INTRODUCTION TO GAME THEORY

Understanding the Game

Game theory: The study of decision-making in situations where one player's outcome depends on the decisions of other players.

A **strategic game** is the fundamental schema for analyzing decision-making via game theory. They have three elements:

1. **Players**: Participants in the game.
2. **Actions:** A player's options for how to behave in the game. Taken together, the players' actions create an outcome.
3. **Payoffs:** The results associated with a given outcome.

In **simultaneous games**, players select their actions at the same time.

Remember: Games can be played sequentially or simultaneously, but the latter lend themselves to more straightforward analysis.

The Normal Form and Nash Equilibria

The **normal form (strategic form)** is a concise method for representing a strategic game.

Best responses (or **strategies**): The actions that maximize a player's payoff given the actions of the other player.

Strategic form is typically drawn as a matrix, with players' best responses marked with stars.

A formal prediction is called a **solution concept**, but one only exists when all players have at least one best response in common.

Nash equilibrium: A set of actions wherein each player's action is a best response given the actions of all other players.

A game may contain one Nash equilibrium (e.g., the matrix with the yellow highlight), or more than one, or none at all.

Nash equilibria can solve **non-cooperative games**, i.e., ones lacking a formal method for ensuring player cooperation.

The **prisoner's dilemma** is an example of a non-cooperative game.

Matrices: Some Examples

Dominance and Rationalization

Strictly dominant: A strategy that always provides higher payoffs than the other(s).

Weakly dominant: A strategy that provides higher payoffs in at least one outcome, and equal payoffs in all others.

Intransitive: No strategy dominates.

Rationalization: A solution concept involving the evaluation of a game's strategies from each player's perspective. The **iterated elimination of strictly dominated strategies** is an example:

1. Examine the game from Player 1's perspective. Remove any of their strictly dominated strategies.
2. Do the same from Player 2's perspective. Repeat until no strictly dominated strategies remain for either player.
3. Any outcomes that survive this process are **rationalizable strategies**. If only one survives, the game is **dominance solvable**.

A game in which the strategies of both players are intransitive.

Tom / Alexa	rock	paper	scissors
rock	0,0	–1,1	1,–1
paper	1,–1	0,0	–1,1
scissors	–1,1	1,–1	0,0

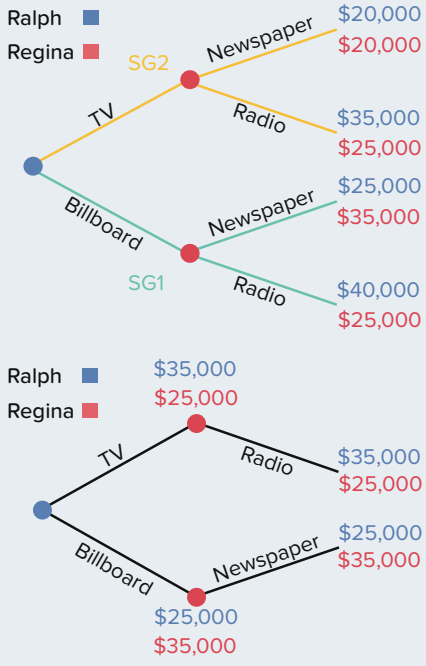
Extensive-Form Games

The extensive form is a means for modeling sequential games. Its major components are:

- **Game tree:** A decision tree used to show players, strategies, and payoffs.
- **Node:** Each circle on the game tree represents a decision point for the respective player.
- **Branch:** Shows the possible strategies at each node.
- **Terminal node:** Shows the payoffs tied to each chain of decisions.

Backward induction: A method for solving extensive-form games.

Examples of exenstive form:



After breaking the game down into smaller **subgames:**

1. Identify Player 2's best responses based on her payoffs at the terminal nodes. Eliminate the remaining strategies.
2. Shift those payoffs forward, creating new terminal nodes. Then select Player 1's best response.
3. The surviving chain of strategies is called a **subgame perfect equilibrium**.