

# **Sustainable Supplier Selection and Order Allocation Problem: A Multi-Period, Multi-Product, Multi-Demand Stochastic Model**

**ME Mechanical Engineering with  
Business**

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## Abstract

This thesis introduces a multi-period, multi-demand, multi-product stochastic model to address the complexities of sustainable supplier selection and order allocation (SSS&OA) within supply chains. Recognising the limitations of deterministic models, this study shifts towards a stochastic approach to better accommodate the uncertainties and dynamic demands of modern supplier selection. The model address the trade-off between cost minimisation and sustainability maximisation, offering a resilient framework for decision-making. Grounded in a robust literature review, the project approaches the solution using both the weighted sum method and the augmented  $\epsilon$ -constraint method to develop a cohesive strategy for practical future applications. Empirical data validates the model's effectiveness, with findings incorporating a balance between economic efficiency and environmental responsibility. The research contributes to sustainable supply chain management by presenting an adaptable tool for procurement in fluctuating markets, with broader implications for theory and practice. Concluding remarks reflect on the model's implications, its limitations, and propose directions for future research.

# 1 Introduction

## 1.1 Background & Context

With every passing year, industries worldwide are becoming more environmentally conscious, largely due to waning resources, environmental pollution and economic globalisation, forcing industry to take responsibility for their sustainability practices [1]. These changes are underscored by alarming statistics indicating that industries contribute significantly to global carbon emissions, with the manufacturing sector alone accounting for approximately 20% of global CO<sub>2</sub> emissions [2]. Additionally, the consumption of non-renewable resources in production processes continues to raise concerns about the long-term viability and environmental impact of these practices.

Companies across various sectors are now re-evaluating their operational strategies to incorporate sustainability into their core activities. Sustainability objectives vary widely but typically include reducing greenhouse gas emissions, ensuring ethical material sourcing, improving workplace conditions, and minimizing travel distances for transporting goods. These considerations are critical as businesses not only strive to reduce their environmental impact but also to enhance their social and economic sustainability. Adopting the Triple Bottom Line (TBL) framework, businesses are increasingly integrating economic goals with environmental and social responsibilities [3]. The TBL framework advocates for a balance between economic growth, environmental stewardship, and social equity, challenging companies to redefine success and broaden their accountability beyond traditional financial gains. This approach has been pivotal in encouraging businesses to conduct comprehensive sustainability assessments and make procurement decisions that align with broader environmental and societal goals [4].

The integration of sustainable practices into procurement strategies involves complex trade-offs, primarily between cost optimization and sustainability objectives [5].

While environmentally sustainable choices may incur higher upfront costs, they can lead to significant long-term savings and benefits. For example, investing in renewable energy sources or sustainable materials often involves higher initial expenditures but can result in lower operational costs due to energy savings and reduced waste [6]. In the context of global supply chains, the selection of suppliers and the allocation of orders are crucial for embedding sustainability into business operations [7]. Effective supplier selection ensures that companies mitigate risks such as regulatory non-compliance and reputation damage, while also capitalising on competitive advantages in industries where consumers demand ethical production practices. Simultaneously, strategic order allocation optimises resource use and minimises waste, significantly reducing a company's environmental impact through smarter logistics and supply chain management [8].

However, sustainable supplier selection and order allocation involve complex assessments of each supplier's environmental, social, and economic practices, necessitating resource-intensive monitoring and continuous improvement [9]. As businesses face increasing competitive pressures and regulatory demands, investing in these areas becomes crucial for maintaining resilience and achieving long-term sustainability goals, making supplier selection and order allocation pivotal in shaping sustainable business strategies.

## 1.2 Problem Statement

The literature on sustainable supplier selection and order allocation (SSS&OA) has significantly advanced our understanding of integrating sustainability criteria into procurement processes. Despite these advancements, a gap persists in specifically addressing the order allocation problem within a sustainability framework [10]. While the framework for selecting suppliers is thoroughly articulated across various industries [11], [12], it often overlooks the dynamic nature of the market and the stochastic nature of demand which are crucial for the order allocation phase. This oversight can render the models less responsive and adaptive to the actual conditions of supply chain environments.

Traditional deterministic models, which have been the backbone of much of the existing research, tend to simplify these complexities by assuming static and predictable demand. These assumptions are not aligned with the realities of modern markets, which often have rapid fluctuations in demand and supply conditions. This discrepancy results in procurement strategies that lack the flexibility to respond to changes in market conditions, leading to inefficiencies and potential disruptions in the supply chain [13]. Deterministic models also often fail to accommodate the potential variations in supplier performance over time, which can significantly impact the sustainability outcomes of procurement decisions. There is a pressing need for models that not only encapsulate the sustainability criteria in supplier selection but also contain more robust and flexible approach to order allocation that accounts for the inherent uncertainties of market dynamics and demand variability. Such models would represent a more holistic approach to SSS&OA, ensuring that sustainability goals are met more consistently and effectively across the supply chain life-cycle.

## **1.3 Aims & Objectives**

This project aims to address the gap in the current research on sustainable supplier selection and order allocation (SSS&OA) by developing a multi-period, multi-demand, multi-product stochastic model. This model will surpass the limitations inherent in deterministic frameworks, providing a resilient and adaptive solution that is better aligned with the dynamic and uncertain nature of modern supply chains. The primary aim is to enhance an existing model, initially introduced by Ghadimi et al. in 2018 [14], evolving it to incorporate stochastic elements that reflect real-world variability and demand fluctuations. This evolution will enable the model to handle unexpected changes and disruptions within the supply chain, offering more realistic and applicable solutions.

Additionally, the project seeks to conduct a comprehensive literature review focusing on the evolution of order allocation methodologies and solution techniques that have been explored in previous research. The objective is to collate these findings to form a cohesive understanding of the state-of-the-art in order allocation strategies, thereby identifying potential areas for innovation and improvement. Through this extensive review, the project will highlight the trends and shifts in methodology that could inform future developments in the field. Ultimately, this will contribute to a broader and more nuanced comprehension of how complex supply chain challenges can be tackled more effectively.

## **1.4 Report Layout**

The layout of the report begins with a comprehensive literature review that covers the order allocation strategies and models which have been proposed in previous research. This is followed by an in-depth exploration of the model development process, illustrating the transition from deterministic to stochastic demands, as well as introducing a time index to the existing constraints, objectives and parameters. The data and implementation chapter explains the practical application of the model, utilising real-world data to validate the proposed framework. Subsequent chapters on results and discussion discuss the flexibility of the model, drawing conclusions from the empirical findings. The report ends with concluding remarks that reflect on the theoretical and managerial implications of the study, acknowledging its limitations, and suggest pathways for future research.

## 2 Literature Review

The pursuit of sustainability goals has become a major factor in business operational decisions. The global marketplace has become more environmentally and ethically conscious as a whole. One of the most sought after parameters to improve the sustainability of a business is the choice of suppliers and order allocation. Supplier choice can be a paramount decision as factors such as material source, labour used, transportation time and so forth can critically effect not only the sustainability of the business but also the economical gain [15]. In previous studies on purchase management in supply chains, supplier selection has been the focus.

Supplier sourcing problems can take two forms: single source or multiple source. In single sourcing problems, one supplier has the capability to meet all of the buyers needs. In multiple source cases, a combination of different suppliers are required to meet all of the buyers' needs [10]. The issue of order allocation arises from multiple sourcing problems. Deciding how to allocate order quantities between an array of different possible suppliers is a crucial managing task which not only has an effect on supplier selection but also has an impact on the company's supplier relationship. To date, there has been a robust amount of literature published on the supplier selection aspect of this problem but order allocation (OA) is often overlooked as it is assumed to follow directly from supplier selection in multi-sourcing problems. However it is often a much more complex issue than this.

Therefore, this literature review aims to expand the review of literature specifically focused on OA in sustainable supply chains. To date, there have been four relevant reviews published on this topic, as can be seen in Table 1:

Table 1: Literature Reviews concerning Order Allocation

Authors and Year	Time Horizon	No. of Articles	Scope of Research
Setak, M., Sharifi, S., Al-imohammadian A. (2012) [16]	2000-2010	170	Supplier Selection (SSP) and Order Allocation (OA) from year 2000 to 2010, reviewing vendor selection criteria and different methods.
Aouadni, S., Aouadni, I. & Rebai, (2019) [17]	2000 - 2017	270	SSP and OA models in which a new structure and classification of the existing research streams & different MCDM methods and mathematical models used for SSP are presented.
Di Pasquale, V., Elena Nenni, M.,&Riemma, S.(2020) [10]	1979 - 2018	113	Expanding the review of scientific literature regarding OA models, identifying research gaps and opportunities, suggesting research agenda for the development of OA models according to current SC trends
Naqvi, M.A., Amin, S.H. (2021) [13]	2015 - 2020	92	SS&OA examined in three subcategories including Literature Reviews (LR), Deterministic Optimisation (DO) models, and Uncertain Optimisation (UO) models.

## 2.1 Research Design

### 2.1.1 Research Questions

The below research questions were carefully considered and chosen with the underlying drive to comprehensively understand the optimisation of supplier order allocation while integrating sustainability considerations:

- RQ1: What are the most common modelling approaches and solution methods?
- RQ2: What are the objective functions chosen?
- RQ3: What industries are being focused on and which sectors aren't included?
- RQ4: What are the environmental and social sustainability aspects addressed?
- RQ5: How have solution methods evolved?

RQ1 and RQ2 sought to find trends in the fundamental methodologies underpinning decision-making processes, probing into the most common modelling approaches, solution methods, and the chosen objective functions. By delving into these aspects, the review aimed to provide a comprehensive overview of the quantitative tools and frameworks currently employed in sustainable supplier order allocation models.

RQ3 explores the contextual backdrop by investigating the industry and sectors considered within the literature. Understanding the industries under scrutiny and identifying sectors that may be underrepresented or overlooked is crucial for contextualising the applicability of existing models. Moreover, RQ4 extended to scrutinise the environmental and social sustainability aspects addressed within the literature. This focus underscores the recognition of the intertwined nature of economic, environmental, and social dimensions within sustainable supply chains. Investigating how these aspects are integrated into order allocation models is essential for an ethically-informed decision-making process.

Finally, RQ5 delves into the evolution of solution methods over time. The aim is to provide insight into the adaptive nature of decision-making frameworks, showcasing trends and advancements that have advanced the progression of sustainable supplier order allocation research. Thus, research gaps were identified and future research opportunities are highlighted.

### 2.1.2 Research Methodology and Boundaries

This review had the aim of providing peer-reviewed insights from quality publications. Therefore the main database used to collect literature was the Scopus database. Overall, 48 papers on SSS&OA published in the Scopus database from 2014 - 2023 were collected. The process of collecting the articles included in the review is as follows.

1. The Scopus database was searched by different combinations of the following keywords “sustainable supplier selection”, “order allocation” by linking them with “and” connectors. The searched combination strings are “sustainable supplier selection and order allocation”, “order allocation and sustainability”.
2. Conference papers, review studies and book chapters were eliminated from the search process, leaving only English articles published in journals.
3. After examining each paper under the criteria of the six research questions, papers which were not able to answer all questions were eliminated. This left 44 research papers and 4 literature reviews to be examined.

