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# ON THE ORIGIN OF THE O- AND B-TYPE STARS WITH HIGH VELOCITIES (THE "RUN-AWAY" STARS), AND SOME RELATED PROBLEMS

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A survey of presently available data on the "run-away" O- and B-type stars is given, and the following theory is considered for the explanation of these objects. It is assumed that these stars, in their earliest stage of formation, were the secondary components of proto-double stars, the more massive primaries of which have shed most of their mass in a violent process; consequently, the secondary component was released as a result of the rapidly diminishing gravitational attraction. The process of the rapid mass loss of the primary may be identified with the vibrational instability occurring in the contraction stage of stars with large masses (> 100 ⊙ mass) or, possibly, with the supernovae of type II. The proposed hypothesis would seem to explain the two principal properties which distinguish the run-away stars from the normal ones: their lack of duplicity and the preponderance of large masses. These two properties are discussed in some detail; the discussion includes an estimation of the fraction of double and multiple stars among the normal objects, which is found to be about 75 per cent.

Numerical computations of the dynamics of a double star with rapid, but not infinitely rapid, mass loss of the primary, made by BOERSMA, are used as a basis for an estimation of the initial properties of the proto-double stars. The original masses of the primaries are found to be of the order of several hundreds of solar masses and the original separations of the order of several to 100 astronomical units. It is pointed out that a tendency towards formation of proto-double stars with these properties is to be expected, in view of the high incidence of duplicity among the most massive main-sequence stars known. The anomalous mass distribution of the run-away stars is related to the differences in contraction times for stars of different masses.

The required time scale for the rate of mass loss of the primary points to the identification with the supernovae of type II. Some aspects of this possible identification are discussed. The admittedly poorly-known frequency of the supernovae of type II appears to agree well with the frequency deduced from the numbers of run-away objects. The identification leads to the consideration of some special aspects of the hypothesis, such as the relation between supernovae and OB associations. We further discuss the problem of the absolute magnitudes of supernovae of type II, the question of the remnants of the proto-primaries, and a possible spread in the contraction times of the early B stars.

## 1. *Introduction and summary*

The early-type stars with high velocities, including the so-called "run-away stars", have been briefly discussed in previous papers (BLAAUW 1956a, 1958b, 1959) dealing with their individual properties and with the characteristics of the group as a whole. Their spectral types range from O5 to B5. Contrary to the generally very low space velocities for stars of these types, the velocities of the stars now under consideration amount to values up to 200 km/sec. The name "run-away" stars was assigned to those members of the group for which the direction of the space velocity indicated that the star must have originated in a known OB association. Since it is very likely that nearly all O- and B-type stars originated in such associations (ROBERTS 1957, BLAAUW 1958b), the "run-away" stars are not to be considered as forming a distinct subdivision of the high-velocity O- and B-type stars; they merely represent those objects for which the direction of the space motion is sufficiently accurately known and for which the distance the star has travelled since it left the association is still small enough to allow the identification of its origin.

Representation of the velocity distributions of the O- and B-type stars with velocities below 30 km/sec either by means of a gaussian curve or by means of an exponential law of the type exp $(-|v|/\eta)$, ($v$ being the velocity, $\eta$ a constant) fails to fit the numbers of

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higher velocities. The high-velocity objects form a distinct excess, and the investigations quoted have shown them to possess two characteristics which distinguish them from the low-velocity objects. These characteristics are:
a) the absence of double or multiple stars—a property markedly contrasting the high frequency of duplicity or multiplicity among the low-velocity objects; and b) their different spectral distribution, which contains a ratio between the number of the O stars and that of the B stars about ten times higher than is found among the "normal" O5-B5 stars. Hence, the high-velocity group must contain a much higher proportion of large masses.

Attempts have been made to explain the expanding motions of stars in OB associations by the process of star formation in the expanding neutral hydrogen gas surrounding HII regions. This theory seems to account satisfactorily for moderate expansion velocities. The higher than average velocities may be explained by the speeding up of the motions of clouds of neutral gas by the "rocket-effect" (OORT and SPITZER 1955; OORT 1954, 1955). However, this process fails to explain the very high velocities of the stars studied in the present paper. That this is so can be readily demonstrated by means of the formula (OORT and SPITZER 1955):

$$\frac{M_0}{M} = e^{(v-v_0)/V},$$

which gives the ratio between the initial mass $M_0$ and the reduced mass $M$ of a cloud by the time the cloud's velocity has increased from $v_0$ to $v$; $V$ being the velocity with which the ionized particles escape from the cloud. It seems reasonable to suppose that for stars of about 20 solar masses to be formed within the accelerated cloud, this cloud should have had at the instant of the star's formation a total mass $M$ of at least a few hundred solar masses, say 200 $\odot$. Taking $V = 20$ km/sec and assuming that the cloud at the beginning of the rocket acceleration may already have acquired by the general expansion a velocity $v_0 = 10$ km/sec, we find that final velocities $v = 100$ km/sec, $v = 150$ km/sec and $v = 200$ km/sec require initial masses $M_0$ of about $5 \times 10^4$, $2 \times 10^5$ and $3 \times 10^6$ solar masses respectively. For more refined calculations we refer to the cited papers by OORT and SPITZER; these give, however, nearly the same results. Such large masses are quite unlikely to occur for individual clouds. Moreover, as OORT and SPITZER have pointed out, the acceleration of these massive clouds would require improbably large collections of O-type stars. AMBARTSUMIAN (1955, 1958) has made the interesting suggestion that the stellar associations, including the run-away stars, were formed, simultaneously with the associated interstellar gas, out of proto-stars in a state of very high density, perhaps of the order of nuclear density. This hypothesis has attractive features, but it also meets with very serious objections. One of the most important of these latter seems to us the fact that it is hard to see how this pre-stellar matter, if it is concentrated in small superdense bodies, could ever have obtained the extremely flat distribution in the galactic plane observed for the associations. The alternative assumption of the formation of the young stars out of the interstellar matter leads, at least qualitatively, to a natural explanation of this flat distribution since this is present in the gas, whereas the flatness of the gaseous layer may be explained by the gradual collapse of the originally more spherically shaped primordial cloud from which the Galaxy was formed. However, bodies of stellar dimension and of stellar or higher densities should not have reached this flat distribution. The observed distribution of the globular clusters confirms this expectation, and a similar spherical distribution should then also be observed for the superdense proto-stars and for the objects recently formed out of them. For this reason, among others, we prefer to adhere to the concept of stars being formed out of the interstellar medium.

In the present paper we wish to propose the following hypothesis for the origin for the high-velocity O and B stars. We suggest that these stars in their earliest stage of development—and as a rule when they were still in the later phases of the contraction—were the secondary components of proto-double stars, the more massive primaries of which have subsequently shed most of their mass, thus releasing the secondary with a velocity equal to a large fraction of its original orbital velocity. This ejection of the mass of the proto-primary must have been rather sudden, as will be shown below, and may have to be identified with the supernovae phenomenon of type II. After the present investigation was concluded, it was kindly brought to our attention by Dr M. SCHMIDT, that several years ago ZWICKY (1957) had already remarked that the supernova phenomenon in binary stars may have been the cause of observed high velocities for released companions.

We have especially in mind proto-primaries with very large masses, ranging from the largest masses actually observed to occur in spectroscopic binaries, say 90 solar masses, up to several hundred solar masses. While it is true that we do not know stars of these masses to occur as luminous and stable bodies unless the super-supergiants (FEAST and THACKERAY 1956, THACKERAY 1958) are identified as such, it does not seem implausible that *proto*-stars with masses of this order do occur in the interstellar medium, and that they may occasionally reach advanced stages of the contraction before instability prevents its completion.

(This suggestion has also been made by GOLD, 1958.) HOYLE (1959) has shown that as far as the ratio between radiation pressure and gravity is concerned, stars with a hydrogen concentration of $X = 0.75$, and masses up to 1000 solar masses may be stable; the instability with which we are dealing here is, however, one of dynamic origin. SCHWARZSCHILD and HÄRM (1959) find that vibrational instability must be expected to occur for stars heavier than 60 solar masses, although in the range between this and the observed upper limit of about 90 solar masses no immediate disruption may take place. For heavier objects, however, sudden disruption due to vibrational instability is indicated. Perhaps this kind of instability is to be identified with the sudden mass loss required to explain the release of the secondary component. Some aspects of the identification with the type II supernovae, an identification which allows some specific tests, are considered in section 7.

The percentage of double and multiple systems among the O- and B-type stars is very high (see also section 4) and it is the rule rather than the exception that a massive star is formed as a binary. It is, therefore, natural to assume that the process of formation which is only partly completed by the very massive proto-stars, will as a rule also tend to produce a binary or a system of still higher order. Our working hypothesis will be that the disruption of the very massive primary may sometimes take place when the system can be described as already consisting of two separate units, moving under their mutual gravitation. The contraction of the two components may yet be far from completed. All we suppose is that their sizes are of an order smaller than the separation between the two components.

For the most probable values of the distance between the two components of the proto-binary, we provisionally adopt the range of the most frequently occurring separations found for double stars in general, i.e. 10 to 100 a.u. (KUIPER 1935). This implies that the size of the massive component of the proto-binary may be as large as one a.u., i.e. about 10 times the main-sequence radius for stellar models of such large masses. (For models of masses of 121 and 218 solar masses, SCHWARZSCHILD and HÄRM (1958) find main-sequence radii of 25 and 47 solar radii respectively.) It may be stressed that we are not thinking in terms of those binary systems which, after the contraction has been completed, belong to the category of the "close binaries".

Denoting the most massive component in a proto-double star and its mass by $M_1$, the proto-secondary (later to be identified as the high-velocity star) by $M_2$, and the radius of the orbit of $M_2$ around the common centre of gravity by $a_2$, the velocity of the secondary component around this centre of gravity, $S_2$, is $30\,[M_1/(M_1+M_2)]\ (M_1/a_2)^{\frac{1}{2}}$ km/sec. For masses $M_1$ of about 200 solar masses, with $M_2 = 20\,\odot$ (so that $M_1/(M_1+M_2) = a_2/(a_1+a_2) \sim 1$), and $a_2 =$ 20 a.u., $S_2$ is about 100 km/sec. Generally, for masses $M_1$ and separations between the components in the ranges of values already mentioned, we find orbital velocities of $M_2$ quite comparable in amount to the speeds of the high-velocity O and B stars.

The amount of reduction of this velocity in the process of the release of $M_2$ by the disruption of $M_1$ depends on the amount of mass left of $M_1$ and on the interval of time during which its mass ejection takes place, or, more properly, on the ratio between this time and the orbital period in the original system. This ratio should be one half or less, according to section 6. The orbital periods of proto-binaries like the one just mentioned are of the order of a few years. The mass loss should therefore almost entirely take place within a year or so—a condition which suggests that we are dealing with the supernova phenomenon. Because of their association with Type I population, only supernovae of type II are considered. We shall show (section 7b) that the frequency of type II supernovae in the galaxy amply suffices for part of them to account for the origin of the high-velocity OB stars.

In section 2 of the present paper we present particulars about presently known high-velocity OB stars. Section 3 deals with their frequency among the various spectral subclasses and with the resulting statistics of the masses of the high-velocity stars. It appears that the incidence of high velocities among the O-type stars (masses $\geq 24\,\odot$) is about ten times higher than among the B types.

In section 4 we discuss provisionally the duplicity and multiplicity among the O to B5 stars. It is found that among the B0 to B5 stars, the occurrence of single stars must be very rare; about 75 per cent appear to be double or multiple with companions brighter than absolute magnitude + 5, and an even larger percentage would be found if fainter companions could also be taken into account. For the O-type stars, the occurrence of single objects must be even more exceptional than among the B types. The marked contrast in this respect of the high-velocity group of objects is shown in section 5; all or nearly all of these are single. This observation forms the main inducement for postulating the proto-binary hypothesis.

The proto-binary hypothesis is discussed in detail in section 6. The kinematic behaviour of a binary with a primary ejecting its mass is considered, first for the simple case of instantaneous mass loss, and next for a finite rate of loss. Results of accurate calculations for the latter case, communicated in a paper by BOERSMA following the present article, are used as a basis for the deduction of the initial properties of the binary systems from which the observed high-velocity objects may have originated (section 6d). The proto-binary hypothesis appears to account satisfactorily for the observed properties of the high-velocity objects provided the rate of mass loss of the primary may be assumed to be as rapid as that observed in the type II supernovae, and provided it may be assumed that the contraction rates of stars of masses 10 ⊙ and lower (types B2 and later) are so slow as to reduce the number of such stars which will survive the supernova explosion of the primary. This latter aspect is discussed in section 6e; current estimates of the contraction rates support the relevant assumption. (See, however, section 8c for possibly longer contraction times for certain B-type stars.)

The masses required for the proto-primaries are found to lie in the range of 100 to 1000 solar masses as was to be expected from the foregoing considerations. A representative description of the proto-binary system at the time of its disruption would be: mass of primary: 250 ⊙, mass of secondary to be released: 10 to 50 ⊙, separation of the centres of the two components: 20 astronomical units, radii of the two components: 1 to 5 astronomical units. Orbital period: 5.4 years. Time during which the mass of $M_1$ is ejected: 3 months, remnant mass of $M_1$: 25 ⊙ or less. Velocity with which $M_2$ will be encountered as a high-velocity object: 90 km/sec.

The proposed theory implies that supernovae of type II occur in OB associations at least as frequently as the high-velocity stars are produced, and probably even more frequently. Thus, the Orion, I Sco and probably also the $h$ and $\chi$ Persei associations must have been the scene of repeated explosions. The consequences which this must have had for the distribution of the interstellar matter in the associations are briefly dealt with in section 7c. Section 8 deals briefly with a few miscellaneous items. The maximum luminosity of type II supernovae should be considerably higher than currently adopted values if these supernovae occur in the regions of heavy interstellar obscuration where massive proto-stars are expected. Section 8b suggests that one remnant of the type II supernovae might perhaps be identified with the irregularly varying subluminous O-type star X Persei in the association II Per.

