l}{g}}$

$\sigma = \frac{\gamma}{m}\sqrt{\frac{l}{g}}$

2.2 f) $\dot{\phi} = w$

$\dot{w} = \sin\phi\left[\cos\phi - \tau - 1\right]$

Hamiltonian System:

$$\begin{cases} \dot{x} = \dfrac{\partial H}{\partial y} \\ \dot{y} = -\dfrac{\partial H}{\partial x} \end{cases}$$

$$H = \int \dot{\phi}\, dw - \int \dot{w}\, d\phi = \frac{w^2}{2} - \left(\frac{1}{2}\sin^2\phi + \tau\cos\phi + \cos\phi\right) + C =$$

$$= \underbrace{\frac{w^2}{2} - \frac{1}{2}\sin^2\phi - \cos\phi(\tau + 1)}\, + C$$

Normalise: multiply with 2

$$\Rightarrow \; -1 = w^2 - \sin^2\phi - \cos\phi(\tau+1) + C$$

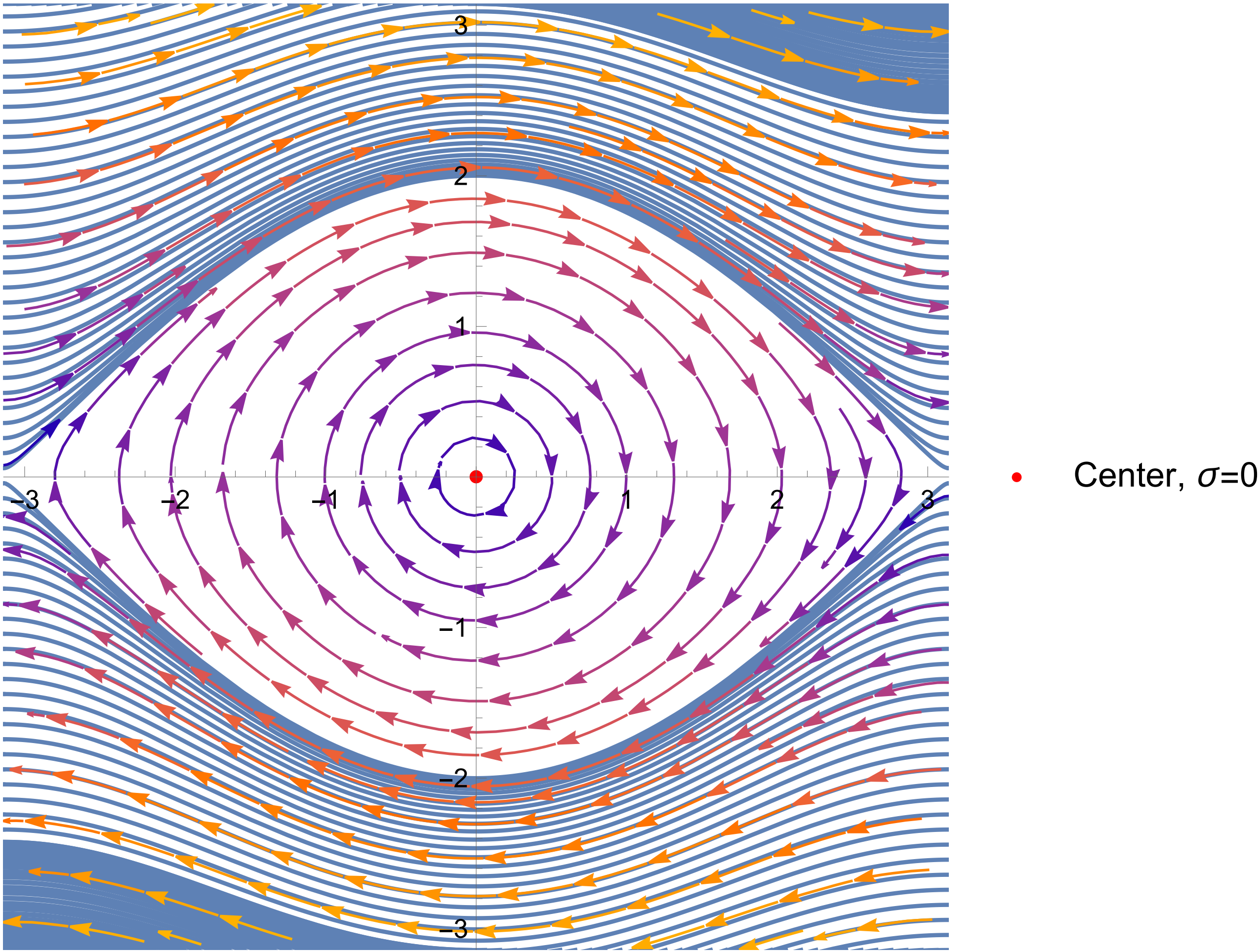
$$\phi = \pi/2, \quad w = 0$$

$$\Rightarrow \; -1 = -1 + C \;\Rightarrow\; C = 0$$

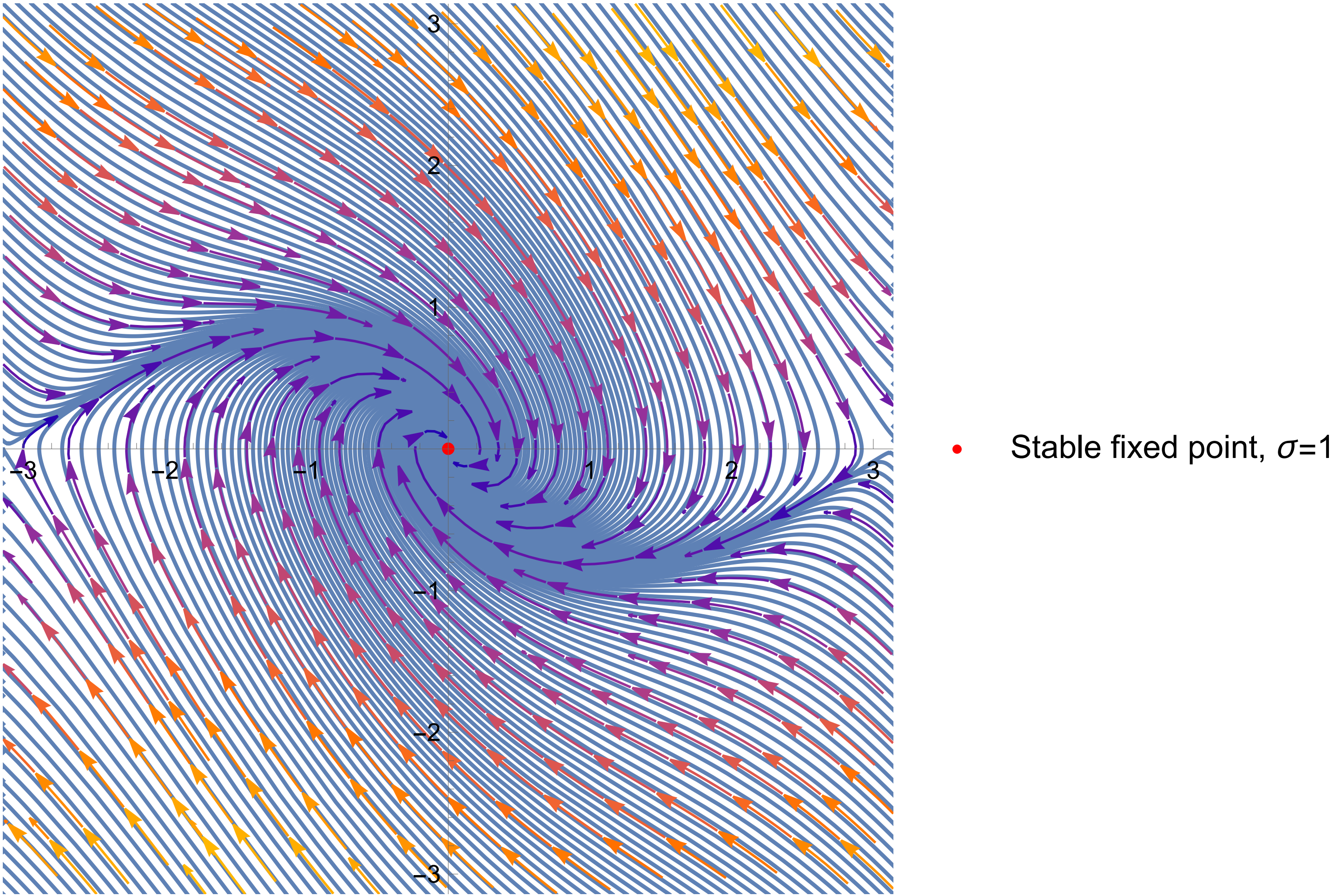
Ans: Integral of motion:

