Draft version November 14, 2016
Preprint typeset using LaTeX style emulateapj v. 12/16/11

STATISTICAL SEARCH FOR INFLATION IN FOUR-YEAR COBE-DMR

Erik A. Levèn

Draft version November 14, 2016

ABSTRACT

A Bayesian likelihood analysis of the four-year COBE-DMR observations published in 1994 of the 53GHz and 90GHz frequency channels with focus on the power-law spectral index and quadrupole normalization: n and Q. The results found are consistent with the $Harrison - Zel'dovich$ model of $n = 1$. Maximum likelihood estimates of n and Q were $n_{53} = 0.93 \pm 0.32$, $Q_{53} = 18.31 \pm 4.40$ and $n_{90} = 1.24 \pm 0.5$, $Q_{90} = 19.58 \pm 6.60$ for the 53Hgz and the 90GHz bands respectively.

Subject headings: cosmic microwave background - cosmology: observations

1. INTRODUCTION

Modern day theory of the early universe tells us all space was filled with plasma of free electrons and protons too energetic to bind together to form heavier elements. As the universe expanded however the temperature dropped. At around 380 000 years after the big bang the universe had cooled down to below 3000K. The energies of the particles were now low enough to bind together to form hydrogen. In this merging photons were released with a temperature of around 3000K. Taking into account that the universe has expanded by a factor of 1100 since the effective CMB photon temperature today should be in the area of 2.7K. When measuring the intensity of CMB radiation for different parts of the sky we can determine the density of photons from this direction when they were released. A CMB picture therefore shows the density of matter in the early universe. The values of the cosmological constants Q and n are vital for contemporary cosmology as they potentially validate modern models for inflation. By determining their observational values and comparing them to theory we can gain a deeper understanding on the evolution of the early universe. Earlier studies (using the first year data; Smoot et al. 1992; Wright et al. 1992; Adams et al. 1992; Scaramella Vittorio et al. 1993; Seljak Bertschinger et al. 1993; Smoot et al. 1994; Bond 1994; using the two-year data; Bennet et al. 1994; Wright et al. 1994a) mostly used Monte Carlo techniques to estimate and calibrate their final results. In order to understand and measure the statistical properties of the measured fluctuations this paper will use a spherical harmonic decomposition via Fourier transformation. Since the harmonics have a Gaussian distribution we can find an exact likelihood function for the parameters. This let's us measure the angular power spectrum as the amplitude of the signal as a function of it's wavelength. We will then make a best fit calculation to find which values of n and Q fits best to the observations.

2. METHOD

I will now describe the method used for handling the data and finding the parameter values.

erikalev@uio.no
[1] Institute of Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, N-0315 Oslo, Norway

To be able to compare the intensity fluctuation measured but the four-year COBE-DMR observations as a function of positions of the sky this project used the spherical harmonics spherical which form a complete basis for the sphere. The spherical harmonic decomposition corresponds to a spherical equivalent of the Fourier transform

$$\Delta T(\hat{n}) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\hat{n}). \tag{1}$$

Here a_{lm} is the wave mode amplitude describing the strength of each individual wave length, l describes effective wavelength and m describes the phase of the mode. We choose to disregard m since we're working with a statistically isotropic and homogeneous field. We're more interested in the angular power spectrum defined as

$$< a_{lm} a^*_{lm'} >= \delta_{ll'} \delta m m' C_l \tag{2}$$

where the shape and amplitude of C_l are the variables often predicted by modern theories. By determining a best fit for these values we can which model best fits with observations. To account for the noise we have chosen different models for different noise sources. As a probability distribution function for the instrumental noise we have chosen Gaussian distribution with a mean value of zero and a standard deviation σ_p where $\sigma_p = \sigma_0/\sqrt{(N_{obs,p})}$ where σ_0 is the noise per observation and $N_{obs,p}$ is how many times the instrument observed pixel p. We have defined the noise covariance matrix as $N_{pp'} = \sigma^2 \delta_{pp'}$ where we assume that the noise is independent between two pixels. To account for the foreground contaminating regions of the sky in the central plane we have chosen a mask which cuts out these regions. This constitutes a removal of 1131 of the original 3072 pixel due to masking process to exclude these regions. When taking into account the point spread function where the COBE-DMR beam covers 7 degrees of the sky, essentially smoothing out the true sky by 7 degrees, this results in a multiplication in harmonic space due to the Fourier convolution theorem. We can therefore rewrite equation (1) to:

$$\hat{T}(\hat{nx}|) = \sum_{l=0}^{l_{max}} \sum_{m=-l}^{l} b_l a_{lm} Y_{lm}(\hat{n}) \tag{3}$$

Taking into account instrumental noise and foreground contamination we can model our data as a sum of a CMD component and an instrumental component:

$$d(\hat{n}) = s(\hat{n}) + n(\hat{n}) + f(\hat{n}) \tag{4}$$

Assuming that none of these values are correlated such as all cross-products have zero mean value we can write our full covariance matrix as

$$C = S + N + F \tag{5}$$

with F being the foreground contamination and N is the noise covariance matrix given by

$$N_{ij} = \sigma_i^2 \delta_{ij}. \tag{6}$$

We can argue that the pixel to pixel covariance can be described as

$$S_{ij} = \frac{1}{4\pi} \sum_{l=0} (2l+1)(b_l p_l)^2 C_l P_l(cos\theta_{ij}) \tag{7}$$

when taking into account the Gaussian and isotropic properties of the CMD field. The log-likelihood can now be described as

$$-2logL(Q,n) = \mathbf{d}^T \mathbf{C}^{-1} \mathbf{d} + log|\mathbf{C}| + constant \tag{8}$$