Figure 1 shows the breakdown of the published year of each of the examined papers:

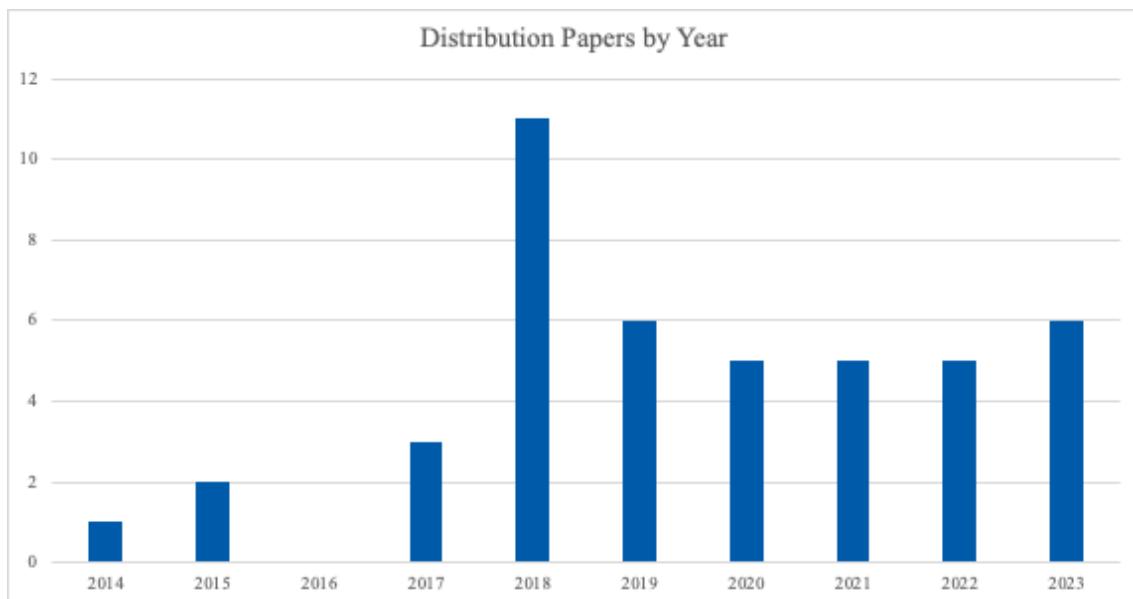


Figure 1: Distribution of Papers by Year

Additionally it is noted that whilst this paper endeavours to focus on order allocation modelling and solution techniques, the existing published literature often covers the topic of SSS&OA simultaneously, and very few examine order allocation in isolation. The vast majority of reviewed papers utilised Multi-Criteria Decision Analysis (MCDMA) methods as a first stage modelling approach before moving onto the order allocation stage. The most common approaches were Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), Analytic Hierarchy Process (AHP) or a hybrid approach which looked at SSS&OA simultaneously. TOPSIS is

a useful technique to evaluate suppliers across economic, environmental and social dimensions simultaneously [18]. While it is more frequently utilised in the supplier selection aspect of decision making, integrated models which dealt with supplier selection and order allocation simultaneously often employed TOPSIS as their selected modelling method. Yadavalli et al. utilised a TOPSIS integrated modelling approach in order to further include the customers' sustainable expectations into the supplier evaluation attributes [19]. Mohammed et al. utilised the TOPSIS technique in their presentation of an integrated MCDM-fuzzy multi objectives optimisation model to solve a two-stage sustainable supplier selection and order allocation problem in the meat industry [20].

AHP is also widely utilised as a decision-making tool in the realm of sustainable supplier selection due to its capability to handle intricate and multi-criteria decision problems. AHP allows decision-makers to systematically evaluate and prioritise suppliers based on various sustainability criteria, encompassing factors such as environmental performance, social responsibility, and economic viability. Its strength lies in providing a structured framework for capturing both qualitative and quantitative aspects, facilitating a comprehensive assessment of supplier sustainability. Kumar et al. acknowledged the applicability of AHP within the broader context of sustainable supply chains by building it into their SSS&OA model [21]. Mohammed et. al integrated both AHP and TOPSIS to make the most informed supplier decision in their model [22].

Other lesser used but still notable selection methods utilised in the studies were the Best Worst Method (BWM), [23] Delphi method, and Combinative Distance based assessment (CODAS) [24]. Li et. al utilised the BWM to consider both environmental and supply risk concerns at the same time [25]. Rabieh et al. meanwhile used the Delphi method to harness expert opinions and build a consensus iteratively. It allowed for the cultivation of diverse perspectives, fostering a more holistic understanding of sustainability criteria and ensuring a robust and informed decision-making process [18]. While these MCDMA methods are not the focus of the discussion of the paper, they are nevertheless important in the context of each order allocation model which was developed by researchers.

## 2.2 Discussion

### 2.2.1 RQ1: Modelling Approaches

The models developed for order allocation were frequently bi-objective or multi-objective decision models. The most commonly encountered modelling approaches were Linear Programming (LP), Multi-Objective Mixed Integer Linear Programming (MOMILP), Fuzzy Multi-Objective Linear Programming (FMOLP), Multi-Objective Optimisation Models (MOOM), as well as mathematical heuristics and meta-heuristic algorithms, such as genetic algorithms. 36% of papers reviewed contained models which were inputted with fuzzy numbers to account for uncertainty in variables such as supply & demand, environmental & social factors and supply risk [26].

Table 2 illustrates the modelling techniques utilised by each paper:

Table 2: Optimisation Methods Used in Literature Models

Programming Used	Paper No.
Fuzzy Mixed Integer Linear Programming (FMILP)	[27],[26]
Dynamic Programming (DP)	[28]
Multi-Objective Metaheuristic Algorithm (MOHEV)	[29]
Fuzzy Multi-Objective Optimisation Model (FMOOM)	[30],[31],[32]
Fuzzy Multi-Objective Linear Programming (FMOLP)	[21],[33],[34],[9],[3], [35],[11]
Multi-Objective Mixed Integer Linear Programming (MOMILP)	[5],[36],[37],[38],[18], [39],[8],[23],[40]
Fuzzy Multi-Objective Optimisation Model w/ Ratio Analysis (FMOORA)	[37]
Multi-Objective Mixed Integer Non-Linear Programming (MOMINLP)	[26],[14],[25]
Multi-Objective Non-Linear Programming (MONLP)	[41], [42]
Fuzzy Multi-Objective Programming Model (FMOPM)	[22],[43],[44]
Mixed Integer Programming (MIP)	[19]
Linear Programming (LP)	[45]
Multi-Objective Optimisation Model (MOOM)	[4],[46],[24],[47],[48], [7],[49]
Possibilistic Multi-Objectives Optimisation Model (PMOOM)	[20]
Multi-Objective Stochastic Programming (MOSP)	[12],[50]

*LP* models stand out for their adeptness in optimising allocation decisions considering various factors like cost, lead time, and environmental impact. Jindal and Sangwan proposed a fuzzy multi-integer LP model in order to optimise a multi-facility closed loop supply chain network, including the order allocation of each supplier [27]. Kumar et. al developed a fuzzy multi-objective LP model for order allocation in a sustainable supply chain, focusing on carbon emissions and taking social welfare impact into consideration [21]. Cheraghaliour and Farsad furthered the trend of LP models by proposing a novel bi-objective mixed integer LP method which

considered quantity discounts under disruption risks [51]. More recently, Tavana et. al introduced a comprehensive framework which integrated a multi-objective MILP model, focusing on minimising CO<sub>2</sub> and maximising job opportunities created [40].

*MOMILP* in sustainable order allocation models offers a multitude of advantages, contributing to its frequent usage - as seen in Table 1, it was the model of choice for almost 25% of the studied papers. MOMILP provides a robust framework enabling decision-makers to optimise multiple conflicting objectives simultaneously, including economic efficiency, environmental impact, and social responsibility [37]. By incorporating MOMILP strategy, Goren et al were able to accommodate the discrete decision variables of both sustainability values but also lost sales, which are intrinsic to order allocation processes [38]. The versatility of MOMILP enables the inclusion of various sustainability criteria, offering a comprehensive method for decision-making that aligns with the complex and interconnected nature of sustainable supply chains. Khalili et. al utilised the modelling technique to create a novel multi-objective circular SSS&OA model for a closed loop supply chain [23]. Furthermore, MOMILP's ability to address multiple objectives concurrently facilitates the identification of Pareto-optimal solutions, empowering decision-makers to perform trade-offs and make informed choices that balance goals. Moheb-Alizadeh et. al utilised this strategy to devise a model which addressed the decision across multiple periods, multiple products and multimodal transportation options in the presence of shortage and discount conditions [39].

*FMOLP* in sustainable order allocation models provides specific advantages that distinguish it from other modelling approaches. 16% of the researched literature utilised this method in their models. FMOLP excels in managing the inherent uncertainty and imprecision associated with sustainability criteria [33]. Its integration of fuzzy logic enables the representation of vague or qualitative information, allowing for a more realistic depiction of the uncertain nature of environmental and social factors. Kumar et. al utilised this technique in the OA model development for a case study in the automobile industry with limited information [21]. This capability is particularly crucial in sustainable supply chain contexts where precise data may be challenging to obtain. Similarly, Tayyab et. al introduced a novel model unique to the textile industry using fuzzy logic [35]. The widespread use of FMOLP in sustainable order allocation models reflects its efficiency in addressing the nuanced challenges posed by uncertain and multifaceted sustainability criteria.

*Hybrid MOOM* are becoming more common in recent years as technology continues to grow and develop, and optimisation techniques improve. It integrates multiple optimisation techniques, combining the strengths of different approaches to enhance the overall modelling capability. This versatility allows decision-makers to address a broader spectrum of sustainability challenges by incorporating complementary methodologies. For example, Kellner and Utz modelled a supply chain using a combination of both integer variables and Markowitz portfolio theory, which captured the discrete nature of decision variables while leveraging the principles of portfolio optimisation [4]. Hybrid MOOM is especially advantageous in scenarios where sustainability criteria involve both quantitative and qualitative aspects, as it can seamlessly handle diverse data types. This is observed in a study published by

Zhao et. al, where they propose an integrated model which can deal with multiple different scenarios [49]. The adaptability of Hybrid MOOM to the complex and evolving nature of sustainable supply chains positions it as an upcoming preferred choice in sustainable order allocation modelling.

### 2.2.2 RQ2: Objective Functions

The most commonly observed objective function were bi- or multi-objective by combining either "Max Profit" or "Min Cost" along with one or more environmental and social criterion i.e. "Maximising Social Impact" [43], "Min Pollution" [3]. Upon review, 88.6% of all the literature had either bi- or multi-objective modelling approaches as described above. In recent years the idea of Sustainable Value Purchase has been introduced, which combines profit and sustainability values with the Triple Bottom Line Ideology (see RQ4). Often the more holistic values relied on numeric weighting systems developed by individual researchers for each paper. See Table 3 for the objective functions utilised and the papers that they are associated with:

Table 3: Objective Functions in Literature Models

Objective Function	Paper No.
Max Profit	[27],[34],[48]
Min Cost	[29],[30],[21],[5],[36],[33],[34],[51],[26],[52],[41],[38],[22],[19],[18],[45],[39],[4],[43],[9],[3],[8],[20],[44],[31],[35],[42],[25],[23],[46],[40],[12],[11],[50],[47],[32],[48],[7],[49]
Min Env. Impact	[29],[30],[21],[22],[43],[20],[42],[23],[46],[48],[49]
Max Supplier Reliability	[30],[8],[49]
Min CO2 Emissions	[21],[39],[31],[40],[11],[32],[49]
Max Social Impact	[21],[43],[20],[46]
Max Sustainable Purchase Value	[5],[33],[51],[38],[22],[14],[19],[43],[8],[31],[42],[12],[47],[49]
Min Lost Sales	[37],[23]
Min Risk	[37],[4],[44],[25],[11],[47]
Max TBL Score*	[26],[52],[41],[14],[18],[45],[4],[44],[50],[7]
Min Travel Time	[22],[20],[42]
Max Job Opp.'s Created	[39],[23],[40],[11],[32]
Min Pollution	[3]
Min Late Deliveries	[3],[50]
Min Defect Rate	[42]
Max Resilience	[47]
Min Travel Distance	[47],[7]
None	[28],[24]

\*Max TBL score implies Max Environmental score, Max Social score and Max Economical score combined.

The objective functions in sustainable supplier selection models are pivotal in defining the focus and effectiveness of the methodologies applied. According to the summary provided in Table 3, while the most common objectives are centered around minimising cost and maximising sustainability performance, there are several notable instances where specific operational objectives centering around sustainability of some description are prioritised. These particular objectives underscore the operational challenges that industries face beyond cost and sustainability.

For instance, the objective of "Minimising Late Deliveries" addressed by Calik [3] and Safari [50] reflects a critical aspect of supply chain efficiency that directly impacts customer satisfaction and retention. In industries where timely delivery is crucial, such as in fast-moving consumer goods or pharmaceuticals, delays can have significant financial and reputation repercussions. Therefore, incorporating such an objective can be essential for models that are tailored to these sectors. Similarly, "Minimising Defect Rate" as explored in paper [42] is particularly relevant in manufacturing contexts where the quality of the product is as critical as the sustainability of the processes used to produce it. High defect rates can lead to increased waste, higher costs, and lower customer satisfaction, which contradicts the principles of sustainability. By including defect rates as an objective, models can provide more holistic solutions that not only focus on environmental and social impacts but also enhance product quality and process efficiency.

These less common objectives reflect a nuanced understanding of the complexities of modern supply chains where operational efficiency, product quality, and punctuality are interwoven with environmental and social responsibilities. The inclusion of such specific operational objectives suggests a shift towards more comprehensive and context-specific models that cater to the particular needs of different industries and operational challenges. This evolution in model objectives is crucial for developing more robust and adaptable strategies that align with the detailed operational realities of businesses.