$$w^2 - \sin^2\phi - \cos\phi(\tau + 1)$$

Out[•]=



Out[ ]=

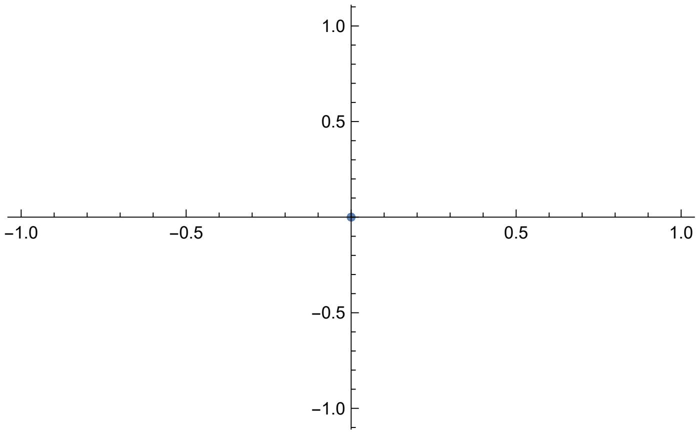


In[35]:=
```
Clear[minx, miny, maxx, maxy]
minx = -π;
miny = -π;
maxx = π;
maxy = π;
```

In[78]:=
```
Clear[sol, x, y, t, mu, tmin, tmax]
tmin = 0;
tmax = 10;
sigma = 0;
sol[x0_, y0_] := NDSolve[
  {x'[t] == y[t],
   y'[t] == -Sin[x[t]] - sigma y[t],
   x[0] == x0, y[0] == y0},
  {x, y}, {t, tmin, tmax}]
```

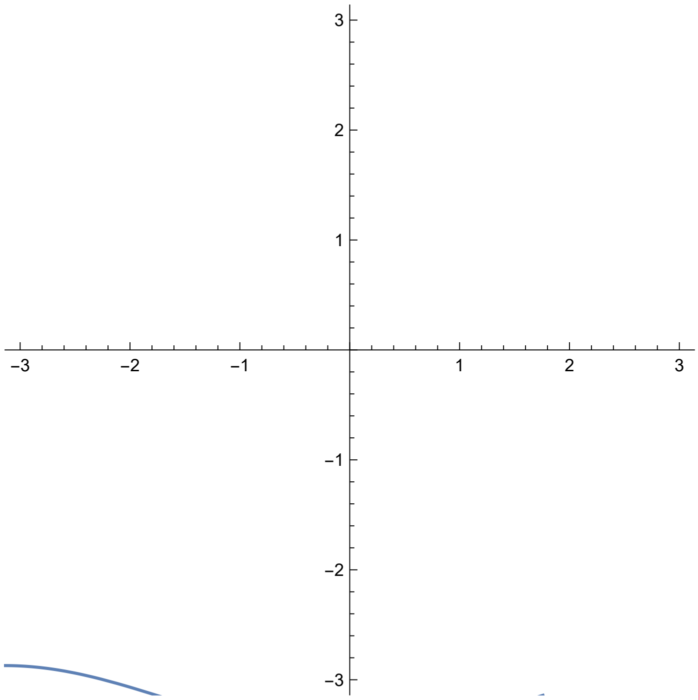
In[66]:=
```
dist = 0.1;
initialCond = Join[
   (*Table[{minx , y } , {y,miny , maxy,0.1}],
   Table[{maxx, y} , {y, miny , maxy,0.1}],
   Table[{x , miny},{x , minx , maxx,0.1}],
   Table[{x,maxy},{x,minx , maxx,0.1}]*)

   Table[{x , miny}, {x , minx , maxx, dist}],
   Table[{x, maxy}, {x, minx , maxx, dist}],
   Table[{minx , y } , {y, miny , maxy, dist}],
   Table[{maxx, y} , {y, miny , maxy, dist}]
  ];
```

