Costs of building Apps and dedicated teams

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Part I

Tides

1 Tidal dynamics

> It does not seem likely that we will ever be able to measure the oceanic tide with any accuracy[1].
>
> G.H. Darwin

1.1 Introduction

The tidal currents are generally weak in the open ocean, but are still important as they indirectly fuel the vertical mixing in the interior (together with the wind) (e.g. **?**)[2]. Near boundaries, especially in shallow shelf seas, volume conservation forces the tides to accelerate and they can easily dominate the dynamics. Large amplitude tidal currents also generate turbulence and can therefore sustain substantial vertical mixing in shallow shelf seas. This can in turn lead to completely vertically well-mixed areas, e.g. large parts of the Irish Sea (e.g. **?**, for a review). Tides can therefore control not only the energy budget in shelf seas, but also influence primary production, sediment transport, pollutant dynamics, and withdrawal of atmospheric CO2 (e.g. **??**).

1.2 The tide-producing force

Tides can be defined as the variation of the sea level, with a period of approximately 12 or 24 hours, caused by the imbalance between centripetal and gravitational forces in the rotating astronomical systems. The tidal range is then the vertical difference between successive high and low waters. The maximum range occurs near to new or full moon and is called spring tide, whereas the minimum range is called neaps and occurs when the moon is in its first or third quarter. There is usually a lag of up to few days between spring tide and full or new moon. This is called the age of the tide and arises because of dissipation of tidal energy, mainly due to bed friction in shelf seas.

The rise (flood) and fall (ebb) of the sea level results in horizontal tidal currents flowing towards or away from a certain point. Depending on the dominating period, tides are either semi-diurnal (two high and two low waters each day), diurnal (one high and one low water a day) or mixed (one or two high and

[1] Darwin was wrong... see e.g. **?**.

[2] For a complete introduction to tidal theory, see **?**. For good reviews of tidal dissipation and its impact on the ocean, see **?** and **?**.

Figure 1.1: The resulting force-field on the earth.

one or two low waters each day). The mixed tides are classified as predominately diurnal or semi-diurnal tides.

1.2.1 The equilibrium tide

The base for the tidal theory is Newton's Law of Gravity, which states that two bodies, having masses m_1 and m_2 respectively, which are separated by a distance r attract each other with a gravitational force according to

$$F = G\frac{m_1 m_2}{r^2} \tag{1.1}$$

where $G = 6.6720 \times 10^{-11}$ Nm^2 kg^{-2} is the gravitational constant.

Let us now investigate a rotating and frictionless earth without topography. When looking at the earth from above the north pole the earth rotates anti-clockwise around its own axis (Fig. 1.1). The earth-moon pair also rotate anticlockwise around the system's center of mass, D. This means that the earth describes a motion where its orientation is fixed but every part of it rotates in a circle with a radius equal to the distance from earth center to D. If all points on the earth are to move in a circle of this size, there must be a centripetal acceleration of the same magnitude and parallel to the earth-moon axis but directed away from the moon at all points on the earth's surface (dashed arrows in Fig. 1.1). There is also a gravitational attraction between the earth and the moon (dotted arrows) which varies in magnitude over the earth's surface because of the different distances between the surface points and the moon. The resultants of the forces (solid arrows) can be decomposed into a normal and a parallel component relative to the surface of the earth. The vertical component of the resulting tide producing forces are so small ($\sim 10^{-5}$ N kg^{-1}) when compared to the gravity (~ 10 N kg^{-1}) that they are neglected without any loss of accuracy.

The horizontal components are of course also small, but they are of the same order of magnitude as the other horizontal forces acting in the sea, and can therefore set up horizontal motions. The horizontal components are thus the effective tide-producing components and they are called the tractive forces.

Note that at the points directly in front of the moon, where the gravitational pull is largest, the tractive force is negligible because of an absent horizontal component. It is instead at the locations perpendicular to the earth-moon center line that the tractive force has its maximum. Somewhat ironically, it is also there the tidal amplitude is smallest, because of the water being pulled away from these locations. Instead the water ends up at the point where there is no tractive forces — at the points on the earth-moon center line — and it is there the tidal bulges appear.

Because of the earth's rotation, each point on the earth's surface will pass through a complete force pattern in one day. Thus, the tractive forces will have a period of half a day despite the daily period of the earth's rotation. The result is a semi-diurnal tide. But since the moon is rotating around the earth with a period of 27.3 days, it will move a short distance while the earth completes one revolution. The earth will then have to rotate a little bit further to "catch up with the moon", and there is a delay of 25.2 minutes between each pass at a fix point on the earth surface. The lunar semi-diurnal tide will therefore have a period of 12.42 hours.

1.2.2 The earth-sun system

Added to the earth-moon-system is the sun-earth-system and co-operation between the sun and the moon as well as effects due to irregularities in the orbits. The effect of the sun, however, is smaller than that of the moon, because of the great distance between the earth and the sun. Nevertheless, the effect of the sun is clearly measurable, and the complex patterns of rotation causes the spring and neap tides, i.e. the monthly variability in the tidal amplitude. When the moon is half, the tidal amplitude is at a minimum, because the sun and moon are opposing each other. When the moon is full, the sun and moon are on opposite sides of the earth, but are still collaborating, and when the moon is new, the sun and moon are at the same side of the earth, resulting in spring tide.

1.2.3 Other constituents

Added to the already mentioned constituents are several others, e.g. the oscillation of the moon over the equator with a period of some 27 days, and the ellipticities and asymmetries of the earth's and moon's orbits. In shallow water areas there are also a number of non-linear effects due to friction, the Coriolis acceleration and bottom topography, which give a local response on the tides-producing forces. Over 600 tidal constituents have been identified, although for reliable predictions we only need to worry about 8–16 of them. The primary tidal constituents are shown in Table 1.1, and discussed further in several references, e.g. **?**. A theoretical tide is presented in Fig. 1.2, using the constituents in Table 1.1.

There are also a number of resonant, higher, harmonics which can be used in tidal analysis. For example, in shallow shelf seas quarter-diurnal constituents can become important (e.g., M4 and S4), and in some case even higher harmonics are used (e.g., M8). It must be noted though, that these are artificial

Table 1.1: The main constituents of the tide and their symbols, relative amplitude, and periods in days.