An emerging trend in the objective functions of sustainable supplier selection models is the explicit inclusion of "Minimising CO2 Emissions," reflecting the growing emphasis on quantifiable environmental impacts in industry practices. The focus on reducing carbon emissions is driven by increased regulatory pressures, consumer awareness of climate change, and the corporate responsibility to mitigate environmental impacts. Unlike broader and often vaguer sustainability metrics, CO2 emissions can be precisely measured and directly linked to specific operational activities, making it an attractive metric for companies aiming to demonstrate tangible improvements in their environmental footprint. This objective is becoming more prevalent as industries seek to align with global sustainability goals such as those outlined in the Paris Agreement. Notable studies that have integrated "Minimising CO2 Emissions" as a key objective include those by authors Kumar [21], Moheb-Alizadeh [39], and Tavan [40]. These studies underscore the relevance of this objective in providing clear, actionable targets that help firms transition towards low-carbon operations, thereby contributing to global efforts in combating climate change.

### 2.2.3 RQ3: Industries Studied

Figure 2 illustrates the breakdown of the case study industries which were highlighted in the reviewed literature. It can be observed that the largest industry researched was Automobile, covered by 27% of papers. Following this was Original Equipment manufacturers, with 20%. The prominence of the automotive and original equipment manufacturers (OEM) industries in academic research on sustainable supplier selection and order allocation can possibly be attributed to their complex and globally significant supply chain structures. Many papers covering these industries dealt with multi-echelon, multi-period, multi-supplier conditions [39], [4], [28]. These industries serve as rich subjects for investigation due to intricate supply chain interactions, significant environmental and social impacts, stringent regulatory pressures, and a culture of continuous innovation and technology integration.

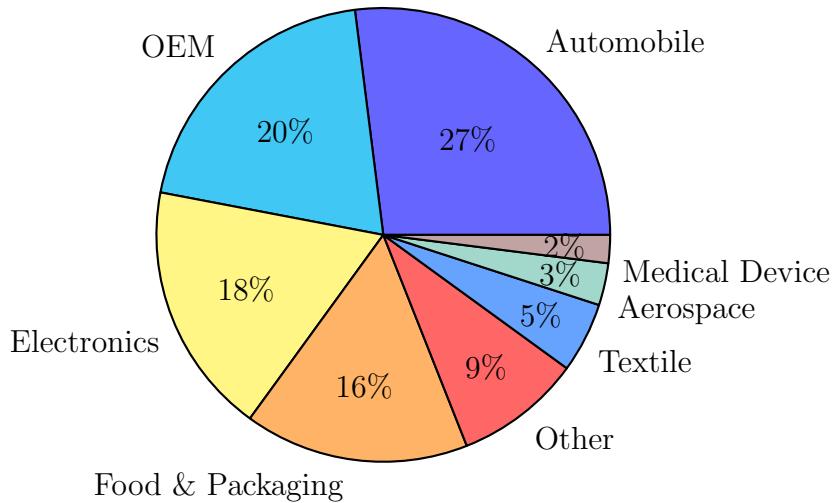


Figure 2: Pareto Front Generated by GPLK Solver

The industries with the least coverage across the research literature were Medical Devices, Aerospace and Textile. The scarcity of case studies in research on sustainable supplier selection and order allocation within the textile, medical devices, and aerospace industries can possibly be attributed to several industry-specific factors. Firstly, the textile industry, characterised by its vast and diverse supply chain, may face challenges in standardising sustainability criteria across the multitude of materials and processes involved. Additionally, the highly competitive nature of the industry might result in a reluctance among companies to share proprietary information in detailed case studies, as mentioned in the Textile case study published by Khalili et. al [23]. Although Tayyab and Sarkar published a case study involving the textile industry, it was mentioned that SME's did not have the facilities to evaluate the origins of the dyes that were supplied, and thus fuzzy assumptions were used [35]. In the medical devices sector, stringent regulatory requirements and the emphasis on product safety and quality may limit the willingness of organisations to disclose sensitive operational details. Furthermore, the critical role played by intellectual property and stringent confidentiality considerations may hinder the comprehensive sharing of sustainable practices. However, one case study was noted in the literature post-COVID-19 pandemic, published by Nayeri et. al [32]. Finally, the aerospace

industry also often operates under classified or proprietary conditions, restricting the availability of detailed case studies. In spite of this, Hosseini et. al procured data for a study in the aerospace industry, driving research for SSS&OA into this sector forward [12]. In all three industries, the intricate and sensitive nature of supply chain operations, coupled with concerns about competitive advantages and confidentiality, may contribute to the relative scarcity of comprehensive case studies in the academic literature.

Over time, there has been a noticeable trend towards expanding the application domains of order allocation models. Researchers are tailoring solutions to industry-specific challenges, recognising that sustainability considerations may vary across sectors. This trend contributes to the development of more contextually relevant and specialised solutions. For instance, Mohammed et al. included product freshness as one of their model parameters as the case study they were dealing with was the fresh meat industry, which is time-sensitive [22]. Similarly, Hosseini et. al included paramters unique to the Aerospace industry as this was the industy case studu which they applied their model to [12].

#### 2.2.4 RQ4: Environmental and Social Issues Addressed

The concept of the triple bottom line (TBL), first coined by Elkington [53] has emerged as a cornerstone framework for assessing the social, environmental, and economic performance of organisations. Within the realm of sustainable supplier selection and order allocation, the TBL framework has been instrumental in guiding decision-makers toward a holistic evaluation that extends beyond traditional profit-centric metrics. This was evident in the literature as a major inspiration when assigning parameters for SSS&OA models. In fact, 29 out of 44 papers, almost 66%, directly included TBL methodologies into their model development

TBL implementation within supplier selection models necessitates a comprehensive examination of a supplier's social impact. Beyond financial viability, factors such as labor practices, working conditions, human rights compliance, and community engagement are integral components of the assessment. Literature in this domain often underscores the importance of supplier diversity, inclusivity, and equitable practices to foster a socially responsible supply chain ecosystem. For example, Lo et. al produced a table within their research highlighting the following key TBL factors: Product Quality, Green Manufacturing, Service Flexibility, Environmental Performance, Innovation Capability, Green Logistics, Labor Intensity, Financial Stability, Supplier Reputation & Information Safety [34]. As companies have become accustomed to environmental and social factors as the mainstream in more recent years, common key-words that were observed in papers were: Environmental Management System (EMS) [42][12], Staff Development [50][47], and Information Disclosure [20].

Although the TBL framework is by far the most common methodology to integrate environmental and social issues, there were some novel approaches observed in the research of the literature. Li et. al examined all environmental and social factors in terms of potential risk (Financial Risk, Manufacturing Risk, Product Quality Risk, Delivery Risk, Cooperative Risk, Service Risk), with the aims of making a dynamic

and resilient supply chain model [25]. Khalili Nasr et. al added a fourth factor to the traditional TBL framework, making it a quadruple framework. The fourth factor was a Circular aspect, with considerations such as utilising eco-friendly and reusable raw materials, using recyclable packaging and designing products to be reused [23]. This, along with other papers published with a focus on circular economy design around SSS&OA models [48] shows a growing trend towards not just a sustainable supply chain but a circular economy.

Additionally, In terms of objective functions, there is a discernible shift towards incorporating explicit sustainability metrics into order allocation models. Instead of treating sustainability as an auxiliary constraint, recent research integrates environmental and social criteria directly into the optimisation objective. From Table 3, it can be observed that 40/44 (90%) of all the reviewed literature had environmental or social values directly built into the objective function. This trend signifies a more holistic approach to decision-making, balancing economic efficiency with sustainability goals. While environmental sustainability has been a focal point, recent trends indicate an increasing emphasis on quantifying and integrating social impact metrics into solution methods [21], [43]. This acknowledges the importance of social responsibility within sustainable supply chain practices.

There has also been a consideration of uncertainty and risk which has been added to some published works. Recognising the inherent uncertainty in supply chain operations, recent literature emphasises the incorporation of uncertainty and risk considerations into solution methods. Fuzzy logic [20] and stochastic [41], [50] optimisation techniques were employed to address uncertainties in demand, supply, and environmental factors, fostering more resilient and adaptive order allocation strategies. Studies which further aimed to improve the resilience of supply chains became more common after the occurrence of the COVID-19 pandemic. Shao et. al in particular published a model designed for multinational enterprises in the wake of the COVID-19 pandemic [47]. In a similar fashion, Nayeri et. al published a resilient SSS&OA model designed for the medical sector [32].

Finally, collaborative decision-making and stakeholder engagement have gained prominence in recent literature. Models which emphasise collaboration among supply chain partners to achieve mutually beneficial outcomes are increasingly explored. This reflects a growing recognition of the interdependence within modern supply chains. For example, Ghadimi et. al explored the idea of using a multi-agent systems approach in order to provide better communication and structured information exchange and promote multi-partnerships [14]. Taking another approach, Yadavelli et. al engaged with consumers as the stakeholder and developed an optimised model based on consumer expectations [19].

### 2.2.5 RQ5: Solution Method Evolution

As the literature on sustainable supplier order allocation has expanded over time, the evolution of solution methods reflect the dynamic nature of this field. Several discernible trends surfaced from the literature on display, as illustrated by Table 4.

Table 4: Solution Methods in Literature Models

Solving Method	Paper No.
Fuzzy Sets Linear Programming	[27], [29], [21], [33], [34], [7]
Multi-Agent System	[28], [14]
Hybrid Metaheuristics	[29], [46]
Max/Min Solution	[29], [5], [26], [20], [31]
Branch & Bound Method	[36]
Multi-Choice Goal Programming	[51], [52], [32]
Augmented $\epsilon$ -Constraint	[41], [22], [18], [39], [4], [43], [42]
Genetic Algorithm	[41], [46], [47], [49]
Hybrid	[43], [39], [50], [32]
Taguchi Loss Functions	[38]
Weighted Goal Programming	[3], [8], [35], [23], [40], [48]
LP-Metrics	[22], [43], [9], [11]
Dynamic Programming	[12]

In more recent years, the focus has shifted towards more integrated and dynamic models. The literature shows a trend towards leveraging computational advancements and new theoretical frameworks. For example, techniques like machine learning and artificial intelligence have begun to find their applications in predictive models and real-time decision-making systems, enabling more agile and adaptive strategies.

*Hybrid Models for Enhanced Performance:* A notable trend involves the development of hybrid models that combine multiple advanced solution methods. Researchers increasingly explore the synergies between different optimisation approaches, such as merging linear programming with meta-heuristic algorithms, to capitalise on the strengths of each method. Zhao et. al published a paper in 2023 which used a novel genetic programming algorithm alongside a linear programming model for a resilient-SSS&OA model [49]. Similarly, Muneeb et. al introduced a hybrid supplier selection-production-distribution model for re-furbished products with the aim of promoting a circular economy future [48]. This trend aims to enhance the overall performance and robustness of order allocation models.

*Meta-Heuristic Solutions:* Aghaei et. al introduced meta-heuristic solution approaches, specifically using the gray wolf and dragonfly optimisation algorithms [46]. This novel method reflects a shift towards bio-inspired algorithms in supply chain management, which are prized for their ability to find optimal solutions in complex landscapes and are particularly effective in handling nonlinear, multi-modal functions. The novel nRa-NSGA-II algorithm proposed by Shao et. al is an advanced form of the Non-dominated Sorting Genetic Algorithm (NSGA-II), tailored for robust multi-objective optimisation. This method enhances the original NSGA-II by

introducing mechanisms to handle noise and uncertainty in evaluation, making it particularly suitable for complex supply chain scenarios where multiple objectives must be optimised simultaneously [47] .

*Dynamic Programming:* Hosseini et. al employed the distance-to-ideal method combined with dynamic programming to optimise the order allocation process [12]. This innovative approach aims to minimize the distance of the current solution to the 'ideal point', which is typically defined in terms of optimal cost and service level, thereby ensuring that the solutions are not only feasible but also as close to optimal as possible under dynamic conditions.

These trends collectively highlight the evolution of order allocation model solution methods towards more sophisticated, integrative, and context-aware approaches. These advancements have significantly enhanced the ability of organizations to make informed decisions in the face of uncertainty, complexity, and competing objectives. The field continues to evolve, driven by the ongoing pursuit of efficiency, resilience, and sustainability in supply chain operations.

### 2.3 Potential for Further Studies

This literature review has provided a comprehensive exploration of sustainable order allocation, shedding light on the multifaceted considerations, models, and methodologies employed in the integration of economic, environmental, and social factors within supply chain decision-making. The adoption of diverse modeling approaches, such as MOMILP, FMOLP, and Hybrid MOOM, reflects the evolving landscape of sustainable supply chain practices. The prominence of the TBL framework has been a key guiding principle, emphasising the interconnected evaluation of social, environmental, and economic dimensions. The analysis of research questions has delved into modeling approaches, solution methods, sustainability aspects addressed, industry focus, supply chain configurations, and objective functions, providing a comprehensive understanding of the current state of the field.

