From theory to observations

Given the stellar mass and chemical composition of a ZAMS, the stellar modeling can, in principle, give the prediction of the stellar radius, bolometric luminosity, effective temperature T_{eff} , and possibly even the surface chemical composition (if it is significantly modified due to the dredge-up) at any age. How to link these parameters to observables?

1 Stellar chemical composition

Not all element abundances can be directly measured, even spectroscopically, for a star. Low mass stars, for example, have typically too cold atmosphere to show any helium spectral lines. The stellar metal abundances, Z, are typically traced with certain spectroscopic indicators and are usually determined differentially wrt the Sun. An element number abundance is defined as

$$[Fe/H] \equiv \log[N(Fe)/N(H)]_* - \log[N(Fe)/N(H)]_\odot \tag{1}$$

Here Fe is chosen partly because its lines are prominent and easy to measure. If the element distribution is assumed to follow the solar mixture, the conversion from Z to [Fe/H] is then given by

$$[Fe/H] = \log\left(\frac{Z}{X}\right)_* - \log\left(\frac{Z}{X}\right)_\odot = \log\left(\frac{Z}{X}\right)_* - 1.61 \tag{2}$$

where $\frac{Z}{X}$ is the metal to hydrogen mass ratio.

X can be considered approximately a constant, because the variations in the helium and heavy element abundances are typically negligible wrt to the hydrogen abundance, which largely dominates the stellar atmospheres apart from specific cases (like non-DA WDs). The above equation can then be simplified into

$$[Fe/H] = \log\left(\frac{Z_*}{Z_\odot}\right) \tag{3}$$

Now the right side is what one needs as the input for a given stellar model. Typical errors of the spectroscopic determinations of [Fe/H] are of the order of at least 0.10 dex.

If the assumption of a universal scaled solar metal mixture needs to be relaxed, the above equations can still be used, but the left side now refers to the ratio of the "total" number abundance of metal to hydrogen, [M/H].

One can often approximately group the metals into two categories: The α-elements and Fe elements. α-elements (mainly O, Ne, Mg, Si, S, Ca, and Ti) are mostly the products of core-collapsed SNe (including Type II and Ib,c). The Fe elements are primarily from Type Ia SNe. The Type Ia SNe started to explore and contribute to the chemical composition of the ISM much later ($\sim$ 1 Gyr) than core-collapsed SNe do. Indeed, old metal-poor stars are typically α-enhanced in the halo and bulge of our Galaxy ([Fe/H]< -0.6; [α/Fe] $\sim$ 0.3 - 0.4) and in the Magellanic Clouds ([Fe/H] < -1.0; [α/Fe] $\sim$ 0.2), i.e., α elements are enhanced by a factor $\sim$ 2 - 3 wrt Fe, compared with the scaled solar metal mixture. For these α-enhanced mixtures, the approximate relationship can be generally given by

$$[M/H] \sim [Fe/H] + \log(0.694 \times 10^{[\alpha/Fe]} + 0.306). \tag{4}$$

On the other hand, younger stellar generations (like our Sun) are characterized by a metal mixture with a small α/Fe ratio wrt the oldest stars.

2 Spectra and magnitudes

The observational counterpart of the HRD is the Color Magnitude Diagram (CMD), i.e., the plot of a star magnitude in a given photometric band vs. a color index. Here we discuss how they are related to the bolometric luminosity and effective temperatures predicted by the stellar models.

The apparent magnitude is defined as

$$m_A = -2.5\log\left(\frac{\int f_\lambda S_\lambda d\lambda}{\int f^0_\lambda S_\lambda d\lambda}\right) + m^0_A \tag{5}$$

where f^0_λ is the monochromatic flux (or spectrum) received at the top of the Earth's atmosphere while f^0_λ denotes the spectrum of a reference star that produces a known apparent magnitude m^0_A, which is called the 'zero point' of the band for a given photometric system. This system specifies a specific choice of the response function, S_λ.

Many photometric systems exist (e.g., Fig. 8.1 of MS). It is thus important to know that the measured magnitudes do depend on both the f_λ and the actual photometric system used (i.e., the shape of the filter); e.g., the 2MASS JHK are different from the HST/NICMOS JHK. The very popular Johnson system, for example, uses the star Vega to fix the zero points. It assumes that $m_V = 0$ and that the color indices are equal to zero for the star, whereas other systems may have slightly different values.