	Name	Symbol	Rel. amplitude	Period
Semi-diurnal	Principal lunar	M2	1.0000	12.4206
	Principal solar	S2	0.4656	12.0000
	Lunar elliptic	N2	0.1915	12.6584
	Lunisolar	K2	0.1267	11.9673
Diurnal	Lunisolar	K1	0.5842	23.9344
	Principal lunar	O1	0.4148	25.8194
	Principal solar	P1	0.1933	24.0659
	Elliptic lunar	Q1	0.0795	26.8684
Long Period	F ortnightly	Mf	0.1722	327.85
	Monthly	Mm	0.0909	661.31
	Semiannual	Ssa	0.0802	4383.05

Table 1.2: The different classes of tides defined from the form factor.

F	category
0-0.25	semidiurnal
0.25-1.5	mixed, mainly semidiurnal
1.5-3	mixed, mainly diurnal
>3	diurnal

constituents which has to be used to compensate for processes not included in the tidal fit itself, e.g. bed friction in shallow waters.

1.2.4 Tidal classification

Because of the large number of constituents, in combination with topographical variations, refraction, reflection, reduction, and amplification of certain tidal constituents, especially the higher harmonics present in shallow water tides, the tides are not symmetrical. Instead, the amplitude of the tides vary under over time, especially on fortnightly, monthly, and yearly scales, but there are also a daily inequality. This means that the two floods or ebbs during a diurnal tidal cycle do not have the same amplitude (see Fig. 1.2 where $F = 0.68$, indicating a semidiurnal mixed tide).

The tides can be classified from the ratio or from factor F between the amplitudes of the major diurnal and the major semidiurnal constituents, or

$$F = \frac{K1 + O1}{M2 + S2} \tag{1.2}$$

The different classes from the form factor are given in Table 1.2.

1.2.5 Self-attraction and Loading

The mass distribution caused by the tidal currents modify the gravitational field of the Earth, and it causes the crust to buckle under the load of the tide. This

Figure 1.2: Top: The tidal constituents from Table 1.1 plotted over a period of 3 days. Bottom: The sum of the constituents plotted over a 31-day period. Note the daily inequality in the data.

causes a feedback mechanism on the tidal forcing known as Self-Attraction and Loading (SAL). The SAL effect can be substantial — in the Pacific it reaches 1/3 of the tidal potential — and has to be included in tidal models. Effectively, SAL enters as a reduction of the surface pressure gradient because the combined SAL force (due to crustal depression and uplift, mass rearrangement, and direct gravitational effects) points away from the tidal bulge.

In its full version the SAL correction consists of convolving a global Green's function with the surface elevation (e.g. **??**). This is often impractical for numerical reasons and instead a constant reduction of typically 10% is used or and iterative scheme is employed (**?**).

1.3 Shelf tides

Fortunately, the dynamics of the tides are well known, and has been for a long time. In combination with recent knowledge of the dynamics of the coastal ocean, and the increased and observational computational capacity, it is possible to make impact analyzes and predictions of tidally and wind forced coastal segments.

Generally, the energy in the coastal tides comes from an input in the deep ocean by the astronomical forces, but the dissipation in the deep ocean is small compared to that on the shelf. The energy is then delivered to the coastal ocean by the energy flux of the tidal waves, and the energy is then dissipated at the shelf. The tidal wave is modified through a number of interacting processes as the wave approaches the shelf, but the most prominent is the amplification of the amplitude because of the conserved energy flux. Typically, the depth changes from a few km in the deep ocean to less than a few hundreds of meters at the shelf. Because of the phase velocity of the long wave, $c_g = \sqrt{gH}$, the amplitude of the wave must increase when the depth decreases. The energy flux in a wave with amplitude a is given by $E_{flux} = \frac{1}{2}\rho g a^2 c_g$, and conservation when the depth changes from H_1 offshore to H_2 at the shelf gives

$$\left(\frac{a_2}{a_1}\right)^2 = \sqrt{\frac{H_1}{H_2}} \tag{1.3}$$

Hence, a wave traveling from 4000 m depth to 100 m depth experience an amplitude increase of a factor 2.51, at least theoretically. Because of wave reflection and dissipation at the shelf, both inwards and outwards, the amplitude may very well be larger or smaller than this. The energy reflected inwards may interact with the coast and form standing waves that further enhance the amplitude.

Because of the Coriolis force, standing waves in an embayment are modified to form amphidromic systems, in which the tides rotate around an amphidromic point where $a = 0$ (e.g. Fig. 1.3). Dynamically such systems cab be described by a number of Kelvin waves propagating along the coastline. There is often an asymmetry in these systems and the amphidromic point is displaced from the center of the basin because of energy losses in the wave. The wave on one side of the basin is thus weaker than on the other, where the phase appears first (often seawards).

1.4 Open ocean tides

The tidal amplitude in the open ocean has only been known for just over a decade on a global scale **?**. The reason for this is that it is very difficult to measure the small amplitude tide (less than a meter) in the open ocean (which is on average some 4000 m deep). In fact, it wasn't until the launch of the TOPEX/Poseidon satellite system that we obtained a high-resolution global picture of the tides. This satellite has now been replaced by Jason and Jason II, which continue to provide high-accuracy topography data. The tidal range in the ice-free ocean is now known globally to within a few cm **?**. In addition, there is the ability to model the global tides to within the same accuracy using only astronomical forcing (**??**, ; 1.4).

Figure 1.3: Amphidromic points for the M2 tidal constituent. Amplitude is indicated by color, and the white lines are cotidal differing by 1 hr. The curved arcs around the amphidromic points show the direction of the tides, each indicating a synchronized 6 hour period. Figure by R. Ray, used by permission from NASA/JPL.

Figure 1.4: a) The tidal amplitude(colour) and co-tidal lines (white lines) for the present from the model by **?**.
b) the same as in a) but for the LGM 22,000 ybp.

2 Tidal prediction: harmonic analysis

There are several methods used to predict the tides at a specific location, see e.g. Chapter 17 in **?**. Here we will focus on the most widely used method: harmonic analysis.