Although the published research is always growing in size, there are still a few *research gaps* which can be addressed in future studies, addressed below:

*Consideration of Circular Economy Principles:* Further research could focus on developing more sophisticated environmental and social metrics for order allocation models. This includes metrics that capture the full life cycle impact of products and the social responsibility aspects of suppliers. There is a gap in research exploring how order allocation models can be aligned with circular economy principles, promoting the reuse and recycling of products. While it is becoming more common in recent years, with a few papers mentioning circularity since 2021, [48] there is certainly more room for further research, in different industries and under different conditions. Investigating how circular economy considerations impact order allocation decisions could be a fruitful area.

*Cross-Industry Comparative Studies:* There is an opportunity for cross-industry comparative studies to understand how order allocation models differ across industries. Every paper that was studied focused on one particular industry sector. Ex-

panding models to potentially cover multiple industries which minor modifications would be a potential large step forward in knowledge-sharing. Examining the applicability and effectiveness of models in diverse sectors could lead to industry-specific best practices. A particular emphasis on expanding case studies for the Medical Device industry, Textile industry and the Aerospace industry should be considered.

*Resilience in Order Allocation:* Investigating how order allocation models can be designed to enhance supply chain resilience, particularly in the face of disruptions such as natural disasters, geopolitical events, or global health crises. A few studies were notably published with a focus after the COVID-19 pandemic [32], [47].

*Integration of Dynamic Considerations:* Many of the reviewed papers focus on static models. Future research could explore the integration of dynamic considerations, such as changes in demand patterns, supplier capabilities, and environmental conditions, into order allocation models to enhance adaptability. There is already a growing trend of more dynamic models being published [50], but there are only a few papers of this kind published so far, to the best of this authors knowledge.

By addressing these potential gaps and exploring these research opportunities, future studies in sustainable supplier selection and order allocation can contribute to the advancement of knowledge and the development of more robust and applicable models in the field. This literature review lays the groundwork for a more holistic understanding of sustainable order allocation within supply chain management. The identified gaps and future research directions provide a road-map for scholars and practitioners to contribute to the ongoing discourse, fostering innovation and sustainability in supply chain decision-making. The integration of economic efficiency with environmental and social responsibility has become not only necessary but also a driving factor for long-lasting positive impacts on global supply chain practices.

### 3 Model Development

The following order allocation model is a multi-period, multi-product, multi-sourcing model and is proposed as a stochastic extension to the deterministic existing model as published by Ghadimi et. al. [5] It operates under the following assumptions, and has the following indices and parameters:

- (a) No discount policies are foreseen.
- (b) Fixed ordering costs are supplier-specific.
- (c) Delivery lead times are supplier-specific.
- (d) There is no item shortage for any suppliers.
- (e) There are no inventories at the beginning and end of planning horizon, and supplier capacity remains constant for each period.
- (f) For each period, demand is known with certainty and remains constant.
- (g) Holding costs are dependant on the purchasing price.

#### Indices

$i$	Product Index
$j$	Supplier Index
$k$	Time Period Index
$m$	No. of Products
$n$	No. of Suppliers
$t$	No. of Time Periods (months)

#### Parameters

$V_{ij}$	capacity of $j$ th supplier for $i$ th product
$P_{ij}$	purchasing price of product $i$ delivered by supplier $j$
$d_{ik}$	total demand of product $i$ in time period $k$
$T_{ij}$	on-time delivery rate of product $i$ offered by supplier $j$
$t_i$	manufacturer's minimum acceptable on-time delivery rate of product $i$
$\eta_{ij}$	defective rate of product $i$ delivered by supplier $j$
$\eta_i$	manufacturer's maximum acceptable defective rate of product $i$
$o_j$	fixed ordering cost for supplier $j$
$oo_j$	variable ordering cost for supplier $j$
$tc_j$	transportation cost of supplier $j$ per vehicle
$n_{jk}$	number of vehicles assigned for supplier $j$ in time period $k$
$v_j$	vehicle capacity for supplier $j$ in KG
$\psi_i$	weight occupied by each unit of product $i$ in KG
$s_i$	space occupied by each unit of product $i$ in $m^3$
$S$	manufacturer's total storage capacity in $m^3$
$h_i$	holding cost ratio of product $i$
$sp_j$	sustainability performance value of supplier $j$
$X_{ijk}$	percentage of product $i$ allocated to supplier $j$ in time period $k$
$Y_{jk}$	1 if an order allocated to supplier $j$ in time period $k$ 0 otherwise for all $j$

### 3.1 Objective Functions

There were two main objective functions examined in the undertaking of this model formulation.

#### 3.1.1 Total Cost Function - $Z_1$

The Total Cost Function,  $Z_1$ , captures the various cost elements involved in the supply chain, aiming to minimise the overall expenditure. It comprises several components, each reflecting a different aspect of the supply chain costs:

##### Purchase Costs

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^t P_{ij} X_{ijk} d_{ik} \quad (1)$$

##### Fixed Ordering Costs

$$\sum_{j=1}^n \sum_{k=1}^t o_j Y_{jk} \quad (2)$$

##### Variable Ordering Costs

$$\sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^t oo_j X_{ijk} d_{ik} \quad (3)$$

##### Holding Costs

$$\sum_{i=1}^m \sum_{j=1}^n h_i P_{ij} \left( \sum_{k=1}^t X_{ijk} d_{ik} - \sum_{k=1}^t d_{ik} \right) \quad (4)$$

##### Transportation Costs

$$\sum_{j=1}^n \sum_{k=1}^t tc_j n_{jk}, \quad (5)$$

where:

$$n_{jk} = \frac{\sum_{i=1}^m \sum_{k=1}^t \psi_i X_{ijk} d_{ik}}{v_j}, \quad \forall j \in n \quad (6)$$

$$\text{Min } Z_1 = \sum \text{ (Equations 1-5)} \quad (7)$$

#### 3.1.2 Sustainability Performance Value Function - $Z_2$

The Sustainability Performance Value Function,  $Z_2$ , focuses on maximising the overall sustainability performance of the supply chain. This function promotes environmentally friendly and socially responsible supply chain practices by preferring suppliers with higher sustainability performance.

$$\text{Max } Z_2 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^t sp_j X_{ijk} d_{ik} \quad (8)$$

The sustainability scores of each supplier, which were calculated in previous research from Ghadimi in 2018, [5] is defined as the summation of the supplier's social

sustainability score, environmental sustainability score, and economic sustainability score, each with equal weighting. Refer to Table 5 for further information on the considerations put towards each supplier in the medical electronics sector regarding their sustainability practices. Each supplier provided information for each topic on a scale of 1-10 which was combined into their final score. Defining the importance in weights of each sub-category is something that can be considered by manufacturers in the future.

Table 5: Supplier Sustainability Dimensions, Sub-Criteria & Influencing Factors

<b>Dimension</b>	<b>Criterion</b>	<b>Influencing factor</b>
<b>Environmental Sustainability</b>	Green image	Market reputation
	Pollution control	Customer reputation
	Green competencies	Use of hazard materials Solid control Green packaging Green process
	Quality	Document control procedure Requirement MDD Medical device vigilance
	Delivery/Service	Internal quality audit Handling and preservation of product Product identification and traceability Customer complaint handling Post market surveillance
<b>Economic Sustainability</b>	Cost	Production Transportation Ordering
		Failure Mode Effects & Critical Analysis Technology level
	Technical capability	
<b>Social Sustainability</b>	Health and safety	Safety audit and assessment OHSAS 18001
	Employment practices	Training Disciplinary and security practices

## 3.2 Constraints

The initial constraints of the previous problem are manufacturer's demand, supplier's production capacity, quality, delivery and manufacturer's storage capacity. They are formulated to capture the added time index as follows:

### 3.2.1 Demand Constraint

This constraint ensures that the total quantity of products delivered from all suppliers to a given destination in each time period exactly meets the demand at that destination. It prevents both shortages and excess supply, aligning supply with demand.

$$\sum_{j=1}^n X_{ijk} d_{ik} = d_{ik}, \quad \forall i \in m, \forall k \in t \quad (9)$$

### 3.2.2 Max Capacity Constraint

This constraint restricts the quantity of products that can be shipped from each supplier to ensure it does not exceed the supplier's maximum capacity. This helps in maintaining a balance between the demand and the supplier's ability to fulfill it.

$$X_{ijk} d_{ik} \leq V_{ij}, \quad \forall i \in m, \forall j \in n, \forall k \in t \quad (10)$$

### 3.2.3 Quality Constraint

This ensures that the products sourced from suppliers meet a certain quality standard applied by the manufacturer. It does this by applying a quality multiplier to the quantity of products shipped, which must not exceed a specified threshold. This helps maintain the overall quality of products within acceptable limits.

$$\sum_{j=1}^n \eta_{ij} X_{ijk} d_{ik} \leq \eta_i d_{ik}, \quad \forall i \in m, \forall k \in t \quad (11)$$

### 3.2.4 Delivery Constraint

This constraint focuses on the reliability and timing of deliveries. It ensures that the adjusted sum of products delivered, taking into account delivery reliability, does not exceed an adjusted demand threshold. This helps in accounting for uncertainties in delivery times and ensures a buffer for reliability.

$$\sum_{j=1}^n (1 - T_{ij}) X_{ijk} d_{ik} \leq (1 - t_i) d_{ik}, \quad \forall i \in m, \forall k \in t \quad (12)$$

### 3.2.5 Storage Capacity Constraint

This ensures that the total size of products stored does not exceed the available storage capacity. This is crucial for preventing overstocking and ensuring that the storage facilities are used efficiently, without exceeding their limits.

$$\sum_{i=1}^m \sum_{j=1}^n s_i X_{ijk} d_{ik} \leq S, \quad \forall k \in t \quad (13)$$

In this current research, Pyomo was utilised to optimise the developed stochastic order allocation bi-objective mathematical model to assign the optimal order quantities in each period to each supplier. Pyomo was utilised within the Python coding framework, using Visual Studio code.

## 4 Data & Implementation

The data used in the implementation of this model was provided by an industrial case study in the electronics sector of the medical device industry. The same data has been used previously in a case study utilising a MAS approach [14] but the model has been further developed to be stochastic and can be altered for differing time periods and a different bi-objective solution method. It is paramount for the SSS&OA process in a situation where different sourcing decisions need to be taken based on variable demands for each planning period.

The actual case study is as follows: The primary focus of the case study is a contract manufacturer that was hired by an Original Equipment Manufacturer (OEM) to provide electronic equipment. The OEM operates a healthcare diagnostics, monitoring, and management division with six divisions as part of a larger product offering. One of these divisions offers nine different product types for diabetes monitoring. One of these monitoring kits served as the case study's end-item model. In addition to wanting to fulfil orders, the contract manufacturer is driven to examine and improve inventory control procedures in order to cut expenses. In one of the company's European production facilities, information was obtained from the inventory planning manager as well as the plant general manager. Regarding the supply side, nine different component types were provided by nine different sources. There were suppliers in places like Taiwan, Germany, and the US. For this unit, each supplier had a single component under contract. The US Food and Drug Administration (FDA) is one of the main regulatory bodies that oversees the medical device market, and both the OEM and contract manufacturer complied with ISO13485. As a condition of agreeing to the deal, the contract manufacturer said that this included approving a list of pre-approved vendors of component parts. The agreement limited the contract manufacturer's capacity to identify component suppliers that it may deem most advantageous for its operations.

Demand was calculated on a weekly basis and averaged at 25,208 units per week. The demand data provided by the contract manufacturer was used to identify the type of distribution. It was indicated that the Gamma distribution was an appropriate probability distribution to model the demand of the manufacturer [54].

