Physics 265 Lab 1 Report

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I. Introduction

As we prepare to launch the Apollo 11 capsule aboard the Saturn V rocket, it is vital that we understand how the spacecraft will be impacted by the variations in gravitational potential and forces within the Earth-Moon system. My team has modeled these potentials and forces to provide a framework for charting the spacecraft trajectory. Additionally, a preliminary assessment of the projected performance of the Saturn V rocket has been conducted, with promising results. This report presents these models' outcomes, significance, and limitations.

II. The Gravitational Potential of the Earth-Moon System

Gravitational potential energy is the energy stored in a system of masses, and gravitational potential maps out how one mass, or a system of massive bodies, influences the energy of surrounding objects. An object like the Saturn V rocket will feel gravitational force in the direction of lower gravitational potential, hence its relevance to planning the spacecraft's trajectory.

Mathematically, gravitational potential Φ can be represented as a function of distance r from a massive body:

$$\Phi(r) = \frac{-GM}{r} \quad (1)$$

where G is the gravitational constant, M is the mass of the body, and r is the distance between the point of interest and the location of the body's center of mass. Gravitational potential is always negative, and objects experience gravitational force in the direction of decreasing gravitational potential, i.e. in the direction of increasing magnitude.

When launching a spacecraft in the Earth-Moon system, we must consider the combined effect of the Earth and Moon's gravitational potential. The gravitational potential of the Earth and the Moon were individually calculated as functions of two dimensional position relative to the center of the Earth. The absolute value of the resulting sum of gravitational potentials, denoting the total gravitational potential of the system, was plotted as a function of two dimensional position in Figure 1 below.

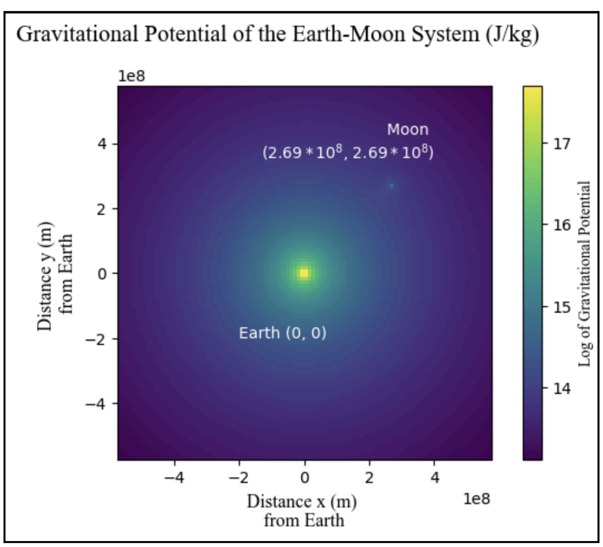

Figure 1: Magnitude of gravitational potential (J/kg) of the Earth-Moon system as a function of two dimensional position relative to Earth's center.

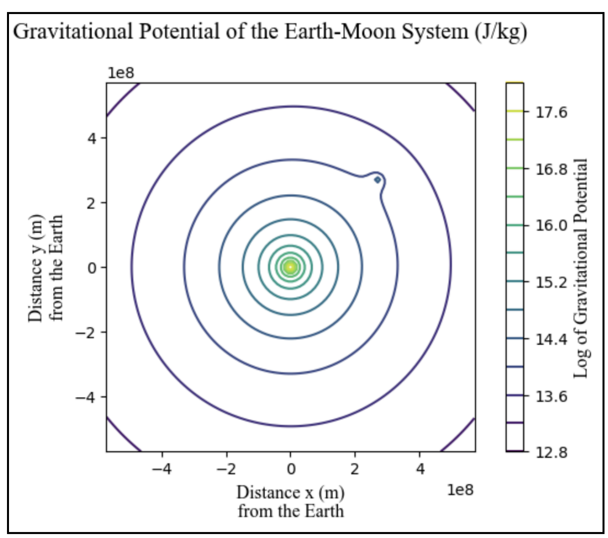

Figure 2: Contour plot of magnitude of gravitational potential (J/kg) of the Earth-Moon system as a function of two dimensional position relative to Earth's center. Earth is at (0,0) and the Moon is at (2.69e8, 2.69e8).

As shown in Figure 1, there exist gravitational potential wells around both the Earth and the Moon, with Earth's potential well having a magnitude almost 100 times greater than that of the Moon. This is because the Earth is 80.82 times as massive as the Moon, causing it to have a significantly greater gravitational potential. The spacecraft will experience a significantly greater gravitational pull towards the Earth than the Moon in most regions of the given system, only experiencing a net force toward the Moon when in close proximity.

The gravitational potential of the system was also plotted in rings of equal gravitational potential corresponding to different orbital radii, shown in Figure 2 above. Constant gravitational potential means that no work is required to move the spacecraft across that trajectory, therefore requiring no net force to be exerted on the spacecraft.