## 2. *Data for individual objects*

Data for nineteen high-velocity objects are assembled in Table 1. Their origin, if known, is given in the fifth column, and in such cases the velocity in the fourth column is with respect to this origin. In the remaining cases the velocity given is the space velocity corrected for differential galactic rotation and standard solar motion. For most of the objects for which the origin is known, the visual absolute magnitude (in the 7th column) could be computed using the distance of this origin from the sun, and the star's proper motion and

TABLE 1

List of high-velocity O-B5 stars

| name | HD | MK spectral type | space velocity (km/sec) | associated with | kinematic age ($10^6$ yrs) | $M_v$ | $M_{bol}$ | mass ⊙ = 1 | $l^{I}$ | $b^{I}$ | distance ps | notes |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | ° | ° | | |
| λ Cep | 210839 | O6 f | 64 | I Cep | ? | −5.9 | −10.4 | 87 | 71.5 | +2.5 | 780 | 1 |
| ξ Per | 24912 | O7 | 50 | II Per | 1.6 | −5.0 | −9.3 | 50 | 128.3 | −12.0 | 430 | 2 |
| | 157857 | O7 f | ≥ 50 | | ? | (−5.5) | (−9.9) | (67) | 340.7 | +11.8 | 2400 | 3 |
| | 152408 | O7-8 fp | ≥ 109 | I Sco | ? | −7.2 | −11.5 | 160? | 311.8 | +0.3 | 2000 | 4 |
| 68 Cyg | 203064 | O8 | 49 | I Cep | 5.2 | −5.3 | −9.7 | 60 | 55.4 | −4.4 | 880 | 1 |
| α Cam | 30614 | O9.5 Ia | 59 | NGC 1502? | 2.0 | (−6.4) | (−10.5) | (90) | 111.4 | +14.9 | 1000 | 5 |
| AE Aur | 34078 | O9.5 V | 106 | I Ori | 2.7 | −3.8 | −7.6 | 24 | 139.8 | −0.9 | 440 | 6 |
| ζ Oph | 149757 | O9.5 | 39 | II Sco | 1.1 | −4.3 | −8.3 | 32 | 334.1 | +22.1 | 170 | 7 |
| μ Col | 38666 | B0 V | 123 | I Ori | 2.2 | −3.6 | −7.3 | 21 | 204.5 | −25.9 | 570 | 6 |
| | 151397 | B0.5 V | ≥ 180 | I Sco | ? | −3.2 | −6.8 | 17 | 312.1 | +2.0 | 2300 | 4 |
| | 149363 | B0.5 III | ≥ 115 | ? | ? | (−4.9) | (−8.6) | (38) | 337.7 | +25.2 | 2600 | 8 |
| 53 Ari | 19374 | B2 V | 59 | I Ori | 4.9 | −2.3 | −5.4 | 10 | 131.3 | −33.0 | 360 | 6 |
| | 97991 | B2 V | 156: | ? | ? | (−2.5) | (−5.7) | (12) | 231.7 | +52.4 | 760 | 9 |
| | 197419 | B2 Ve | 23: | I Lac | 10 : | −2.8 | −6.2 | 14 | 44.4 | −5.0 | 540 | 10 |
| 72 Col | 41534 | B3 V | 191 | I Sco | 14 | −1.5 | −4.1 | (8) | 205.7 | −22.2 | 260 | 4 |
| | 214930 | B3 V | 73 | ? | ? | (−1.5) | (−4.1) | (8) | 56.8 | −30.7 | 440 | 11 |
| | 216534 | B3 V | 82 | ? | ? | (−1.5) | (−4.1) | (8) | 72.1 | −8.8 | 830 | 12 |
| | 4142 | B5 V | 74 | ? | ? | (−0.7) | (−2.6) | (5) | 89.8 | −14.7 | 170 | 11 |
| | 201910 | B5 V | 58: | I Lac | 2.7: | −1.5 | −4.1 | 8 | 52.5 | −5.4 | 480 | 10 |

radial velocity; reference to the quantities used in these computations is made in the notes below pertaining to the individual objects. The reference numbers of these notes are given in the last column of Table 1. Assumed visual absolute magnitudes are in parentheses. Kinematic ages, given in the 6th column, are the times the stars must have travelled with the present relative speed to reach the present distances from their origin. These ages can be given only if the proper motion is known with sufficient accuracy.

The list should be considered as a sample and is by no means complete, not even for the brighter objects because the southern sky has not yet been searched as thoroughly as the northern sky. There are, obviously, strong selection effects connected with the differences in absolute magnitude. Nevertheless the relatively large number of O-type stars is striking, and this will be discussed further in section 3.

The bolometric absolute magnitudes in the 8th column have been obtained by adding to the visual absolute magnitudes the bolometric corrections tentatively adopted by LIMBER (1960) on the basis of a compilation of available data. The next column gives the masses, derived by means of the theoretical relations between bolometric absolute magnitude and mass as computed by SCHWARZSCHILD and HÄRM (1958) and by HENYEY, LELEVIER, and LEVEE (1959). The masses found, while quite uncertain—especially for the very luminous objects—are usually larger than 10 solar masses and may in some cases be as much as 100 solar masses or more.

Table 2 gives, for four intervals of values of these masses, the number of objects and the space velocities for the various objects occurring in each interval of mass, and the mean values. There is little evidence for a dependence of the space velocity on the mass within the range of velocities considered.

We have also investigated whether there is a preferential direction of the velocities, particularly whether the motions parallel to the galactic plane are different from those perpendicular to it. No such difference exists. We use only the objects which originated in the nearer associations because inclusion of the remaining ones would imply selection of objects moving parallel to the galactic plane (and towards the sun). Denoting by $U$, $V$, and $W$, the components of the velocities in the directions $l^{\rm I}=0°$, $b^{\rm I}=0°$; $l^{\rm I}=90°$, $b^{\rm I}=0°$; $b^{\rm I}=90°$, expressed as fractions of the total velocity, we find for the mean values from seven stars:

$$\overline{|U|}=0.40; \quad \overline{|V|}=0.62; \quad \overline{|W|}=0.44.$$

The stars used are: AE Aur, μ Col, 53 Ari, ζ Oph, 68 Cyg, ξ Per, and α Cam.

TABLE 2

Distribution of masses and space velocities for the objects of Table 1

| range of mass (in units of solar mass) | < 20 | 20 to 50 | 50 to 80 | ≥ 80 |
|---|---|---|---|---|
| number of stars | 9 | 4 | 3 | 3 |
| space velocities (km/sec) | 23:<br>58:<br>59<br>73<br>74<br>82<br>156:<br>≥ 180<br>195 | 39<br>106<br>≥ 115<br>123 | 49<br>50<br>≥ 50 | 64<br>59<br>≥ 109 |
| mean | ≥ 100 | ≥ 77 | | |

*Notes concerning the objects in Table 1*

1. λ *Cep and 68 Cyg*

λ *Cep* is located near the border of the association I Cep, and according to the direction of its proper motion its origin must be somewhere in this association. The available data do not allow a significant estimate of the kinematic age. The space velocity with respect to I Cep given in the table is based on a radial component of −54 km/sec and a tangential component of 35 km/sec. The distance modulus of the association is assumed to be 9.6 (distance 830 ps), and in estimating the distance of λ Cep we have assumed it to be about 50 ps in front of the association, such as would correspond to a kinematic age of $1 \times 10^6$ yrs. Its distance modulus accordingly is 9.5 and its visual absolute magnitude −5.9.

The earlier results for *68 Cyg*, which also is supposed to have originated in I Cep (BLAAUW 1956b), have been slightly revised, using for the association I Cep the distance modulus 9.6 instead of the previous value 9.3. The distance of 68 Cyg which best fits the observed radial velocity and proper motion is then found to be about 880 ps, and the luminosity, the kinematic age and the space velocity are found as given in the table.

2. ξ *Per*

The distance modulus of the association II Per has been assumed to be 7.7 (JOHNSON 1959), corresponding to a distance of 350 ps. The space velocity of ξ Per has a radial component of +48 km/sec and a tangential component of 14 km/sec. This motion and the position of ξ Per relative to II Per lead to an estimated kinematic age of $1.6 \times 10^6$ yrs, almost the same as the value of $1.5 \times 10^6$ yrs derived for the association as a whole (DELHAYE and BLAAUW 1953). With this age and the relative radial velocity just mentioned we find ξ Per to be about 80 ps more distant than the association, and hence its distance modulus 8.2. This was used for the computation of the visual absolute magnitude $M_v = -5.0$.

3. *HD 157857*

For this star no reliable proper motion is available, hence its origin cannot be established. Assuming an absolute magnitude of $-5.5$, the distance modulus is found to be 11.9 and the distance 2400 ps. The radial velocity due to differential galactic rotation and solar motion would be about + 2 km/sec. The observed radial velocity is + 59 km/sec (WILSON 1953) and we assume that at least 50 km/sec of this represents motion of the star with respect to its origin. This is also taken to be the lower limit of the space velocity with respect to the star's origin. The latitude, $b^1$, is $+11°.7$, and the distance from the galactic plane, 490 ps.

4. *HD 151397, HD 152408 and 72 Col*

*HD 151397* is situated about 2° north of the cluster NGC 6231 which forms part of the association I Sco. According to HOUCK (*thesis*, unpublished) the distance modulus of I Sco is 11.6 and the distance 2100 ps. The apparent visual magnitude of HD 151397 corrected for interstellar absorption as estimated from OOSTERHOFF's (1951) photometry is 8.6. Using JOHNSON's (1958) value of $-4.0$ for the visual absolute magnitude we find for HD 151397 the distance modulus 12.6. This is not so different from the value for I Sco as to throw serious doubt on I Sco as the origin of the star; the more so because there are no other associations in the area concerned which might be considered as a likely origin of the star. The proper-motion data do not allow the direction of the motion of HD 151397 to be determined with sufficient accuracy.

The radial velocity of HD 151397 is + 151 km/sec (FEAST, THACKERAY, and WESSELINK 1957) and that of NGC 6231 is $-29$ km/sec (STRUVE 1944). The difference, + 180 km/sec, is a lower limit of the space velocity of HD 151397 with respect to NGC 6231. Assuming the kinematic age of the star to be $1 \times 10^6$ yrs, we find that it must be located 180 ps beyond NGC 6231, so that its distance modulus is 11.8 and hence its absolute magnitude $-3.2$.

*HD 152408* is situated in the immediate surroundings of NGC 6231 and its radial velocity differs from it by $-109$ km/sec (STRUVE 1944). The radial velocity requires further confirmation, but pending this we tentatively assume HD 152408 to be a run-away star from NGC 6231. From WALRAVEN's (1960) photometry we find the visual magnitude corrected for interstellar absorption to be 4.3, and hence the absolute magnitude $M_v = -7.2$, using the distance modulus 11.5.

*72 Col.* Attention was drawn to the high space velocity of this star by BIDELMAN (1948). Its orbit and origin are investigated by T.S. VAN ALBADA in another article in the present *Bulletin.* It appears that this star must almost certainly have originated in the I Sco association 14 million years ago, with a space velocity of 147 km/sec with respect to the association. The velocity quoted in Table 1 is with respect to the standard of rest at the star's present location.

5. *α Cam*

Assuming an absolute visual magnitude $-6.4$ for the spectral type O9.5 Ia (JOHNSON 1958), we find for α Cam the distance modulus 10.0, and a distance of 1000 ps. In view of the short lifetime of a star of such high luminosity we should expect it to be still rather close to its origin, and the origin should be the more easily identified because of the location of α Cam at the relatively high galactic latitude of $+15°$, in a region of the Milky Way ($l^{\mathrm{I}} = 111°$) which is only scarcely populated by early-type stars. Yet there are no associations in this area. The proper motion in latitude is $+0''.005$, and the same value is found after correction for differential galactic rotation and solar motion. During a lifetime of one million years the displacement on the sky would be only 1°.4.

We suggest that α Cam was formed in the cluster NGC 1502. Data about the position and motion of both objects are given in Table 3. We have confined ourselves to the proper motions derived from meridian observations. The distance modulus and other quantities for NGC 1502 have been derived from its four brightest members, HD 25443 (B0.5 III), 25090

TABLE 3

NGC 1502 and α Camelopardalis

| | $m_0 - M$ | $\mu_l \cos b$ | $\mu_b$ | p.e. | radial velocity (km/sec) | $l^{\mathrm{I}}$ | $b^{\mathrm{I}}$ |
|---|---|---|---|---|---|---|---|
| NGC 1502 (4 brightest stars) | 10.3 | $-0''.007$ | $-0''.007$ | $\pm 0''.003$ | $-8$ | 111°.1 | $+8°.4$ |
| α Cam | 10.0 | $-0''.005$ | $+0''.005$ | $\pm 0''.001$ | $+6$ a | 111.4 | $+14.9$ |

(B1 II), 25638 (B0 II-III) and 25639 (B0). Their mean distance modulus, 10.3, agrees well with that of α Cam. The components of proper motion of NGC 1502 and α Cam in galactic longitude, $\mu_l \cos b$, are practically the same, but those in latitude differ by $0''.012 \pm 0''.003$. The difference is in the sense of increasing the angular distance of α Cam from NGC 1502. Notice that the galactic longitudes of the two are the same but that the latitudes differ by 6°.5. This situation suggests that the origin of α Cam was in NGC 1502. The kinematic age of α Cam as derived from the angular distance and the relative proper motion is then $2.0 \times 10^6$ yrs with an uncertainty of about 30 percent due to the still poorly-determined relative proper motion. The space velocity of α Cam with respect to NGC 1502 is 59 km/sec ± 14 (p.e.), of which 57 km/sec is in the tangential direction and 14 km/sec in the line of sight.

We do not want to suggest that the kinematic age for α Cam also represents the age of NGC 1502. The present case, as well as that of ζ Oph described below, may belong to the category of early-type stars in, or associated with, clusters, which occupy a position on the HR diagram corresponding to a younger age than the run of the main sequence for the bulk of the stars indicates. Another typical case of this category may be the A0 star in NGC 752 (ROMAN 1955).

### 6. *AE Aur, μ Col, and 53 Ari*

The cases of *AE Aur* and *μ Col* have been discussed by BLAAUW and MORGAN (1953, 1954) and *53 Ari* was discussed more recently by BLAAUW (1956b). A diagram showing the positions and motions of these three stars with respect to the association I Ori was published by BLAAUW (1956c). In these investigations the association I Ori was assumed to be at a distance of 500 ps, and the distance modulus used was 8.5. A smaller distance modulus, 8.1, is given by JOHNSON'S (1959) rediscussion of the recent photometric data. Using this, we have made a new computation of the space velocities with respect to I Ori and of the visual absolute magnitudes of these three stars from the radial velocities and the proper motions The results are listed in Table 1. In this revised computation, AE Aur and μ Col have been treated independently, i.e. the solution is not based on the hypothesis of equal age and space velocity as in the 1954 paper by BLAAUW and MORGAN. This hypothesis does not fit the observational data as well with the revised distance of I Ori as with the previously used value. The almost equal and opposite velocities of these two very similar stars remain none the less a remarkable phenomenon, to which further reference is made in section 7c.

### 7. *ζ Oph*

ζ Oph moves away from the northern part of the Scorpio-Centaurus association, the part also referred to as the nebula region (BLAAUW 1959). The position of ζ Oph with respect to the Sco-Cen association may be seen from Figure 2 of the author's paper at the Cosmic Distance Scale Conference (BLAAUW 1958a), where ζ Oph is indicated by the cross at $l^{\mathrm{I}} = 334°.1$, $b^{\mathrm{I}} = +22°.1$. Reference to the relation between ζ Oph and the Sco-Cen association has been made earlier (BLAAUW 1952). The distance of the nebula region of the Sco-Cen association was assumed to be 180 ps in accordance with the investigation by BERTIAU (1958). The basic data for ζ Oph are $\mu_l \cos b = +0''.025$, $\mu_b = +0''.005$, Rad. Vel. $= -19 \pm 8$ km/sec. Taking into account the space motion of the Sco-Cen association also, we find the kinematic age, the relative space velocity, and the visual absolute magnitude as listed in Table 1.

The proper-motion and radial-velocity data on ζ Oph and on the Sco-Cen association leave little doubt about the genetic relation between the two. The spectral type of ζ Oph, O9.5, is, however, earlier than the earliest type found among the members of the association, which is B0V, and the kinematic age of ζ Oph, $1.1 \times 10^6$ yrs, is much shorter than the age of $12.5 \times 10^6$ yrs estimated from the position of the main sequence (HOYLE 1960) or the age estimated from dynamical considerations, which is at least $20 \times 10^6$ yrs (BERTIAU 1958) for the non-nebular region and somewhat shorter for the nebula region (BLAAUW 1959).