### 4.1 Demand Forecasting Using the Gamma Distribution

The Gamma distribution is often employed in demand forecasting due to its flexibility and ability to model a wide range of distribution shapes [55]. The parameters for the Gamma distribution were derived from the mean demand and the variability in the demand, expressed through the coefficient of variation (CV). The correlations between a distribution's mean and standard deviation are referred to as CV [56]. According to Ghadimi et al. [14], the probability density function for the Gamma distribution is as follows:

$$f(x) = \frac{(\alpha x)^{\beta-1}}{\Gamma(\beta)} \alpha e^{-\alpha x}, \quad 0 \leq x \leq \infty \quad (14)$$

where

$$\Gamma(\beta) = \int_0^\infty t^{\beta-1} e^{-t} dt \quad (15)$$

where  $\alpha$  is the scale parameter,  $\beta$  is the shape parameter and  $\Gamma(\beta)$  is the gamma function around the shape parameter. Using the coefficient of variation, the standard deviation  $\sigma$  can be calculated as:

$$\sigma = \mu \times CV \quad (16)$$

Based on the method of moments, the Gamma distribution's shape parameter  $\beta$  and the scale parameter  $\alpha$  were calculated using the following formulae [57]:

$$\beta = \frac{\mu^2}{\sigma^2} \quad (17)$$

$$\alpha = \frac{\sigma^2}{\mu} \quad (18)$$

Using these parameters, demand scenarios were generated for each product. The demand values are generated using the numpy library's 'np.random.gamma' function, with the shape and scale parameters. In this analysis, the demand was estimated for 12 periods. The mean monthly demand for the products was denoted by  $\mu = 25208$ . The coefficient of variation, a measure of relative variability, was assumed to be  $CV = 0.1$  so as not to exceed the maximum supplier demand in generating random demand values. To align with practical considerations, such as the impossibility of partial product demand, the continuous values obtained from the Gamma distribution were rounded to the nearest whole number. This resulted in a discrete set of demand scenarios for each month and each product, facilitating more realistic and applicable demand planning. Table 6 shows the set of demand data generated in one instance for the purposes of examining results and order allocation for the remainder of this paper:

Table 6: Monthly Demand for Products as generated by Gamma Distribution

<b>Month</b>	<b>Component A</b>	<b>Component B</b>
1	21376	24057
2	24381	20405
3	24164	28402
4	27952	26407
5	27043	24615
6	27552	26469
7	22812	29540
8	25789	25163
9	28603	26435
10	24415	21067
11	27506	24575
12	19688	26521

## 4.2 Experimental Scenario

For the purposes of this model, the following assumptions were made:

- There are two components needed to be procured per period (Components A & B) and there are three potential suppliers (S1, S2, S3) which can potentially meet each demand.
- The contract manufacturer placed an order in 12 weekly planning periods, over a 3 month planning horizon.
- All supplier and manufacturer parameter values remained constant for the duration of the planning periods.
- The demands for each period follow a Gamma distribution as described in Section 4.1.
- The sustainability performance of each supplier was assumed to remain consistent for the duration of the 12 weeks.

Table 7 describes to the parameters of the suppliers and components.

Table 7: Data related to products, suppliers, and manufacturer.

	Suppliers				Manufacturer	
	Product (i)	S1	S2	S3	Product (i)	
$V_{ij}$	Component A	9500	9500	10500	$t_i$	Component A 0.92
	Component B	10000	10000	10000		Component B 0.92
$P_{ij}$	Component A	15	18	10	$\eta_i$	Component A 0.02
	Component B	12	13	12		Component B 0.02
$\eta_{ij}$	Component A	0.02	0.01	0.025	$\psi_i$	Component A 0.38
	Component B	0.02	0.01	0.025		Component B 0.101
$T_{ij}$	Component A	0.95	0.90	0.92	$s_i$	Component A 0.0012
	Component B	0.95	0.90	0.92		Component B 0.0005
$o_j$		15	15	15	$h_i$	Component A 0.01
						Component B 0.021
$oo_j$		0.1	0.12	0.09	S	100
$tc_j$		215	230	190		
$v_j$		480	480	480		
$sp_j$		0.499	0.613	0.705		

## 4.3 Software Used

### 4.3.1 Pyomo

Pyomo, an open-source Python-based software, is a library used for formulating, solving, and analysing complex optimisation problems. Pyomo was the main computational tool used to optimise the proposed stochastic model in this paper. The decision to employ Pyomo was due to its methodical approach to decompose the model into sets, parameters, variables, objectives, and constraints [58]. This method not only enhanced model clarity but also aided in debugging and allowed for a modular approach to building complex models.

Pyomo's versatility is further showcased through its ability to interface with numerous solvers, both open-source and commercial, offering a breadth of algorithmic strategies for problem-solving. The choice to utilise Pyomo was also informed by its Pythonic integration, and rich library support. Additionally, Pyomo's capacity for model extension and adjustment made it an excellent fit for the iterative nature of developing a stochastic model, where modifications and refinements were integral to the research process. By using Pyomo's "SolverFactory", many types of solvers could be experimented with to optimise for the best outcome. The chosen solver to be represented in this paper is elaborated on in section 4.3.2.

Pyomo provided the necessary computational efficiency and flexibility to manage the stochastic elements of the supply chain model effectively, ensuring robust and reliable optimisation outcomes.

### 4.3.2 GLPK

In the undertaking of solving the linear programming (LP) and mixed-integer programming (MIP) problems presented in this study, the GNU Linear Programming Kit (GLPK) was utilised as the primary solver. GLPK is an open-source solver capable of solving large-scale linear, integer, and mixed-integer linear programming problems efficiently through its provision of simplex and branch-and-cut methods [59]. The selection of GLPK was informed by its accessibility, given that as open-source software, it is freely available and conducive to the principle of open research. The performance of GLPK is noteworthy; it is well-regarded for its efficiency in handling problems of moderate size and complexity, aligning well with the scale of the optimisation tasks faced within this research. Its compatibility with various programming languages and extensive support for problem formulations made it a natural fit for integration with the Python-based optimisation framework used in this study. The GLPK solver was activated using Pyomo's SolverFactory.

## 5 Results

Access to the coding files created for this project for the purposes of the following results can be found at the following Google Drive link:

[Link to Python Code](#)

In order to compare results of the model, both a weighted sum method and an augmented  $\epsilon$ -constraint method were used to optimise order allocation. The use of both of these methods in this study was strategically chosen to leverage their respective strengths in identifying a comprehensive set of Pareto-optimal solutions. The WSM is particularly advantageous for its simplicity and the intuitive appeal of converting a multi-objective problem into a single-objective one by assigning weights to each criterion, reflecting their relative importance to the decision-maker [60]. It is a well-suited approach for scenarios where a balanced trade-off between objectives is desired. On the other hand, the Augmented  $\epsilon$ -Constraint Method is renowned for its ability to systematically generate a diverse set of Pareto-optimal solutions by varying constraints on the objectives [61]. This method provides a more complete picture of the trade-off surface, which is invaluable for decision-makers who need to understand the full spectrum of the feasible solution space.

By employing both methods, this research hopes to use the WSM to yield a single, aggregate solution that considers all objectives simultaneously, and on the Augmented  $\epsilon$ -Constraint Method to map out the range of possible solutions within the constraints of the model. This dual-method approach ensures that the model does not just seek a compromise solution, but also respects the integrity of the multi-objective nature of the problem, presenting a list of solutions from which decision-makers can choose based on their strategic priorities and constraints of the real-world system. This comprehensive approach is essential in addressing the stochastic nature of the problem where variability and uncertainty are inherent in the system. The steps of each solution method are as follows:

### 5.1 Weighted Sum Method

The optimisation problem involves two conflicting objectives: the cost  $Z_1$  and sustainability  $Z_2$ . In the weighted sum method, these objectives are normalised and combined into a single objective function. To begin, each objective is solved separately to obtain the negative ideal solution (worst solution) and positive ideal solution (best solution) of each function. Since the values of objective functions vary in different scales, Equations 19 & 20 were used to normalise the objective functions. where,  $Z_{i_{\text{norm}}}$  is the normalised value of the  $i$ th objective function,  $Z_{i_{\text{worst}}}$  is the negative ideal solution of  $i$ th objective function and  $Z_{i_{\text{best}}}$  is the best solution or the positive ideal solution of  $i$ th objective function. Then, a weight  $\omega_i$  is assigned to each normalised objective

$$Z_{i_{\text{norm}}} = \frac{Z_{i_{\text{worst}}} - Z_i}{Z_{i_{\text{worst}}} - Z_{i_{\text{best}}}}, \quad \text{for minimisation} \quad (19)$$

$$Z_{i_{\text{norm}}} = \frac{Z_i - Z_{i_{\text{worst}}}}{Z_{i_{\text{best}}} - Z_{i_{\text{worst}}}}, \quad \text{for maximisation} \quad (20)$$

The normalised objectives are then combined into a single weighted objective function:

$$\text{Max}(Z) = \sum_{i=1}^n \omega_i \cdot Z_{i_{\text{norm}}} \quad (21)$$

where  $\omega_1$  and  $\omega_2$  are the weights assigned to the cost and sustainability objectives, respectively. In the case of this paper, both weights were set to 0.5, indicating an equal trade-off between cost and sustainability. Table 8 describes the best and worst possible values for each objective function. Note that the "best" value for the cost objective,  $Z_1$  is the minimum possible value and the "best" value for the sustainability objective,  $Z_2$  is the maximum possible value. This is in line with what each objective is attempting to accomplish. Table 9 shows the results for the WSM, with a final combined sustainable purchasing value of 0.6293. This is further discussed in Section 5.3.

Table 8: Best & Worst Results for Objective Functions

Metric	Best Value	Worst Value
$Z_1$ (Cost)	7,955,516.60	8,656,193.06
$Z_2$ (Sustainability)	387,063	367,823

Table 9: WSM Results

Metric	Value	Normalised Value
Optimal $Z_1$ (Cost)	8,069,073.12	0.8379
Optimal $Z_2$ (Sustainability)	376,297.55	0.4405
Optimal Combined Total	-	0.6392

## 5.2 Augmented Epsilon-Constraint Method

The epsilon constraint method was also utilised to generate a Pareto front for multi-objective optimisation problems. In this approach, one objective is optimised while the other is constrained to a certain limit, termed as  $\epsilon$ . This process is repeated by varying the  $\epsilon$  value systematically to obtain different Pareto optimal solutions. Augmented  $\epsilon$ -Constraint Method avoids inefficient solutions and produces equally optimal solutions [61].

The method was applied as follows:

The general  $\epsilon$ -constraint model to minimise the cost objective function ( $Z_1$ ) while taking the other objectives into account can be shown as follows:

$$\text{Min}(Z_1(x) - \delta * (\frac{c_2}{r_2} + \frac{c_3}{r_3} + \dots + \frac{c_n}{r_n})) \quad (22)$$

where

$$Z_n(x) - c_n = \epsilon_n \quad (23)$$

and

$$i \in [2, n] \quad (24)$$

$$c_i \in R^+ \quad (25)$$

Best and worst values for each objective function were defined as in Table 8, Section 5.1. Optimal solutions of the model are achieved through parametric variation of the  $\epsilon$ -constraint sub-equations. The Pareto-optimal solutions are obtained, where  $r_i$  is the range of the  $i$ th objective function,  $\delta$  is a small number between 0.001 and 0.000001, and  $c_i$  is a non-negative slack variable. The range of the  $i$ th objective function was calculated by:

$$r_i = Z_{2\text{best}} - Z_{2\text{worst}} \quad (26)$$

The range,  $r_i$  was divided into equal intervals,  $n_i$ . For the purposes of the results of this study, an arbitrary number of  $n_i = 30$  intervals were chosen. The  $\epsilon$ -constraint was then systematically varied via the following equation:

$$\epsilon_i^n = Z_{2\text{worst}} + \frac{r_i}{n_i} \quad (27)$$

This method resulted in a set of  $(Z_1, Z_2)$  values that form the Pareto front, illustrating the trade-off between the cost and sustainability objectives. For the 30 attempted solutions, the first 13 were feasible to optimise both objectives. The Pareto front of solutions that was generated, along with the accompanying values of the objective functions can be seen in Figure 3.

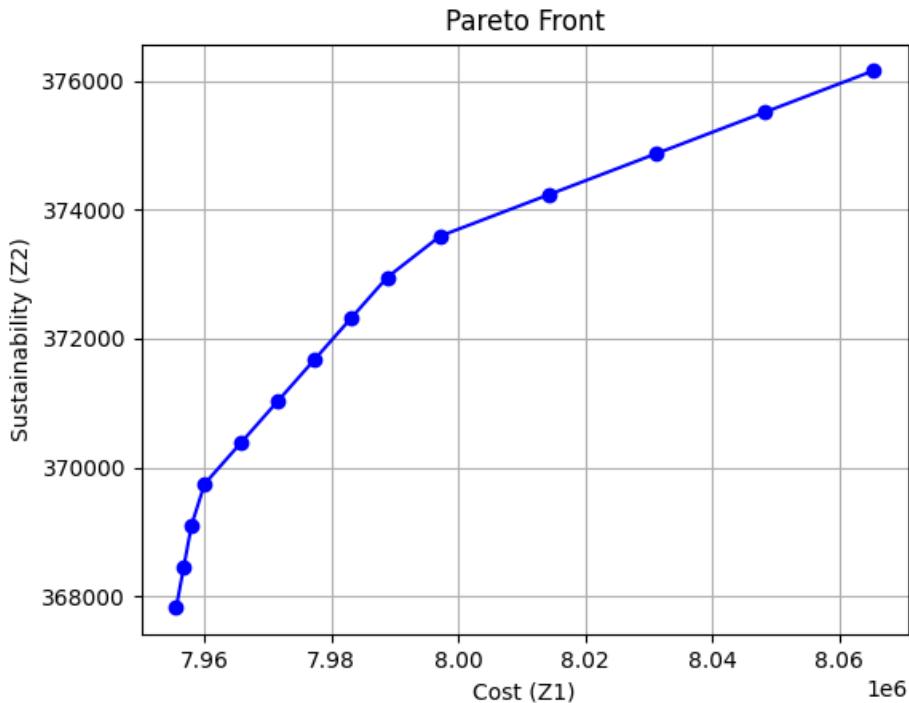


Figure 3: Pareto Front Generated by GPLK Solver

It can be observed from the Pareto front of solutions in Figure 2 that initial slight increases in cost offer large returns in improving the sustainability score of the order. However, this rapid growth in sustainability does not sustain itself as costs continue to increase with each iteration. As the model progresses and cost

increments lead to smaller gains in sustainability, a zone of diminishing marginal returns is encountered. This is an area of interest as it marks the threshold beyond which the pursuit of sustainability can become a question of diminishing economic rationality. It is here that the concept of a 'sustainability ceiling' may be considered, representing the point where the incremental costs of improved sustainability outpace the benefits. Decision-makers need to balance the goals of sustainability with pragmatic constraints of budgetary limitations and diminishing returns.