III. The Gravitational Force of the Earth Moon System

As discussed in section II, variations in gravitational potential within a system result in gravitational forces exerted on objects interacting with that system. When launched into the Earth-Moon system, the trajectory of the Apollo 11 command module will be influenced by gravitational forces from the Earth and Moon.

Gravitational force is the force between two massive bodies (M_1 and m_2) as a result of their gravitational attraction, represented by equation 2:

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|}\hat{r}_{21} \quad (2)$$

where r_{21} is representative of the displacement between the two bodies.

The gravitational force experienced by the Apollo 11 command module (mass $m_1 = 5500\ kg$) in the Earth-Moon system was plotted as a function of location relative to the Earth's surface in Figure 3 below.

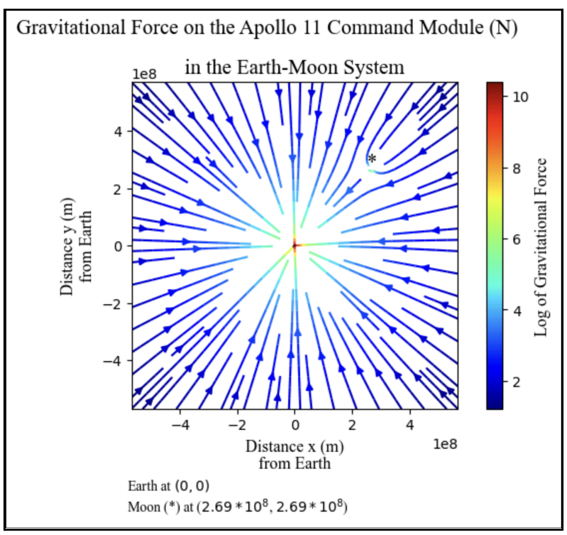

Figure 3: Gravitational force (N) exerted on the Apollo 11 command module as a function of two dimensional position relative to Earth's center.

The command module will experience a net force toward the Earth at most points in the defined system, with gravitational contributions from the Moon appearing to be largely negligible. This imbalance is due, again, to the Earth being over 80 times as massive as the Moon, making its maximum gravitational force exerted on the command module almost 100 times that of the Moon. However, when closer to the Moon, there is a distinct curvature in the force field that will accelerate the command module toward the Moon, disrupting the otherwise Earth-centered force field. When the command module is close to the Moon, it will be essential to take this force into account to keep the craft on its intended trajectory.

IV. Projected Performances of the Saturn V Stage 1

Based on the prototype of the Saturn V rocket, calculations can now be made about its projected performance. The Tsiolkovsky rocket equation,

$$\Delta v(t) = v_e ln(\frac{m_o}{m(t)}) - gt \quad (3),$$

was used to calculate Saturn V's projected velocity changes as it burns fuel in the first stage of its journey. In our calculations, $v_e = 2.4 * 10^3\ m/s$ is the fuel exhaust velocity, $g = 9.8\ m/s^2$ is the gravitational acceleration, $m_o = 2.8 * 10^6\ kg$ is the initial combined mass of the fuel, rocket, and payload, and $m(t)$ is the mass at time t. Mass was defined as a function of time by the following equation: $m(t) = m_o - m't$, where $m' = 1.3 * 10^4\ kg/s$ is the fuel burn rate. The total burn time (T) of the first stage was calculated based on the rate of fuel consumption and the final dry mass, $m_f = 7.5 * 10^5\ kg$ represented in equation 4:

$$T = \frac{m_o - m_f}{m'} \quad (4).$$

This calculation yielded a projected burn time of 157.692 seconds. The altitude h at the time of burnout was then calculated using equation 6,

$$h = \int_0^T \Delta v(t)dt \quad (6),$$

projecting a burnout altitude of 74.094 km.

V. Discussion and Future Work

As these are preliminary models, notable approximations were made in the above calculations. In modeling the gravitational potential and gravitational force fields of the Earth-Moon system, the Earth and Moon were both approximated as point masses at their respective coordinates. Though this approximation effectively models gravitational potential at large distances, an accurate depiction of gravitational potential and force close to either body's surface requires more intricate calculations taking into account the inhomogeneity of their mass distribution. Future work will take these complexities into account to create a more accurate model of gravitational potential and forces closer to these bodies.

Additionally, in calculating the burn time of the Saturn V rocket, the fuel burn rate m' was approximated to be constant. This approximation, projecting a 157.64 second burn time, is consistent with the recent test results of the Saturn V prototype, which recorded a burn time of about 160 seconds. Additional approximations were made in calculating the rocket's altitude at burnout. In addition to approximating a constant burn rate, gravitational acceleration was taken to be $g = 9.8\ m/s^2$, ignoring variations in gravitational force with increased altitude. This calculation also neglected the effects of drag. In the recent prototype test, the burnout altitude was calculated to be about 70 km, which is about 4 km less than projected. This discrepancy is likely due to the neglection of air resistance, indicating that future models should account for drag.

Moving forward, our team will create more robust mathematical models of the forces experienced by the Saturn V rocket and Apollo 11 command module, with the end goal being a highly accurate representation of spacecraft trajectories throughout this mission.