Periodic data can quite easily be simulated through a special regression technique called harmonic analysis, see e.g. **?** or **?** for detailed descriptions. A number of sine- or cosine functions with specific periods are then adopted to the data set through a version of the least squares method. The result is the amplitude and phase lag for each component. For tides, the constituents are known (although there are more than 600 of them) and their contribution to the total series can easily be calculated through a fourier series. For data with unknown components, one has to calculate with as many constituents as possible, which gives large systems, but it is possible to solve them on modern computers. A record of the observed tide is needed and it must be long compared with the tidal components concerned: To accurately simulate the spring-neap modulation we need 15 days of data, and to obtain longer periods we strive to get 28 days of data.

Assume that we have a sea-level data series, z_i, that consists of N observations sampled regularly *without any gaps*. We want to rewrite it in K known constituents according to

$$\zeta(t) = \sum_{k=1}^{K} A_k \cos\big(\omega_k(t-\tau)\big) \tag{2.1}$$

where A_k, ω_k and τ_k are the constituents' phase, frequency, and phase lag, respectively. Eq. (2.1) can be written as

$$\zeta(t) = \sum_{k=1}^{K} [a_k \cos(\omega_k t) + b_k \sin(\omega_k t)] \tag{2.2}$$

Figure 2.1: The black line is an 11-day sea-level record from the western Irish Sea consisting of 60-second averages. The heavy grey line is the resulting time-series after a tidal fit of 9 constituents using t_tide. The fit describes 92% of the variance in the original record.

where

$$
\begin{aligned}
a_k =& A_k \cos(\omega_k \tau_k) && (2.3)\\
b_k =& A_k \sin(\omega_k \tau_k) && (2.4)\\
A_k =& \sqrt{a_k^2 + b_k^2} && (2.5)\\
\tau_k =& \arctan\left(\frac{b_k}{a_k}\right) && (2.6)
\end{aligned}
$$

If this is used on every observation, we have

$$
z_i = \sum_{k=1}^{K} (a_k \cos(\omega_k t_n) + b_k \sin(\omega_k t_n)) \tag{2.7}
$$

This is N algebraic equations in the $2K$ unknowns a_k and b_k. By rewriting the system on matrix form, it can be solved numerically. Note that it is strongly over-determined and must be solved numerically using a least-squares method. There are some packages available for tidal prediction in Matlab (although solving the system above is a fairly easy task). The most widely used is `t_tide`, which is freely available on the internet (see also **?**).

An example of a tidal fit to an observed sea-level series is shown in Fig. 2.1 (the calculations are made by a single, one-line, call to t_tide). The fit is obviously quite good, describing over 90% of the variance in the original data set, but why is the spring-neap modulation not fully resolved in the resulting time-series?

3 Tidal dissipation

This section (which is rather brief) connects the theory of the tides to that of turbulence. Tidal dissipation is a massive subject, and we will barely scratch the surface in this text. The reader is directed to the literature for more information.

3.1 Cause and effect

The gravitational torque between the Moon and the tidal bulge of the Earth causes the Moon to move into a higher orbit, slowing down, and the Earth's rotation to decelerate (see e.g. **?**, for a good description). The tidal bulges on Earth is dragged ahead of the Moon by the Earth's much faster rotation, and tidal friction is required to drag and maintain the bulge ahead of the Moon. As a consequence, there is a dissipation taking place of the excess energy supplied by the moon. If the friction and heat dissipation were not present, the Moon's gravitational force on the tidal bulge would rapidly (within a couple of days) bring the tide back into synchronization with the Moon, and the Moon would no longer recede. Most of the dissipation occurs in a turbulent bottom boundary layer in shallow seas such as the European shelf around the British Isles, the Patagonian shelf off Argentina, and the Bering Sea, but there is also substantial dissipation near topography in the deep ocean (**?**).

The motion of the Moon can be followed with an accuracy of a few centimeters by lunar laser ranging (LLR). Laser pulses are bounced off mirrors on the surface of the moon. Measuring the return time of the pulse yields a very accurate measure of the distance. These measurements show that the recession rate at them moment is 3.84 ± 0.07 cm/year, which means that the dissipation of energy by tidal friction averages about 3.75 terawatt in the present ocean (**?**). Some 2.7 TW dissipate in shelf seas, whereas the remaining terawatt dissipates in the open ocean. As a consequence, the day on Earth is currently getting longer by 2.3 ms/century, although this has varied substantially over geological time.

3.2 Historic dissipation rates

This mechanism has been working for long periods of time (since the earth-moon system was formed). There is geological evidence that the Earth rotated faster and that the Moon was closer to the Earth in the remote past (**?**). These records indicate that 620 million years ago the day was 21.9 ± 0.4 hours, and there were 13.1 ± 0.1 months/year and 400 ± 7 solar days/year. The length of the year has remained virtually unchanged during this period because no evidence exists that the gravitation has changed. The average recession rate of the Moon between then and now has been 2.17 ± 0.31 cm/year, which is about half the present rate.

Ancient observations of solar eclipses give fairly accurate positions for the Moon, and they are consistent with those above (**?**). However, the dissipation rate during the last glacial maximum (22,000 ybp) was at least 50% larger than today, indicating that the recession rate has varied over time (**?**, ; Fig. 1.2). This is contradicted by some investigations, where growth rates of invertebrates have been used to achieve some level of correlation between a constant increase in day length, i.e. a recession rate (indicating a constant tidal dissipation). However, the data is highly scattered and several data sets don't fit in at all, and the theory can be seriously questioned (see **?**, for a discussion).

Part II

Turbulence

4 An introduction to turbulence in geophysics

Big whirls have little whirls which feed on their velocity
while little whirls have lesser whirls
and so on to viscosity

Lewis Richardson

4.1 About turbulence

Turbulence remains one of the unsolved problems in physics, and yet it is incredibly important for sustaining and controlling a variety of processes. These range from the dynamics and mixing of shelf seas, via controlling marine biogeochemical cycles, to sustaining the climate controlling meridional overturning circulation in the global ocean. An increased understanding of turbulence and its implementation in models is of practical application because of the importance of management of the coastal seas and the prediction of responses to climate change.