### 8. *HD 149363*

With an assumed visual absolute magnitude $M_v = -4.9$ (JOHNSON 1958) the distance modulus of this star is found to be 12.1, hence the distance 2600 ps. At its latitude $b^{\mathrm{I}} = +25°.2$, this corresponds to a distance of 1100 ps above the galactic plane. Unfortunately, no reliable proper motion is available, so that the direction of the star's motion on the sky cannot be established. The radial velocity is + 115 km/sec (WILSON 1953). Differential galactic rotation and solar motion would give rise to a radial velocity of about 0 km/sec, so that the space velocity should be at least 115 km/sec.

### 9. *HD 97991*

The apparent visual magnitude of this star, corrected for interstellar absorption, is 6.9; hence, with an assumed absolute magnitude of $-2.5$, the distance modulus is found to be 9.4 and the distance 760 ps. The proper motion, corrected for solar motion and for differential galactic rotation, is $\mu_l \cos b = +0''.015$, $\mu_b = +0''.040$ with a probable error of $\pm 0''.006$. This corresponds to a total tangential space-velocity component of 155 km/sec ± 22 (p.e.). The radial velocity, corrected similarly, is + 19 km/sec, hence the total space velocity, 156 km/sec.

The uncertainties in the distance of the star and in the proper motion make confirmation of this result desirable, but there can be little doubt that it has a high space velocity.

10. *HD 197419 and HD 201910*

These two stars appear to move away from the association I Lac. The data in Table 1 are quoted from a rediscussion of the association I Lac by BLAAUW, DELHAYE, and ROEMER (in press). There seems to be little doubt that these stars originated from I Lac, but the uncertainties in the amount of the proper-motion components away from the association cause large uncertainty in the values of the relative space velocities and, hence, in the kinematic ages.

11. *HD 4142 and HD 214930*

The data for these stars are taken from a study of the motions of the nearer northern O- and B-type stars (BLAAUW 1956b). Their origin has not been identified. Most likely, both were formed about 40 to 50 million years ago in the Perseus arm about 3000 ps from their present position.

12. *HD 216534*

The high space velocity of this star was confirmed by means of new observations of the radial velocity (BLAAUW, DELHAYE, and ROEMER, in press). The star occurs near the association I Lac, but does not belong to it. The radial velocity, corrected for solar motion and differential galactic rotation, is $-82$ km/sec; the tangential component, according to the proper motion given in the paper just quoted, is probably less than 30 km/sec. No identification of the origin of this star has been possible so far.

### 3. *Frequency of high-velocity objects among the O to B5 spectral types*

The percentage of high-velocity stars has been estimated as follows.

a. For types B1 to B5, we consider the stars north of declination $-20°$ in the region between galactic longitudes $l^I = 340°$ and 200°, and within 600 ps. Data for these stars are taken from a basic list, used for a discussion of the motions of the northern O to B5 stars published earlier (BLAAUW 1956b). Space velocities have been computed with the spectroscopic parallaxes resulting from that investigation in so far as the kinematic data are known with relatively high accuracy. The most important criterion is the accuracy of the proper motion; as a rule we confine the choice to stars with probable errors of the proper motion below $\pm 0''.0060$. At an average distance of 400 ps this limit corresponds to a tangential velocity of $\pm 11$ km/sec. For the stars in, or originating from, the associations I Lac, I Ori, II Per, and those in the Cas-Tau group or the $\alpha$ Per cluster, we use the space velocities relative to the mean motion of these groups, whereas for the remaining stars the velocities corrected for solar motion and differential galactic rotation are used. The selection of the objects for the statistical discussion on the basis of the precision of the kinematic observational data does not, of course, affect the percentage of high-velocity objects to be determined; it merely somewhat reduces the number of later spectral types used in our discussion as compared to the earlier types, as a consequence of the generally lower luminosity of the later types and their consequently less accurate proper motions. (The relative numbers of the spectral subclasses in Table 4a therefore do not represent their relative frequencies in the volume around the sun within 600 ps.) The selection of the material used here has been somewhat more stringent than that applied in a previous, more preliminary, discussion (BLAAUW 1958b), and we also put the limit for the definition of high-velocity objects at 40 km/sec instead of the previously used 30 km/sec. This eliminates a few doubtful objects around 30 km/sec.

The results are given in Table 4a, where the total numbers of stars per subclass selected for the analysis are in the second column and the numbers of high-velocity objects in the third column. The four objects in the latter category, all of which occur in Table 1,

TABLE 4a

Percentage of B1 to B5 stars with space velocities > 40 km/sec among the objects with accurately determined velocities, within 600 ps, in the region Decl. > $-20°$, $l^I = 340°$ to 200°

| spectral type MK | total number | number with space velocity > 40 km/sec | percentage |
|---|---|---|---|
| B1 | 28 | 0 | |
| B2 | 101 | 1 | |
| B3 | 94 | 1 | |
| B5 | 47 | 2 | |
| B1 to B5 | 270 | 4 | 1.5% |

TABLE 4b

Percentage of O5 to B0.5 stars with space velocities or residual radial velocities > 40 km/sec, within 1000 ps in the region Decl. > $-20°$, $l^I = 340°$ to 200°

| spectral type MK | total number | number with space velocity or resid. rad. vel. > 40 km/sec | percentage |
|---|---|---|---|
| O5,6 | 5 | 1 | 21 % (O5 to O9.5) |
| O7,8 | 9 | 2 | |
| O9,9.5 | 15 | 3 | |
| B0,0.5 | 39 | 1 | 2.5% |

represent 1.5 per cent of the total sample. Nearly all the stars considered belong to luminosity class V and most of the remaining ones to luminosity class IV.

b. For the spectral types O5 to B0.5, the region within 600 ps is too scarcely populated to provide meaningful statistics. We therefore consider all O5 to B0.5 stars with MK classification, within 1000 ps and north of $-20°$ declination, for which radial velocities are available. For only part of these do we have reliable proper motions and, hence, space velocities; for the remaining stars the evidence for high space velocity rests mainly on the radial velocity. The proper-motion data for many of the stars of this first category are still rather weak: a probable error of $\leq \pm$ 10 km/sec in the tangential velocity for a star at 800 ps requires a probable error of the proper motion $\leq \pm''.0024$, an accuracy which is generally not available. By putting, as for the B1 to B5 stars, the limit for high-velocity objects at 40 km/sec we exclude a number of B0 and B0.5 stars in the region of the association I Cep with velocities around 30 km/sec which were included in our previous statistics, but for which the kinematic data are somewhat dubious.

The results are contained in Table 4b. The total numbers of stars are in the second column, and in the third column are those for which either the space velocity or only the radial component exceeds 40 km/sec. We find that six O-type stars, or about one fifth of all O stars, are in the high-velocity category, and this must be considered as a minimum because there may be some unrecognized high-velocity objects without reliable proper motions for which the radial components are below 40 km/sec. (Statistically, for an isotropic velocity distribution, the mean of the radial components of the space velocities, without regard to sign, is one half of the mean of the space velocities.) Of the seven high-velocity objects included in Table 4b, six occur in Table 1.

The high percentage of high velocities among the O types as compared to that for the B types is undoubtedly a real phenomenon. If there were more high-velocity objects among the 270 B1 to B5 stars used for our present selection, they certainly could not have escaped attention. On the other hand, the low percentage of high velocities actually found for the B1 to B5 stars would correspond to only one object among the 29 O-type stars. The six or more objects which actually occur exceed this expectation significantly, even if allowance is made for statistical fluctuation.

We shall adopt a proportion of 0.2 for the high velocities among the O stars, and a proportion of 0.02 for the B types, although there is evidence for an intermediate proportion among the B0 and B0.5 types. A comparison with the numbers of stars formed per unit of volume according to the Initial Luminosity Function is of interest. Using this function in the form given by SANDAGE (1957) or by LIMBER (1960) we find that the total number of B0 to B5 stars formed in a given interval of time is 8 times larger than the number of O stars. This implies that the number of *high-velocity* B0 to B5 stars formed in a given lapse of time is about equal to or slightly less than the number of high-velocity O stars formed. The total mass carried by these O stars is about 5 times larger than the mass carried by the B0 to B5 stars.

## 4. *Duplicity among the O5 to B5 stars in general*

In this section we shall provisionally discuss the evidence for the high frequency of duplicity and multiplicity among the stars of types O5 to B5 in general, as a basis for the comparison with the high-velocity objects to be made in the following section. The discussion will be partly based on preliminary results of a special study of the duplicity and multiplicity properties of young stars for which radial-velocity observations have been collected at McDonald Observatory since 1950, a more extensive account of which will be published separately.

### a. *The spectral types B0 to B5*

We first consider the types B0 to B5. As a representative sample we use the objects in the associations Scorpio-Centaurus, II Per, I Ori, the Cassiopeia-Taurus stars, and the field stars within 500 ps in the region between galactic longitudes $l^{I} = 340°$ and $50°$. (For these latter stars, data were also collected earlier in connection with the luminosity calibration and other studies.) The total number of these objects is 210.

Particulars about the *visual double stars* were taken from AITKEN's (1932) double star catalogue. We limit the choice of companions to be used for the statistical

TABLE 5

Observed numbers of visual double stars and expected numbers of optical pairs

| limits of separation | companions brighter than 10th magnitude | | companions between magnitudes 10 and 12 | |
|---|---|---|---|---|
| | observed | expected optical pairs | observed | expected optical pairs |
| 1″ to 4″ | 19 | 0.0 | 5 | 0.0 |
| 5 to 9 | 4 | 0.0 | 1 | 0.1 |
| 10 to 14 | 5 | 0.0 | 1 | 0.2 |
| 15 to 19 | 1 | 0.0 | 1 | 0.3 |
| 20 to 29 | 4 | 0.1 | 4 | 0.8 |
| 30 to 39 | 3 | 0.2 | 5 | 1.2 |
| 40 to 49 | 1 | 0.2 | 2 | 1.5 |
| 50 to 59 | 2 | 0.3 | 2 | 1.9 |
| 60 to 69 | 0 | 0.3 | 2 | 2.2 |
| 70 to 79 | 1 | 0.4 | 0 | 2.6 |
| 80 to 89 | 0 | 0.5 | 0 | 2.9 |

discussion to those with angular distances between 1″ and 40″ from the primary component. This lower limit of 1″ has been adopted because at still smaller separations only pairs with small magnitude differences are recorded. The upper limit of 40″ has been chosen in order to reduce the influence of optical pairs. It was fixed on the basis of a comparison between the observed distribution of angular separations and the distribution to be expected purely on the basis of a random distribution of the stars in the sky and star counts at latitude 15°, this latter being the mean latitude of the stars considered. Table 5 gives, separately for companions brighter than 10th magnitude and for those between magnitudes 10 and 12, the observed numbers between the limits of separation in the first column, and the corresponding expected numbers of optical pairs. It appears that beyond about 40″ the majority of the observed companions fainter than magnitude 10 must be optical. The double star observations are practically limited to companions brighter than 12th magnitude. This implies that only companions brighter than about absolute magnitude + 5 are included in the discussion.

Of the 210 objects, 39 are found to be visual doubles and 9 visual triples; hence 48 objects, or 22 percent, have at least one companion in the range 1″ to 40″. All of these stars are within 500 ps, and their mean parallax is 0″.0044. Since the spread of the individual parallaxes is small, we shall tentatively assume that this mean parallax applies to all double stars. The separations of 1″ and 40″ then correspond to projected distances between primary and companion of 226 and 9100 astronomical units. We shall also tentatively assume that this is the range of the semi-major axes of the systems considered. Some justification for this assumption lies in the fact that the projected distances are smaller than the true distances, whereas the semi-major axes *on the average* will also be smaller than the actual distances between primary and companion.

Particulars about the *spectroscopic binaries* are taken from existing catalogues of spectroscopic binaries and from the preliminary results of the current investigation already referred to. Table 6 gives a survey of the available data, together with references to some unpublished material (in the notes to the table). The various columns give, for each group or association, the total number of stars, the number of spectroscopic binaries with determined orbital elements, the number of objects for which no determination of the orbital elements is available yet but for which the variations in the radial velocity leave little doubt as to the binary nature, and, in the last column, the percentage of spectroscopic binaries in each group based on the sum of the two preceding columns. Among the 210 stars of all groups together, there are

TABLE 6

Numbers of spectroscopic binaries in the various groups of B0 to B5 stars

| group | total number of stars | number of spectroscopic binaries | | percentage of spectroscopic binaries % |
|---|---|---|---|---|
| | | with known elements | without elements | |
| Cas-Tau | 49 | 15 | 4 | 39 |
| $l^I = 340°$ to $50°$ | 50 | 6 | 6 | 24 |
| II Per | 15 | 5 | 1 | 40 |
| Sco-Cen | 53 | 6 | 7 | 25 |
| Orion, ε Ori sub-group | 26 | 5 | 0 | 19 |
| Orion, Outer Nebula group | 17 | 0 | 0 | 0 |
| all groups | 210 | 37 | 18 | 26 |

*Notes to Table 6*

The following unpublished elements were incorporated in the statistics:

| group | star HD No. | period (days) | $e$ | $K_1$ | $K_2$ (km/sec) |
|---|---|---|---|---|---|
| Cas-Tau | 1976 | 27.8 | 0.20 | 30 | |
| ,, | 11241 | 15.6 | 0.45 | 10 | |
| ,, | 23466 | 2.408 | 0.1 | 23 | |
| ,, | 30211 | 7.33 | 0.39 | 23 | |
| ,, | 34759 | 35.5 | 0 | 28 | |
| ,, | 37367 | 6.5: | | 10: | |
| ,, | 42545 | 19 | | 10: | |
| II Per | 23625 | 2.002 | | 80 | 80 |
| ,, | 24190 | 26.1 | 0.1 | 14 | |
| ,, | 25799 | 10.3 | [1] | 25 [1] | |
| ε Ori sub-gr. | 35588 | 2.885 | 0.05 | 69 | |

37 spectroscopic binaries with known orbital elements, and 18 without element determination, so that the total incidence of spectroscopic binaries is 26 percent. A discussion of possible differences between the percentage of spectroscopic binaries in the various groups is beyond the scope of the present article. We shall confine ourselves to the results as found for the 210 objects together.

Among the 37 spectroscopic binaries with known elements, there are eleven for which both components have been measured so that we know the product $(a_1 + a_2) \sin i$. The distribution of these values is in the third column of Table 7. According to HYNEK (1952), visibility of both components in the spectrum may be expected to occur for main-sequence stars if the mass ratio $M_1/M_2$ is smaller than about 2.5.

[1]) Strong perturbations in the velocity curve, probably due to gas streams in this system, make the determination of $e$ and $K_1$ uncertain.

TABLE 7

Distributions of the values of the quantity $(a_1 + a_2) \sin i$ for the spectroscopic binaries with known elements of Table 6

| $(a_1 + a_2) \sin i$ (astr. units) | $\log (a_1 + a_2) \sin i$ | both components measured | one component measured | sum |
|---|---|---|---|---|
| < 0.010 | < − 2.0 | 0 | 0 | 0 |
| 0.010 to 0.032 | − 2.0 to − 1.5 | 1 | 1 | 2 |
| 0.032 to 0.100 | − 1.5 to − 1.0 | 5 | 7 | 12 |
| 0.10 to 0.32 | − 1.0 to − 0.5 | 4 | 6 | 10 |
| 0.32 to 1.00 | − 0.5 to 0 | 0 | 8 | 8 |
| 1.0 to 3.2 | 0 to + 0.5 | 1 | 4 | 5 |

For the remaining 26 objects with element determinations, we know only the quantity $a_1 \sin i$. For these we shall assume the average mass ratio $M_1/M_2 = 5$, as found by HYNEK (1952) from statistical considerations. For a mean value of about 8 solar masses of the primary component, such as may be adopted for a main-sequence B2 star, this corresponds to $M_2 = 1.6 \odot$ on the average. We now multiply all values of $a_1 \sin i$ found by the factor

$$\frac{a_1 + a_2}{a_1} = \frac{M_1 + M_2}{M_2} = 6.$$

The hypothetical values of $(a_1 + a_2) \sin i$ thus obtained have the distribution shown in the fourth column of Table 7. The sum of the two distributions is in the last column.

For our present purpose we shall assume that this distribution of the quantities $(a_1 + a_2) \sin i$ may be considered equivalent to the distribution of the quantities $a_1 + a_2$. The average value of $\sin i$ would be $\pi/4 = 0.78$ for an isotropic distribution of the poles of the orbital planes, and the observational selection of high values of $\sin i$ will have tended to make the true average still closer to unity.

The decrease of the numbers in the last column of Table 7 with increasing value of $(a_1 + a_2) \sin i$ must be spurious. KUIPER's (1935) analysis of the frequency of the major axes of double stars indicates an increase of the frequency of systems with increasing $a_1 + a_2$ in this range of $a_1 + a_2$. The decrease in Table 7 must be due to the decreasing degree of completeness of the material with increasing $a_1 + a_2$, i.e. with decreasing amplitude of the velocity variation and increasing period. The semi-amplitude, $S_1$, of the velocity variation of the primary, and the orbital period, for circular motion, are given by

$$S_1 = 30 \left(\frac{M_2}{M_1 + M_2}\right)^{\frac{1}{2}} \left(\frac{M_2}{a_1 + a_2}\right)^{\frac{1}{2}} \sin i ,$$

$$P = \frac{(a_1 + a_2)^{3/2}}{(M_1 + M_2)^{\frac{1}{2}}} \text{ yrs} ,$$

where $M_1$ and $M_2$ are expressed in solar masses and $a_1 + a_2$ in astronomical units. For $M_2 = 1.6$ solar masses, $M_1/M_2 = 5$ and $(a_1 + a_2) > 1$ a.u., we get $S_1 < 15 \sin i$ km/sec and $P > 118$ days. Such objects have only rarely been identified so far, and no significant part of them may be expected to occur among the recognized binaries, not even among those for which the available radial-velocity observations show definite variations but for which no orbital elements are yet determined.

It follows that for $(a_1 + a_2) < 1$ a.u. our material will also be incomplete. For our present analysis we shall nevertheless tentatively assume that the binaries with $(a_1 + a_2) < 1$ a.u. are approximately known. We shall then be dealing with a minimum estimate of the duplicity. We assume that the 18 objects with unknown elements in Table 6 have a distribution of the quantity $(a_1 + a_2) \sin i$, similar to that of the 37 with known elements. This assumption is supported by the provisional results of our current analysis of hitherto neglected spectroscopic binaries. We then find that at least 48 objects out of the 210 are binaries with $(a_1 + a_2) < 1$ a.u., that is, 23 percent.

We shall further assume that these spectroscopic companions occupy the same range in absolute magnitude as that found for the companions of visual binaries, i.e. values of $M_v$ brighter than + 5. This corresponds to solar masses and greater. The assumption appears reasonable in view of the estimated *mean* value of 1.6 solar masses for the invisible secondaries referred to before, and the already-quoted upper limit for invisible companions of about $8/2.5 = 3.2$ solar masses. We estimate that the difference between the lower limit to which our observational data refer for the spectroscopic and for the visual binaries may well be one magnitude, but that it is unlikely to amount to more than two magnitudes.

As a basis for the estimation of the frequency of duplicity and multiplicity we now use the data as arranged in the first two columns of Table 8. This table gives the numbers of systems among the 210 objects of the basic list for seven categories as specified in the first column and in the heading of the table. This subdivision takes into account the fact that some spectroscopic binaries are at the same time visually

TABLE 8

Observed and predicted numbers of the B0 to B5 stars in the various categories of duplicity or multiplicity

SB = spectroscopic binary, and ST = spectroscopic triple, with $(a_1 + a_2) < 1$ a.u.;

VD = visual double, and VT = visual triple, with separations between 1″ and 40″

| observed character | observed number | computed numbers $\beta = 1.0$, $f_2 = 0.158$ | $\beta = 0.5$, $f_2 = 0.40$ | $\beta = 0.2$, $f_2 = 0.73$ |
|---|---|---|---|---|
| single | 127 | 123 | 124 | 127 |
| SB only | 34 | 26 | 31 | 36 |
| VD only | 29 | 29 | 34 | 40 |
| ST only | 1 | 6 | 3 | 1 |
| SB and VD | 10 | 12 | 8 | 3 |
| VT only | 7 | 7 | 4 | 2 |
| SB and VT | 2 | 3 | 1 | 0 |
| sum of these categories | 210 | 206 | 205 | 209 |

double or multiple. Thus, of the 48 spectroscopic binaries with $(a_1 + a_2) < 1$ a.u., ten are also visual doubles between 1″ and 40″, whereas two of these objects have two visual companions in this range. In one case we are dealing with a spectroscopically triple system (VV Ori, a,b and ab,c); this was counted as two spectroscopic binaries in the preceding counts. In the case of the spectroscopic binaries without determined orbital elements, one does not know which of the visual binaries among them fall in the class of $(a_1 + a_2) \sin i < 1$ a.u. We have again assumed a similar fraction to have $(a_1 + a_2) < 1$ a.u. as was found for the binaries with known elements in Table 7.

For the interpretation of the numbers in the second column of Table 8 we shall take into account the occurrence of single, double, triple, quadruple, and quintuple systems. The highest-order systems observed are two spectroscopic binaries with two visual companions each. We shall denote by

$f_1$, the fraction of all 210 objects which are single,
$f_2$, the fraction which are double,
$f_3$, the fraction which are triple, etc.

We shall assume, that the fractions $f_2, f_3, f_4, f_5$ decrease monotonously with increasing order, and we therefore put

$$f_3 = \beta f_2, f_4 = \beta^2 f_2, f_5 = \beta^3 f_2 \qquad (1)$$

with $\beta \leq 1$.

We shall further tentatively assume that the frequency distribution of the distances between the companions and the primary is given by the curve derived by KUIPER (1935) for double stars of spectral types A and later. Thus, for a triple system with its two companions at distances $(a_1 + a_2)'$ and $(a_1 + a_2)''$ from the primary, we assume that the chances for $(a_1 + a_2)'$ to lie in a certain interval are exclusively given by KUIPER's frequency curve, and independent of $(a_1 + a_2)''$, and *vice versa*.

In order for a system to be considered as a spectroscopic double in our analysis, the value of $a_1 + a_2$ should be smaller than 1 astronomical unit. The chance that such a value will occur will be denoted by $F_{sp}$. In order for a companion to be counted as a visual component, its separation $d$ should be between 230 and 9100 a.u. (log $d$ between 2.36 and 3.96); the chance that this will occur will be denoted by $F_{vis}$.

According to KUIPER's curve, we have

$$F_{sp} = 0.18\,;\; F_{vis} = 0.20. \qquad (2)$$

Using these values we now try to reproduce the numbers in Table 8 by a suitable choice of $f_2$ and $\beta$. Thus the observed number of systems with only one companion, in the spectroscopic binary range, gives the relation

$$34 = 210\,[f_2 F_{sp} + 2 f_3 F_{sp} (1 - F_{sp} - F_{vis}) + 3 f_4 F_{sp} (1 - F_{sp} - F_{vis})^2 + 4 f_5 F_{sp} (1 - F_{sp} - F_{vis})^3]\,,$$

in which the relations (1) and (2) are to be substituted, so that we get:

$$0.89 = f_2\,[1 + 2\beta \times 0.62 + 3\beta^2 \times 0.62^2 + 4\beta^3 \times 0.62^3]\,.$$

Similarly, the objects with one companion in the spectroscopic-binary range and one in the visual-duplicity range give the relation

$$10 = 210\,[2!\,f_3 F_{sp} F_{vis} + 3!\,f_4 F_{sp} F_{vis} (1 - F_{sp} - F_{vis}) + \frac{4!}{2} f_5 F_{sp} F_{vis} (1 - F_{sp} - F_{vis})^2]\,,$$

etc.

The last three columns of Table 8 show the computed numbers in the various categories for the three values of $\beta$: 1.0, 0.5 and 0.2, and the corresponding values of $f_2$ which were found to give the best fit for each of these values of $\beta$. A quite satisfactory representation of the observed numbers is obtained for $\beta = 0.5$, $f_2 = 0.40$. Values of $\beta$ larger than 1 are not taken into account since this would imply increasing fractions of higher-order systems, which evidently do not exist; for this reason $\beta = 1$ is also not a plausible solution.

Adopting as a reasonable solution $\beta = 0.5$, $f_2 = 0.4$, we get $f_1 = 0.25$, $f_2 = 0.40$, $f_3 = 0.20$, $f_4 = 0.10$, $f_5 = 0.05$.

For a proper discussion of the best fit we should also take into account the lack of observed cases consisting of, for instance, a spectroscopic triple with one visual companion, or the lack of cases with three visual components (visual quadruples) for both of which the predicted numbers are not entirely negligible. However, the foregoing results suffice to show that a satisfactory representation can be obtained on the basis of the assumptions made.

In connection with the discussion of the high-velocity stars in the next section we shall be particularly interested in the evidence found here with regard to the scarcity of single objects, i.e. the fraction $f_1$, which was 0.25 for $\beta = 0.5$. For the cases $\beta = 1.0$ and $\beta = 0.2$, we find $f_1 = 0.38$ and $f_1 = 0.08$ respectively. Evidently the single objects form a minority of the order of 0.25 only among the B0 to B5 stars in general. If we further take into account the fact that the foregoing analysis was confined to companions brighter than absolute magnitude + 5 only, it is clear that *the occurrence of a single B0 to B5 star is a rather rare phenomenon.*

b. *The O-type stars*

The general tendency of O-type stars to occur in multiple systems like the Orion trapezium is well known. For investigations of the nature of such, as a rule unstable, systems, we refer to papers by AMBARTSUMIAN (1950) and by SHARPLESS (1954). In the context of the present article, a brief discussion along lines comparable to those of the foregoing treatment of the B0 to B5 stars is in order. For this purpose we select the same objects as those used for the investigation of the frequency of high-velocity O stars in the preceding section. The same limits of $(a_1 + a_2) \sin i$ ($<$ 1 a.u.) and of the angular separation (1″ to 40″) are adopted as for the B0 to B5 stars. Although the number of objects, 29, is small, the evidence points rather convincingly towards a still higher incidence of multiplicity than was found for the B stars. The situation may be described as follows:

1. Seven objects, or 24 percent of the 29, are spectroscopic binaries; these have up to 5 visual companions. For the B0 to B5 stars we found that 23 percent were spectroscopic binaries, with up to 2 visual companions.

2. Among the 29 objects, 10 are a visual double or multiple and these contain altogether 22 visual companions. If the numbers for B0 to B5 are reduced to a total of 29 objects, we find that there are 6.6 visual doubles or triples, with altogether 7.9 visual companions.

The high frequency of visually multiple stars among the O types as compared to the B stars is the more striking because their average distance from the sun is about three times larger than that of the B0 to B5 stars studied, so that the adopted limits of angular separation correspond with larger linear distances where, according to KUIPER's curve, the frequencies become lower. Moreover, the larger distances from the sun limit the visual companions to a brighter absolute magnitude (about $M_v = + 3$) than the value $M_v = + 5$ which holds for the B stars.

A rough estimate shows that the frequencies of the O-type stars in the various categories of Table 8 require a larger value of $\beta$, approaching 1.0, than was found for B0 to B5. This result is, of course, not surprising in view of the already-mentioned tendency of O-type stars to belong to multiple systems of high order.

In the present discussion we have made no distinction between the high-velocity O stars and the remaining ones. We shall show in the next section that duplicity and multiplicity among the high-velocity objects is very rare, and perhaps even completely absent. Of the 29 objects just discussed, 6 belong to the high-velocity group. None of these is a visual double or multiple, or a spectroscopic binary. If these objects are eliminated from the preceding discussion, the incidence of duplicity and multiplicity is found to be so high that the occurrence of single, low-velocity O stars must be exceedingly rare.

A more precise discussion of the multiplicity among the low-velocity O stars, with an estimate of the values of $\beta$ and $f_2$, would require a more elaborate investigation than falls within the scope of the present article.

## 5. *Lack of duplicity among the high-velocity O5 to B5 stars*

The very low frequency of double and multiple systems among the high-velocity objects can be demonstrated by a discussion of the properties of the 19 stars of Table 2. We first consider, separately, the visual and the spectroscopic evidence.

a. *None of the stars* of Table 2 is a visual binary. The only object occurring in AITKEN's (1932) catalogue is HD 34078 (AE Aur), but the information given there is erroneous. One observation is listed which refers to SCHEINER's (1908) compilation of double stars in the *Astrographic Zone Catalogue* for the epoch 1895. The companion was obviously spurious. Visual inspection does not show it, and this explains why AITKEN's catalogue contains no entries for this object later than 1895.

b. *None* of the stars in Table 2 is known as a spectroscopic binary with determined orbital elements. For 13 objects, the constancy of the radial velocity may be considered well established in view of the judgement of the observer or of the whole of the available data. For 5 of the remaining 6 objects, the scatter in the measurements has sometimes suggested variability, but these are objects which could not be accurately measured and for which new, extensive series would be desirable. They are: HD 24912 ($\xi$ Per), HD 152408, HD 203064 (68 Cyg), HD 149757 ($\zeta$ Oph), and HD 149363. The star HD 19374 (53 Ari) is a $\beta$ Canis Majoris star according to MÜNCH and FLATHER (1957) and probably should not be classified as a binary.

With regard to the lack of visual duplicity we note that all but 4 objects in Table 2 are at distances of

1000 ps or nearer. On the basis of the statistics of the preceding section we should expect that for the O to B5 stars in general at these distances, about 4 out of 19 should be visual doubles. Spectroscopic binary character should be observed for 4 or 5 out of 19 such objects.

In order to arrive at an upper limit of the duplicity among the high-velocity objects, we assume that there is a maximum of two spectroscopic binaries among the objects in Table 2. This, together with the absence of visual doubles, leads to a maximum value $f_2 = 0.10$ for $\beta = 0.5$ and $f_2 = 0.19$ for $\beta = 0.2$. The corresponding minimum values of the fraction of single objects are $f_1 = 0.82$ and $f_1 = 0.76$ respectively.

We conclude that, among the high-velocity objects, at least about 80 per cent are single, and that it is more likely that *the high-velocity objects are only very rarely double or multiple*. It seems quite possible that in actuality *all* of them are single.

The remarkable contrast in duplicity properties between the normal and the high-velocity objects forms the main inducement for the proto-binary hypothesis. Its explanation of the two principal characteristics of the high-velocity OB stars is further discussed in section 6.

## 6. *The proto-binary hypothesis*

### a. *Exploratory considerations*

In discussing the consequences of the mass loss of the heaviest component, $M_1$, of a proto-binary, we are primarily interested in those cases in which this mass loss is sufficiently large to completely release $M_2$, i.e. the mass loss should be at least half of the total mass (see below) and for comparison with observations we want to know the velocity of $M_2$ once it has reached distances where the gravitational attraction of $M_1$ has become negligible. It will be assumed that the ejection of mass from $M_1$ occurs isotropically and at a uniform rate during a certain interval of time, and that the velocity of the ejected mass well exceeds the original orbital velocity of $M_2$ with respect to $M_1$. The orbital velocities relevant to our analysis will be a few hundred km/sec at most; the velocities of expansion observed in supernovae are of the order of several thousand km/sec.

The mass shed by $M_1$ may be considered as a continuous series of expanding homogeneous shells. As long as these shells have radii smaller than the orbital radius of $M_2$, their gravitational effect on $M_2$ will be practically indistinguishable from that of a point mass at $M_1$ with the shell's mass. As soon as a shell has passed beyond the orbit of $M_2$ it ceases to gravitationally affect $M_2$. There will be some interaction between the shells and $M_2$ and this may affect the motion of $M_2$. Such interaction is the more likely because we should expect $M_2$ still to be in the contraction stage. For the present reconnaissance, however, we merely assume $M_2$ to survive the event and the consequences of this interaction for the motion of $M_2$ will be ignored (see also section 6e).

We thus assume $M_2$ to move in a gravitational field equivalent to that of a continuously decreasing mass $M_1$. It will be shown below that in the case of sufficiently rapid mass loss of $M_1$ the hyperbolic velocity of $M_2$ with respect to the original centre of gravity, i.e. the velocity to be compared with the observed velocity, may be of the order of one half or more of the original orbital velocity. In the following sub-sections we shall consider in more detail which hyperbolic velocities are to be expected under certain conditions with regard to the rate of mass loss of $M_1$, the initial mass ratio between $M_2$ and $M_1$, and other initial properties of the proto-binary system. First, however, we shall briefly discuss the gravitational influences to which $M_2$ may be subjected other than those due to $M_1$.

After the termination of the mass ejection, the accumulated shells will be decelerated by the accretion of interstellar matter outside the proto-binary, and eventually the ejected mass will be overtaken again by $M_2$. The forces to which $M_2$ will then be subjected in addition to the attraction by $M_1$ are:

a) The attraction due to the shell.

b) The gravitational forces acting within the cloud complex at large in which we assume the proto-binary to have been formed.

c) The deceleration by the attraction of the cloud complex as a whole, once $M_2$ has left it.

In order to estimate the significance of the gravitational force due to the accreting shell, once $M_2$ has again reached and traversed it, we first estimate the total mass the shell has obtained by that time. The ratio between this total mass and the initial mass of the shell will depend on the initial velocity of expansion and the velocity of $M_2$.

We denote by

$m_\circ$, the initial mass of the shell,
$m_a$, the accreted mass at time $t$,
$v_\circ$, the initial velocity of expansion of the shell,
$v$, the velocity of the expansion at time $t$,
$r$, the shell's radius at time $t$,
$\rho$, the density of the interstellar medium outside the shell.

We assume the accretion to start at $r = 0$, at $t = 0$. We then have:

$$v\,(m_\circ + m_a) = v_\circ m_\circ$$

with

$$m_a = \tfrac{4}{3}\pi\rho r^3\,;\; v = \frac{dr}{dt},$$

hence

$$(\tfrac{4}{3}\pi\rho r^3 + m_\circ)\,dr = v_\circ m_\circ\,dt$$

and

$$r\,(\tfrac{1}{3}\pi\rho r^3 + m_\circ) = v_\circ m_\circ t\,. \qquad (3)$$

We denote by

$\overline{v_2}$ the mean radial component of the speed of $M_2$ with respect to the centre of the shell, averaged over the time it takes $M_2$ to reach the shell, and by

$D_o$, the initial distance between $M_1$ and $M_2$. We then have approximately

$$r - D_o = \overline{v_2}\, t. \tag{4}$$

Assuming tentatively $r \gg D_o$, we derive from relations (3) and (4) for the radius at which $M_2$ overtakes the shell:

$$\tfrac{1}{3}\pi\rho r^3 + m_o = \frac{v_o}{\overline{v_2}} m_o,$$

and hence

$$m_a = 4\left(\frac{v_o}{\overline{v_2}} - 1\right) m_o \sim 4\frac{v_o}{\overline{v_2}} m_o. \tag{5}$$

For $v_o$, we assume 5000 km/sec (see, for instance, Payne-Gaposchkin 1957) and for $\overline{v_2}$, 100 km/sec. Hence, $v_o/\overline{v_2} = 50$ and $m_a \sim 200\ m_o$. Even if the initial velocity, $v_o$, were only 1000 km/sec, we still would have $m_a \sim 40\ m_o$.

It thus appears that for the velocities concerned the accreted mass is always very large compared to the initial mass. This implies that once $M_2$ has traversed the shell the force it experiences from the direction of the shell's centre is nearly the same as that to which it would have been subjected at that position if the region inside the shell had been filled only with the accreted interstellar mass and the mass of $M_1$ had not existed. Two kinds of circumstances may be envisaged at this instant: *a*) The interstellar medium around the proto-binary may be dense, for instance one or two powers of 10 lower than the smeared-out density which would result from distributing the mass of the proto-binary over the spherical volume containing the system, and which is of the order of $10^{-14}$ g cm$^{-3}$. This situation may arise if we are dealing with the formation of the proto-binary out of a local condensation in an interstellar cloud complex in which other mass fluctuations occur with density maxima of the same order as that in the proto-binary region. The forces due to these other density maxima will then be comparable in amount to the force due to the mass inside the shell—including that of $M_1$—and there is no reason to consider this latter force separately. We shall assume that the stochastic forces which will be exerted by this kind of interstellar medium do not cause an appreciable increase or decrease of the star's velocity. *b*) The interstellar density in the surrounding cloud may be low, for instance of the order of $10^{-20}$ g cm$^{-3}$, i.e. $10^{-6}$ times the smeared-out density just referred to. The distance at which $M_2$ overtakes the shell is then of the order of $(10^6 m_a/m_o)^{1/3}$ times the original dimension of the proto-binary and the decelerating force exerted by the mass of $M_1$ will be negligible.

We finally consider the deceleration $M_2$ may experience when leaving the cloud complex. If this has a total, spherically distributed, mass of $m_c$ solar masses and an average density of $\rho$ solar masses per cubic a.u., then the velocity of escape at its border is approximately

$$v_{esc} = 42\, m_c^{1/3} \rho^{1/6}\ \text{km/sec}.$$

In order that $v_{esc}$ be sufficiently low so that the star $M_2$ will not be appreciably decelerated after leaving the cloud, it will suffice that $m_c^{1/3}\rho^{1/6} \leq \frac{1}{4}$. For $m_c = 10^4\ \odot$ masses this means $\rho \leq 2.4 \times 10^{-12}\ \odot$ masses (a.u.)$^{-3}$ or $1.4 \times 10^{-18}$ g cm$^{-3}$. This is about $10^6$ times the average interstellar density and similar to the estimated density of the densest type of globules according to Bok (1946). It thus appears justified to assume that the released star $M_2$ experiences little or no deceleration upon leaving the cloud.

In view of these considerations we shall henceforth assume that the actually observed velocities of the high-velocity stars, identified with the released secondary components $M_2$, can be directly compared to the hyperbolic velocities at infinity as computed by exclusively taking into account the effect of the decrease of mass of the primary $M_1$.

b. *Instantaneous mass loss of $M_1$; motion of $M_2$ relative to $M_1$*

Earlier investigations of the dynamics of binaries with decreasing mass (Jeans 1929, Su-Shu Huang 1956, and others) have dealt with the effects due to a slow loss of mass, i.e. cases in which the mass loss per orbital period is a small fraction of the total mass of the components.

For our present problem, only the case of rapid loss of mass is relevant. Furthermore, we are primarily interested in estimating the effects on the velocity of $M_2$ with respect to the original centre of gravity of the system (which we assume to be at rest in the medium in which the proto-binary is formed) rather than in the effects on the velocity with respect to $M_1$.

Yet, for the sake of the argument, let us first consider the relative motion of $M_2$ with respect to $M_1$. The relative velocity of $M_2$, $S_{rel}$, at an arbitrary instant is given by

$$S_{rel}^2 = 30^2\,(M_1 + M_2)\left(\frac{2}{R} - \frac{1}{a}\right)(\text{km/sec})^2,$$

where

$R$ is the distance between $M_1$ and $M_2$,

$a$ is the increasing semi-major axis of the elliptic orbit if we are dealing with the first stages of the mass loss; once the orbit has become hyperbolic this is also characterized by the quantity $a$, which then, however, assumes negative values from $-\infty$ upwards,

$M_1$ is the decreasing mass of the primary component,
$M_2$ is the mass of the secondary component,
$a$ and $R$ are expressed in astronomical units.

The quantity $a$ is given by

$$a = -\frac{M_1 + M_2}{2E} \; ; \; E = T + \Omega \, ,$$

where

$E$ is the total energy of $M_2$ with respect to $M_1$,
$T$ is the kinetic energy of $M_2$ with respect to $M_1$,
$\Omega$ is the potential energy of $M_2$ with respect to $M_1$.

For $E < 0$ we have elliptic motion; for $E > 0$ hyperbolic motion.

For the hyperbolic velocity of $M_2$ with respect to $M_1$ at infinity, or, for practical purposes, when $R$ has become large and $M_2$ practically beyond the influence of $M_1$, we have

$$S^2_{rel,\,\infty} = -\frac{M_1 + M_2}{a} \, 30^2 \, (\mathrm{km/sec})^2 \, .$$

We shall for all calculations assume the initial orbit to be circular. We then have:

$$S^2_{rel,\,\circ} = 30^2 \, (M^\circ_1 + M_2)/R_\circ \text{ and } E_\circ = -\frac{M^\circ_1 + M_2}{2R_\circ} \, ,$$

where

$S_{rel,\,\circ}$ is the initial relative velocity of $M_2$,
$R_\circ$ is the radius of this initial orbit,
$M^\circ_1$ is the initial mass of $M_1$,
$E_\circ$ is the original total energy of $M_2$ with respect to $M_1$;

$$E_\circ = \Omega_\circ + T_\circ \text{ and } \Omega_\circ = -\frac{M^\circ_1 + M_2}{R_\circ} \, .$$

It is useful first to consider the case of instantaneous mass loss of the primary. Its reduced mass will be denoted by $q \, M^\circ_1$, $q$ being the reduction factor. The initial kinetic energy, $T_\circ$, of $M_2$ with respect to $M_1$ will start to change with time, but not discontinuously at the moment of the mass loss. The potential energy, $\Omega_\circ$, does change abruptly at that moment and becomes

$$\Omega = -\frac{q \, M^\circ_1 + M_2}{R_\circ} \, .$$

Hence, right after the mass loss of $M_1$, the total energy of $M_2$ with respect to $M_1$ is

$$E = T_\circ + \Omega_\circ + \frac{(1-q)\, M^\circ_1}{R_\circ} = \frac{M^\circ_1 (1-2q) - M_2}{2\, R_\circ} \, .$$

It follows immediately that the orbit of $M_2$ will become hyperbolic if the numerator is positive, i.e.,

$$q < \frac{M^\circ_1 - M_2}{2\, M^\circ_1} \, .$$

The mass of $M_1$ should therefore be reduced to at least one half of the original value, and the higher the ratio $M_2/M^\circ_1$, the smaller $q$ has to be. For $M_2 > M^\circ_1$ the relative orbit always remains elliptic.

If the mass loss does not occur instantaneously, this limiting value of $q$ decreases with increasing duration of the time over which the mass loss takes place. This point will be discussed in more detail in the following sub-section.

The fact that only $q < 0.5$ will cause hyperbolic relative orbits, and, hence, complete release of $M_2$ from $M_1$, does not mean that the cases $q \geq 0.5$ are without interest for the present problem. In such cases the binary character will be maintained, at least as long as the system is not dissolved due to perturbations, which may have to be taken into account if large separation of the components results. But the centre of gravity will assume the velocity, $S_g$, relative to the original centre of gravity, according to

$$S_g = \frac{M^\circ_1 \, M_2 \, (1-q)}{(M^\circ_1 + M_2)\,(q\, M^\circ_1 + M_2)} \, S_{rel,\,\circ} \, .$$

For instance, for $q = 0.7$ and $M_2/M^\circ_1 = 0.2$, we get

$$S_g = \tfrac{1}{18} \, S_{rel,\,\circ} \, .$$

Returning now to the consideration of cases of large instantaneous mass loss, we want to compare the hyperbolic relative velocity of $M_2$ to the initial relative velocity. This is given by

$$\frac{S^2_{rel,\,\infty}}{S^2_{rel,\,\circ}} = \frac{q\, M^\circ_1 + M_2}{M^\circ_1 + M_2} \cdot \frac{R_\circ}{a} = \frac{M^\circ_1 (1-2q) - M_2}{M^\circ_1 + M_2} \, .$$

Thus, for $q = 0.2$ and $M_2/M^\circ_1 = 0.2$, we get

$$S_{rel,\,\infty} = 0.58 \, S_{rel,\,\circ} \, .$$

In order to find the velocity of $M_2$ with respect to the original centre of gravity, we have to add vectorially to $S_{rel,\,\infty}$ the velocity of $M_1$ with respect to this centre of gravity. It appears from more detailed calculations referred to below that for the present case this vectorial addition is insignificant. We thus find that the hyperbolic velocity of $M_2$ with respect to the original centre of gravity will be about $0.6 \, S_{rel,\,\circ}$.

c) *Finite time of mass loss*

The problem of the motion of $M_2$ relative to the original centre of gravity for continuous mass loss by $M_1$, is treated in an exact manner by Boersma in the article following the present paper in this *Bulletin*. The notations for the quantities common to the two articles are the same; a list of them is given in Boersma's paper. The numerical computations were carried out with the ZEBRA electronic computer for a variety of values of the parameters involved. These are:

the initial mass ratio $M_2/M^\circ_1$,
the remnant mass, $M^f_1$, to which $M_1$ is reduced,
the rate of mass loss of the primary;

$\tau$ denotes the time which would be required for the original mass $M^\circ_1$ to reduce to zero, expressed in units of the orbital period of the original circular motion. In BOERSMA's article, the rate of mass loss is denoted by $C = 1/\tau$. The mass loss is always supposed to take place uniformly: $dM_1/dt = -CM^\circ_1$.

We further denote by

$\tau_f$, the time elapsed when $M_1$ is reduced to the remnant mass, $M^f_1$; $\tau_f = \tau \; (M^\circ_1 - M^f_1)/M^\circ_1$; it is expressed in the same unit as $\tau$.

For a number of values of the three dimensionless parameters: $M_2/M^\circ_1$, $M^f_1/M^\circ_1$, and $\tau_f$, the following quantities were computed:

A. *In case the motion remains elliptic*—that is, always for $M^f_1/M^\circ_1 \geq 0.5$, and in addition for sufficiently large values of $\tau$ and of $M_2/M^\circ_1$—:

1) the velocity, $s_g$, of the centre of gravity of the resulting binary, expressed in units of the original velocity of $M_2$ with respect to the original centre of gravity. This latter velocity will be denoted by $S^\circ_2$;

2) the semi-major axis, $a_e$, of the resulting binary, expressed in the radius of the original relative circular orbit as a unit.

B. *In case hyperbolic motion is reached*—that is, for sufficiently small values of the three parameters just mentioned—:

1) the velocity, $s_2$, of $M_2$ at infinity, or, for practical purposes, when $M_2$ is beyond the sphere of attraction of $M^f_1$. $s_2$ is taken with respect to the original centre of gravity, and expressed in the original velocity, $S^\circ_2$, as a unit;

2) the angle, $\psi$, between the direction of the motion of $M_2$ at infinity and the direction of the motion of $M^f_1$ at infinity, both with respect to the original centre of gravity.

It is shown by BOERSMA that the velocity of $M^f_1$ at infinity, to be denoted by $s_1$ (also expressed in $S^\circ_2$ as a unit), is independent of $M^f_1/M^\circ_1$ and of $\tau$ and only depends on $M_2/M^\circ_1$:

$$s_1 = M_2/M^\circ_1 .$$

The results of the computations are arranged in BOERSMA's tables pertaining to the original mass ratio's $M_2/M^\circ_1 = 0.8$, 0.6, 0.4, 0.3, 0.2, 0.1, 0.05, and 0.00, respectively (this last refers to infinitesimally small $M_2$). In each table, the part above the heavy line refers to the elliptic cases and the part below to the hyperbolic cases. For instance, consider the case of $M_2/M^\circ_1 = 0.2$ (BOERSMA's Table 4). We find that for $\tau = 0.5$, (mass loss of $M_1$ at the rate of its total mass in half of the original orbital period), mass reduction down to $M^f_1 = 0.5 \; M^\circ_1$ results in a binary with a semi-major axis three times larger than the original value, and the centre of gravity of this binary has a speed with respect to the original centre of gravity equal to 0.13 times the original velocity of $M_2$ with respect to this centre. For the remnant mass $M^f_1/M^\circ_1 = 0.4$, these quantities become $a_e = 9.0$ and $s_g = 0.18$. For $M^f_1/M^\circ_1 = 0.3$, the system is dissolved, $M_2$ now has a velocity at infinity, $s_2$, of 0.38 times its original velocity with respect to the common centre of gravity, and its direction deviates by 59° from the direction of motion of $M^f_1$. The latter's velocity with respect to the original centre of gravity is $s_1 = M_2/M^\circ_1 = 0.20$. For a reduced mass $M^f_1/M^\circ_1 = 0.1$, the velocity of $M_2$ becomes $s_2 = 0.69$, and $M^f_1$ moves in a direction making an angle of 73° with respect to $M_2$, at the speed $s_1 = 0.2$.

Inspection of the tables shows the following features:

1. For the large values of $M_2/M^\circ_1$ (0.8, 0.6) only very rapid mass loss of $M_1$ can cause release of $M_2$, and even then the remnant mass $M^f_1$ must be 0.2 $M^\circ_1$ or less. $M_2$ and $M^f_1$ move with respect to the original centre of gravity in directions making an angle of 50° or less. For the small values of $M_2/M^\circ_1$ (0.10, 0.05, 0.00) we get resolution of the system for $M^f_1/M^\circ_1 = 0.4$ or 0.3 unless $\tau$ becomes larger than 1.0.

2. The semi-major axis of the binaries, obtained for a series of decreasing values of $M^f_1$ for a given combination of $M_2/M^\circ_1$ and $\tau$, increases only slowly until resolution is approached. For almost all listed cases of elliptic orbits, $a_e$ is below 10. This implies that, roughly speaking, the mass ejection of $M^\circ_1$ either results in complete separation, or only slightly or moderately increases the separation of the components — as a rule by a factor of less than 10.

3. The velocity, $s_2$, which $M_2$ acquires in cases of complete release from $M_1$ is usually between 50 and 100 percent of the original velocity of $M_2$. It depends strongly on the rate of mass loss, i.e. on the time $\tau_f$ during which the mass loss takes place. For $\tau > 1.0$, $s_2$ is always smaller than 0.5.

d) *Interpretation of the observed velocities*

For the deduction of the initial properties of the proto-binaries from the observed characteristics of the high-velocity objects we shall use as a representative sample of these latter, four imaginary stars: two with an observed velocity of 150 km/sec and masses of 50 and 10 solar masses, and two with a velocity of 50 km/sec and the same two values of the mass. We want to investigate under which initial conditions such objects may have been formed.

The conditions are:

the original mass, $M^\circ_1$, of the primary,
the remnant mass, $M^f_1$, of the primary,
the rate of decay of the primary from $M_1$ to $M^f_1$,
the original separation $R_\circ$ of the proto-binary, circular orbits being assumed throughout.

The original period will be denoted by $P_o$.

Starting from specified values of $M^\circ_1$, $M^f_1$, $M_2$, and $\tau$ we find from BOERSMA's tables the corresponding quantity $s_2$. This, combined with the observed velocity $s_2 S^\circ_2$, gives $R_o$ and $P_o$ by means of the formulae

$$R_o = s^2_2 \left(\frac{30}{s_2 S^\circ_2}\right)^2 M_2 \frac{(M^\circ_1/M_2)^2}{1+M^\circ_1/M_2},$$

$$P_o = s^3_2 \left(\frac{30}{s_2 S^\circ_2}\right)^3 M_2 \frac{(M^\circ_1/M_2)^3}{(1+M^\circ_1/M_2)^2}.$$

From $\tau$, $M^\circ_1$, and $M^f_1$ we know $\tau_f$ and hence also $\tau_f P_o$, the decay time, expressed in years.

As an example of the results of these calculations we show in Table 9 the values of $R_o$ and of $\tau_f P_o$ corresponding to the production of released secondaries characterized by $M_2 = 50$ ⊙ and by $M_2 = 10$ ⊙, both with the "observed" velocity $s_2 S^\circ_2 = 150$ km/sec, in case such secondaries are the consequence of a decay from $M^\circ_1 = 1000$ ⊙ and from $M^\circ_1 = 250$ ⊙ or 200 ⊙. Results referring to $M^\circ_1 = 1000$ ⊙ are given only for $M_2 = 50$ ⊙, since practically the same values of $R_o$ and of $\tau_f P_o$ are obtained for $M_2 = 10$ ⊙. That the masses $M^\circ_1$ are chosen to be 250 ⊙ and 200 ⊙ for $M_2 = 50$ ⊙ and $M_2 = 10$ ⊙ respectively, reflects the fact that BOERSMA's tables offer data for $M_2/M^\circ_1 = 0.2$ and 0.05 respectively, and not, for instance, $M_2/M^\circ_1 = 0.04$ as would be required for the combination $M^\circ_1 = 250$, $M_2 = 10$. It did not seem worth-while to interpolate into BOERSMA's tables for this purpose. For each of the three sets of parameters, $M^\circ_1 = 1000$, $M_2 = 50$; $M^\circ_1 = 250$, $M_2 = 50$; and $M^\circ_1 = 200$, $M_2 = 10$, the resulting values of $R_o$ and of $\tau_f P_o$ are arranged according to the remnant mass $M^f_1$, and within these sub-divisions according to the dimensionless rate of decay, $\tau$. This is also an arrangement according to decreasing values of $R_o$.

It is of some interest to consider the run of the values of $R_o$ and of the decay time for a certain choice of $M^\circ_1$, $M_2$ and $M^f_1$ in Table 9. Only a certain range of values of these two quantities is allowed—for obvious reasons. But within this range, a certain decay time, $\tau_f P_o$ (expressed in years), occurs for two different values of $R_o$. Thus, for $M^\circ_1 = 250$ ⊙, $M_2 = 50$ ⊙, and $M^f_1 = 25$ ⊙, the speed 150 km/sec of $M_2$ may be the result of a decay time of 0.1 year for both $R_o$ around 6 a.u. and around 1 a.u. The explanation here is that for large $R_o$ the decay time of 0.1 year is a small

TABLE 9

Examples of values of the original radius, $R_o$, of the proto-binary system and of the decay time of the primary, as required for the production of released secondaries characterized by the velocity $s_2 S^\circ_2 = 150$ km/sec, $M_2 = 50$ solar masses and $M_2 = 10$ solar masses, for $M^\circ_1 = 1000$ solar masses and for $M^\circ_1 = 200$ or 250 solar masses

| | | | $M^\circ_1 = 1000$ ⊙ $M_2 = 50$ ⊙; $s_2 S^\circ_2 = 150$ km/sec | | | $M^\circ_1 = 250$ ⊙ $M_2 = 50$ ⊙; $s_2 S^\circ_2 = 150$ km/sec | | | $M^\circ_1 = 200$ ⊙ $M_2 = 10$ ⊙; $s_2 S^\circ_2 = 150$ km/sec | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{M^f_1}{M^\circ_1}$ | $\tau$ | $\tau_f$ | remnant mass, $M^f_1$ (⊙ masses) | original separation $R_o$ (astr. units) | decay time $\tau_f P_o$ (years) | remnant mass, $M^f_1$ (⊙ masses) | original separation $R_o$ (astr. units) | decay time $\tau_f P_o$ (years) | remnant mass, $M^f_1$ (⊙ masses) | original separation $R_o$ (astr. units) | decay time $\tau_f P_o$ (years) |
| | 0.0 | 0.00 | | 14.2 | 0 | | 2.39 | 0 | | 2.83 | 0 |
| | 0.2 | 0.14 | | 12.8 | 0.20 | | 2.12 | 0.024 | | 2.56 | 0.040 |
| 0.3 | 0.5 | 0.35 | 300 | 9.1 | 0.30 | 75 | 1.22 | 0.027 | 60 | 1.83 | 0.060 |
| | 1.0 | 0.70 | | 3.4 | 0.14 | | no release of $M_2$ for $\tau \geq 1.0$ | | | 0.69 | 0.027 |
| | 2.0 | 1.40 | | no release of $M_2$ for $\tau = 2.0$ | | | | | | no release of $M_2$ for $\tau = 2.0$ | |
| | 0.0 | 0.00 | | 22.0 | 0 | | 4.40 | 0 | | 4.40 | 0 |
| | 0.2 | 0.16 | | 20.3 | 0.45 | | 4.05 | 0.073 | | 4.06 | 0.090 |
| 0.2 | 0.5 | 0.40 | 200 | 14.6 | 0.69 | 50 | 2.67 | 0.098 | 40 | 2.93 | 0.138 |
| | 1.0 | 0.80 | | 7.4 | 0.49 | | 0.99 | 0.044 | | 1.48 | 0.098 |
| | 2.0 | 1.60 | | 1.2 | 0.07 | | no release of $M_2$ for $\tau = 2.0$ | | | 0.25 | 0.013 |
| | 0.0 | 0.00 | | 30.2 | 0 | | 6.43 | 0 | | 6.04 | 0 |
| | 0.2 | 0.18 | | 27.5 | 0.80 | | 5.86 | 0.143 | | 5.50 | 0.160 |
| 0.1 | 0.5 | 0.45 | 100 | 19.7 | 1.22 | 25 | 4.05 | 0.206 | 20 | 3.95 | 0.243 |
| | 1.0 | 0.90 | | 10.7 | 0.97 | | 1.96 | 0.139 | | 2.14 | 0.194 |
| | 2.0 | 1.80 | | 4.2 | 0.47 | | 0.38 | 0.023 | | 0.83 | 0.094 |
| | 0.0 | 0.00 | | 38.1 | 0 | | 8.50 | 0 | | 7.62 | 0 |
| | 0.2 | 0.20 | | 34.4 | 1.24 | | 7.51 | 0.231 | | 6.87 | 0.249 |
| 0.0 | 0.5 | 0.50 | 0 | 24.4 | 1.86 | 0 | 5.30 | 0.342 | 0 | 4.88 | 0.371 |
| | 1.0 | 1.00 | | 14.2 | 1.65 | | 2.76 | 0.257 | | 2.83 | 0.329 |
| | 2.0 | 2.00 | | 6.7 | 1.07 | | 1.10 | 0.130 | | 1.34 | 0.215 |

TABLE 10

Limiting values of $R_\circ$ and of the decay time $\tau_f P_\circ$ for the release of high-velocity secondaries with masses $M_2 = 10 \odot$ and $50 \odot$ and velocities $s_2 S^\circ_2 = 50$ km/sec and 150 km/sec. The allowed values range from zero to the values given in the table.

| $s_2 S^\circ_2$ | $M_2$ | remnant mass, $M^f_1$ ($\odot$ masses) | limit of $R_\circ$ (astr. units) | limit of decay time (years) | remnant mass, $M^f_1$ ($\odot$ masses) | limit of $R_\circ$ (astr. units) | limit of decay time (years) | remnant mass, $M^f_1$ ($\odot$ masses) | limit of $R_\circ$ (astr. units) | limit of decay time (years) | remnant mass, $M^f_1$ ($\odot$ masses) | limit of $R_\circ$ (astr. units) | limit of decay time (years) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 150 km/sec | 50 $\odot$ | | $M^\circ_1 = 83 \odot$ | | | $M^\circ_1 = 250 \odot$ | | | $M^\circ_1 = 1000 \odot$ | | | $M^\circ_1 = 5000 \odot$ | |
| | | 25 | — | — | 75 | 2.39 | 0.027 | 300 | 14.2 | 0.30 | 1500 | 79 | 1.82 |
| | | 17 | 0.75 | 0.00 | 50 | 4.40 | 0.098 | 200 | 22.0 | 0.64 | 1000 | 117 | 4.12 |
| | | 8 | 1.40 | 0.027 | 25 | 6.43 | 0.206 | 100 | 30.2 | 1.22 | 500 | 157 | 6.86 |
| | | 0 | 2.08 | 0.057 | 0 | 8.50 | 0.342 | 0 | 38.1 | 1.86 | 0 | 198 | 10.05 |
| | 10 $\odot$ | | $M^\circ_1 = 83 \odot$ | | | $M^\circ_1 = 200 \odot$ | | | $M^\circ_1 = 1000 \odot$ | | | $M^\circ_1 = 5000 \odot$ | |
| | | 25 | 0.96 | 0.016 | 60 | 2.83 | 0.060 | 300 | 15.7 | 0.36 | 1500 | practically the same values as for $M_2 = 50 \odot$ | |
| | | 17 | 1.62 | 0.043 | 40 | 4.40 | 0.138 | 200 | 23.5 | 0.82 | 1000 | | |
| | | 8 | 2.29 | 0.085 | 20 | 6.04 | 0.243 | 100 | 31.4 | 1.37 | 500 | | |
| | | 0 | 2.96 | 0.130 | 0 | 7.62 | 0.371 | 0 | 39.6 | 2.01 | 0 | | |
| 50 km/sec | 50 $\odot$ | | $M^\circ_1 = 83 \odot$ | | | $M^\circ_1 = 250 \odot$ | | | $M^\circ_1 = 1000 \odot$ | | | $M^\circ_1 = 5000 \odot$ | |
| | | 25 | — | — | 75 | 22 | 0.72 | 300 | 128 | 8.1 | 1500 | 707 | 49 |
| | | 17 | 6.8 | 0.00 | 50 | 40 | 2.64 | 200 | 198 | 18.6 | 1000 | 1057 | 111 |
| | | 8 | 12.6 | 0.72 | 25 | 58 | 5.54 | 100 | 272 | 32.9 | 500 | 1411 | 185 |
| | | 0 | 18.8 | 1.54 | 0 | 76 | 9.25 | 0 | 343 | 50.2 | 0 | 1782 | 271 |
| | 10 $\odot$ | | $M^\circ_1 = 83 \odot$ | | | $M^\circ_1 = 200 \odot$ | | | $M^\circ_1 = 1000 \odot$ | | | $M^\circ_1 = 5000 \odot$ | |
| | | 25 | 8.7 | 0.43 | 60 | 26 | 1.61 | 300 | 141 | 9.9 | 1500 | practically the same values as for $M_2 = 50 \odot$ | |
| | | 17 | 14.6 | 1.17 | 40 | 40 | 3.72 | 200 | 211 | 22.2 | 1000 | | |
| | | 8 | 20.7 | 2.30 | 20 | 54 | 6.57 | 100 | 282 | 37.1 | 500 | | |
| | | 0 | 26.7 | 3.52 | 0 | 69 | 10.00 | 0 | 356 | 54.2 | 0 | | |

fraction of the orbital period and therefore the amount of the retardation of the velocity in the release of $M_2$ is relatively small. For the alternative small value of $R_\circ$, the orbital velocity is large and therefore requires considerable deceleration, but this does indeed happen, because the chosen decay time is now large compared to the orbital period.

Table 10 summarizes the results of the calculations in a form more suitable for the purpose of the interpretation of the observed characteristics of the high-velocity objects. Here, we do not specify the various pairs of values of $R_\circ$ and $\tau_f P_\circ$ obtained for different decay rates, but we merely give for each chosen remnant mass the range of values of $R_\circ$ and of the decay time within which these quantities are allowed to lie. Thus, in the case of $s_2 S^\circ_2 = 150$ km/sec, $M_2 = 50 \odot$, $M^\circ_1 = 250 \odot$, and $M^f_1 = 25 \odot$ just referred to, we only quote the extreme values $R_\circ = 6.4$ a.u. and $\tau_f P_\circ = 0.206$ yrs. The top half of Table 10 refers to the production of the high-velocity objects with $s_2 S^\circ_2 = 150$ km/sec, the bottom half to those with $s_2 S^\circ_2 = 50$ km/sec. Within each of these, the upper division refers to $M_2 = 50 \odot$ and the lower one to $M_2 = 10 \odot$. From this table we derive the following information.

1. We first compare the results for $s_2 S^\circ_2 = 150$ km/sec with those for $s_2 S^\circ_2 = 50$ km/sec. The release of the former from proto-primaries with $M^\circ_1$ up to 1000 solar masses *always* requires decay times of two years or less. For instance, for $M^\circ_1$ around 200 solar masses, the mass loss must occur within four months if the desintegration of $M_1$ is complete, and within three months if $M_1$ reduces to 25 solar masses only. For $s_2 S^\circ_2 = 50$ km/sec, the permitted decay times are 27 times longer; moreover, the allowed range in $R_\circ$ is larger. The observed fact, that the number of high-velocity objects with $s_2 S^\circ_2 = 150$ km/sec is not much lower than the number with $s_2 S^\circ_2 = 50$ km/sec may be taken as evidence that the actually occurring decay times are as a rule of the order of one year or less.

2. For $s_2 S^\circ_2 = 150$ km/sec and $M^\circ_1$ up to 1000 solar masses, the allowed range of $R_\circ$ is roughly between 1 and 40 astronomical units, as compared to a range of from several to about 400 astronomical units for $s_2 S^\circ_2 = 50$ km/sec. Using KUIPER's frequency curves again for an estimation of the expected frequencies —however uncertain its validity for these massive binaries may be—we find that the integrated frequencies between 1 and 32 a.u. and between 10 and 320 a.u. are practically equal. We therefore do not expect these different allowed ranges of $R_\circ$ to cause a very large difference in the observed numbers for the two velocity groups. For $M_1 = 5000 \odot$ the allowed $R_\circ$ are mostly beyond 100 a.u.—here the expected frequencies rapidly decrease, but the applicability of KUIPER's curve becomes even more doubtful.

3. We consider finally the circumstances leading to

the release of masses $M_2$ of 50 ⊙ on the one hand, and 10 ⊙ on the other. Except for $M°_1$ about 80 solar masses, the differences are negligible. This implies that if for given values of $M°_1$ and $R_o$ there is a preponderance of secondaries with $M_2 = 10$ as compared to $M_2 = 50$, as one would expect if the ordinary initial luminosity function holds for these secondaries, this preponderance should be observed to occur in the same ratio among the high-velocity objects—unless another agency enters into its determination. The fact that, as was shown in section 3, the masses around 50 ⊙ are relatively more frequent by a factor ten as compared to those of 10 ⊙, shows that such an agency must indeed be active. In the next sub-section this will be identified with the different contraction times for the two kinds of objects.

e) *Influence of differences in contraction times*

According to current views on the process of star formation, the contraction of stellar bodies from the interstellar medium proceeds the faster as the contracting masses are larger. Accordingly, we must expect the massive primary of a proto-binary to have reached the stage of instability with which we have dealt in the preceding considerations, at a time when the stars of intermediate and small masses in the surrounding medium are still in earlier phases of contraction. It seems plausible to look for an explanation of the relatively large number of large masses among the released secondaries in terms of this effect. For its proper evaluation we require a better knowledge of the earliest history of the double and multiple stars than is available at present. Broadly speaking, two possibilities may be distinguished:

1) The prospective double or multiple system was, from the outset, dynamically isolated from other systems in the process of formation. This is the case, for instance, with the solar system as it developed according to the monistic theories, which find their most recent development in KUIPER's work (see, for instance, KUIPER 1951). In this case, primary and secondary components originated simultaneously.

2) Double or multiple systems are the result of capture as a consequence of multiple encounters. To what extent this process may have been active deserves further investigation, particularly in view of recent numerical calculations on the dynamics of clusters within which double stars appear to result in this way. For our present considerations it is important that we need not think in terms of bodies whose contraction has already been completed; the capture theory must also be applicable to stellar bodies in the early phase of contraction, as soon as they have emerged from the interstellar medium as independently moving systems. (The investigation of the capture effect in the still earlier phase, when the braking effects of the interstellar medium on the stellar condensations are not yet negligible, may also be promising for the understanding of the formation of double and multiple stars.) In these cases we are dealing with a primary and a secondary which did not originate strictly simultaneously.

With these two possibilities in mind, let us first consider the contraction times as a function of the stellar masses. These times, $t_c$, are given in the fifth column of Table 11 for bodies of 250, 50, 10, 5 and 2 solar masses. They are based on the formula

$$t_c = 10^8 \left(\frac{M}{M_\odot}\right)^2 \frac{R_\odot}{R} \frac{L_\odot}{L} \text{ years},$$

where the constant factor, $10^8$, has been chosen somewhat larger than the minimum value $6.3 \times 10^7$ which holds in the absence of burning of light elements (SANDAGE 1958). The radii $R$ and the luminosities $L$ for stars which have just reached the main sequence are based on the model computations by SCHWARZSCHILD and HÄRM (1958) and by HENYEY, LE LEVIER, and LEVEE (1959). Those given by the former authors, referring to the largest masses, were slightly modified in order to obtain agreement with the results of the latter authors in the overlapping region around masses of 30 ⊙.

We are primarily interested in the ratios of these contraction times for the various masses. The con-

TABLE 11

Contraction times for masses 2 to 250 solar masses

| mass (⊙ masses) | adopted luminosity $L/L_\odot$ | adopted main-sequence radius $R/R_\odot$ | adopted main-sequence radius $R_{ms}$ (astr. units) | contraction time, $t_c$ (years) | $R_x$ (astr. units) | $\frac{M_2}{250} \cdot R_x^{-3}$ |
|---|---|---|---|---|---|---|
| 250 | $5.0 \times 10^6$ | 26 | 0.121 | $0.48 \times 10^5$ | 1.00 | 1.0 |
| 50 | $4.0 \times 10^5$ | 10 | 0.046 | 0.63 × ,, | 0.56 | 1.14 |
| 10 | $7.25 \times 10^3$ | 4.3 | 0.020 | 3.2 | 1.08 | $3.2 \times 10^{-2}$ |
| 5 | $7.4 \times 10^2$ | 2.7 | 0.013 | 12.6 | 2.07 | $2.2 \times 10^{-3}$ |
| 2 | $2.1 \times 10$ | 1.7 | 0.0079 | 115 | 3.99 | $1.3 \times 10^{-4}$ |

traction times are found to be about the same for masses of 250 ⊙ and 50 ⊙. For masses of 10 ⊙ the contraction time is already seven times longer than that of 250 ⊙, and for 5 ⊙ it becomes about 26 times longer. These figures suggest in a general way the explanation of the large fraction of large masses among the released secondaries: if we suppose the massive proto-binary to have been formed by the capture process at the earliest stage of condensation in the interstellar medium, then the supply of sufficiently stable secondaries of masses 10 ⊙ and smaller may not yet have been as large as would correspond with the numbers of such masses according to the initial luminosity function. Some evidence for even larger contraction times for the less massive B stars than the values of Table 11 will be mentioned in section 8c.

A somewhat more detailed consideration deserves to be mentioned, especially in connection with the monistic hypothesis. Let us consider a model in which at a certain stage a proto-binary with $M_1 = 250$ ⊙ has formed, with both components having radii of about 5 astronomical units, and let us suppose that these are both contracting according to the rate which holds for isolated objects. Let us further assume that the sudden mass ejection of $M_1$ occurs when it has contracted down to a radius of 1 a.u.—i.e. about 8 times its main-sequence radius as given in Table 11. We now consider the condition of the secondary at this instant, for various masses $M_2$: 50 ⊙, 10 ⊙, 5 ⊙, and 2 ⊙. The time elapsed during the contraction of $M_1$ from 5 to 1 a.u. is given by

$$\frac{\triangle t}{t_c} = \left(\frac{1}{R_2} - \frac{1}{R_1}\right) \Big/ \frac{1}{R_{ms}},$$

where for $R_2$ and $R_1$ we must substitute 1 and 5 astronomical units respectively, and $R_{ms}$ is the main-sequence radius. We find:

$$\triangle t = 0.0465 \times 10^5 \text{ years.}$$

The radius $R_x$ of the secondary $M_2$ will then be given by

$$\frac{R_{ms}}{R_x} - 0.2\, R_{ms} = 0.0465 \times 10^5 / t_c\,,$$

where $t_c$ now refers to the contraction time of $M_2$. The values of $R_x$ are in the 6th column of Table 11. Note that $R_x$ for $M_2 = 50$ ⊙ is smaller than that for $M_1 = 250$ ⊙, but that for the smaller values of $M_2$, $R_x$ is larger.

The last column of Table 11 gives the average density of $M_2$, in units of the density of $M_1$ at the instant of the explosion. We find that the density of a proto-secondary with mass 50 ⊙ is about the same as that of $M_1 = 250$ ⊙, but that the densities of the lighter secondaries are considerably lower.

This model points to an alternative way of accounting for the scarcity of low masses among the released secondaries: it suggests that, while $M_2 = 50$ ⊙ will be sufficiently dense to survive the explosion of $M_1$, masses $M_2$ of the order of 10 ⊙ or less will have a much smaller chance of survival. If the distance between the centres of $M_1$ and $M_2$ is of the order of 20 astronomical units, the density of the expanding shell when it reaches $M_2$ will be of the order of $20^{-2}$ to $20^{-3}$ in units of the original density of $M_1$, i.e. $2.5 \times 10^{-2}$ to $1.2 \times 10^{-3}$. This is low compared to the density of $M_2 = 50$ ⊙ in the last column of Table 11, but it is of the same order as that of $M_2 = 10$ ⊙, so that we should expect the latter and secondaries of lower mass to be disrupted in the process.

## 7. *Relation with supernovae of type II*

### a) *Identification with supernovae of type II*

According to the preceding sections, the hypothesis of the proto-binary origin of the high-velocity objects implies that as a rule the mass ejection of the proto-primary should take place within a lapse of time not exceeding the orbital period; for instance, for proto-primaries of around 200 solar masses it should happen within several months to provide that a secondary be released with a velocity of 150 km/sec. We shall consider some aspects of the possible identification of the process involved with that of the supernovae—and particularly with those of type II because of their pronounced association with Population I. Type II supernovae are found mostly in spiral arms (see, for instance, ZWICKY 1958) and according to PAYNE-GAPOSCHKIN (1957) all nine supernovae of type II which have been identified as such occur in spirals of types Sc, SBc, Sb or SBb. (Of 16 objects identified as type I, 9 occur in systems of these types.) The close association of supernovae type II with Population Type I fits entirely, of course, with our hypothesis which connects the phenomenon directly with the process of the star formation. Also, the rate at which the supernova phenomenon proceeds seems to meet our requirement well. We find from the composite light-curve of three well observed supernovae of type II, reproduced by PAYNE-GAPOSCHKIN, that brightness within five magnitudes from the maximum lasts only for about four months. Further, it seems plausible that the bulk of the ejected material has passed beyond the orbit of the secondary within this lapse of time. Matter with ejection velocities ranging from 5000 to 500 km/sec will have passed beyond 20 a.u. from the primary in the period between 7 and 70 days after the onset of the explosion if all these velocities are imparted to the material simultaneously. The decay time of $M_1$ as experienced by $M_2$ in the sense of the preceding kinematic considerations is then of the order $\tau_f P_0 = 70$ days or less.

b) *Comparison with frequency of supernovae*

The proposed identification of the origin of the high-velocity objects with the supernovae of type II implies, among other tests, that the observed number of high-velocity objects can be related to the frequency of the occurrence of supernovae. As a basis for the estimated rate of production of high-velocity objects we take those of Table 1 within 1000 ps from the sun and with kinematic ages below 5 million years. For some of the objects in the table, no ages are known. An estimate may be made on the basis of the ages found for objects of similar luminosity. We shall assume that 10 of the objects in Table 2 fall within the limits just mentioned. Even within these limits of age and distance, the table is incomplete. Objects of the lower luminosities are only partly known at distances between 500 and 1000 ps. Further, from the statistics of the recorded ages, it appears that those between 3 and 5 million years are less completely represented than the younger ones. An, admittedly rather uncertain, factor of 4 will be tentatively adopted to take account of these two causes of incompleteness. We further notice that no objects younger than one million years occur in the table. Apparently, these do not yet show within the limits of spectral type and luminosity considered (see also section 8c). We thus are led to the estimate that within 1000 ps from the sun, 40 high-velocity O to B5 objects occur with kinematic ages of between 1 and 5 million years.

The origin of the high-velocity objects may be supposed to lie close to the galactic plane. The majority of them will be sufficiently close to the galactic plane within the first 5 million years to remain within the distance limit of our selection. We are therefore justified in assuming that the number of high-velocity objects which is *formed* per 4 million years within 1000 ps is also given by the above figure. Hence the number of events which release O to B5 high-velocity objects within 1000 ps from the sun is one per $10^5$ years. In extending this to the Galaxy as a whole, we assume a factor $10^2$, which leads to one such event per $10^3$ years. If the event is identified with the type II supernovae, the true number of these supernovae will be larger, since only a fraction of them may lead to the release of secondaries in the range of masses encountered among the high-velocity objects. This factor is very hard to estimate, but we are inclined to tentatively estimate it to be of the order of 10—assuming a distribution of masses of the proto-secondaries similar to that of the initial luminosity function in general, and taking into account the tendency of massive objects to have more than one companion. These very rough estimates then lead to a prediction of one type II supernova in the Galaxy per 100 years, and one per $10^4$ years within 1000 ps from the sun.

These figures compare reasonably well with current estimates of the actual occurrence of type II supernovae, which are about 1 per 40 years for the Galaxy as a whole (ÖPIK 1953, PAYNE-GAPOSCHKIN 1957).

c) *Supernovae in associations*

If the proposed identification of the origin of the high-velocity objects with the supernovae is accepted, it leads to a number of interesting conclusions with regard to the phenomena in stellar associations. The possibility of a close relation between the structure of stellar associations and the supernova phenomenon has been discussed before. Thus, ÖPIK (1953, 1955) has suggested that the stars observed in the associations were formed in the interstellar matter after this was compressed by an expanding supernova shell. SHKLOVSKY (1960), too, has suggested that type II supernovae occur in OB associations, but this author rather considers them as the product of the later evolution of these stars. The mechanism proposed by ÖPIK might also explain the expanding motions of the stars observed in some associations. We shall not discuss in the present context the extent to which the supernova phenomenon may have been responsible for subsequent star formation in the association in which the supernova occurs, but we shall discuss in a provisional way a few other aspects of the relation between supernovae and stellar associations as inferred from the present hypothesis.

The fact that a number of high-velocity objects have originated from the same association but with different kinematic ages shows that in some associations supernovae of type II must have occurred repeatedly. In general, theories accounting for the structure of the OB associations should then take into account the fact that series of supernova explosions may have taken place, spread over an interval of time which may be as long as 10 million years and more.

*The Orion association*

Thus, according to Table 1, supernova explosions must have occurred in the Orion association at epochs 4.9, 2.7, and 2.2 million years in the past. We might perhaps identify the latter two as one explosion in which the two objects, AE Aur and $\mu$ Col, were released from one proto-multiple star; the individual age determinations could well be reconciled with a common age of 2.5 million years.

A farther-reaching hypothesis involving simultaneous origin with equal and opposite motions for these two stars was advanced by BLAAUW and MORGAN (1954). This hypothesis would be satisfied somewhat better on the basis of the then deduced distance of the Orion association, 500 ps, than with the now assumed distance of 420 ps. But even with this latter distance, these two stars as a pair remain remarkable because of their great similarity in mass, spectral type, and space velocity, and the approximately opposite directions of these velocities. Such a system would be produced by the explosion of a proto-primary accompanied by two proto-companions symmetrically placed with respect to the primary. Whether or not such a configuration is as unlikely as it may seem to be at first sight deserves further investigation.

In any case, there must have been at least two supernova explosions in Orion and probably more because not all of them might have given rise to observed high-velocity objects. ÖPIK (1953) has suggested that the huge Barnard ring in Orion results from a supernova explosion. SAVEDOFF (1956) considered this unlikely because the momentum which the ring carries according to MENON's (1958) 21-cm analysis is about $10^6$ ⊙ mass km/sec; with an initial velocity of expansion of several thousands km/sec this would correspond to an ejected shell of several hundred solar masses, which SAVEDOFF considered unlikely. However, we notice that this is the same order of mass as that deduced for the proto-primaries in general in the preceding section. Moreover, we should now consider the observed velocity of the matter in the BARNARD ring as the accumulated effect of a series of explosions, each of which must have contributed momentum to the ring. This makes the requirements for the individual supernovae less demanding.

The repeated injections of high-velocity supernova shells may also have interesting implications for the state of motion in the dense parts of the Orion nebula. Shock waves, travelling the ionized inner portions of the nebula with a velocity of 10 km/sec will travel 10 ps in $10^6$ years. This is of the order of the size of the dense part of the Orion nebula. The total frequency of supernovae in Orion as inferred from the 2 or 3 identified cases may be estimated to be of the order of at least one per $10^6$ years. Thus the state of motion in this nebula may be governed to a large extent by these shock waves, and it is interesting to notice that this finds some support by the provisional analysis of radial-velocity observations by WILSON, MÜNCH, FLATHER, and COFFEEN (1959).

### *The association I Scorpii*

Three stars of Table 2 originated from the association I Sco, which contains the open cluster NGC 6231. One of them, 72 Col, has an expansion age of 14 million years. For the other two the expansion ages are not known, but they probably are much less, especially for the O7-8 star HD 152408. The cluster NGC 6231 cannot be older than a few million years, as judged from its HR diagram.

The true number of high-velocity objects which have been emitted by this association is probably much larger—considering the large distance of the association from the sun (2100 ps) as a result of which many of the objects must have passed beyond detection. Also, O-type stars which may have left the association during the first half of the period of about 15 million years for which it must have existed probably will no longer appear as bright O-or early B-type stars. Ten explosions over the elapsed period of the existence of the association may be a conservative estimate.

The association has been described by MORGAN, GONZÁLEZ, and GONZÁLEZ (1953). Attention is drawn by these authors to the interesting structure, with NGC 6231 as a nucleus near one of the extremes of the aggregate, and to the relation with the emission nebula IC 4628. Figures 2 and 3 of the paper by MORGAN *et al.* are particularly revealing. The position and form of the emission nebula strongly suggest that we are dealing with remnants of one or more supernova explosions which in that particular region of the interstellar medium have been slowed down by local interstellar matter. The structure of the bright rims in the nebula suggests its being composed of a number of sections of ejected shells. Their centres of curvature all lie in the direction towards the centre of the association, which is also approximately the direction towards NGC 6231. The bright rims are all located

TABLE 12

Present velocity of the shells, possibly ejected from NGC 6231, as recognized in IC 4628, and interstellar density along their path, as inferred for an assumed mass of 200 solar masses of the entire nova shell and an initial shell velocity of 5000 km/sec

| | time elapsed since the explosion | | |
|---|---|---|---|
| | $10^5$ yrs | $10^6$ yrs | $5 \times 10^6$ yrs |
| mean velocity, $\bar{v}$ (km/sec) | 650 | 65 | 13 |
| present velocity (km/sec) | 158 | 16.2 | 3.2 |
| angular velocity (″/yr) | 0″.016 | 0″.0016 | 0″.00032 |
| original interstellar density (gm/cm³) | $3.6 \times 10^{-24}$ | $3.6 \times 10^{-23}$ | $1.8 \times 10^{-22}$ |

at the opposite side of these shell fragments, suggesting collision with the interstellar medium. The projected distance of the emission nebula from the centre of the aggregate is about 34 ps, and that from NGC 6231 about 65 ps.

It is of some interest to estimate the present velocity of the emission nebula and the inferred original density of the interstellar medium it has swept up for various assumptions of the time elapsed since the explosion(s) occurred which ejected this shell. The results are given in Table 12. They are based on the assumption that a supernova occurred in the region of NGC 6231, and that the original mass of the entire nova shell was 200 solar masses and the ejection velocity 5000 km/sec. The adopted age and the present position of the shell give its mean velocity $\bar{v}$ over the time elapsed since the outburst, and this, combined with the shell's initial mass and velocity, determines the accreted mass, $m_a$, according to formula (5). For elapsed times of $10^5$ and $10^6$ yrs, we find an original smeared-out interstellar density along the path of the shell fragments now in IC 4628, equal to 4 and 36 times the average interstellar density of $10^{-24}$ respectively. A lapse of time of $10^6$ yrs or less since the outburst appears then quite reasonable. The fact that we do not recognize other shell remnants in the area may imply that in these regions the interstellar density was lower, perhaps due to cleaning by previous supernova shells.

### *The association I Persei (h and χ Persei)*

The absence of gas both around the nuclei *h* and χ of the large Perseus association and from the general area of the association, as revealed by the absence of absorption and emission nebulosities (see, for instance, the Palomar charts) has been a puzzling phenomenon in view of the fact that the association should be a relatively young, rich formation. We should like to suggest that this is due to the cleaning effect of a large number of supernova outbursts which may have originated in this association.

The fact that a large number of early-type supergiants occur in this association suggests that conditions for the formation of objects with large masses must have been exceptionally favourable. Thus, JOHNSON and MORGAN's (1955) study of photometry and spectral classes in a restricted region around the double cluster gives 39 stars brighter than visual absolute magnitude $-4$ if the distance modulus is 11.8 (JOHNSON and IRIARTE 1958) and JOHNSON and HILTNER's (1956) list of objects in a more extended region around the double cluster even contains twice as many such objects. On the other hand, the corresponding number in the associations II Per, I Lac, I Ori, and Sco-Cen taken together is only 17 (BLAAUW 1958b). Judging from the number of high-velocity objects ejected by these latter associations, it seems not unlikely that at least 15 such objects have been ejected from the *h* and χ Per association, and that the true number of supernova explosions which have occurred in it is several times larger, at least 50. This should have caused most of the interstellar matter in the area of the association to be swept outwards. Current 21-cm radio observations may perhaps throw more light on this hypothesis.

### *The association II Scorpii*

One high-velocity object, ζ Oph, is known to have originated from this association about one million years ago. More of them may have been ejected earlier, especially from the southern part of the Scorpio-Centaurus association, the age of which is at least 10 million years (BLAAUW 1959, HOYLE 1960), although, on the other hand, this association, contrary to *h* and χ Per, contains only a few supergiants brighter than $M_v = -4$.

A peculiar feature of the youngest, most northern part of the association, centred around ρ Ophiuchi, are the long dark lanes which seem to have been stretched outwards from a central mass in the upper Scorpio region (see, for instance, photographs Nos. 13 and 14 of BARNARD's (1927) atlas). It is tempting to think that these structures have resulted as a consequence of the outflow of matter from one or more supernova outbursts in this region.

## 8. *Miscellaneous remarks*

### a) *The absolute magnitudes of type II supernovae*

A photographic absolute magnitude of $-14$ at maximum is usually adopted for type II supernovae and the total amount of energy emitted in the outburst in photographic and photovisual light is given as about $10^{47}$ ergs (see, for instance, PAYNE-GAPOSCHKIN 1957). It has been felt as a serious objection to the assumption of large masses of supernovae that the kinetic energy of the ejected mass is several orders of 10 larger than this value. This objection might be raised especially against the very large masses proposed in the present paper. For instance, if a shell of 200 solar masses is ejected with an average velocity of 3000 km/sec, its kinetic energy is $1.8 \times 10^{52}$ ergs. However, the quoted absolute magnitude of the supernova does not take into account the interstellar absorption in the region surrounding the nova. If the present hypothesis is correct and the type II supernovae are to be identified with the supermassive proto-stars in their contraction stage, then we should expect them to occur in heavily absorbing surroundings. The amount of absorption to be expected may be estimated from the densest dark clouds. Typical examples would seem to be the dense nebulae near ρ Ophiuchi and in Taurus. For the former, Bok (1956), from counts on Palomar 48-inch plates, finds absorptions up to 8 magnitudes and estimates its central density to be about 1000 times the average density near the galactic plane. It would not seem excessive to assume that the massive proto-binaries occur within clouds absorbing 10 magnitudes and more, and that the true absolute magnitudes of the type II supernovae are that much brighter than the value quoted. The total energy emitted in visual and photographic light then becomes comparable to the kinetic energy.

b) *The remnants of the proto-primaries*

In our discussion of the supernova process, we have reckoned with the possibility that the proto-primary may not be entirely annihilated and a remnant mass $M^f_1$ remains. It would be most desirable, of course, for testing the present hypothesis, if such remnants could be identified. It follows from the simplified kinematic considerations of section 6, that the remnant mass should have a velocity with respect to the original centre of gravity of the proto-binary, equal to $S^\circ_2 M_2/M^\circ_1$, and particulars about its direction follow from Boersma's tables. The remnant will not be located in the centre of the supernova shell. According to current views on the evolution of supernovae, these remnants are probably subluminous stars of early spectral type.

One star which may perhaps have to be identified with such a supernova remnant is X Persei (HD 24534) in the association II Per. Its MK spectral type is Ope. $U,B,V$ photometry has been published by Harris (1956) and by Hiltner and Johnson (1956), who give $U-B = -0.82$, $B-V = +0.29$ and $U-B = -0.82$, $B-V = +0.31$ respectively. We adopt $U-B = -0.82$, $B-V = +0.30$. These colours do not fit the regular reddening curve of O-type stars, which should give values like $U-B = -0.82$, $B-V = +0.12$ or $U-B = -0.70$, $B-V = +0.30$. For an evaluation of the visual absorption we therefore adopt the $B-V$ excess of the neighbouring star, ζ Per, $E_{B-V} = 0.38$, which gives for X Per the visual absolute magnitude $M_V = -2.3$. This is about the absolute magnitude of a B2V star, and 2.5 magnitudes below the main-sequence luminosities of the late O-type stars. X Per is, moreover, an irregular variable star with a range of variability of 0.5 magnitude. It is also a spectrum variable (McLaughlin 1932). Its position in the II Per association as well as its radial velocity leave little doubt about its membership of the group. It is tempting to think that X Per is connected with the high-velocity star ξ Per on the proto-binary hypothesis. Unfortunately, its proper motion and, hence, its space motion, is not well determined due to a discrepancy between the old and modern meridian observations (Delhaye and Blaauw 1953). This makes it impossible to infer the original characteristics of the proto-system.

Another interesting case is the O-type subdwarf GS 259-8 in the region of the I Lac association, described by Münch and Slettebak (1959), which according to these authors might well belong to the association as judged from the radial velocity. No proper motion is available for this star.

c) *Contraction times of early B stars*

According to Table 11, the contraction times for the early B stars are $3 \times 10^5$ yrs for mass 10 ⊙ (approximate MK type B2) and $1.2 \times 10^6$ yrs for mass 5 ⊙ (approximate MK type B5). We want to draw attention to an apparent discrepancy between these figures and the evidence from Table 1.

Of the 8 B2—B5 stars in Table 1, two have well-established kinematic ages of 4.9 and $14 \times 10^6$ yrs, two have rather uncertain values of 10 and $2.7 \times 10^6$ yrs, and for four objects these ages are not known. Preliminary investigation of the possible origin of these latter stars indicates that their ages must be rather high, of the order of $10^7$ yrs. It is surprising that no case of a B-type high-velocity object is known with a well-established kinematic age below $4 \times 10^6$ yrs, in view of the fact that 5 such cases are known among the O to B0 stars. The shorter the kinematic ages, the easier the objects are to relate to their origin. Thus, B stars ejected from the Orion association within the last 4 million years can hardly have escaped attention, and the same holds for the still nearer II Scorpii association. The statistics of section 3 suggests that we should have encountered at least two such young B stars.

The interpretation of the lack of such objects would seem to be that the contraction age—or more precisely, the stage during which these stars do not present themselves as early B stars with the regular luminosities—*may* be considerably longer than the quoted contraction times. That this is not a general rule, and that B2—B5 stars of ages about one million years do occur elsewhere, follows from their presence in the young II Per association.

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