### 5.3 Total Value of Sustainable Purchasing

In order to select the most preferred Pareto-optimal solutions, as well as compare the  $\epsilon$ -Constraint Method results directly to the Weighted Sum Method results, the Total Value of Sustainable Purchasing (TVSP) was calculated. The TVSP serves as a decisive metric for comparing the outcomes of the two methods by providing a singular value that encapsulates the trade-offs between cost efficiency and sustainability. TVSP is particularly useful in multi-objective optimisation scenarios because it offers a scalar representation of performance across diverse objectives, enabling a clear, quantitative comparison between different methods [34]. This value is especially critical when decision-makers are confronted with a spectrum of Pareto-optimal solutions, each with its own set of trade-offs. By synthesising the multiple objectives into a unified index, the TVSP allows for an immediate assessment of which method aligns more closely with the strategic goals of sustainability and cost-effectiveness.

The TVSP integrates the concept of fuzzy membership values for each objective to reflect the degree of satisfaction with the achieved values of objectives [26]. The membership value for each objective function,  $\alpha_i^l$ , is calculated using:

$$\alpha_i^l = \begin{cases} 1 & Z_i \leq Z_{i\min} \text{ for minimisation} \\ \frac{Z_{i\max} - Z_i}{Z_{i\max} - Z_{i\min}} & f_i^{\min} < f_i^l \leq f_i^{\max} \text{ for minimisation} \\ 0 & Z_i > Z_{i\max} \end{cases} \quad (28)$$

$$\alpha_i^l = \begin{cases} 0 & Z_i \leq Z_{i\min} \text{ for maximization} \\ \frac{Z_i - Z_{i\min}}{Z_{i\max} - Z_{i\min}} & Z_{i\min} < Z_i \leq Z_{i\max} \text{ for maximization} \\ 1 & Z_i > Z_{i\max} \end{cases} \quad (29)$$

where  $Z_i$  represents the value of the  $i$ th objective function in the  $l$ th Pareto-optimal solution. The  $Z_{i\min}$  and  $Z_{i\max}$  are the best and worst values for the  $i$ th objective function, respectively. These values are obtained from calculating each objective function separately, without considering the other.

The TVSP for each Pareto-optimal solution was then calculated as a weighted sum of the membership values:

$$\text{TVSP} = \sum_{i=1}^n w_i \alpha_i^l \quad (30)$$

Here,  $w_i$  is the weight corresponding to the  $i$ th objective function, reflecting its relative importance in the overall evaluation. As in the Weighted Sum Method solution, equal weights of 0.5 were assigned to each objective function. The solution with the highest TVSP is selected as the most preferred solution. In this case study, the

highest TVSP for  $\epsilon$ -constraint method was found to be for the 13th iteration, with a TVSP value of 0.6390.

Table 10 displays all of the associated TVSP's with each Pareto solution that was generated as in Section 5.3.

Table 10: Pareto solution & TVSP's obtained using augmented  $\epsilon$ -constraint method.

<b>Sub-problem no.</b>	<b>Cost (Z<sub>1</sub>)</b>	<b>Sustainability(Z<sub>2</sub>)</b>	<b>TVSP</b>
1	7956725.29	368464.00	0.5158
2	7957934.95	369105.00	0.5316
3	7959994.37	369746.00	0.5468
4	7965747.38	370387.00	0.5593
5	7971500.39	371028.00	0.5719
6	7977253.40	371669.00	0.5844
7	7983006.41	372310.00	0.5970
8	7988759.42	372951.00	0.6095
9	7997117.94	373592.00	0.6202
10	8014165.59	374233.00	0.6247
11	8031213.23	374874.00	0.6292
12	8048260.88	375515.00	0.6337
13	8065308.53	376156.00	0.6390

In the most preferred iteration of all the generated solutions, it can be observed that the cost function (Z1) remained relatively low as the possible cost range was  $7,955,516.6 \leq Z_1 \leq 8,656,193.06$ . Whereas the sustainability function (Z2) was in the middle-to-low range of possible values, the sustainability function range being  $367,823 \leq Z_2 \leq 387,063$ . This is likely due to intrinsic costs associated with sustainability being maximised, not directly tied to the economic purchasing costs of operation. This could lead to a plateau in sustainability gains as costs increase, which represents inherent limitations within the supply chain, such as a lack of available sustainable options or increasing incremental costs for further sustainable improvements that the model could not justify within its parameters.

## 5.4 Final Allocations & Discussion

Table 11 indicates that the most preferred supplier in the optimal Pareto solution was Supplier 3, as all orders in all time periods were first allocated to this supplier. This is in line with expectations, as Supplier 3 had the highest sustainability performance value,  $sp_j$ , of 0.705, and the cheapest price per unit. The additional allocations between the other two suppliers represents a trade-off of the model between cost and sustainability. Supplier 1 had the cheaper component cost per unit but a worse sustainability performance value in comparison to Supplier 2. The model displayed a slight preference in allocation to Supplier 2, indicating that the improvement in sustainability score per order was valued higher than the relative cost savings of ordering with Supplier 1.

The allocation pattern across the time periods showcases the model's dynamic response to fluctuating demands and supplier performances, highlighting its robustness in adapting to multi-dimensional optimisation in a real-world context. It also highlights that despite equal weightings in cost & sustainability score, the most preferred optimal solution values sustainability performance as a major driving factor in its decision-making process, which was a main objective of this study.

Table 11: Order Quantity Allocation

Period 1		Period 2		Period 3		Period 4		Period 5	
$x_{111}$	5,592	$x_{112}$	5,552	$x_{113}$	5,465	$x_{114}$	7,952	$x_{115}$	7,043
$x_{121}$	5,284	$x_{122}$	8,329	$x_{123}$	8,199	$x_{124}$	9,500	$x_{125}$	9,500
$x_{131}$	10,500	$x_{132}$	10,500	$x_{133}$	10,500	$x_{134}$	10,500	$x_{135}$	10,500
$x_{211}$	5,623	$x_{212}$	4,162	$x_{213}$	8,402	$x_{214}$	6,563	$x_{215}$	5,846
$x_{221}$	8,434	$x_{222}$	6,243	$x_{223}$	10,000	$x_{224}$	9,844	$x_{225}$	8,769
$x_{231}$	10,000	$x_{232}$	10,000	$x_{233}$	10,000	$x_{234}$	10,000	$x_{235}$	10,000
Period 6		Period 7		Period 8		Period 9		Period 10	
$x_{116}$	7,552	$x_{117}$	4,924	$x_{118}$	6,115	$x_{119}$	8,603	$x_{1110}$	5,566
$x_{126}$	9,500	$x_{127}$	7,388	$x_{128}$	9,174	$x_{129}$	9,500	$x_{1210}$	8,349
$x_{136}$	10,500	$x_{137}$	10,500	$x_{138}$	10,500	$x_{139}$	10,500	$x_{1310}$	10,500
$x_{216}$	6,588	$x_{217}$	9,540	$x_{218}$	6,065	$x_{219}$	6,574	$x_{2110}$	4,427
$x_{226}$	9,881	$x_{227}$	10,000	$x_{228}$	9,098	$x_{229}$	9,861	$x_{2210}$	6,640
$x_{236}$	10,000	$x_{237}$	10,000	$x_{238}$	10,000	$x_{239}$	10,000	$x_{2310}$	10,000
Period 11		Period 12							
$x_{1111}$	7,506	$x_{1112}$	3,675						
$x_{1211}$	9,500	$x_{1212}$	5,513						
$x_{1311}$	10,500	$x_{1312}$	10,500						
$x_{2111}$	5,830	$x_{2112}$	6,608						
$x_{2211}$	8,745	$x_{2212}$	9,913						
$x_{2311}$	10,000	$x_{2312}$	10,000						

Figures 4 and 5 illustrate the allocated order percentages and quantities over time for the two components the three suppliers. They offer a more detailed visualisation of the model's operational dynamics. In Figure 4 which presents absolute order quantities, we see that orders to Supplier 3 consistently hit the maximum capacity for both components across all time periods. This trend not only reaffirms Supplier 3's favored status due to superior sustainability performance and cost-effectiveness but also indicates a capacity constraint that the model consistently pushes against.

Figure 5 brings focus to allocated order percentages, revealing the proportional distribution of orders among suppliers over time. This percentage-based perspective reveals the model's adaptability, as it adjusted allocations among the suppliers in response to changing circumstances across different periods. While Supplier 3 retained a dominant share, the fluctuations in allocations to Suppliers 1 and 2 captured the model's sensitivity to other underlying factors beyond cost and sustainability, such as risk distribution, supply reliability, or perhaps lead times.

Both figures collectively demonstrate the stochastic nature of the model, which does not rigidly adhere to a static allocation strategy but rather exhibits a responsive allocation pattern. This reflects real-world scenarios where demand variability and supplier performance can shift dynamically over time. The consistency in hitting the maximum allocation for Supplier 3 across all periods could suggest a possible need for revisiting the supplier capacity constraints or expanding the supplier base to mitigate risks of over-reliance on a single supplier, while the proportional allocation trends highlight the model's balancing of competing objectives.

