Computer exercise 1

1 Preparations

Study the steepest descent method and the different versions of Newton's method in the textbook.

2 Unconstrained optimization

In this part of the computer exercise, you will solve some unconstrained problems using steepest descent and Newton's method. The latter exists in three different versions, (1) with a unit step length (the classic version), (2) Newton's "modified" method, which includes a line search, and (3) the Levenberg–Marquardt modification, where diagonal terms are added, if necessary, such that the eigenvalues of the resulting matrix are positive. The methods are implemented in MATLAB. The purpose of this lab is to graphically illustrate the methods in order to give an insight into their behaviour.

To start, download the zip-file from the course homepage. Start MATLAB and go to the directory `Files_computer_exercise_1`, then type ilpmeny in the command window.

The computer exercise is menu-driven and mostly self-explaining. The following selections may be done:

Setting	Default value
Starting point	0 0
termination criterion	Gradient length
Function to be minimized	Function 1
Maximal number of iterations	200
Printing of iteration data	On
Method	Steepest descent

You may choose to take 1, 10 or 100 iterations at a time and follow the algorithm search path in the graph.

3 Exercises

In all these exercises, the function is to be **minimized**. For the motivations, it is not enough to just look at the graphs. Answer all questions, the motivations are often more important than the numerical values.

1. Study **function 1**

$$f(x_1, x_2) := 2(x_1 + 1)^2 + 8(x_2 + 3)^2 + 5x_1 + x_2.$$

(a) Solve the problem by using steepest descent and Newton's method (unit step). Start at the points $(10, 10)^{\mathrm{T}}$ and $(-5, -5)^{\mathrm{T}}$ as well as in some starting point of your own choice. Toward which point do the methods converge? How many iterations are required?

(b) Is the point obtained an optimal point (globally or locally)?

(c) Why does Newton's method always converge in *one* iteration?

2. Study **function 2** (Rosenbrock's function)

$$f(x_1, x_2) := 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

(a) Solve the problem by using steepest descent and Newton's method (all versions). Start at the point $(-2, -1)^{\mathrm{T}}$. Towards which point does the methods converge? How many iterations are required?

(b) Is the function convex? Is the obtained point a global optimum?

(c) Choose some starting points on your own to study the methods' behaviour.

3. Study **function 4**

$$f(x_1, x_2) := x_1^3 + 2x_2^3 - 27x_1 - 6x_2.$$

(a) Start at the point $(0, 0)^{\mathrm{T}}$ and solve the problem using steepest descent and Newton's method (unit step). Why does not Newton's method work? Try the Levenberg–Marquardt modification and study that method's behaviour.

(b) Start at some arbitrarily chosen points. How many stationary points do you find? Which kinds of stationary points?

Those who wish to play further may test the other functions. They are:

(3) $f(x_1, x_2) := -5e^{-\frac{x_1^2+x_2^2}{10}} + 4e^{-\frac{x_1^2+x_2^2}{100}} - 5e^{-\frac{(x_1-5)^2+(x_2+4)^2}{10}} - 5e^{-\frac{(x_1+4)^2+(x_2-5)^2}{10}} - 4e^{-\frac{(x_1+4)^2+(x_2-5)^2}{100}}$

(5) $f(x_1, x_2) := -4e^{-\frac{(x_1+2)^2+(x_2+1)^2}{10}} + 4e^{\frac{(x_1+2)^2+(x_2+1)^2}{100}} + 0.01((x_1+2)^2 + (x_2+1)^2 + 0.01x_1$

(6) $f(x_1, x_2) := (x_1^3 - x_2)^2 + 2 * (x_1 - x_2)^4$

(7) $f(x_1, x_2) := -5\cos(0.2(1 + \dfrac{x_2^2 - 0.5}{x_1^2 + 0.5})) + 0.001x_2^4 + 0.003x_1^4 + 2x_1$

(8) $f(x_1, x_2) := 2(x_2 - x_1^3)^2 + 0.1(x_1 + 1)^2 + 0.5x_2^2 + x_1^2x_2^2$