Historic studies of turbulence in the ocean suggested that the ocean is everywhere turbulent and parameterizations in terms of the eddy viscosity were common. However, viewed on a small scale, the ocean is not everywhere turbulent. For example, in the deep ocean the turbulence is patchy and intermittent and occurs in elongated, thin layers, while the levels of fluctuations through most of the water volume are usually small for most of the time. This is because of the stable density gradients, which significantly limit the vertical extent of the mixing and thereby keep the vertical Reynolds number small.

The oceanic turbulence can be classified according to several methods. The most obvious is the difference between mechanically generated turbulence, i.e. turbulence caused by the kinetic energy in the motions (such as interactions in shear flows), and convective turbulence, produced by unstable stratification. Another classification is based on the location of the source of the turbulence. Externally generated turbulence of course originates at the boundary, e.g. mixing generated by the wind. Internal turbulence, on the other hand, is produced and used within the same volume of fluid.

All turbulent flows have several specific characteristics, some of which are

- Randomness and chaos.

- Nonlinearity: Turbulent flows are highly nonlinear. This means that the Reynolds, Rayleigh, and inverse Richardson numbers all exceed their critical values and, hence, small perturbations may grow spontaneously and the flow may become unstable. Also, the nonlinearities introduce vortex stretching.
- Diffusivity: Turbulent flows are characterized by a large diffusion of momentum and heat.
- Vorticity: Turbulent flows include a large range of eddy sizes. The larger ones contain the most energy, but because of the nonlinear interactions this energy is handed down to smaller scales until it is dissipated by viscosity at the length scales of millimeters. This is known as the turbulent cascade.
- Dissipation: The vortex stretching mechanism transfers energy and vorticity to smaller scales until the gradients become so large that they are smeared out by molecular viscosity.

4.2 The role of turbulence in the ocean

Molecules in a fluid close a solid boundary sometimes strikes the boundary and transfer momentum to it. Molecules away from the boundary then strike the molecules that have struck the boundary and transfer momentum to them. In this way, the change in momentum is transferred inwards, from the the boundary, by molecular viscosity. The scale of this motion is only a few centimeters at the most, and molecular viscosity is only important within a few millimeters from the boundary. But the boundaries still have effects on the large scale oceanic flows, because the change of momentum at the boundaries are transferred to the interior of the ocean through turbulence.

5 Diffusion and Advection—non-turbulent flows

5.1 Diffusion

Diffusion is a (slow) transport phenomenon on molecular level, in which separate particles (smoke, dust, water molecules, salt...) can mix by randomly "walking" in its environment (this is known as Brownian motion). It is diffusion which is responsible for smoke to fill a room or for a spoon to become warm even if only part of it is submerged in a warm liquid.

If we look at an entire system, or group, instead of individual of particles the continuity equation can be written as

$$\frac{\partial c(\mathbf{x}, t)}{\partial t} = -\nabla \mathbf{q}(\mathbf{x}, t) \tag{5.1}$$

where $\mathbf{q}$ is the mass (or volume) flux and $c(\mathbf{x}, t)$ is the concentration. Following Fick's law the flux of material is proportional to the spatial gradient of the concentration. This arises from nature's strong dislike of vacuum, leading to a transport attempting to fill out any voids or to reduce gradients. In mathematical terms we have

$$\mathbf{q}(\mathbf{x}, t) = -k\nabla c(\mathbf{x}, t) \tag{5.2}$$

where k is a constant diffusion coefficient depending on the substances under investigation. Combining these two equations gives the diffusion equation:

$$\frac{\partial c(\mathbf{x}, t)}{\partial t} = k\nabla^2 c(\mathbf{x}, t) \tag{5.3}$$

It is left as an exercise for the reader to derive Eq. (5.3) if $k = k(\mathbf{x})$.

5.2 Advection

Advection is a transport mechanism of a substance or conserved property with a moving fluid, and it is generally orders of magnitudes faster than diffusion. An example of advection is the transport of heat in the Gulf Stream: the motion of the water carries the heat away from the Gulf of Mexico into the North Atlantic. In fact, any substance or conserved property can be, and is, advected in the ocean by the oceanic currents, e.g. pollutants, salt, and plankton (momentum is advected as well, causing nonlinearities to occur in the equations of motion). The advection equation for a property ψ is given by

$$\frac{\partial \psi(\mathbf{x},t)}{\partial t} + \mathbf{u} \cdot \nabla \psi(\mathbf{x},t) = 0 \tag{5.4}$$

where $\mathbf{u}$ is the velocity field. Eq. (5.4) states that the rate of change of a property depends on the product between the gradient of the property and the velocity field, i.e. the rate at which the velocity transports the gradient. Note that if $\mathbf{u} = 0$, $\partial\psi/\partial t = 0$, i.e., without if there are no motions in the fluid we don't have any advection. On the other hand, if for some reason there is a rate of change of a property (e.g. by heating through the sea surface) that change can set up a current system and changes in gradients (the heat is transported away from the region of heating).

6 The averaged equations of motion

The following discussion is mainly based on the chapter on turbulence in **?**, with some additional material presented by **?**, **?**, and **?**.

Consider a non-rotating, nonlinear, Boussinesq fluid with density as a function of temperature only. The instantaneous parameters, $\tilde{\eta}$, of the flow may be written as a sum of a time-averaged part η and a fluctuating or turbulent part η' by the relation $\tilde{\eta} = \eta + \eta'$. If this is introduced into the equations of motion and the results are time-averaged some special results arise.

First, the continuity equation shows that both the average and the turbulent flows are non-divergent, just as for the instantaneous flow, i.e. $\nabla \cdot \boldsymbol{u} = \nabla \cdot \boldsymbol{u'} = 0$. This is not too surprising, because why should the introduction of turbulence produce or disintegrate fluid?