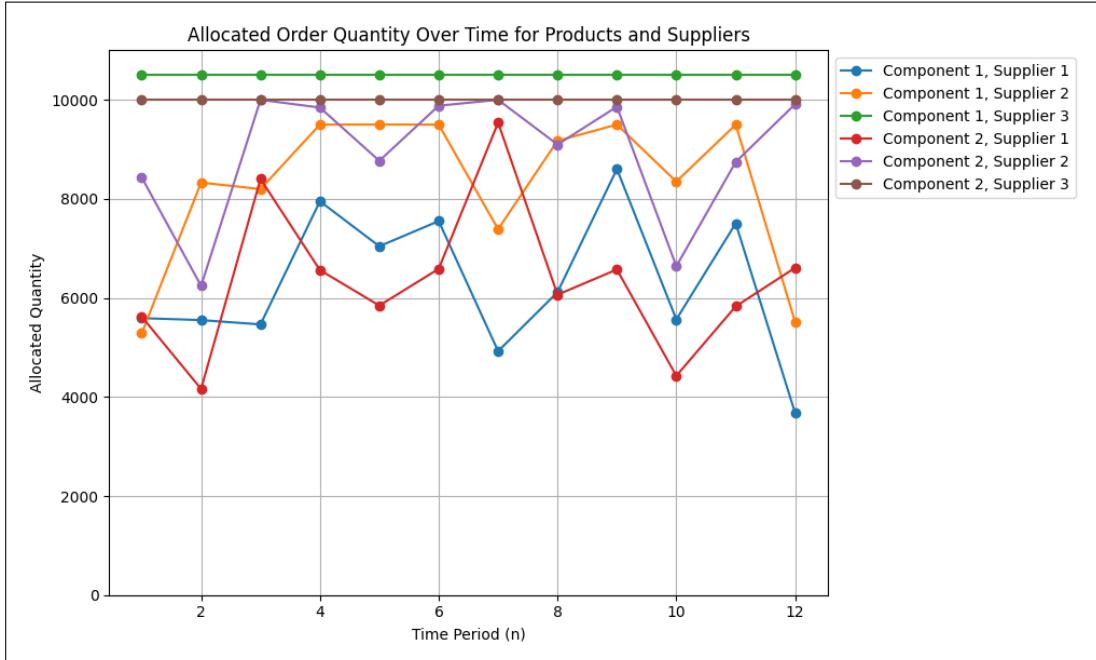


Figure 4: Quantity Order Allocation of Optimal Solution

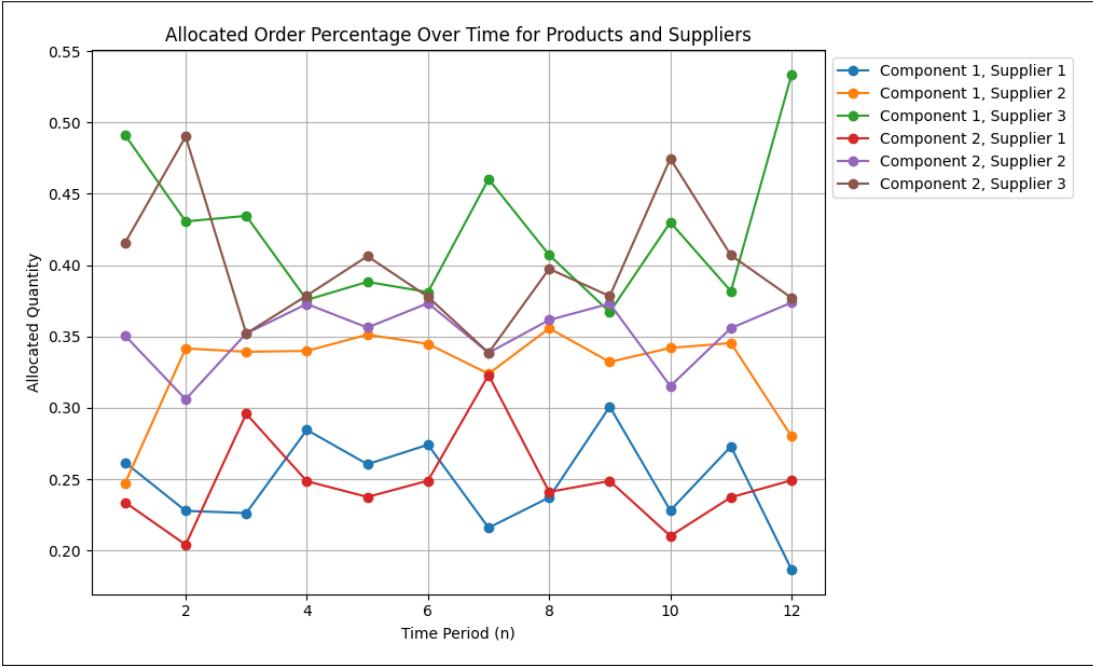


Figure 5: Percentage Order Allocation of Optimal Solution

The comparison presented in Table 12 between the Weighted Sum Method (WSM) and the  $\epsilon$ -constraint method reveals an interesting result in the total value of sustainable purchasing (TVSP). The TVSP is marginally higher for the WSM, indicating a slightly more balanced trade-off between cost and sustainability compared to the  $\epsilon$ -constraint method.

Table 12: Comparison WSM vs  $\epsilon$  - constraint Results

Metric	Z1: Cost	Z2: Sustainability	TVSP
WSM	8,069,073.12	376,297.55	0.6392
$\epsilon$ - constraint	8,065,308.53	376,156.00	0.6390

The WSM inherently seeks a balanced solution by assigning equal weights to both objectives, which could explain the slight edge in TVSP. It operates by converting a multi-objective problem into a single-objective one, with the aim to find a solution that is as close as possible to the ideal point [18]. This method, therefore, tends to produce a solution that might not be the absolute best in terms of individual objectives but achieves an aggregate optimum considering all objectives.

On the other hand, the  $\epsilon$ -constraint method emphasises feasibility and attainability of solutions within the constraints defined by the  $\epsilon$ -levels. By iterating over different levels of  $\epsilon$ , it may sacrifice some potential gains in one objective to find feasible solutions in the objective space, leading to a collection of solutions on the Pareto front. The Pareto front of solutions is further influenced by the number of iterations chosen by the user, which in the case of this study and presentation of results was 30 iterations. It is possible that by increasing the number of iterations, the  $\epsilon$ -constraint solution method would offer a set of results closer to the best TVSP of 0.6392, produced by the WSM.

## 6 Concluding Remarks

### 6.1 Theoretical Implications

This thesis applies a novel application of a stochastic multi-period, multi-demand model for sustainable supplier selection and order allocation (SSS&OA). The study addresses the complexities of real-world supply chain dynamics, which are marked by uncertainty and fluctuation in supply and demand, by switching from deterministic models to stochastic demand, specifically defined by a Gamma-distribution. The application of this kind of model is consistent with the theoretical claims made in earlier research [21], [54] that precise and adaptable response mechanisms are essential to improving SCM performance. This is so that more dependable and durable solutions may be provided by stochastic models, which are good at taking into account the unpredictable character of supply chain elements. Stochastic models' innate ability to accommodate unpredictability accurately captures the realities of supply chain operations, including shifting supplier performance and shifting market demands.

By showing how the model may balance the opposing goals of cost minimisation and sustainability maximisation, the solution approach used — integrating stochastic optimisation within a sustainable procurement context — contributes to advancement in research. There is also potential for introducing daily, or even hourly periods due to the introduction of a general time index to the model. By converting these conceptual qualities into quantitative decision-making processes, this method supports results in the literature that highlight the importance of strategic communication and co-operation in SCM. This approach provides a methodical and measurable way to strike a balance between environmental and economic factors, making way for more sophisticated supply chain management techniques.

This model captures the diversity and risk in the supply chain and allows decisions to be informed by a wide range of data inputs, which is consistent with accepted ideas that robust information exchange leads to better supply chain outcomes [41]. This work has theoretical relevance because it confirms the critical role that sophisticated mathematical modelling tools play in helping supply chain organisations establish long-term alliances based on mutual commitments to efficiency and sustainability.

### 6.2 Managerial Applications

The managerial implications of this study extend into applications within the supply chain sector, particularly concerning the strategic engagement between suppliers and manufacturers. The stochastic model developed provides manufacturers with a quantifiable and strategic tool to navigate the interplay of cost, demand variability, and sustainability in supplier selection and order allocation decisions.

A particularly noteworthy observation from this model is how the Pareto curve of solutions varies when you pit sustainability and cost against one another as objectives. Figure 2 revealed that initial cost increases present opportunities for large jumps in sustainability improvements, where the pursuit of TBL goals can still drive cost-effectiveness. However, as the cost increments increased in the Pareto front, the

sustainability returns diminished. This observation may signal underlying inefficiencies or capacity constraints within the supply chain that could be targeted for improvement. It prompts a critical analysis of the supply chain processes to identify and address the bottlenecks or barriers that prevent cost-effective sustainability enhancements. Understanding the nature of this trade-off curve could inform the development of incentives or regulations designed to shift the balance toward more sustainable outcomes.

The model's capacity to integrate sustainability performance into procurement decisions empowers managers to make choices that are not solely driven by cost but are reflective of a company's broader environmental and social commitments. This is increasingly pivotal in sectors where consumers and regulatory bodies are demanding greater accountability and transparency in sustainable practices. The collaborative relationship between suppliers and manufacturers is another focal point where the model has significant managerial applications. It provides a data-driven approach to long-term partnerships, whereby suppliers are evaluated and chosen not just on the basis of immediate cost benefits, but on their sustainability performance and reliability over time. This gives suppliers incentive to invest in sustainable practices, knowing it will influence their selection and the order volume they receive.

As the sector continues to evolve under the pressures of both market forces and sustainability mandates, this model provides managers with a dynamic tool for decision-making. By embracing such advanced optimisation techniques, managers can ensure more resilient and sustainable supply chain operations, which are critical for maintaining robust supplier-manufacturer relationships and ensuring the long-term viability of their operations.

### 6.3 Limitations

The current iteration of the model presented in this thesis has limitations that warrant consideration for future enhancements and applications.

Firstly, the static nature of the suppliers' sustainability score presents a limitation in terms of dynamic data validation. The model assumes these scores remain constant over time, which is a simplification that overlooks the possibility of suppliers improving or degrading their sustainability practices. This limitation impedes the model's ability to accurately reflect changes in the environmental and social performance of suppliers and, as a consequence, could affect the long-term validity of the data. Also, the model's time measurement in fixed weekly periods may not fully capture the fluidity of real-world supply chain operations. A more real-time approach could potentially provide a more accurate reflection of supply chain dynamics and allow for more responsive decision-making in the face of rapid market changes.

In terms of limits imposed by the demand conditions, the necessity for known demand, within fixed periods imposes a constraint on the model's flexibility. While this is a common practice in modeling, it does not account for unexpected demand variability and unexpected demand shocks, which are critical factors in supply chain

management. Additionally, this model could only be applied to demand data which has previously assessed and found to follow a Gamma-distribution of demand. Although this still verifies it as a stochastic model, it does not account for other types of common demand distribution.

Additionally, the model's approach to sustainability measurement could benefit from a subtler framework. Currently, it utilises an aggregate sustainability score for suppliers, which consolidates various factors into a single metric. This could obscure the individual contributions of different sustainability aspects, thereby providing a less detailed view of supplier performance in this critical area. As observed in Table 4, the sustainability metric used doesn't consider some important environmental and social factors which have come up in other literature, such as re-usability of design, environmental management systems & certifications, and information disclosure [43]. The model also does not account for downstream supply chain activities such as distribution and customer service. These elements are integral to the overall sustainability and cost-effectiveness of the supply chain, suggesting that a more holistic approach could yield a more comprehensive understanding of the supply chain's performance.

Lastly, the models sustainability metric is subject to its current application within the medical device industry. Transferring this model to other industries with different regulatory environments, market dynamics, and supply chain structures would likely necessitate considerable adjustments. Each industry presents unique challenges and characteristics that must be reflected in the modeling approach to ensure both accuracy and applicability.

Addressing these limitations could significantly extend the model's utility and accuracy, enabling it to serve as a more versatile tool in the strategic planning of sustainable supply chains across various sectors.

## 6.4 Conclusion and Future Work

This study has successfully further developed and validated a multi-period, multi-demand stochastic model for sustainable supplier selection and order allocation (SSS&OA), incorporating both cost minimisation and sustainability maximisation within the supply chain management framework. Through the application of the Weighted Sum Method and the Augmented Epsilon Constraint Method, the model has demonstrated its effectiveness in performing trade-offs between economic efficiency and environmental responsibility. The results highlight the practical implications of implementing a stochastic approach to SSS&OA, where the incorporation of probabilistic demand forecasting and dynamic sustainability evaluations can significantly enhance decision-making processes. This approach not only aligns with current sustainability trends and economic demands but also offers a robust framework for supply chain managers to optimise their supplier relationships and operational strategies in a sustainable manner.

The findings of this study make way for several avenues of future research and development:

- (a) Dynamic Sustainability Scoring - Future research could focus on developing a more dynamic model for evaluating supplier sustainability scores. Incorporating real-time data and feedback mechanisms could allow the model to adapt to changes in supplier performance, providing a more accurate and responsive basis for decision-making.
- (b) Integration with Downstream Supply Chain Activities - Extending the model to include downstream activities such as distribution, logistics, and customer service could offer a more comprehensive view of the supply chain. This integration would help in understanding the full impact of supplier selection and order allocation decisions on the overall supply chain sustainability and cost.
- (c) Validation for Other Industries - Further studies could explore the application of the model in different industries with varying characteristics and constraints. This would help in identifying industry-specific adaptations required to enhance its effectiveness.
- (d) Investigating more advanced optimisation techniques and algorithms, such as machine learning models for predictive analytics or genetic algorithms, could enhance the model's predictive accuracy and operational efficiency. These techniques could provide better handling of the complexities and variation inherent in large-scale supply chains.

By addressing these areas, future research can continue to build on the foundational work of this thesis, driving forward the integration of sustainability into the core strategic frameworks of sustainable supply chain management.

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# *Appendix 1 (Condensed Research Paper)*

## Multi-Period, Multi-Demand, Multi-Product Stochastic Model for Sustainable Supplier Selection & Order Allocation

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### **Abstract**

This paper introduces a multi-period, multi-demand, multi-product stochastic model to address the complexities of sustainable supplier selection and order allocation (SSS&OA) within supply chains. Recognising the limitations of deterministic models, this study shifts towards a stochastic approach to better accommodate the uncertainties and dynamic demands of modern supplier selection. The model address the trade-off between cost minimisation and sustainability maximisation, offering a resilient framework for decision-making. Grounded in a robust literature review, the project approaches the solution using both the weighted sum method and the augmented  $\epsilon$ -constraint method to develop a cohesive strategy for practical future applications. Empirical data validates the model's effectiveness, with findings incorporating a balance between economic efficiency and environmental responsibility. The research contributes to sustainable supply chain management by presenting an adaptable tool for procurement in fluctuating markets, with broader implications for theory and practice. Concluding remarks reflect on the model's implications, its limitations, and propose directions for future research.

