Computer exercise 2

Nonlinear optimisation (TMA947)

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1 Part I

1. Using the interior point method, solve the LPs 01-03. Do we always find an optimal extreme point (problem 03)? Notice how the algorithm follows closely the central path and goes "directly" to the global minimum point (i.e., it skips visiting the extreme points), if you change the penalty parameter smoothly. Compare with the Simplex method.

Ran until $\rho < 0.001$

Problem 1: (2, 0)
Problem 2: (6.54, 0.55)
Problem 3: the line $x_2 = 0$ for $10 \leq x_1 \leq 20$ (18.74, 0)

The interior penalty method does find an optimal point for problem 3, but it's not an extreme point. There is a line of optimal points for this problem where the optimal line is $10 \leq x_1 \leq 20$, $x_2 = 0$. The method converges to one optimal point and fulfils the minimisation, however the converging point is not always an extreme point. The converging point could be dependant on the initial condition. Compared to the Simplex method, Simplex visits the extreme points, rather than approaching the optimal point directly. Simplex is an "extreme point"-based method, unlike the penalty method. We would have found the extreme point in problem 3 using that method.

2. Find the KKT points of the nonlinear problems. Are the KKT conditions sufficient for the global optimality (problem 03)? Are they necessary (problem 04)? Hint: There is a built-in tolerance in the graphical interface that sometimes fools you. Make sure to analytically verify the results. This hint is especially important for the problem 04.

Problem 1:
(2, 1) is the only point where the negative gradient can be expressed as a non-negative linear combination of the active constraints. The problem is convex so KKT is sufficient, and the constraints are affine so Abadie's hold, hence KKT is also necessary.

Problem 2:
There are KKT-points all long the line $x_1 = 2$, $1 \leq x_2 \leq 4$ because any point on this line is where the negative gradient can be expressed as a non-negative linear combination of the active constraints. The problem is convex so KKT is sufficient, and the constraints are affine so Abadie's hold, hence KKT is also necessary.

Problem 3:
(0.700597, 0.767873),
(3.18299, 0.762443),
(2.58119, 2.57602),
(0.437313, 3.15158),
(2.02776, 0.751584),
(0.986716, 1.8674)

The problem is non-convex so KKT is not sufficient. One constraint is non-convex so Slater CQ does not hold, however LICQ hold at the solutions hence Abadie's hold at these points so KKT is necessary.

Problem 4:
The point (0.370896, 0.00395928) is the only point shown to be a KKT point in the program, however this is not true because when the constraints $g(x) = x_2 \geq 0$ and

$g(x) = 1/8x^3 - x_2$ intersect and are active is in origo, not in the suggested point. The gradients of the active constraints in origo are $(0,-1)^T$ and $(0,1)^T$ respectively, meaning that they are vertical to each other. The negative gradient in origo is $(3{,}0)^T$, so it is impossible to express this negative gradient as a non-negative combination of the active constraints in this point. Thus, there are not KKT-points in this problem.

The problem is non-convex so KKT is not sufficient. One constraint is not convex so Slater CQ does not hold. LICQ does not hold either, hence Abadie's does not hold so KKT is also not necessary.

3. Using the exterior penalty algorithm, solve the nonlinear and linear problems. Can you get different "optimal" solutions by changing the penalty parameter in a different manner or by starting from different points (problem 03)? Tricky: Can you think of a reason for the slow convergence in problem 04 (hint: KKT)?

LP1: (2,0) Ran until $\mu > 80$.
LP2: (6.54,0.55) Ran until $\mu > 10{,}000$
LP3: (20,0) Ran until $\mu > 500$

We can get different results by changing the initial value for μ. For example, setting $\mu = 0.97$ gives the result (15.37,0). Setting $\mu = 5.08$ gives the result (11.13,0).

NLP1: (2,1) Ran until $\mu > 700$.
NLP2: (2,2) Ran until $\mu > 500$
Can get different results by changing the starting point.

NLP3: (0.68, 0.75) Ran until $\mu > 10{,}000$
Can converge to other KKT-points for other starting points and larger penalty parameter.

NLP4: (0,−0.05) Ran until $\mu \sim 10^8$

Slow convergence in problem 4 is due to not having any KKT points.

2 Part II

Problem 1: 1.5625 at (0.6250, 1.2500)

The objective function is concave because the Hessian of the negative function is positive definite (the minimisation of the function is convex). The two constraints are convex as well (linear is obvious and the non-linear gives a positive semi-definite matrix). Therefore, the problem is convex so KKT is sufficient. This implies that the maximum solution found is indeed a global maximum.

Problem 2:
(1, 1): 3.4 at (3.4000, 1.2000)
(0, 0): -3 at (-3.0000, -2.0014)
(3.7, 0): 3.4 (3.4000, 1.2000)
(-1, -1): -3 at (-3.000, -2.0014)
(1,-1): -3 at (-3.000, -2.0014)

The best points are (0, 0), (-1, -1) and (1,-1) because the model converge to the global minimum for these points whereas for the other points the model converge to local minimum.

The objective function and the first constraint are convex, but not the second constraint. Therefore, we don't have a convex problem. Hence, KKT is not sufficient so no global optimum can be guaranteed for any found solution. However, in the point (-3.000, -2.0014) we have that the negative gradient can be expressed as a non-negative linear combination of the gradients of the active constraints. Thus, this is a KKT point. The point (3.4000, 1.2000) of the other solution is not a KKT point because the negative gradient cannot be expressed as a non-negative linear combination of of the gradients of the active constraints.