Assignment 6

Erik Norlin, CID: norliner

October 17, 2023

Problem 13

Let the Subset Sum problem be problem X and the Half-Half Subset Sum problem be problem Y. It is known that X is NP-complete and we prove here that Y is also NP-complete by reducing from X to Y in polynomial time.

Y is in NP because if given a subset of positive integers w_i that add to W, it is possible to check in polynomial time if they add to W.

X can be reduced to Y in the following way. Let an instance of X be a set S of n integers and W an integer where a subset of S adds to W. We have

$$S = \{w_1, ..., w_n\}$$
$$w_i + ... + w_k = W$$

To reduce X to Y we add $\sum_i^n w_i - 2W$ to both sides in X.

$$w_i + ... + w_k + \sum_i^n w_i - 2W = W + \sum_i^n w_i - 2W$$

Let $\sum_i^n w_i - 2W$ be w_{n+1}, and let the set S' of a new instance for Y be S with an additional term w_{n+1}. We have

$$S' = \{w_1, ..., w_n, w_{n+1}\}$$

Y becomes

$$w_i + ... + w_k + w_{n+1} = \sum_i^{n+1} \frac{w_i}{2} = \sum_i^n \frac{w_i}{2} + \frac{w_{n+1}}{2}$$
$$w_i + ... + w_k + \sum_i^n w_i - 2W = \sum_i^n \frac{w_i}{2} + \sum_i^n \frac{w_i}{2} - W$$
$$w_i + ... + w_k = W$$

Thus, X is true when Y. X is therefore reducible to Y, and is reduced in polynomial time because adding the term w_{n+1} requires n operations to perform the sum, so we have that $X \leq_p Y$. Since X is NP-complete and Y is in NP we also have that Y is NP-complete.