## 1 Introduction

As industries globally confront the pressing challenges of environmental degradation and resource scarcity, the imperative for sustainable supply chain management (SSCM) has never been greater. This growing environmental consciousness, driven by economic globalisation and diminishing natural resources, has compelled industries to re-evaluate their operational strategies to incorporate sustainable practices effectively. The Triple Bottom Line (TBL) framework has emerged as a pivotal approach, urging businesses to integrate economic, environmental, and social dimensions to redefine success beyond financial gains [1]. However, despite extensive research and development in sustainable supplier selection, significant gaps remain, particularly in adapting order allocation frameworks to the stochastic nature of supply chain demands and the dynamic market environment [2].

The literature reveals that while the supplier selection component of the TBL framework is well-articulated across multiple industries, the order allocation phase frequently lacks consideration of the market's dynamic and stochastic characteristics [3]. This oversight renders many existing models less effective, as they fail to capture the real-time variation and complexities of modern supply chains, often relying on traditional deterministic approaches that are increasingly misaligned with the volatile market conditions [4]. These deterministic models, while

foundational, often do not account for the potential variations in supplier performance over time, nor do they accommodate the unpredictable fluctuations in demand that typify today's economic landscapes [5].

This project addresses these critical gaps by proposing a multi-period, multi-demand, multi-product stochastic model that extends beyond the deterministic confines, offering a more resilient and adaptive framework for SSS&OA. It aims to enhance an existing model developed by Ghadimi et al. (2018), which will be evolved to incorporate real-world uncertainties, thereby providing a more robust tool for making informed and sustainable decisions within the SCM domain [6].

## 2 Methodology

The methodological foundation of this project is encapsulated by the development and application of a multi-period, multi-demand stochastic model designed to tackle the complexities inherent in sustainable supplier selection and order allocation (SSS&OA). This innovative approach was necessitated by the limitations of traditional deterministic models, which often fall short in capturing the dynamic and unpredictable nature of modern supply chains. The stochastic model introduced in this research incorporates variability directly into its structure, using probabilistic methods to better reflect the real-world unpredictability of demand and supply.

The model is formulated around dual objective functions: cost minimisation (Z1) and sustainability maximisation (Z2). These objectives encompass various supply chain costs such as purchase, ordering, holding, and transportation costs, alongside a sustainability performance value that quantifies the environmental and social impact of supplier practices [4]. This dual-objective approach is crucial as it allows the model to address the often conflicting goals of reducing costs while enhancing sustainability, providing a more balanced and realistic decision-making framework.

The inclusion of stochastic elements, specifically the use of a Gamma distribution to model demand variability across different time periods, represents a significant advancement in the model's capability to simulate real-world conditions more accurately [7]. This methodological choice not only enhances the realism of the model but also its applicability to a variety of industrial scenarios where demand uncertainty is a major concern. Overall, the methodological innovations of this study lie in its integration of stochastic modelling techniques with robust multi-objective optimisation methods, paving the way for more effective and realistic approaches in the field of sustainable supply chain management. This model provides a significant tool for managers and decision-makers, enabling them to navigate the complexities of modern supply chains with greater precision and foresight.

## 3 Model Development

The developed stochastic multi-period, multi-demand model for sustainable supplier selection and order allocation (SSS&OA) incorporates both cost minimisation and sustainability maximisation. The model's novelty lies in its stochastic nature, addressing the dynamic and uncertain elements of supply chain management.

### 3.1 Objective Functions

The model utilised dual objective functions to capture the trade-offs between cost and sustainability. For all parameter definitions see the published work by Ghadimi et. al [4]. The demand  $d_{ik}$  was modelled as a stochastic variable with a  $\gamma$ -distribution to account for demand uncertainty, enhancing the model's responsiveness to real-world variability:

- **Total Cost Function** ( $Z_1$ ), aiming to minimise:

$$Z_1 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^t (P_{ij} X_{ijk} d_{ik} + o_j Y_{jk} + oo_j X_{ijk} d_{ik} + h_i P_{ij} (X_{ijk} d_{ik} - d_{ik}) + t_{cj} n_{jk}) \quad (1)$$

where  $n_{jk} = \frac{\sum_{i=1}^m \sum_{k=1}^t \psi_i X_{ijk} d_{ik}}{v_j}$  for all  $j \in n$ .

- **Sustainability Performance Value Function** ( $Z_2$ ), aiming to maximise:

$$Z_2 = \sum_{i=1}^m \sum_{j=1}^n \sum_{k=1}^t s p_j X_{ijk} d_{ik} \quad (2)$$

### 3.2 Constraints

The model is subject to various constraints ensuring practical and realistic supply chain operations:

- **Demand Constraint** ensures demand is exactly met without excess:

$$\sum_{j=1}^n X_{ijk} d_{ik} = d_{ik}, \quad \forall i \in m, \forall k \in t \quad (3)$$

- **Capacity Constraint** limits the quantity to the supplier's capacity:

$$X_{ijk} d_{ik} \leq V_{ij}, \quad \forall i \in m, \forall j \in n, \forall k \in t \quad (4)$$

- **Quality Constraint** ensures products meet a predefined quality threshold:

$$\sum_{j=1}^n q_{ij} X_{ijk} \geq Q_i, \quad \forall i \in m, \forall k \in t \quad (5)$$

- **Delivery Constraint** guarantees delivery within stipulated timelines:

$$\sum_{j=1}^n X_{ijk} \leq D_{ik}, \quad \forall i \in m, \forall k \in t \quad (6)$$

- **Storage Capacity Constraint** limits the inventory based on available storage:

$$\sum_{i=1}^m \sum_{j=1}^n X_{ijk} s_i \leq S, \quad \forall k \in t \quad (7)$$

## 4 Solutions & Results

This section presents the key findings from the application of the Weighted Sum Method and the Augmented Epsilon-Constraint Method, and the calculation of the Total Value of Sustainable Purchasing (TVSP).

### 4.1 Weighted Sum Method

The Weighted Sum Method (WSM) was employed to handle the multi-objective nature of the model by normalising and combining the cost and sustainability objectives into a single objective function. The method involved normalising each objective relative to their ideal and worst values, and assigning equal weights to both objectives, reflecting a balanced trade-off strategy. The results, as shown in Table 1, indicate an optimal combined sustainable purchasing value of 0.6392, suggesting a balanced approach to cost minimisation and sustainability maximisation.

Table 1: WSM Results

Metric	Value	Normalised Value
Optimal Z1 (Cost)	8,069,073.12	0.8379
Optimal Z2 (Sustainability)	376,297.55	0.4405
Optimal Combined Total	-	0.6392

### 4.2 Augmented Epsilon-Constraint Method

The Augmented Epsilon-Constraint Method was applied to generate a Pareto front, revealing the trade-offs between cost and sustainability. By systematically varying the epsilon constraints, a series of Pareto optimal solutions were generated, mapping out the decision space and allowing for an understanding of the compromises between objectives. The method highlighted the diminishing returns on sustainability with increased costs, as illustrated in the generated Pareto front (Figure 1), which showed initial significant improvements in sustainability for slight increases in cost, followed by smaller gains as costs escalated.

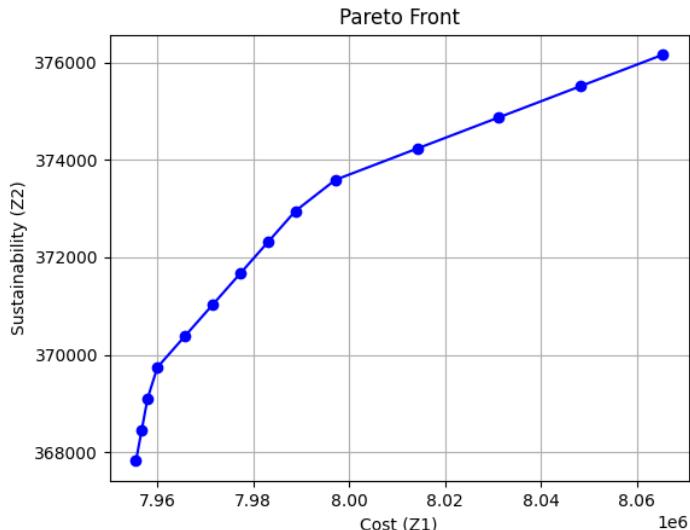


Figure 1: Pareto Front Generated by GPLK Solver

### 4.3 Total Value of Sustainable Purchasing

The Total Value of Sustainable Purchasing (TVSP) served as a decisive metric for comparing the outcomes of the two methods, encapsulating the trade-offs between cost efficiency and sustainability. Calculated using fuzzy membership values to represent satisfaction levels across the objectives, TVSP provided a scalar representation of performance, facilitating a clear comparison. As seen in Table 2, the WSM showed a slightly higher TVSP of 0.6392 compared to the Augmented Epsilon-Constraint Method's 0.6390, indicating a marginally more balanced trade-off under the WSM. This highlights the utility of TVSP in multi-objective optimisation, offering a direct measure of overall solution quality relative to strategic goals.

Table 2: Comparison WSM vs  $\epsilon$  - constraint Results

Metric	Z1: Cost	Z2: Sustainability	TVSP
WSM	8,069,073.12	376,297.55	0.6392
$\epsilon$ - constraint	8,065,308.53	376,156.00	0.6390

Overall, the results demonstrate the model's capability to effectively balance economic efficiency and environmental sustainability, providing valuable insights into the strategic management of sustainable supply chains.

## 5 Discussion

This research demonstrates the effectiveness of stochastic modelling in sustainable supplier selection and order allocation (SSS&OA), addressing the dynamic and uncertain nature of supply chains. The findings underline the model's ability to navigate the complex trade-offs between cost efficiency and sustainability goals. The use of the Weighted Sum and Augmented  $\epsilon$ -Constraint methods revealed significant insights into how strategic decision-making can balance these often conflicting objectives.

The discussion highlights the limitations inherent in traditional deterministic models, which often fail to capture the fluctuations and uncertainties typical of modern supply chains. By incorporating stochastic elements, this model offers a more adaptive and realistic framework, potentially leading to more robust supply chain management strategies [1]. The results emphasize that while initial costs of sustainable practices can be higher, the long-term benefits, including reduced environmental impacts and enhanced social responsibility, justify these investments. The model's capacity to adapt to varying demands and supplier capabilities suggests its applicability across different industrial contexts, albeit with necessary changes to address specific characteristics and constraints of each sector [6]. This adaptability is crucial for businesses facing rapid market changes and increased regulatory requirements for sustainability.

## 6 Conclusion and Future Work

This study concludes by emphasising the robust applicability and theoretical relevance of a multi-period, multi-demand stochastic model in sustainable supplier selection and order allocation (SSS&OA), seamlessly integrating cost minimisation and sustainability maximisation within the supply chain management framework. By transitioning from deterministic models to those accommodating stochastic demands—specifically defined by a Gamma-distribution—this research aligns with previous claims that precise and adaptable mechanisms significantly enhance SCM performance by providing more reliable and sustainable solutions [8], [9].

The dual objectives of cost and sustainability are intricately balanced through the application of the Weighted Sum Method and the Augmented Epsilon Constraint Method. This strategic approach not only underscores the importance of robust communication and cooperation in SCM but also showcases how quantitative methods can effectively manage the inherent trade-offs between economic and environmental goals. The potential expansion to more granular time indexing, from daily to hourly periods, further illustrates the model's capacity to adapt to fast-paced market changes and supply chain dynamics, emphasising its readiness for future scalability and detail-oriented optimisation [10].

In terms of managerial implications, the findings highlight how strategic supplier engagement and enhanced decision-making tools can lead businesses to not only meet cost and efficiency goals but also uphold environmental and social standards, crucial in sectors where sustainability is increasingly demanded by both consumers and regulatory frameworks. The observations from the Pareto curve analysis provide actionable insights into potential inefficiencies and areas for targeted improvement within the supply chain processes.

While acknowledging its current limitations—such as the static nature of supplier sustainability scores and the lack of integration with downstream activities—the study sets a firm foundation for future research aimed at refining and expanding the model's capabilities. Future work may explore more dynamic sustainability scoring, broader industry applications, and integration of advanced optimisation techniques to bolster the model's accuracy and operational efficiency. By continuing to build on this groundwork, future research can further advance the integration of sustainability into strategic supply chain frameworks, enhancing the long-term viability and resilience of supply chain operations across various sectors.

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