The momentum equations are a bit trickier. The result is, in the x-direction,

$$\begin{aligned} \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} &= \\ &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} + \nu \nabla^2 u + \frac{\partial}{\partial x}\overline{u'u'} + \frac{\partial}{\partial y}\overline{u'v'} + \frac{\partial}{\partial z}\overline{u'w'} \qquad (6.1) \end{aligned}$$

where bars denote time-averages. The terms on the right hand side contain-

Figure 6.1: Sketch of the mechanism behind the Reynolds stresses. The fluctuating term $u'v'$ can moves of x-momentum in the y-direction and vice versa. Here $dU/dy > 0$, so a particle (black dot) moved upwards along the dashed line will enter a region with a larger background velocity (at the position of the gray dot). This means that moving the particle will force it to decelerate and $u'v'$ can be physically interpreted as a stress term.

ing averaged products of fluctuation describe fluxes of momentum in different directions. For example, $\overline{u'v'}$ states that x-momentum is transferred in the y-direction, whereas $\overline{u'u'}$ denote x-momentum being transferred in the x-direction. In the following we focus on $\overline{u'v'}$.

Transferring x-momentum in the y-direction means that the flow looses x-momentum, because if we move x-momentum in the positive direction of the y-axis, $\frac{\partial u}{\partial t} < 0$ (see Fig. 6.1). Hence, $\overline{u'v'}$ acts in the same manner as an additional stress term τ_x. Consequently, the product terms are called the Reynolds stresses. They usually dominate over the viscous terms ($\nu \nabla^2 u$), except close to solid boundaries where the fluctuations must be small. Note that if the turbulence is isotropic, i.e. without a preferred direction, only terms on the form $\overline{u'u'} = \overline{u'^2}$ remain. The Reynolds stress terms arise from the nonlinearity of the equations of motion and describe the stress exerted on the average flow by the turbulence. Physically, they describe the rate of mean momentum transfer to the turbulent fluctuations from the mean flow.

It is time to define the all-important Reynolds number, Re. It is merely the quotient between the inertia and viscous force:

$$Re \equiv \frac{\rho u \frac{\partial u}{\partial x}}{\mu \frac{\partial^2 u}{\partial x^2}} \propto \frac{Ul}{\nu} \tag{6.2}$$

Typical values in the open ocean are $U = 0.1$ m s^{-1} and $l = 1000$ km, so $Re \sim 10^{11}$. As the Reynolds number increases above some critical value, usually taken to be $Re > 2000$, the flow becomes more and more turbulent. Worth noting is also that all flows with the same geometry and the same Reynolds number have the same flow pattern. Thus flow around all circular cylinders, whether 1 mm or 1 m in diameter, look the same if they have the same Reynolds number.

6.1 TKE

An equation for the turbulent kinetic energy in a flow is obtained by taking an equation for the total flow minus an equation for the mean flow to obtain an equation for the turbulent flow. The TKE-equation can then be derived by multiplication with u (note that primes have been dropped) and averaging. The result is, again for the equation in x only,

$$\frac{D}{Dt}\overline{\left(\frac{u^2}{2}\right)} = \text{transport} + \text{shear prod.} + \text{buoyant prod.} + \text{viscous diss.} \tag{6.3}$$

The right-hand side terms are, with U denoting the background state,

$$\begin{aligned}
\text{transport} &= \frac{\partial}{\partial x}\left(\frac{1}{\rho_0}\overline{pu} + \frac{1}{2}\overline{u^2 u} - 2\nu \overline{u\frac{\partial u}{\partial x}}\right) + \\
&+ \frac{\partial}{\partial y}\left(\frac{1}{\rho_0}\overline{pv} + \frac{1}{2}\overline{u^2 v} - \nu \overline{u\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)}\right) + \\
&+ \frac{\partial}{\partial z}\left(\frac{1}{\rho_0}\overline{pw} + \frac{1}{2}\overline{u^2 w} - \nu \overline{u\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)}\right) \qquad (6.4)\\
\text{shear prodction} &= \overline{u^2}\frac{\partial U}{\partial x} + \overline{uv}\frac{\partial U}{\partial y} + \overline{uw}\frac{\partial U}{\partial z} \qquad (6.5)\\
\text{buoyant production} &= g\alpha\overline{wT'} \qquad (6.6)\\
\text{viscous dissipation} &= \nu\left(2\overline{\left(\frac{\partial u}{\partial x}\right)^2} + \overline{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2} + \overline{\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2}\right) \qquad (6.7)
\end{aligned}$$

Similar expressions are of course available for the component in $y-$ and z-directions as well.

6.2 Turbulence production, cascading, and dissipation

In order to be sustained turbulence must become anisotropic or be overtaken by dissipation. The reason is that in an isotropic flow, there can be no production of turbulence because terms on the form $\overline{uv}$ etc. and the vertical heat flux are zero.

The largest eddies in a turbulent flow have the same length-scale as that of the flow, e.g. the thickness of the boundary layer at the bottom. These eddies feed on the mean field, and eddies somewhat smaller then feed at the larger ones by straining from the larger flow field. The smaller eddies thus extract energy from the larger ones by vortex stretching. Much smaller eddies do not feel the larger ones but are merely advected in the larger flow field, because the strain rate scale of the larger eddies are much larger than the size of the smaller. The cascading therefore occurs in several small steps, and the process is essentially inviscid since the vortex stretching terms arises from the nonlinearities of the equations motion and not from the stress terms. Therefore, in fully developed turbulent flows, viscosity has no effect on the production of shear, but only the rate of loss of energy, or dissipation ϵ. The dissipation occurs only at very small scales where the fluctuating strain rates are high and the filaments become thin enough for molecular diffusion to the smear out the velocity gradients. This scale may be described by the Kolmogorov microscale given by

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{\frac{1}{4}} \tag{6.8}$$

Eq. (6.8) also shows that the dissipation rate is independent of the viscosity; a change in ν only changes the scale at which dissipation takes place, not the rate of dissipation. Note that $\eta \sim \mathcal{O}(0.001)$ m in the ocean.

The dissipation of TKE in the ocean is very patchy, both in the vertical and in the horizontal. In the ocean interior, $\epsilon \sim 10^{-6}$ W m^{-3} (10^{-9} W kg^{-3} or m^2 s^{-3}), which is the equivalent of some ten kitchen blenders stirring 1 km^3 of water. In the bottom boundary layer in Liverpool Bay, on the other hand, we find $\epsilon \sim 10^{-1}$ W m^{-3} or even larger. Large values of dissipation are usually found in thin layers near boundaries — mainly near the sea-bed, around topographic features, and in the pycnocline (e.g. **?**).