Out[ ]=



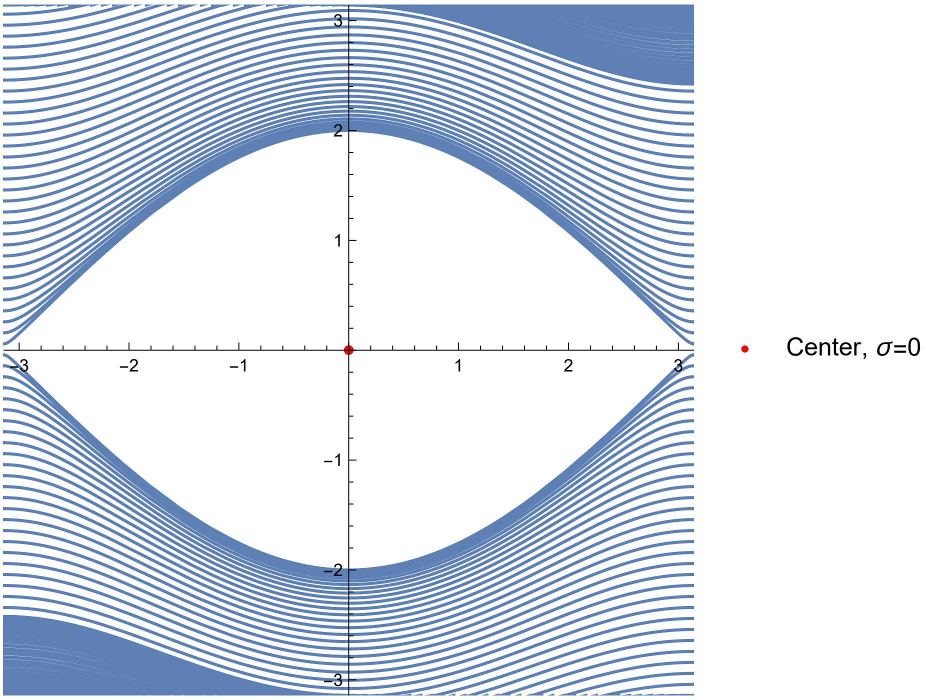
In[83]:=
```
ParametricPlot[
 Evaluate[{x[t], y[t]} /. sol[initialCond[[50, 1]], initialCond[[50, 2]]]],
 {t, tmin, tmax}, PlotRange → {{minx, maxx}, {miny, maxy}}]
```

Out[83]=



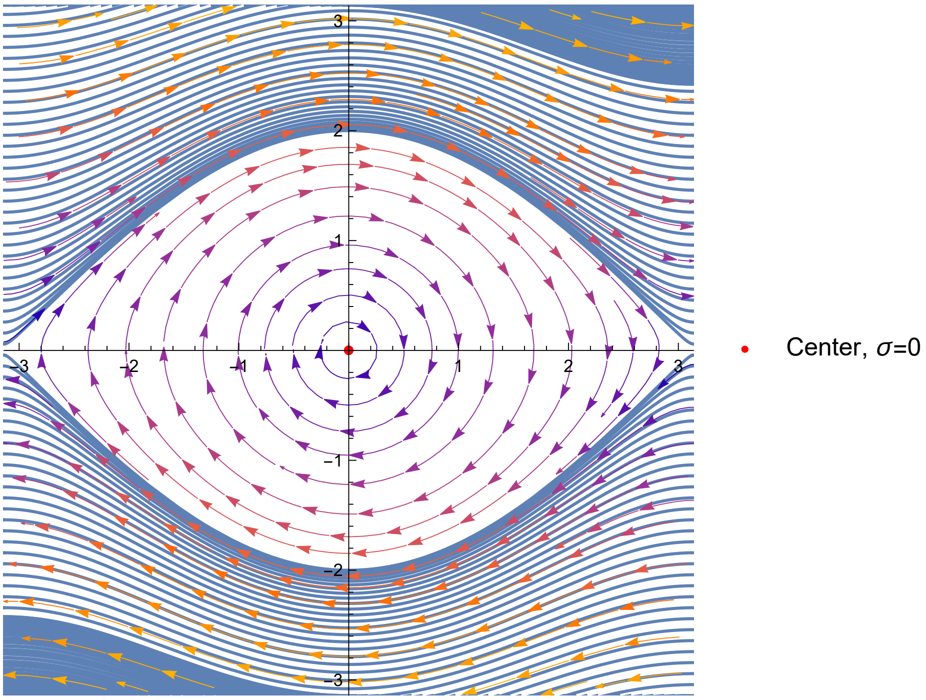
In[86]:=
```
p2 = Show[
  Table[
   ParametricPlot[
    Evaluate[{x[t], y[t]} /. sol[initialCond⟦i, 1⟧, initialCond⟦i, 2
       ⟧]], {t, tmin, tmax}, PlotRange → {{minx, maxx}, {miny, maxy}}],
   {i, 1, Length[initialCond]}],
  ListPlot[{{0, 0}}, PlotStyle → {PointSize[0.1], Red},
   PlotMarkers → {"•", Large}, PlotLegends → {"Center, σ=0"}]
 ]
```

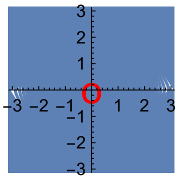
Out[86]=



In[87]:=
```
Show[p2, StreamPlot[{y, -Sin[x] - sigma * y}, {x, -π, π}, {y, -π, π}],
 PlotRange → {{minx, maxx}, {miny, maxy}}]
```

Out[87]=




In[∘]:=
```
Show[
```


```
, StreamPlot[{y, -Sin[x] - sigma * y}, {x, -π, π}, {y, -π, π}],
 PlotRange → {{-π, π}, {-π, π}}]
```