Assignment 4

Erik Norlin, CID: norliner

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Problem 8

Let i be the house the vehicle starts at and visits first, S be the set containing all houses to visit (including house i), t the total time taken to travel between houses in S; initially zero at house i. Then $OPT(i, S, t)$ is the shortest time taken to visit the houses in S such that all houses are visited before their respective deadlines, assuming there exists such a solution (or solutions). Defining the following recursive formula,

$$OPT(i, S, t) = \begin{cases} \infty \text{ if } t > d_i \\ \min_{j \in S, j \neq i}\{t_{ij} + OPT(j,\ S \backslash \{i\},\ t + t_{ij})\} \text{ if } t \leq d_i \\ 0 \text{ if } S = \{i\} \text{ (only house } i \text{ left in the subset) and } t \leq d_i \end{cases}$$

let j be the next house to travel to from i, t_{ij} the time it takes to travel from house i to j, i.e., $|s_i - s_j|$, and d_i the deadline of house i. If t exceeds d_i we set $OPT(i, S, t)$ to a positive infinite time as penalty because no lateness is acceptable. This makes sense because we are dealing with a minimization problem, thus paths with infinite time will never be an optimal solution. On the other hand if t does not exceed the deadline d_i we continue to search for an optimal solution but now recursively computing the travel time of paths starting from each $j \in S$ such that $j \neq i$. This recursive fashion can be interpreted as branches branching out from other branches, which is analogous to an exponential running time. When only one house i is left in the subset and t meets d_i, the optimal travel time is set to zero to indicate a valid solution. When the algorithm is done with all recursive calls, any solution that that equals to zero is a valid solution that visits all houses before their respective deadlines, and any solution that equals to infinity is an invalid solution.