6.3 The inertial subrange

Let's look at the spectra of nearly isotropic turbulence (Fig. 6.3). We begin by associating, somewhat vaguely, a wavenumber K with the length scale of the eddies, $1/K$. Hence, small eddies have large wavenumbers. Now assume that the length scale of the larger eddies is l, e.g. the scale of the boundary layer. At the smaller scales, i.e. where $K \gg 1/l$, there is no interaction between the turbulence and the motions of the large eddies that contains the most energy. The reason is the cascading, during which information is lost through each step. As a result, the small-scale eddies are unaware of the direction of the mean gradients and hence are (nearly) isotropic. The small-scale spectrum does thus not depend on the energy at the larger scales, but only on the parameters that determine the nature of the small-scale flow. Altogether, we may write the spectrum as $S = S(K, \epsilon, \nu)$, $K \gg 1/l$. The range of wavenumbers $K \gg 1/l$ is known as the equilibrium range. The high-end range of the equilibrium range is located where the wavenumbers are of the Kolmogorov microscale, or $K \sim 1/\eta$. The other end of the spectrum is called the inertial subrange because the only transfer of energy present there is forced by vortex stretching (inertia forces). Therefore, both the dissipation and the production of TKE are small in the

Figure 6.2: The observed rate of dissipation of TKE in the western Irish Sea. Note the log scale of the data.

inertial subrange. This range can be found in the interval $1/l \ll K \ll 1/\eta$.

The production of energy thus produces a large peak of S at a certain $K \simeq 1/l$, and the dissipation causes a sharp drop of S for $K > 1/\eta$ (e.g. Fig. 6.3). Following Kolmogorov, it can be argued that S is independent of ν in the inertial subrange, or $S = S(K, \epsilon),\ 1/l \ll K \ll 1/\eta$. The dependence of ϵ is necessary to transfer energy down across the inertial subrange. Dimensional reasoning (remember that $S \sim m^2/s^2$ and $\epsilon \sim m^2/s^3$) then gives Kolmogorov's 5/3-law:

$$S = A\epsilon^{2/3}K^{-5/3}, \quad 1/l \ll K \ll 1/\eta \tag{6.9}$$

Note that the Kolmogorov law has been verified using observations in the ocean.

6.4 Eddy viscosity

In a laminar flow, the shear stress may be described as

$$\frac{\tau}{\rho} = \nu \frac{dU}{dy} \tag{6.10}$$

where ν is the molecular diffusivity. One is tempted to assume that the diffusive term in a turbulent flow may be described in the same way as that of molecular diffusion, but with a much larger diffusivity. Hence,

$$\frac{\tau}{\rho} = A_z(z)\frac{dU}{dy} \equiv -\overline{uv} \tag{6.11}$$

where $A_z(z)$ is the eddy viscosity, a property that depends on *the state of the flow, not on the fluid.* The drawback with this formulation is that it is only valid for eddies smaller than the curvature of the profile. In the cases where it might work, it is reasonable to assume that $A_z \sim u'l_l$, i.e. the eddy viscosity is

Figure 6.3: A hypothetic wavenumber spectrum with the ranges marked.

proportional to the typical scale of the fluctuating velocity u' and the mixing length l_m. Note that the stress terms in the equations of motion have the form of differentials, e.g. $F_x = \partial\tau/\partial z$, leading to a quite complicated expression if A_z is a function of depth.

I have only discussed vertical derivatives above, but there are of course analogous expressions for horizontal stresses as well, but then with a different eddy viscosity (usually denoted A_H).

The mixing-length theory has one problem: it is not correct in principe (but it is correct dimensionally) because it requires a single length and times-scale for the turbulence; this is hardly realized in geophysical flows. Nevertheless, it is often used because of its relative simplicity (and because there are few other formulations available).

7 Flows near walls

7.1 Wall-free flows—entrainment

Consider a nearly parallell shear flow far away from any boundaries. The part of the fluid that is not turbulent is considered irrotational. The turbulent flow, e.g. in a plume or jet, may slowly pull the surrounding fluid inwards due to frictional effects. This is known as entrainment and is an important process in, e.g., river mouths with negligible tidal mixing. The frictional processes behind the flow are viscosity (laminar flows) and inertia (turbulent flows). Once the new fluid is drawn into the turbulent flow, it must also become turbulent. This is initially done by small viscous eddies acting at the boundary between the fluids (which by the way has a thickness on the order of the Kolmogorov microscale).

7.2 Wall-bounded shear flows

The larger scales of wall-free flows are independent of viscosity. This is not the case when solid boundaries become present, because viscosity then plays a significant role near the wall, in the frictional boundary layer. There, the drag at the wall is always depending on the Reynolds number Re, even when $Re \to \infty$.

Consider a fully developed turbulent flow in a channel. The flow is no longer dependent on x, meaning that $0 = -\partial P/\partial x + \bar{\partial}\tau/\partial y$ (subscripts here denote derivation), where $\bar{\tau} = \mu(\partial U/\partial y) - \rho\overline{uv}$. By principles of separation, the pressure gradient and the stress gradients must both be constants, requiring the stress term to be linear. Near the wall, the stress term is dominated by viscosity because the fluctuations required for the Reynolds stresses vanish, whereas the latter dominate the inner part of the fluid. Within the viscous sublayer, there is no pressure gradient and the mean flow may be described by

$$\rho U \frac{\partial U}{\partial x} + \rho V \frac{\partial V}{\partial y} = \frac{\partial \bar{\tau}}{\partial y} \tag{7.1}$$

7.2.1 Inner layer: Law of the wall

Consider the flow near a smooth channel wall or boundary layer. We denote the free-stream velocity or center velocity with U_∞ and the width of the flow with δ, e.g. the boundary layer thickness or the channel half width. Near the wall we expect the velocity only to depend on the parameters of importance near

Figure 7.1: The law of the wall illustrated and the different subregions and layers marked.

