

Assignment 3

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Problem 7

Let $OPT(i)$ be the optimal payment from a set of jobs up to job $i \in \{1, \dots, n\}$ such that any two chosen jobs do not happen on two consecutive days. Let p_i be the payment of job i , and set $OPT(0) = 0$ and $OPT(1) = p_1$. The following recursion formula maximizes the total payment up to job i .

$$OPT(i) = \max\{OPT(i-2) + p_i, OPT(i-1)\}$$

If we choose job i we cannot choose job $i-1$ because these two jobs happens on two consecutive days so we must choose the most optimal payment up to job $i-2$ and add p_i . Consequently, if the optimal payment is $i-1$ we cannot choose job i , thus we must maximize between $OPT(i-2) + p_i$ and $OPT(i-1)$. Performing this n times we end up with the optimal solution $OPT(n)$. We can then trace back to get the set of jobs that maximizes the total payment. The running time for this algorithm is in $O(n)$ because we choose the max between two values for every i which is one operation for each i up to n . We can then back trace to find the set of jobs that maximizes payment, and this does not make the order of running time more expensive.