The absolute magnitude is defined as the apparent magnitude a star would have at a distance of 10 pc (if the radiation travels undisturbed from the source to the observer):

$$M_A = m_A - 5(\log(d) - 1) = -2.5\log\left[\left(\frac{d}{10\ \mathrm{pc}}\right)^2 \frac{\int f_\lambda S_\lambda d\lambda}{\int f^0_\lambda S_\lambda d\lambda}\right] + m^0_A \qquad (6)$$

where d is the distance in parsec.In the cosmological context, the distance is so called luminosity distance d_L. The difference $(m_A - M_A) = (m - M)_A$ is called the *distance modulus*.

Assuming $S_\lambda = 1$ at all wavelengths and adopting the Sun as reference star, we introduce the **absolute bolometric magnitude** of a star as

$$M_{bol} = M_{bol,\odot} - 2.5\log(L/L_\odot) \qquad (7)$$

where $L_\odot = 3.826 \times 10^{33}$ ergs s^{-1} and $M_{bol,\odot} = 4.75$. The bolometric correction to a given photometric band A is defined as

$$BC_A \equiv M_{bol} - M_A. \qquad (8)$$

If the stellar bolometric luminosity is known (from a model), one can predict the corresponding M_A. In practice, tables of bolometric corrections and color indices are available, for a grid of gravities and T_{eff} that cover all the major phases of stellar evolution, and for a number of chemical compositions. Interpolations among the grid points provide the sought BC_A for the model.

It is important to select the proper filters so that they are sensitive to the desirable measurements, particular T_{eff}.

2.1 Theoretical vs. empirical spectra

Two main shortcomings of current theoretical model atmosphere:

- Many spectral lines predicted by the models are not observed in the Sun. Or the relative strengths of many lines are not well reproduced.
- Existing convective model atmosphere is still treated with the MLT. More sophistical models are needed that cover all the relevant evolutionary, mass, and chemical ranges.

The resultant uncertainties are about several % in magnitude. An alternative solution is to use empirical spectra of a sample of stars with independently determined T_{eff}, gravities and chemical compositions. If we know the angular diameter of a nearby star, θ (via interferometric observations), we can infer the T_{eff} from the bolometric flux, F_{bol} and the relation

$$T_{eff} = \left(\frac{F_{bol}}{\sigma}\right)^{1/4} \left(\frac{d}{R}\right)^{1/2} \tag{9}$$

where $d/R = 2\theta^{-1}$. This expensive approach can be applied only to some of nearby stars, which cover a narrow range of the composition, but is certainly useful to test the stellar models.

3 The effects of interstellar extinction

If the extinction is important, the observed flux is then

$$f_\lambda = f_{\lambda,0} e^{-\tau_\lambda} = f_{\lambda,0} 10^{-0.4A_\lambda} \tag{10}$$

If A_λ is constant in the effective wavelength range of a filter A, then

$$m_A = m_{A,0} + A_A \tag{11}$$

The extinction-corrected magnitude, $m_{A,0}$, should then be used to calculate the absolute magnitude. The effect of extinction on a color index $(A - B)$ is

$$(A - B) = (A - B)_0 + E(A - B) \tag{12}$$

where $E(A-B) = A_A - A_B$ is called the **color excess** or reddening. What is usually determined empirically is the so-called extinction law, A_λ/A_V, where

A_V in the extinction in the Johnson V band. The law applies to a broad range of wavelength, but can be rather different from one galaxy to another. The ratio $R_V = A_V/E(B-V)$ is nearly a constant and equal to 3.12 for stars in the Galaxy. More accurately, $E(B-V)$ can be determined in a color-color plot, e.g., $U-B$ vs. $B-V$.

4 Review

Key concepts: apparent and absolute magnitudes, photometric system, zero points, distance modulus, absolute bolometric magnitude, bolometric correction, color excess, extinction law

Why are old metal-poor stars in the halo and bulge of our Galaxy typically α-enhanced?

What is the reference star used for defining an absolute bolometric magnitude? Is it the same as for a magnitude in the Johnson photometric system?

How can the apparent K-band magnitude of a star in the Galaxy be corrected if we know the color excess $E(B-V)$ along the line of sight?

How may the effective surface temperature of a star, T_{eff}, be determined from the absolute bolometric magnitude of a nearby star and its angular diameter?

What may be the procedure to infer the apparent magnitude of a star of a certain type in the Galactic center from a stellar model?