the wall, or $U = U(\rho, \tau_0, \nu, y)$, where τ_0 is the shear stress at the wall. From dimensional arguments we may define the frictional velocity

$$u_* \equiv \sqrt{\frac{\tau_0}{\rho}} \tag{7.2}$$

Using Buckingham's pi-theorem we obtain the law of the wall:

$$\frac{U}{u_*} = f\left(\frac{yu_*}{\nu}\right) = f(y_+) \tag{7.3}$$

where $y_+ \equiv yu_*/\nu$ is the distance non-dimensionalised with the viscous scale ν/u_*. The innermost part of the wall layer is dominated by viscosity and is hence called the viscous sublayer. Because of its limited extent, the stress can be assumed uniform within this layer and thus put equal to a wall stress τ_0. This means that, using a no-slip condition at the wall,

$$\mu\frac{dU}{dy} = \tau_0 \Leftrightarrow U = \frac{y\tau_0}{\mu} \Leftrightarrow \frac{U}{u_*} = y_+ \tag{7.4}$$

within the viscous sublayer. The linear distribution holds to about $yu_*/\nu \sim 5$ (see Fig. 7.1).

7.2.2 Outer layer: Velocity defect law

Outside the viscous sublayer we encounter a flow where the turbulence is grossly inviscid and of the Reynolds stress type. These stresses generate a drag on the flow and generates a velocity defect $U_\infty - U$ that is expected to be proportional to the wall friction. Thus, the velocity distribution in the outer region must have the form

$$\frac{U - U_\infty}{u_*} = F\left(\frac{y}{\delta}\right) = F(\xi) \tag{7.5}$$

This is the velocity defect law.

7.2.3 Overlap layer: Logarithmic law

We have a slight inconsistency here: the inner and outer layer are scaled by different parameters (δ in the outer layer and ν/u_* in the inner). The two layers must therefore meet in a transition region, known as the overlap layer, where $y_+ \to \infty$ and $\xi \to 0$ simultaneously. However, it is in fact more convenient to match the gradients instead of the velocities, among other things to avoid discontinuities on the velocity curve. The gradients can be written

$$\frac{dU}{dy} = \frac{u_*^2}{\nu}\frac{df}{dy_+} \qquad \text{(inner)} \tag{7.6}$$

$$\frac{dU}{dy} = \frac{u_*^2}{\delta}\frac{dF}{d\xi} \qquad \text{(outer)} \tag{7.7}$$

and matching gives

$$\xi\frac{dF}{d\xi} = y_+\frac{df}{dy_+} = \frac{1}{k} \tag{7.8}$$

where $k \approx 0.41$ is von Karman's constant. By integrating Eq. (7.8) and introducing integration constants based on laboratory experiments, we get

$$\frac{U}{u_*} = \frac{1}{k}\ln\frac{yu_*}{\nu} + 5.0 \tag{7.9}$$

$$\frac{U - U_\infty}{u_*} = \frac{1}{k}\ln\frac{y}{\delta} - 1.0 \tag{7.10}$$

which describe the properties in the overlap layer. This layer is also known as the inertial sublayer or logarithmic layer. Note that Eqs. (7.9)-(7.10) are only valid for large y_+ and small y/δ.

The linear distribution in the viscous sublayer in Eq. (7.3) is valid when $y_+ < 5$, meaning that the layer has a thickness $\delta_\nu \simeq 5\nu/u_*$- The logarithmic

Figure 7.2: Sketch of the velocity profile over a smooth (left) and rough (right) surface, the latter with with roughness length y_0. The dashed line in the left figure is the continuation of the logarithmic profile.

layer in Eq. (7.9), on the other hand, is valid for $30 < y_+ < 300$, meaning that there is a buffer layer between $5 < y_+ < 30$ where both the viscous and Reynolds stresses are important. In this layer, the turbulent production term reaches its maximum due to the large velocity gradients.

If the surface is not perfectly smooth we expect another logarithmic distribution because of the roughness of the surface. The viscous sublayer breaks down if the roughness length scale y_0 is larger than the thickness of the layer, see Fig. 7.2. From dimensional arguments, it can be shown that the velocity distribution within a rough logarithmic layer is the well known

$$\frac{U}{u_*} = \frac{1}{k} \ln \frac{y}{y_0} \tag{7.11}$$

Eq. (7.11) states that the velocity near the wall actually goes to zero a distance y_0 from the wall, i.e. there is no viscous sublayer but a mere continuation of the logarithmic layer.

8 Turbulence and stratification

8.1 Mixing and turbulence

The loss of energy to turbulence may be used to force irreversible (vertical) mixing in the ocean. If the fluid is homogenous the energy supply need only be large enough to overcome mechanical friction. However, if the fluid is stratified work must be done against the buoyancy forces as well, and the energy demand is hence larger.

Let us consider a two-layer shear flow. In the initial state the center of mass of the system must lie below middepth, because the lower layer is denser than the upper. If the fluid becomes completely mixed, the potential energy must increase because the center of mass is relocated upwards. The potential energy in the unmixed system is

$$PE_i = 0.5\left(\rho_2 g \frac{H^2}{4} + \rho_1 g \frac{3H^2}{4}\right) \tag{8.1}$$

whereas in the final, fully mixed state,

$$PE_f = 0.5\rho g H^2 \tag{8.2}$$

The difference between these two systems show that there has been a *gain* in PE equal to $1/8(\rho_2 - \rho_1)gH^2$. In a geophysical system with no external sources the added energy must come from the kinetic energy, and it is an easy assignment to show that the loss of KE in the system is equal to the gain in PE above, the requirement being that there is a shear present in the initial state. It is also required that the initial density difference is sufficiently small unless the pycnocline is to act as a gravitational barrier. This may of course be counteracted by a sufficiently large velocity shear, and is the topic of the next section.

