8.2 Halton numbers

An elementary low discrepancy sequence

d = dimension, n = n'th draw,

$\gamma_i : \mathbb{Z}_+ \ni n \mapsto \gamma_i(n) \in \mathbb{Z}_+$, $\mathbb{Z}_+ = \{1, 2, 3, \dots\}$ outcome of n'th draw along coordinate i, $i = 1, \dots, d$.

For each i, $i = 1, \dots, d$, choose a prime number p_i.

Express $\gamma_i(n)$ in the p_i base,

$$\gamma_i(n) = \sum_k a_{ki} p_i^{k-1}.$$

$$\gamma_i(n) = a_{1i} a_{2i} \dots a_{mi}$$

Let

$$u_i(n) = \sum_k a_{ki} p_i^{-k}$$

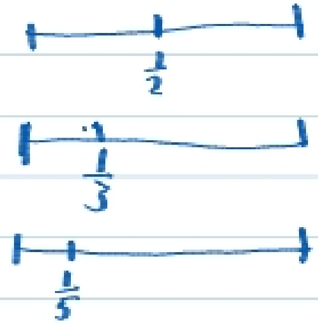
$$u_i(n) = 0.a_{mi} \dots a_{2i} a_{1i}$$

Ex $d = 3$, $\gamma_1(n) = \gamma_2(n) = \gamma_3(n) = n$, $p_1 = 2$, $p_2 = 3$, $p_3 = 5$

$n = 1$: $p_1 = 2$, $\gamma_1(n) = n = 1 = 1 \cdot 2^0 = 1_2$, $u_{n1} = 0.1_2 = 1 \cdot 2^{-1} = \frac{1}{2}$

$p_2 = 3$, $\gamma_2(n) = 1 = 1 \cdot 3^0 = 1_3$, $u_{n2} = 0.1_3 = 1 \cdot 3^{-1} = \frac{1}{3}$

$p_3 = 5$ $\gamma_3(n) = 1 = 1 \cdot 5^0 = 1_5$, $u_{n3} = 0.1_5 = 1 \cdot 5^{-1} = \frac{1}{5}$



$$\gamma_1 = \begin{pmatrix} 1/2 \\ 1/3 \\ 1/5 \end{pmatrix}$$

$n=2, \quad p_1=2, \quad \gamma_1(2)=2=1\cdot 2^1+0\cdot 2^0=10_2, \quad u_{21}=0.01_2=0\cdot 2^{-1}+1\cdot 2^{-2}=\frac{1}{4}$

$p_2=3, \quad \gamma_2(2)=2=2\cdot 3^0=2_3, \quad u_{22}=0.2_3=2\cdot 3^{-1}=2/3$

$p_3=5, \quad \gamma_3(2)=2=2\cdot 5^0=2_5, \quad u_{23}=0.2_5=2\cdot 5^{-1}=2/5$

$$r_2=\begin{pmatrix}1/4\\2/3\\2/5\end{pmatrix}$$

$n=3, \quad p_1=2, \quad \gamma_1(3)=3=1\cdot 2^1+1\cdot 2^0=11_2, \quad u_{31}=0.11_2=1\cdot 2^{-1}+1\cdot 2^{-2}=3/4$

$p_2=3, \quad \gamma_2(3)=3=1\cdot 3^1+0\cdot 3^0=10_3, \quad u_{32}=0.01_3=0\cdot 3^{-1}+1\cdot 3^{-2}=1/9$

$p_3=5, \quad \gamma_3(3)=3=3\cdot 5^0=3_5, \quad u_{33}=0.3_5=3\cdot 5^{-1}=3/5$

$$r_3=\begin{pmatrix}3/4\\1/9\\3/5\end{pmatrix}$$

r_1, r_2, r_3:

$i=1$:

$i=2$:

$i=3$: