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Wolf–Rayet Stars with Relativistic Companions

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Abstract—The evolution of close binary systems containing Wolf–Rayet (WR) stars and black holes (BHs) is analyzed numerically. Both the stellar wind from the donor star itself and the induced stellar wind due to irradiation of the donor with hard radiation arising during accretion onto the relativistic component are considered. The mass and angular momentum losses due to the stellar wind are also taken into account at phases when the WR star fills its Roche lobe. It is shown that, if a WR star with a mass higher than $\sim 10 M_{\odot}$ fills its Roche lobe in an initial evolutionary phase, the donor star will eventually lose contact with the Roche lobe as the binary loses mass and angular momentum via the stellar wind, suggesting that the semi-detached binary will become detached. The star will remain a bright X-ray source, since the stellar wind that is captured by the black hole ensures a near-Eddington accretion rate. If the initial mass of the helium donor is below $\sim 5 M_{\odot}$, the donor may only temporarily detach from its Roche lobe. Induced stellar wind plays a significant role in the evolution of binaries containing helium donors with initial masses of $\sim 2 M_{\odot}$. We compute the evolution of three observed WR–BH binaries: Cyg X-3, IC 10 X-1, and NGC 300 X-1, as well as the evolution of the SS 433 binary system, which is a progenitor of such systems, under the assumption that this binary will avoid a common-envelope stage in its further evolution, as it does in its current evolutionary phase.

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1. INTRODUCTION

Tutukov and Yungelsona [1] proposed a general scenario for the evolution of massive close binary systems describing both the primary and secondary mass transfer between the components. The secondary mass transfer should occur in a common-envelope regime. This process leads to the formation of either a short-periodic close binary composed of a Wolf–Rayet (WR) star and a relativistic object (WR + C, where "C" stands for a compact object), or a Thorne–Żytkow object [2] (after the merger of the close-binary components). The original binary explodes as a supernova, possibly resulting in a high spatial velocity for the WR + C binary, which can achieve appreciable distances from the Galactic plane, where these close binaries arise.

First attempts to search for WR + C binaries among WR stars lying high above the Galactic plane and localized at the centers of ring nebulae (see, e.g., the catalog of highly evolved close binaries [3]) were not successful, since these binaries had very low X-ray luminosities ($L_x \sim 10^{31}$–10^{33} erg/s), as was shown in subsequent studies. This did not correspond well with a WR + C binary model, in which a compact relativistic companion accreting matter from the intense stellar wind from a WR star should be a strong source of X-rays ($L_x > 10^{36}$ erg/s). However, three reliably identified WR + C binaries were subsequently found—one in our own Galaxy (Cyg X-3) and two in other galaxies (IC 10 X-1 and NGC 300 X-1).

The question why there are so few such binaries is of considerable interest. This problem has been considered in a number of studies (see, e.g., [4, 5]). The results of population syntheses [4] have indicated that $\sim$200 WR + C binaries containing black holes and $\sim$500 WR + C binaries containing neutron stars may exist in our Galaxy at the current epoch. However, after analyzing the possible number of WR + C binaries with parameters similar to those of Cyg X-3, Lommen et al. [4] concluded that only one such system may exist in the Galaxy now. The discrepancy between the large number of O/B+C binaries that could be progenitors of WR + C systems and the small number of observed WR + C binaries is analyzed in [5]. The major factor hindering an O/B + C binary from being transformed into a WR + C system is believed to be a merger of the components in the common-envelope stage [5]; however, studies of this process are complicated by the lack of detailed knowledge of the physics relevant for this stage.

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Lommen et al. [4] and Linden et al. [5] also note that a considerable number of WR + C binaries cannot be discovered, since they do not have high X-ray luminosities due to the absence of accretion disks (in particular, the propeller effect can hinder accretion in binaries with neutron stars).

The characteristic properties of the evolution of WR + C binaries with various component masses and initial orbital semi-major axes are of interest. The purpose of our current study is to elucidate these properties using numerical modeling and, particularly, to determine the effect of hard radiation that arises during accretion onto the relativistic object and irradiates the donor star. We restrict our consideration to close binaries containing BHs. We consider both massive WR stars and lower-mass helium stars with masses of $2-5M_\odot$ as donors in these systems. Numerical modeling of the binary evolution begins when a helium star burning helium in its central region forms at a some distance from the BH. Evolutionary scenarios for binaries containing WR stars and BHs are compared to those for binaries of the same type whose donors are main-sequence or slightly evolved stars.

Section 2 describes the properties of observed WR + C binaries. Section 3 presents the method used to compute the evolution of close binaries with relativistic components. The results of numerical computations of WR + C binary evolution in various cases are given in Section 4. Section 5 considers the evolutionary statuses of observed WR + C binaries.

2. OBSERVED WR + C BINARIES

The first close binary that was reliably identified with the WR + C stage was the peculiar, very short-period (the orbital period is $P \approx 4.8^{\mathrm{h}}$) X-ray binary Cyg X-3, which is a strong source of X-rays ($L_x \approx 10^{38}$ erg/s) and contains an optical WR donor of spectral type WN 3–7 [6]. The main parameters of this binary are available in [3]. It is striking that Cyg X-3 not only confirms the correctness of the theory of massive close-binary evolution [1], but also clearly indicates that the WR star in this binary is a helium star with a small radius ($R < 3.2-5.6R_\odot$), since the short orbital period (4.8^{h}) corresponds to a radius for the relative orbit of $a < 5.6R_\odot$ [7]. Cyg X-3 is currently the only WR + C binary identified in our Galaxy.

The entry into operation of new, large 8–10 m optical telescopes has stimulate optical studies of X-ray binaries in other galaxies. Two new WR + C binaries that are sources of strong X-rays ($L_x \approx 10^{38}$ erg/s) were recently identified. These are the systems IC 10 X-1 ($P \approx 1.46^{\mathrm{d}}$) [8] and NGC 300 X-1 ($P \approx 1.35^{\mathrm{d}}$) [9], n the galaxies IC 10 and NGC 300. Therefore, three WR + C binaries have been discovered thus far, providing reliable observational data to verify the theory of the late stages of the evolution of massive close binaries [1].

Studies of the unique X-ray binary—microquasar SS 433 [10] have also contributed to observational investigations of late stages of the evolution of massive close binaries. This binary is in the final stage of the secondary mass transfer; however, no common envelope has formed, and the binary is losing its excess mass, angular momentum, and energy via a supercritical accretion disk and jets (see [3]).

The table presents the parameters of four well-known massive close binaries that are in the final stage of their secondary mass transfer (SS 433) or beyond the termination of this stage (Cyg X-3, IC 10 X-1, and NGC 300 X-1). The mass function of the optical star $f_v(m)$ in Cyg X-3 was derived from the radial-velocity curve, which was constructed using an absorption feature in the IR spectrum of the WN 3–7 star [11]. The passage of this radial-velocity curve through the γ-velocity (the velocity of the binary barycenter) coincides with the minima of the X-ray and IR light curves. This confirms that the radial-velocity curve of the optical star in Cyg X-3, based on an absorption feature, reflects the orbital motion of the WN 3–7 star.

The radial-velocity curves of the optical stars in the binaries IC 10 X-1 and NGC 300 X-1 were based on the HeII λ4686 emission line. For these binaries, there is no reliable information about whether or not the passages of the radial-velocity through the γ-velocity coincide with X-ray minima. Therefore, we cannot be completely sure that the radial-velocity curves of the optical stars in these systems reflect the orbital motions of the optical stars.

In all three cases (Cyg X-3, IC 10 X-1, and NGC 300 X-1), the component masses have not yet been determined very reliably. However, the huge X-ray and bolometric optical (for SS 433) luminosities ($L_x \sim 10^{38}-10^{39}$ erg/s) undoubtedly indicate that there are accreting relativistic objects in these binaries, most likely a BH in SS 433 [10]. BHs are also preferred in the other cases, although neutron stars cannot be completely ruled out so far.

3. METHOD FOR COMPUTING THE EVOLUTION OF CLOSE BINARIES WITH RELATIVISTIC COMPONENTS

We have numerically studied the evolution of binary systems that are composed of WR stars (or lower-mass helium stars) and BHs. For comparison, we have also considered binaries containing BHs

The parameters of the (WR + C) and SS 433 binaries

Binary	Spectral type of the optical star	Orbital period, days	e	L_x, erg/s	$f_v(m)$, $M_\odot$	i, deg	M_v, $M_\odot$	M_x, $M_\odot$
Cyg X-3	N 3-7	0.19968462(6)	0	10^{38}	0.027	>60	<70:	$\sim$10
IC 10 X-1	WNE	1.4554(2)	0	10^{38}	7.64 ± 1.26:	$\sim$90	26 ± 9	28 ± 5
NGC 300 X-1	WN5	1.346(8)	0	2×10^{38}	2.6 ± 0.3	60–75	15–26	14.5–20:
SS 433	A7I	13.0821(1)	0	10^{36} ($L_{bol} = 10^{39}$)	0.268	78.81	$\sim$15	$\sim$5

and main-sequence (or evolved) stars, in order to show how their parameters may differ from the main characteristics of WR binaries. The initial time was taken to be the moment when a helium star at a some distance from the BH arises, with helium burning at its center, and can become a donor. As the binary evolves, helium burn in the stellar core, and afterwards in a thin layer above the carbon–oxygen core. The evolution of a binary with a massive WR star was usually computed until the relative mass of the donor's carbon–oxygen core reached 0.5–0.6; i.e., to the evolutionary stage when the stellar radius begins to rapidly increase. We do not consider here scenarios for the formation of the binaries studied, and only study specific sets of initial parameters.

We have numerically studied the evolution of detached and semi-detached close binaries with neutron stars or BHs in the framework of the hypothesis that hard radiation arising during the accretion of matter onto the relativistic component irradiates the donor, leading to intense mass loss from its surface, giving rise to an induced stellar wind (ISW). The intrinsic stellar wind from the donor is also included in the computations. The ISW strengthens the total mass loss of the donor, increasing the orbital semi-major axis. Earlier, we used this hypothesis to study the evolution of binaries containing neutron stars or BHs [12–17]. The results of those studies indicated that the hypothesis of an ISW helps us explain some observed properties of these binaries, for example, to match the observed and theoretical X-ray fluxes from binaries in our Galaxy and, probably, M31 [13].

The method used to compute the ISW is described in detail in [12]. Here, we present the resulting formula for the mass-loss rate of the donor due to the ISW:

$$\dot{M}_{\rm ISW} = 2.47 \times 10^{-10} C \frac{\alpha_{\rm ISW}^2 M_2^2}{R_2} \tag{1}$$
$$\times \left[\left(1 + 9.53 \times 10^{13} f \frac{R_2^4}{\alpha_{\rm ISW}^2 M_2^3 a^2} \dot{M}_{acc}\right)^{1/2} - 1\right].$$

Here, M_2 and R_2 are the mass and radius of the donor, $\dot{M}_{acc}$ the accretion rate onto the surface of the accretor, f the fraction of the hard radiation energy incident on the donor surface that is responsible for the mass loss via ISW, and $\alpha_{\rm ISW}$ the ratio of the donor's stellar-wind velocity to the escape velocity at the donor surface. The coefficient C in (1) allows for uncertainty in the adopted formalism. We adopted for our computations $f = 1$, $\alpha_{\rm ISW} = 1$, and $C = 0.5$.

In addition to mass transfer and the loss of matter and angular momentum from the binaries via the stellar wind, the computations take into account the loss of momentum due to gravitational-wave radiation and the donor's magnetic stellar wind. The method used to compute the evolution of the binaries and the software used in the evolutionary computations are described in detail in [12, 13]. It is important that the intrinsic and induced stellar winds are also taken into account in stages when the donor fills its Roche lobe. This is of great importance for studies of the evolution of binaries with massive WR stars, since these components have appreciably stronger stellar winds than do main-sequence stars [18]; a WR binary loses mass and angular momentum via this wind, leading to an increase in the semi-major axis of the orbit, potentially appreciably decreasing the mass loss through the Lagrangian point L1, detaching the donor star from its Roche lobe.

One of the most important parameters to affect the evolution of the binaries analyzed is the mass-loss rate of the WR stars via their stellar wind. A number of analytical formulas for this rate have been proposed [19–23]. According to [5], the formulas of [23] provide the best fit to the observational data, although they yield far lower mass-loss rates for lower-mass helium stars than other analytical approximations. The formulas of [23] were applied to a detailed analysis of the binary Cyg X-3 [4].

We mainly calculated the stellar-wind mass-loss rates of the WR stars using the formulas of [23]:

$$\dot{M} = 2.8 \times 10^{-13} (L/L_\odot)^{1.5} M_\odot/\mathrm{yr} \tag{2}$$

$$\text{for } \log(L/L_\odot) \geq 4.5,$$

$$\dot{M} = 4.0 \times 10^{-37}(L/L_\odot)^{6.8} M_\odot/\text{yr} \quad (3)$$
$$\text{for } \log(L/L_\odot) < 4.5.$$

For comparison, the following empirical formula [21] was also used for lower-mass helium stars in some cases:

$$\dot{M} = 5 \times 10^{-13}(L/L_\odot)^{1.4}(Z/Z_\odot)^{0.7} M_\odot/\text{yr}. \quad (4)$$

According to these formulas, the intensities of the stellar winds of WR stars depend mainly on their luminosities. Comparison with observations indicates that the observed stellar-wind intensities of WR stars are estimated fairly well by these formulas. For example, observations of the WR6 star (EZ CMa, HD 50896) give a luminosity of $10^{5.6} L_\odot$, and its mass-loss rate is estimated to be $2 \times 10^{-5} M_\odot$/yr [18]. On the other hand, formula (2) yields $7.0 \times 10^{-5} M_\odot$/yr, and formula (4) yields $3.5 \times 10^{-5} M_\odot$/yr. These results are in satisfactory agreement, allowing for remaining uncertainties in the theory and the inhomogeneity of teh stellar winds from WR stars, which was first noted by Cherepashchuk [24]. This last circumstance suggests that empirical estimates of mass-loss rates of WR stars may be overestimated.

The total mass-loss rate of the donor is the sum of the mass-loss rates due to the intrinsic and induced stellar winds, as well as mass loss through L1, if the donor fills its Roche lobe. When considering the accretion of matter from the onto the BH, we calculated the fraction of the stellar-wind material captured by the accretor using the Bondi—Hoyle formula, assuming the stellar-wind velocity of the donor to be equal to the escape velocity at its surface. We assumed that the matter-accretion rate onto the BH cannot exceed its Eddington limit. If the flow of matter reaching the accretor exceeds this limit, the matter that is not captured escapes from the binary and carries away the specific angular momentum of the accretor, since the flow of the escaping matter is generated in the accretion disk around the BH. These assumptions are simplified and likely to describe the real processes only to within a factor of 2—3; however, their use is partly justified by the comparable uncertainty in the actual stellar-wind intensities of WR stars, which is a key factor in the evolution of the analyzed binaries.

Note that the loss of mass and angulr momentum from the binary via the donor's stellar wind is the major factor determining changes in the orbital parameters in the detached evolutionary phase, since the orbital period is long enough for gravitational-wave radiation to be insignificant. In this case, the change in the orbital semi-major axis a is determined by the classical Jeans invariant: $a(M_1 + M_2) = \text{const}$, where M_1 and M_2 are the masses of the binary components. However, although a changes in a relatively simple way, it is necessary to perform numerical simulations of the donor evolution, and to compute its stellar-wind mass loss in detail, when studying the evolution of such a binary system.

4. CHARACTERISTIC PROPERTIES OF THE EVOLUTION OF BINARIES WITH WR STARS AND BLACK HOLES

4.1. Evolution of Binaries that are Initially Semi-detached

Since a helium star can expand as it evolves, we cannot exclude the possible existence of semi-detached binaries with WR stars and BHs [4]. For example, Cyg X-3 may be a semi-detached binary [4]. To demonstrate the characteristic properties of such binaries, we present evolutionary tracks for binaries containing BHs with masses of $10 M_\odot$ (the accretors) and WR stars or lower-mass helium stars (the donors) with initial masses of 2, 5, 10, and $20 M_\odot$. These tracks were computed assuming that the WR star fills its Roche lobe at the initial time, when helium burning begins in its core. Figure 1 shows a plot of the logarithm of the orbital period versus the logarithm of the donor mass-loss rate for these tracks.

The initial orbital periods of the analyzed binaries are 1.3, 2.0, 3.5, and 3.2 hr for donors with masses of 2, 5, 10, and $20 M_\odot$, respectively. The loss of angular momentum from the binary due to gravitational-wave radiation (GWR) can play an important role in the evolution of close binaries with orbital periods of less than 10 hr. However, this only occurs in binaries with low-mass helium stars. The intense stellar wind and rapid evolution of massive donors suggest that the effect of GWR is considerably less than the effect of the stellar wind in binaries with massive donors. The ratio of $(da/dt)_{\text{GWR}}$ to $(da/dt)_{\text{SW}}$ in the initial stage of binary evolution is 17, 1.5, 0.07, and 0.04 for donors with masses of 2, 5, 10, and $20 M_\odot$, respectively (a is the semi-major axis, and $(da/dt)_{\text{GWR}}$ and $(da/dt)_{\text{SW}}$ are the components of da/dt due to GWR and the stellar wind). Thus, GWR is important only for binaries with the lowest-mass donors, regardless of the short orbital periods of all the analyzed close binaries. The determining process for massive donors is the loss of mass and angular momentum via the stellar winds of their WR stars.

The ISW due to the hard radiation of the accreting BH is of great importance for a low-mass helium donor. The initial ratios of the hard-radiation energy incident on the donor surface to the stellar luminosity are 2.3, 0.66, 0.23, and 0.08 for donors with masses of 2, 5, 10, and $20 M_\odot$, respectively. The initial fractions of the total stellar wind of the donor made up by the ISW are 0.79, 0.32, 0.06, and 0.01, respectively. Therefore, the initial stellar-wind mass-loss rates of low-mass helium donors are almost completely determined by the irradiation of the donor by hard radiation from the BH, but this does not play as large a role for massive WR stars.

Numerical simulations of the evolution of semi-detached binaries containing donors with initial masses of 10 and $20M_\odot$ indicate that a strong stellar wind and the associated loss of mass and angular momentum from the binary rapidly lead to the donor detaching from the Roche lobe, which makes the binary becomes semi-detached. A donor with a mass of $10M_\odot$ detaches from its Roche lobe 560 years after the onset of the binary's evolution, and a donor with a mass of $20M_\odot$ detaches from its Roche lobe in 90 years. In the subsequent evolution, the mean radius of the donor Roche lobe increases rapidly due to the intense loss of mass and angular momentum via the stellar wind of the WR star, and the ratio of the donor radius R_2 to the mean radius of the donor Roche lobe R_R decreases monotonically. However, there is intense mass transfer between the components in this detached binary, and the accretion rate onto the BH exceeds the Eddington limit until the mass of the donor decreases to 7.3 or $9.3M_\odot$ (for an initial donor mass of 10 and $20M_\odot$, respectively).

At the same time, donors with initial masses of 2 and $5M_\odot$ detach from their Roche lobes only temporarily; after a while, their binaries again become semi-detached due to the increase in the radius of the helium star during its evolution.

A donor with an initial mass of $2M_\odot$ detaches from its Roche lobe 5.8 million years after the onset of its evolution, after it has lost $0.68M_\odot$ (Fig. 1). The minimum ratio of the donor radius R_2 to the mean radius of its Roche lobe R_R, which is reached when the donor temporarily loses contact with the Roche lobe, is 0.66. The semi-detached stage lasts 10.1 million years, during which the donor loses $0.40M_\odot$. It is important that mass transfer between the components does not stop in the semi-detached stage, since the BH continues to accrete the stellar wind from the helium star. When the donor again fills its Roche lobe, the donor mass is 45% of its initial mass. The donor continues to fill its Roche lobe in its subsequent evolution, in the stage of helium core-burning and shell-burning.

Computation results obtained for the evolution of this binary based on formula (4) for the stellar wind (i.e., with a more intense stellar wind) differ from those described above only slightly, since the ISW continues dominate the total stellar wind of the donor. The strengthening of the total wind means that the semi-detached phase becomes somewhat longer (by a factor of 1.2), and the minimum ratio of R_2/R_R decreases by 10%.

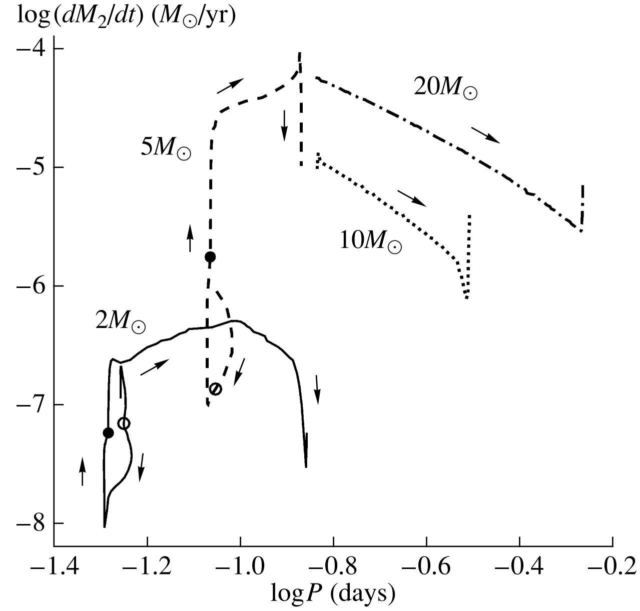

Fig. 1. Theoretical evolutionary tracks of binaries that are semi-detached at the initial time and contain BHs with masses of $10M_\odot$ (accretors) and WR stars or low-mass helium stars (donors) with initial masses of 2, 5, 10, and $20M_\odot$, on a plot of the logarithm of the orbital period versus the logarithm of donor mass-loss rate. Different tracks are shown by different kinds of lines. The donor masses are indicated next to their tracks. The hollow circles on the tracks of donors with initial masses of 2 and $5M_\odot$ mark times when the donors detach from their Roche lobes, while the black circles show times when the donors fill their Roche lobes again. The arrows indicate in which direction the evolution proceeds.

A donor with an initial mass of $5M_\odot$ evolves similarly, but has a shorter semi-detached phase. The donor detaches from its Roche lobe 4.5 million years after the onset of its evolution, after it has lost $1.61M_\odot$ (Fig. 1). The minimum ratio R_2/R_R, which is reached when the donor temporarily loses contact with its Roche lobe, is 0.74. The semi-detached stage lasts 0.81 million years, during which the donor loses $0.10M_\odot$. The donor mass when it again fills its Roche lobe is 66% of its initial mass. In the subsequent evolution, in the stage of helium burning in the donor core and in a thin shell, the donor continues to fill its Roche lobe until the mass of the CO core exceeds 60% of the initial mass.

4.2. Evolution of Binaries that are Initially Detached

Here, we consider binaries that are detached, but fairly close, at the initial time; the initial extent of Roche-lobe filling by the donor, i.e., the ratio R_2/R_R, was taken to be 0.5 at the epoch of helium ignition

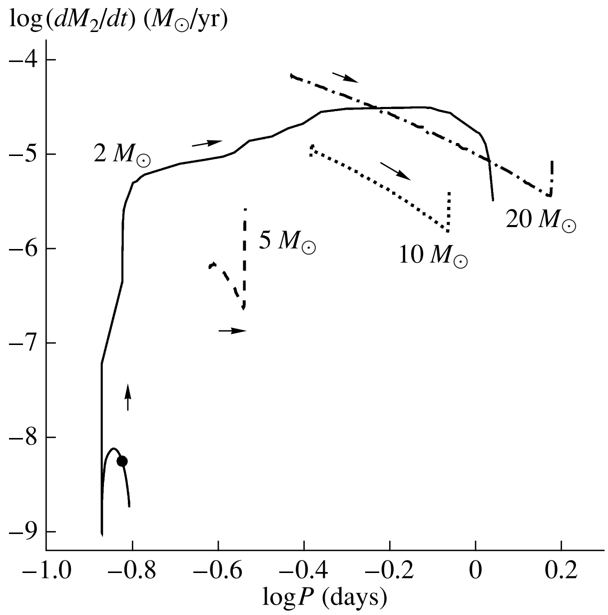

Fig. 2. Aame as Fig. 1 for binaries that are detached at the initial time, with the initial ratio of the donor mass to the mean radus of the Roche lobe being 0.5. The black circle on the track for a donor with an initial mass of $2M_{\odot}$ marks the time when the donor fills its Roche lobe.

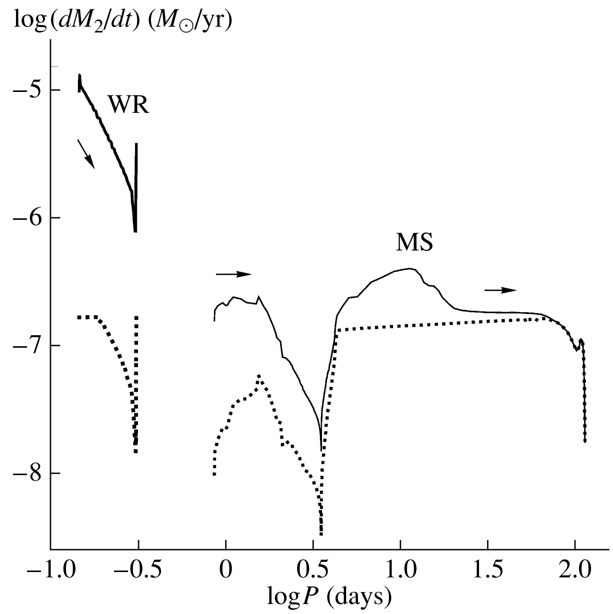

Fig. 3. Theoretical evolutionary tracks of binaries that are semi-detached at the initial time on a plot of the logarithm of the orbital period versus the logarithm of donor mass-loss rate. The bold solid line at the left of the figure shows the track for a binary containing a BH with a mass of $10M_{\odot}$ and a WR star with a mass of $10M_{\odot}$. The thin solid line at the right of the figure shows the track for a binary containing a BH with a mass of $10M_{\odot}$ and a main-sequence star with a mass of $10M_{\odot}$. The dotted lines show the accretion rates for the corresponding tracks, and the arrows indicate the direction of the evolution. The accretion rate at the initial phase of the evolution of a binary containing a main-sequence star is less than the Eddington rate, since the donor loses a smaller amount of matter via the ISW through the Lagrangian point L1 than it does via the total stellar wind.

in the stellar core (smaller ratios were also considered for a donor with a mass of $2M_{\odot}$). In this case, the evolution of low-mass and massive donors proceeds differently. To illustrate the characteristic properties of these systems, we present theoretical evolutionary tracks of such binaries comprised of the same components as those described above. Figure 2 shows the logarithms of the orbital periods plotted versus the logarithms of donor mass-loss rates for these tracks.

The initial periods of these binaries were 3.8, 5.8, 9.9, and 8.9 hr, and the initial donor masses 2, 5, 10, and $20M_{\odot}$, respectively. The computations indicate that binaries containing donors with masses of 5, 10, and $20M_{\odot}$ remain detached during their evolution, while a donor with a mass of $2M_{\odot}$ fills its Roche lobe. This occurs 11.7 million years after the onset of the evolution, when the donor mass is $1.53M_{\odot}$, and the relative mass of its carbon—oxygen core is 0.28. This scenario for the evolution of a binary with a low-mass donor is partly due to GWR by the binary. The initial ratio of $(da/dt)_{\rm GWR}$ to $(da/dt)_{\rm SW}$ for a donor with a mass of $2M_{\odot}$ is 2.3, while it is only 0.11 for a donor with a mass of $5M_{\odot}$. At lower initial ratios R_2/R_R of 0.4, 0.2, and 0.05, a donor with an initial mass of $2M_{\odot}$ also fills its Roche lobe, when its mass is close to $1.6M_{\odot}$ and the relative mass of its CO core is 0.34, 0.40, and 0.48, respectively.

Therefore, if the initial component separation in a WR + C binary increases only slightly, the range of initial masses for donors capable of filling their Roche lobes in their subsequent evolution becomes far narrower.

5. COMPARISON BETWEEN WR + C BINARIES AND BINARIES WITH MAIN-SEQUENCE DONORS

It is interesting to compare the characteristic properties of binaries composed of WR stars and BHs with binaries whose donors are main-sequence or slightly evolved stars (MS + C, where MS denotes a main-sequence star). Here, we confine our consideration to comparing the evolution scenarios of binaries whose initial donor and accretor masses are $10M_{\odot}$. Our computations of the evolution of MS + C binaries (at all evolutionary phases) took into account the ISW and the donor's intrinsic stellar wind, whose rate was calculated using the formula for O stars from [25].

Let us first consider binaries with the shortest orbital periods, in which either a helium star (WR + C) or an unevolved main-sequence star (MS + C) fills its Roche lobe at the initial time. With the adopted component masses, the initial orbital periods are 3.5 and 20.7 hr, respectively. Figure 3 depicts the donor mass-loss rate and the corresponding accretion rate onto the BH as functions of the orbital period for both kinds of binaries. As was noted above, the donor is not able to retain contact with the Roche lobe in a WR + C binary, while a MS + C binary remains semi-detached during most of its evolution (although a main-sequence donor detaches from its Roche lobe for a period of 3.7×10^7 yrs due to the ISW, losing in the initial stage of evolution a much smaller amount of matter through L1 than via the total stellar wind). However, the accretion rates in WR + C binaries remain fairly high due to the very strong stellar wind from the WR star. It is interesting that the accretion rates in these two binaries are nearly the same: equal to or slightly below the Eddington limit. Therefore, both types of binaries can be bright X-ray sources. However, the donor mass-loss rates are appreciably different: the intensity of the stellar wind from the WR star is approximately two orders of magnitude higher than the mass-loss rate of the main-sequence star (filling its Roche lobe) through L1. Accordingly, the characteristic evolution time scale is $\sim 2 \times 10^6$ yrs for WR + C and $\sim 8 \times 10^7$ yrs for MS + C binaries.

Let us now consider WR + C and MS + C systems, as longer-period binaries in which the ratios R_2/R_R for the helium star or main-sequence star are 0.1 at the initial time. Figure 4 shows the donor mass-loss rate and corresponding accretion rate as functions of the orbital period for both kinds of binaries. The initial periods of these binaries are 4.6 days for WR + C and 27.3 days for MS + C. Subsequently, the WR + C binary remains detached in the analyzed stage of evolution, while the donor in the MS + C binary fills its Roche lobe 2.6×10^7 yrs after the beginning of the evolution, when the relative mass of the helium core reaches 0.14. In this case, the rate of matter transfer in the semi-detached stage of the MS + C binary is two orders of magnitude higher than the intensity of the stellar wind from the WR star, suggesting that the accretion rate in the MS + C binary remains close to the Eddington rate. In the WR + C binary, the rate of accretion of the stellar-wind from the WR star is two or three orders of magnitude lower, implying that the long-period WR + C binary should be a far less luminous X-ray source than the long-period MS + C binary. The evolution time scale (in the analyzed stages) will be approximately twice as long for the WR + C binary as for MS + C.

Therefore, short-period binaries with WR stars lose much more matter than binaries with ordinary stars; both kinds of binaries are bright X-ray sources.

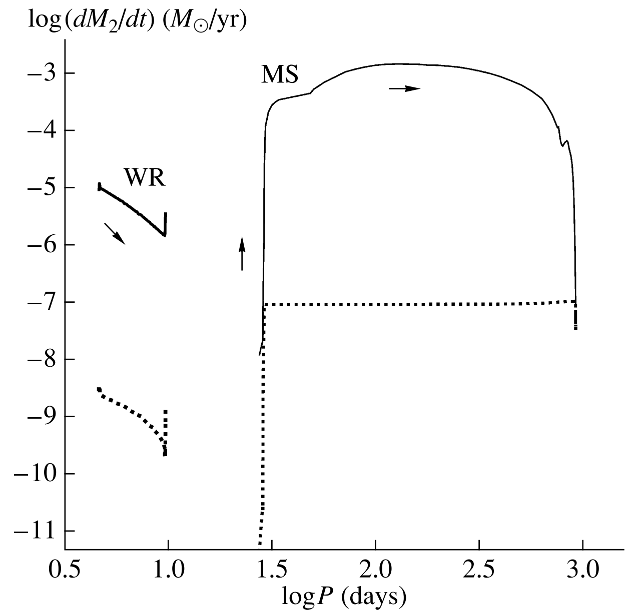

Fig. 4. Same as Fig. 3 for binaries that are detached at the initial time and have an initial ratio of the donor radius to the mean Roche-lobe radius of 0.1.

However, for binaries with longer orbital periods, the X-ray intensity and mass-loss rate are far higher from binaries with ordinary stars than from binaries with WR stars.

6. THE EVOLUTIONARY STATUS OF KNOWN WR + C BINARIES

6.1. Cyg X-3

The binary Cyg X-3, which has the shortest orbital period among all known WR + C binaries ($P \approx 4.8^{\rm h}$), has been intensively studied for a long time (see, e.g., [4, 26]). Nevertheless, there remain appreciable uncertainties in the binary parameters. A detailed analysis of a range of possible component masses in Cyg X-3 was carried out in [4], based on observational estimates of the rate of change of the orbital period and the mass-loss rate. Earlier estimates of the component masses given in [4] indicate a very wide range of possible masses: up to $40M_\odot$ for the relativistic component and up to $70M_\odot$ for the WR star. The estimated rates of increase of the orbital period $\dot{P}/P$, obtained in various studies and collected in [4], lie in the range 1.2×10^{-6}–4.0×10^{-6} yr^{-1}. Estimated mass-loss rates calculated using various methods range from 5×10^{-7} to $3 \times 10^{-4} M_\odot$/yr [4]. The results of [4] indicate that the observed values of $\dot{P}/P$ and mass-loss rates in the binary correspond to moderate component masses. If the WR donor does not fill its Roche lobe, the mass of the relativistic star could lie in the range 7–$20M_\odot$, while the mass of the

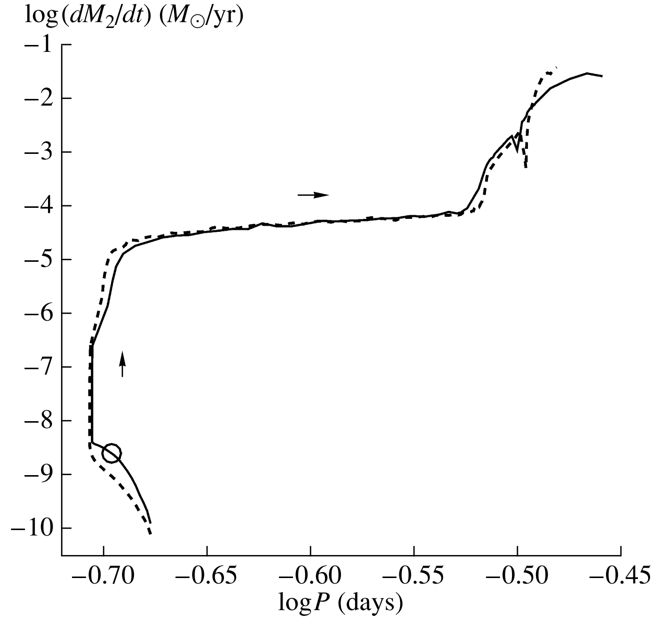

Fig. 5. Theoretical evolutionary tracks of a binary composed of a helium star with a mass of $3M_\odot$ and a BH with a mass of $5M_\odot$ on a plot of the logarithm of the orbital period versus the logarithm of donor mass-loss rate. These tracks represent the binary Cyg X-3 in the initial evolutionary phase for one possible pair of component masses for which the binary is semi-detached. The solid and dashed curves show the tracks without and with the ISW. The circle marks the position of Cyg X-3. The arrows indicate the direction of the evolution.

WR star could be 5–$14M_\odot$. If the donor fills its Roche lobe (which occurs in the core helium-burning stage), these ranges become far narrower. If the accretor is a BH, its mass is 3–$5M_\odot$, and the mass of the WR star lies in the range 4–$6M_\odot$. If the accretor is a neutron star, the component masses are close to 1.5–$2M_\odot$, according to [4, Fig. 2].

Note that, in principle, the possibility of high masses for WR stars reaching several dozen solar masses is consistent with the short orbital periods of the binaries, since the radii of (even massive) helium stars are comparatively small. For example, in binaries with component masses of $30 + 10$ and $50 + 10M_\odot$ (where the former is the mass of the helium donor, and the latter the mass of the BH), the orbital semi-major axes for $P = 4.8^{\mathrm{h}}$ are 4.9 and $5.6R_\odot$ and the donor radii 1.9 and $2.3R_\odot$, respectively. Thus, R_2/R_R is close to 0.8 for both binaries. However, our test computations indicate that the rate of the period increase $\dot{P}/P$ in a $30 + 10M_\odot$ binary is $8.7 \times 10^{-6}\mathrm{yr}^{-1}$ for $P = 4.8^{\mathrm{h}}$, which is approximately twice the upper limit of the range of possible values in Cyg X-3. Moreover, the mass-loss rate in this binary is close to $1.8 \times 10^{-4} M_\odot$/yr. Although within the range of possible estimates, this value is near its upper bound. This supports the conclusion of [4] that the components in Cyg X-3 have moderate masses.

We have studied how the irradiation of the donor and the resulting ISW affect the evolution of a low-mass binary system corresponding to one of the possible pairs of component masses in Cyg X-3. One such binary, in which a donor with an initial mass of $3M_\odot$ fills its Roche lobe, and the mass of the BH is $5M_\odot$, was analyzed in detail in [4]. We computed the evolution of this binary both with and without the ISW. Figure 5 shows theoretical evolutionary tracks of the binary on a plot of the logarithm of orbital period versus the logarithm of donor mass-loss rate. These computations assumed that the donor fills its Roche lobe for a period close to the orbital period of Cyg X-3, which occurs at a relative mass for the CO core of 0.3. At this phase of the evolution, the mass-loss rate via the donor's intrinsic stellar wind is $2.6 \times 10^{-7} M_\odot$/yr, and the ISW mass-loss rate is comparable to this value ($2.3 \times 10^{-7} M_\odot$/yr). However, the mass-loss rate of the donor through L1, once it fills its Roche lobe, is initially $1.2 \times 10^{-6} M_\odot$/yr, and later increases rapidly. As a result, the ISW does not have an appreciable effect on the subsequent evolution of the binary, although it does affects the binary parameters to some degree before the stage when the donor fills its Roche lobe.

6.2. IC 10 X-1

A complete evolutionary track of the IC 10 X-1 binary was studied in detail in [27] using the "Scenario Machine" software, beginning with two massive main-sequence stars and ending with a single

BH, which forms as a result of the merging of the two BHs that are the final products of the components' evolution.

We considered the current stage of the evolution of this binary, when it is a WR + C system. We adopted $26M_\odot$ for the WR star and $28M_\odot$ for the BH for the current component masses in IC 10 X-1 (see the table). To illustrate one possible scenario for the evolution of this binary, we computed the evolution of a binary composed of a BH with a mass of $28M_\odot$ and a helium star with an initial mass of $30M_\odot$, slightly exceeding the current mass of the donor. The initial orbital period of the binary is 30.3^h, the orbital semi-major axis is $19.0R_\odot$, and R_2/R_R is 0.25 at the initial time. Theoretical evolutionary tracks of this binary on a plot of the logarithm of the orbital period versus the logarithm of the donor mass-loss rate are shown in Fig. 6, which also presents the accretion rate onto the BH.

The major factor determining the binary evolution is the loss of mass and angular momentum via the WR stellar wind. Accordingly, the orbital period increases during the evolution. When the evolutionary track corresponds to the current parameters of IC 10 X-1 (a donor mass of $26M_\odot$ and an orbital period of 34.9^h), the semi-major axis increases to $20.4R_\odot$ and R_2/R_R is 0.23. The binary orbital period increases at a rate of $\dot{P}/P = 4.7\times10^{-6}$ yr^{-1} at this time. The donor is still almost a pure helium star at this time, since the helium concentration at its center has only decreased by 8%. The stellar-wind mass-loss rate calculated using (2) is $1.3\times10^{-4}M_\odot$/yr, and the matter captured by the accretor is $2.6\times10^{-7}M_\odot$/yr, which is approximately half the critical Eddington accretion rate. This accretion rate corresponds to a luminosity of $L_x = 1.5\times10^{39}$ erg/s if the accretion of one gram of matter onto the BH releases an energy of $0.1c^2$ (where c is the speed of light). In the subsequent evolution of the binary, L_x decreases by an order of magnitude in 6.1×10^5 years, while the mass of the donor decreases to $7.2M_\odot$.

However, note that this value of L_x, calculated using an approximate method for estimating the accretion rate of matter from the donor onto the BH exceeds the corresponding observational estimate by an order of magnitude. The accretion rate onto the BH depends strongly on v_w, the velocity of the donor stellar wind: in our formalism, $\dot{M}_{acc} \sim v_w^{-4}$. These computations assume that v_w for a WR star is equal to the escape velocity at its surface. This may be justified, for example, by the conclusion of [20] that all WR stars have stellar-wind velocities close to 2000 km/s. The escape velocity at the donor surface on our track is close to this value—2400 km/s at

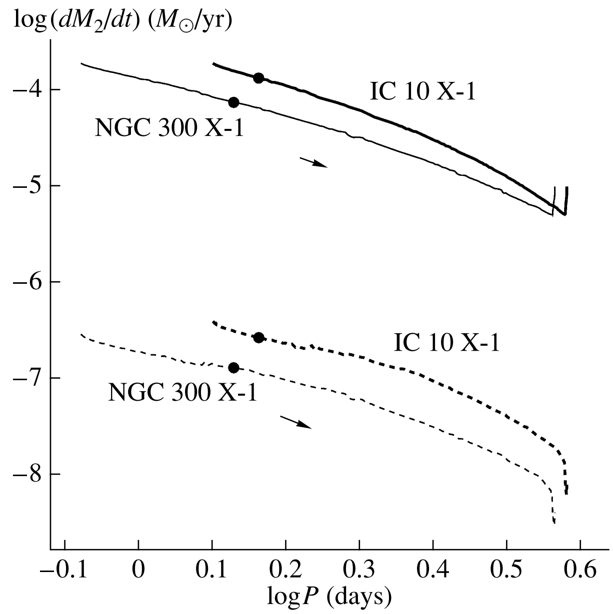

Fig. 6. Theoretical evolutionary tracks for binaries that reproduce the initial phases of evolution of IC 10 X-1 and NGC 300 X-1 on a plot of the logarithm of the orbital period versus the logarithm of the donor mass-loss rate. The dashed lines show the accretion rates of donor material onto the BH. The black circles mark times corresponding to the parameters of these binaries. The arrows indicate the direction of the evolution.

the phase resembling the current state of IC 10 X-1. However, there remain significant uncertainties about the values of v_w for WR stars. A fairly wide range of possible v_w values from 1000 to 1900 km/s is considered in [28], where accretion onto the BH in NGC 300 X-1 (whose parameters are close to IC 10 X-1) is analyzed in detail. However, a decrease in v_w in our computations would yield even greater theoretical values of L_x.

Discrepancies between theoretical and observational values of L_x could also be due to other factors, such as possible overestimation of the estimated mass and stellar-wind intensity of the donor. To study the effect of decreasing the stellar-wind intensity on theoretical parameter estimates for this binary, we carried out test computations with the donor mass-loss rate reduced by a factor of ten compared to the rate derived from (2). In this case, L_x is 2.5×10^{38} erg/s at the evolutionary phase that resembles IC 10 X-1, with the BH accreting $4.5\times10^{-8}M_\odot$/yr — only 9% of the critical Eddington accretion rate. Accordingly, the rate of increase of the orbital period decreases by almost an order of magnitude: $\dot{P}/P = 5.8\times10^{-6}$ yr^{-1}. Therefore, matching the theoretical and observational values of L_x requires a very considerable decrease in the stellar-wind velocity of the WR star calculated

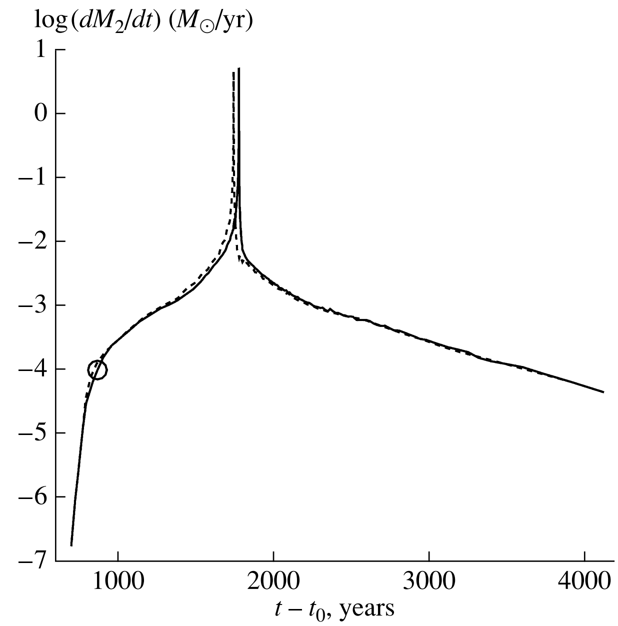

Fig. 7. Mass-loss rate of the donor as a function of time elapsed since the filling of its Roche lobe in a binary that resembles SS 433 in the initial phase of evolution. The circle marks the time corresponding to the observational estimate of the donor mass-loss rate. The dashed curve shows the track computed taking into account the ISW.

using the formulas of [23]. Observational estimates of $\dot{P}/P$ could be helpful in resolving this problem.

6.3. NGC 300 X-1

The parameters of the binary NGC 300 X-1 are close to those of IC10 X-1. We took the current component masses to be $20M_\odot$ for the WR star and $17M_\odot$ for the BH (the mean values within the ranges given in the table).

We took the initial mass of the WR star to be $30M_\odot$ when computing the evolutionary track to illustrate a possible evolutionary scenario for this binary. The initial orbital period of a binary composed of a helium star with this mass and a BH with a mass of $17M_\odot$ is $20.0^{\rm h}$, the semi-major axis of the orbit is $13.4R_\odot$, and R_2/R_R is 0.32 at the initial time. Theoretical evolutionary tracks on a plot of the logarithm of the orbital period versus the logarithm of donor mass-loss rate are shown in Fig. 6, which also presents the matter accretion rate onto the BH.

As in the evolutionary track for IC10 X-1 described above, the orbital period increases as the evolution proceeds. At the time corresponding to the current parameters of NGC 300 X-1 (a mass for the WR star of $20M_\odot$ and a period of $32.3^{\rm h}$), the orbital semi-major axis increases to $17.1R_\odot$ and $R_2/R_R = 0.32$. The donor is nearly a helium star at this time, with the helium concentration at its center reduced by only 22%. The rate of increase of the the orbital period $\dot{P}/P$ is 3.8×10^{-6} yr^{-1} at this time. The stellar-wind mass-loss rate calculated using (2) is $7.0 \times 10^{-5} M_\odot$/yr, and the mass captured by the accretor is $1.3 \times 10^{-7} M_\odot$/yr. This is 40% of the critical Eddington accretion rate. The amount of energy released by accretion can be calculated as above: $L_x = 7.1 \times 10^{38}$ erg/s. As the binary evolves, its luminosity L_x decreases by an order of magnitude in 5.9×10^5 yrs, while the mass of the donor decreases to $6.8M_\odot$.

However, as for IC 10 X-1, the value of L_x obtained for this track exceeds observational estimates. As above, we analyzed only the effect of a decrease in the stellar-wind rate on the value of L_x, due to the uncertainty in the WR stellar-wind velocity. In test computations performed with a stellar-wind velocity reduced by a factor of 10, the BH accretes $3.1 \times 10^{-8} M_\odot$/yr at the evolutionary phase resembling NGC 300 X-1, which is 11% of the critical Eddington accretion rate, and $L_x = 1.8 \times 10^{38}$ erg/s, which agrees better with the observational estimate of this value. The corresponding rate of increase of the orbital period $\dot{P}/P$ decreases by more than an order of magnitude, and is equal to 5.8×10^{-6} yr^{-1}. As for IC 10 X-1, observational estimates of $\dot{P}/P$ would help in determining the actual intensity of the stellar wind from the WR star.

6.4. SS 433

As was noted above, SS 433 is a unique microquasar binary in a stage of secondary mass transfer, which can be considered a precursor of a WR + C binary. SS 433 has been studied extensively for many years. We illustrate a possible scenario for the subsequent evolution of this binary, and investigate to what extent the ISW and the donor's intrinsic stellar wind could affect this evolution.

We computed a track for a binary with an ordinary evolved donor with a mass of $15M_\odot$ and a BH accretor with a mass of $5M_\odot$, in which the donor fills its Roche lobe for the orbital period of SS 433. At this time, the relative mass of the helium core of the donor is 0.17. The track was computed applying the standard assumptions described above. Figure 7 shows the theoretical dependence of the donor mass-loss rate on the time elapsed since the donor filled its Roche lobe.

The computations show that the donor mass-loss rate $\dot{M}$ increases rapidly over time. In less than 1000 yrs after the onset of mass transfer, $\dot{M}$ increased to $\sim 10^{-4} M_\odot$/yr (the current estimate of the mass-transfer rate in SS 433), and increased to $\sim 10^{-3} M_{\odot}$/yr in the subsequent $\sim$400 years. The maximum value of $\dot{M}$, $\sim 5.5 M_{\odot}$/yr, is reached $\sim$1700 years after the onset of mass transfer, with the donor mass decreasing to $8.3 M_{\odot}$ and the orbital period decreasing to $3.2^{\rm d}$. Further, a rapid drop in $\dot{M}$ begins. In $\sim$500 years, $\dot{M}$ decreases to $\sim 10^{-3} M_{\odot}$/yr. At this time, the donor mass is already $4.2 M_{\odot}$ and the orbital period has increased to $4.4^{\rm d}$. As the binary evolution proceeds, $\dot{M}$ gradually decreases, but more slowly, while the period increases.

In total, the donor loses approximately $11 M_{\odot}$ at a rate of more than $10^{-3} M_{\odot}$/yr on the computed track. Our computations do not consider the possible formation of a common envelope in the binary, since the presence of jets in SS 433, which carry away matter from the binary, may lead to the absence of a common-envelope stage in the subsequent evolution of this binary. As a result, our track may be able to provide insight into the actual evolution of SS 433. Note that the stage corresponding to the current estimate of the mass-transfer rate is short-lived, according to these results.

The dashed curve in Fig. 7 shows the computed results obtained when the ISW and the donor's intrinsic stellar wind [calculated using (2)] are included. We assumed the amount of hard radiation incident on the donor surface to be equal to the maximum upper limit, i.e., we took the accretion rate onto the BH to be equal to the Eddington limit. The results of this computation indicate (Fig. 7) that the irradiation of the donor does not appreciably effect the evolution of the binary.

7. CONCLUSIONS

Current views on the evolution of massive close binaries provide a firm foundation for theoretical analyses of the evolutionary statuses of observed close binaries in late stages of their evolution. Analysis of the observed parameters of three binaries composed of WR stars and relativistic objects can be used to establish their evolutionary status and determine their previous and subsequent evolution.

We have numerically studied the evolution of close binaries composed of WR stars and black holes. Our computations have considered both the intrinsic stellar wind of the donor star and the induced stellar wind due to the irradiation of the donor by hard radiation arising during the accretion of matter by the relativistic component. We have also taken into account the loss of mass and angular momentum via the donor wind in the phases when the WR star fills its Roche lobe.

The evolution of binaries containing WR stars with masses higher than $\sim 10 M_{\odot}$ are mainly determined by the intense stellar winds from the donors. In particular, a donor (WR star) that fills its Roche lobe in the initial evolutionary phase eventually loses contact with its Roche lobe as a result of the loss of mass and angular momentum via stellar wind, leading to an increase in the semi-major axis of the orbit, so that the initially semi-detached binary should become detached. This star remains a bright source of X-rays, since the accretion of some of the stellar wind by the BH ensures a near-Eddington accretion rate. Alternatively, the semi-detached binary cannot become detached if the initial mass of the helium donor is comparatively low (less than $\sim 5 M_{\odot}$); in this case, the donor only temporarily detaches from its Roche lobe. The evolution of a binary that contains a helium donor with an initial mass of $\sim 2 M_{\odot}$ is significantly affected by the ISW due to the irradiation of the donor and by the loss of angular momentum due to gravitational-wave radiation by the binary.

A comparison of the characteristic properties of close binaries composed of WR stars and BHs with those of binaries containing massive main-sequence (or slightly evolved) stars instead of WR stars indicates that short-period binaries with WR stars lose far greater amounts of matter than analogous binaries with ordinary stars, with both kinds of binaries being bright X-ray sources. However, if the orbital periods are long, the X-ray intensities and mass-loss rates of binaries with ordinary stars are much higher than those of WR binaries.

We have briefly considered the evolutionary status of the binary systems Cyg X-3, IC 10 X-1, and NGC 300 X-1. If the WR star in Cyg X-3 were to have a very high mass (several tens of solar masses), the orbital period will increase too rapidly, at a rate beyond the range spanned by observational estimates. The irradiation of the moderate-mass donor by hard radiation will not appreciably affect the evolution of the binary. The use of theoretical formulas for calculating stellar-wind velocities in IC 10 X-1 and NGC 300 X-1 may lead to overestimated values of L_x (for current estimates of the component masses in these binaries). This problem may be resolved if the stellar-wind rate is reduced by an order of magnitude; however, this is challenging, since the accretion rate depends strongly on the stellar-wind velocity, which is very uncertain.

The evolution of the binary system SS 433—a precursor of a WR + C binary—was computed assuming that this binary will not undergo a common-envelope stage in its evolution due to the mass lost via its jets taking place at the current evolutionary phase. In this case, the maximum mass-loss rate of the donor,

$\sim 5.5 M_{\odot}$/yr, can be reached $\sim$1700 years after the onset of mass transfer.

Further progress in studying late evolutionary stages of massive close binaries requires enlarging the number of observed binaries with well determined parameters and developing a well-based theory for the stellar winds from WR stars.

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