8.2 The Richardson numbers

Let us now turn to the important problem with stratification and turbulence. In this discussion, z is the vertical coordinate. We then return to the equation for the turbulent kinetic energy q ($q^2 = (\overline{u^2} + \overline{v^2} + \overline{w^2})/2$). Without viscous transports and no horizontal variations, we have

$$\frac{D}{Dt}q^2 = \underbrace{-\frac{\partial}{\partial z}\left(\frac{1}{\rho_0}\overline{pw} + \overline{q^2 w}\right)}_{\text{transport}} - \underbrace{\overline{uw}\frac{dU}{dz}}_{\text{prod. shear}} + \underbrace{g\alpha\overline{wT'}}_{\text{prod. buoy.}} - \underbrace{\epsilon}_{dissipation} \tag{8.3}$$

The transport term arises because of a fluctuating w. The first production term arises because of interaction between Reynolds stresses and the mean flow shear (almost always positive), whereas the second production term comes from the vertical heat flux (e.g., it is positive in an unstably stratified environment where convection takes place). The flux Richardson number is basically defined as the quotient between the buoyant destruction and the shear production terms (or mechanical production terms), or

$$R_f = \frac{g\alpha\overline{wT'}}{\overline{uw}dU/dz} \tag{8.4}$$

Because of the positive shear production term, $Rf < 0$ in an unstable environment, e.g. with an upward heat flux. If $R_f > 1$ the buoyant destruction removes turbulence at a higher rate than it is produced by the shear. However, the critical value is practically $R_f \simeq 0.25$ because dissipation is usually a large fraction of the shear production.

One may also define the gradient Richardson number by introducing the buoyancy frequency $N^2 = -g/\rho_0 \partial\rho/\partial z$:

$$R_i = \frac{N^2}{(dU/Dz)^2} = \frac{\alpha g(d\bar{T}/dz)}{(dU/dz)^2} \tag{8.5}$$

The presence of a stable stratification hence damps the vertical transports of heat and momentum. However, the momentum flux is damped less because the then present internal waves transfer momentum but not heat. When R_i is based on parameters over the entire depth of the ocean, it is usually very large. This implies that the associated flows are indeed very stable. But the stratification in the ocean is nonuniform, and across the pycnocline the gradients are of course larger. It may be shown that the discontinuities across the pycnocline are less stable and thereby allow turbulence to exist. Following **?**, we define a discrete version of R_i for density through $R_{i0} = g'd/u^2$, where d (u) is the overall length (velocity) scale of the motions. It can now be shown that one can chose nonuniform profiles for velocity and density such that any value of R_i can be found everywhere in the interior, regardless of the value of R_{i0}. The reason is that $(\Delta U/\Delta z)^2 \leq \overline{(\partial u/\partial z)^2}$, and hence a redistribution of properties can always reduce R_i to a value where turbulence can be maintained. However, there is not enough kinetic energy available in the local mean flow to produce the nonuniform profiles, even when $\epsilon = 0$. The conclusion must be that energy propagates inwards, as internal or gravity waves, from the boundaries in order to maintain the mixing in the ocean.

The Richardson number describe more than just the condition of stability. Because of its definition, it describes the time-scales of the buoyancy and shear.

The buoyancy promotes the propagation of waves, whereas the shear suppresses them. This is because the shear tends to continuously rotate the material lines until they become horizontal and thereby infinitely stretched. The same thing must happen to phase-lines of internal waves propagating through the fluid, and if $R_i < 1/4$, the shear is large enough to completely suppress wave propagation. In other words, *when $R_i < 1/4$ instabilities may spontaneously grow and generate shear-driven turbulence* (**?**).

8.3 The Monin-Obukhov length

The Richardson numbers compare the ratio between buoyant and mechanical turbulence. This may also be done using the Monin-Obukhov length defined by

$$L_M \equiv -\frac{u_*^3}{k\alpha g\overline{wT'}} \left(= -\frac{u_*^3}{kB} \right) \tag{8.6}$$

where $k = 0.41$ is the von Karman constant, $u_*^2 = C_d \rho U^2$ is the frictional velocity, α is the thermal expansion coefficient, and B is the buoyancy flux (see e.g. **?**). In freshwater influenced regimes, α and T can readily be changed for β and S, describing effects due to salinity changes.

The Monin-Obukhov length can thus be determined from the conditions on drag and heat flux at the surface. Within the logarithmic layer, $\overline{uv} = u_*^2$ and we have $R_f = z/L_M$. This means that L_M is the height above (or below for the sea surface) the boundary at which the buoyant destruction and mechanical production are of the same magnitude. L_M is therefore the length-scale within the boundary for which buoyancy forces are important, and it sets one limit of the thickness of the surface mixed layer (the Ekman layer is the other). The effects of the stratification is therefore small if $z <\ll |L_M|$ because at these small heights the velocity profile is logarithmic and we have a forced convection region. On the other hand, if $z \gg |L_M|$ stratificational effects dominate, the shear production terms are negligible and we find a zone of free convection with thermal plumes.

8.4 Ocean mixing

The turbulence in the ocean obviously plays an enormously important role in the dynamics of the ocean circulation and structure. Because of the ocean's stable stratification, any (vertical) mixing must work against the buoyancy forces and, as a result, the horizontal mixing is several orders of magnitude larger than the vertical. However, the vertical, diapycnal, mixing is very important due to its obvious effect on the vertical structure of the ocean. Actually, mixing is horizontally two-dimensional only for horizontal scales larger than $NH/(2f) \sim$ 100 km in the open ocean.

The interior vertical mixing in the ocean is partly forced by breaking internal waves, mainly at topographic obstacles, and by shear instabilities at the boundaries (**???**). In order to sustain the open ocean mixing rates an energy supply of 2 TW is necessary (**?**). About half of this has been shown to come

Figure 8.1: Shown is $\log_{10}$ of the energy flux of the global internal tide. Note the patchiness and the focus of the flux at topography (figure from **?**).

from the internal tide, and the remaining terawatt is supplied by the wind, mesoscale eddies and high-frequency internal waves (e.g. **???**). This dissipation takes place at localized topography, leading to a very patch picture in open ocean dissipation (Fig. 8.1).

It has long been speculated that the meridional overturning circulation (MOC) cannot be sustained without open ocean mixing (e.h. **???**). There simply is no other mechanism for bringing deep water back towards the surface. However, by introducing enormous quantities of freshwater very rapidly in the Arctic, it has been shown that the MOC can temporarily shut down, or even switch to a different state (e.g. **?**), but more work has to be done before we can say for certain what effects changes in mixing have on the deep ocean and the MOC.