Computer Exercise 1

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October 19, 2023

Problem 1

a)

Steepest descent:

$(10, 10)^T$:
Function value: -11.1562
Solution point: $(-2.2491, -3.06255)^T$
No. iterations: 9

$(-5, -5)^T$:
Function value: -11.1562
Solution point: $(-2.25225, -3.06234)^T$
No. iterations: 9

$(10, -5)^T$:
Function value: -11.1562
Solution point: $(-2.24885, -3.06266)^T$
No. iterations: 16

Newton (unit step):

$(10, 10)^T$:
Function value: -11.1563
Solution point: $(-2.25, -3.0625)^T$
No. iterations: 1

$(-5, -5)^T$:
Function value: -11.1563
Solution point: $(-2.25, -3.0625)^T$
No. iterations: 1

$(10, -5)^T$:
Function value: -11.1563
Solution point: $(-2.25, -3.0625)^T$

No. iterations: 1

b)

The obtained point is first and foremost local. Because the objective function is convex, by the fundamental theorem of global optimality we have that every local optimum is a global optimum. See calculation.

c)

With the gradient and the Hessian (second order approximation) we recover information about the quadratic curvature. Since the function is quadratic we therefore recover all information about the objective function.

Newton's method converges quadratically meaning that the error reduces quadratically. If the descent is steep smaller steps will be taken and larger steps for flater descents. If the function is perfectly quadratic the Newton's method will converge in one iteration. See calculation.

Problem 2

a)

Steepest descent:

$(-2, -1)^T$:
Function value: 0.058814
Solution point: $(0.757571, 0.573264)^T$
No. iterations: 201 (max iterations)

Newton, modified:

$(-2, -1)^T$:
Function value: 2.65683-11
Solution point: $(0.999995, 0.999991)^T$
No. iterations: 14

Newton (Marquardt):

$(-2, -1)^T$:

Function value: 2.6568-11
Solution point: $(0.999995, 0.999991)^T$
No. itrations: 14

Newton (unit step):

$(-2, -1)^T$:
Function value: 2.0031e-27
Solution point: $(1, 1)^T$

No. iterations: 5

b)

The function is not convex. The eigen values of the Hessian are dependent in x_1 and x_2. The hessian is therefore not positive definit nor positive semi-definite for all x_1 and x_2, only for certain values. Thus, Rosenbrock's function is not convex. See calculation.

c)

$(0, 2)^T$:
Function value: 4.1501e-09
Solution point: $(100006, 1.00013)^T$
No. iterations: 8

An observation was that Newton's method takes larger steps for steeper slopes and smaller steps for gentler slopes. Say that the gradient has quadratic terms, then the Hessian must have linear terms. The gradient will therefore tend to get much larger than the Hessian for large values and tend to get much smaller than the Hessian for smaller values. Since we have f'/f" this will result in the observation that was stated.

Problem 3

a)

Steepest descent:

$(0, 0)^T$:
Function value: -58
Solution point: $(2.9978, 0.99993)^T$
No. iterations: 4

Newton (unit step):

$(0, 0)^T$:
Function value: 2.0031e-27
Solution point: $(1, 1)^T$
No. iterations: 5

b)

If we start on $(0, 0)^T$ we have a singularity in f'/f'' because the Hessian is 0 at this point. See calculation.

$(1, 1)^T$: minimum
Function value: -58
Solution point: $(3.00009, 1)^T$

No. iterations: 4

$(-3, 3)^T$: saddle point
Function value: 50
Solution point: $(-3, 1.00003)^T$
No. iterations: 4

$(-3, -1.5)^T$: maximum
Function value: 58
Solution point: $(-3, -1.00001)^T$
No. iterations: 3

$(3, -1.5)^T$: saddle point
Function value: -50
Solution point: $(3, -1.00001)^T$
No. iterations: 3