We have chosen to sample each C_l for $l = 0, 1, 2....47$ with

$$C_l = \frac{4\pi}{5} Q^2 \frac{\Gamma(l + \frac{n-1}{2})\Gamma()\frac{9-n}{2}}{\Gamma(l + \frac{5-n}{2})\Gamma(\frac{3+n}{2})} \tag{9}$$

3. RESULTS

Plotting out the data with the covariance taken into account the following contour plots were produced:

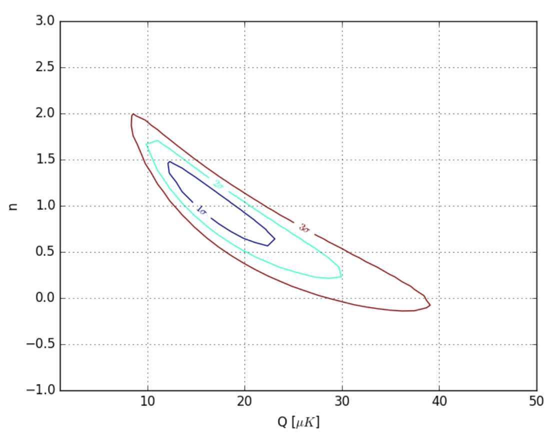

FIG. 1.—: Contour of the CMD data results with 53HGz frequency with a certainty up to 3 σ

When running the best fit algorithm we achieved the following values for Q and n for the 53GHz and 90GHz frequencies:

TABLE 1: Best fit values for Q and n for the specified frequancies with the error stated as the standard deviation

Frequencies	Q	n
53GHz	0.928 ±0.320	18.314 ±4.409
90 GHz	1.242 ±0.508	19.583 ±6.603

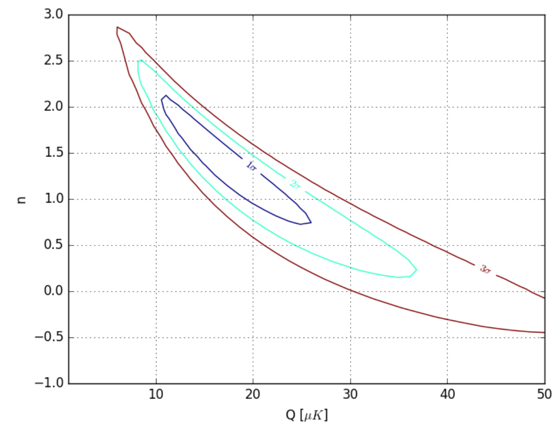

FIG. 2.—: Contour of the CMD data results with 90HGz frequency with a certainty up to 3 σ

Comparing the probability distribution for both frequencies the following plot was produced:

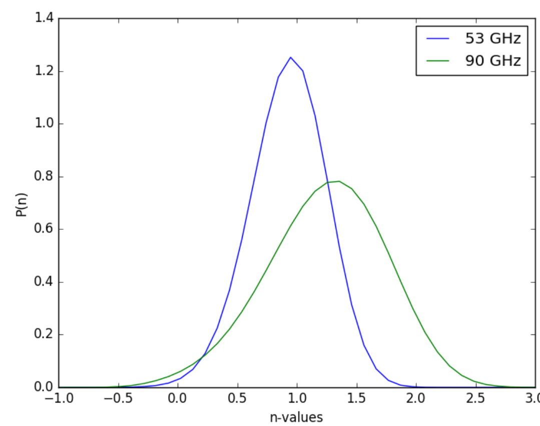

FIG. 3.—: Probability distribution for both frequencies as a function of n

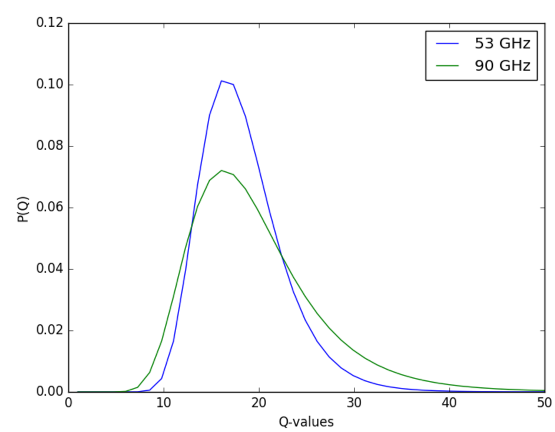

FIG. 4.—: Probability distribution for both frequencies as a function of Q

4. CONCLUSIONS

The results seems to be consistent with the Harrison-Zel'dovich $n = 1$ spectrum with an amplitude of primordial inhomogeneity $Q \approx 20\mu K$. A more thorough investigation of more frequencies would have to be made to find a precise probability distribution, but when regarding figures 3 and 4 we already see a clear tendency of a best fit value of n and Q in the area around one and 19 respectively. Even though these probability-plots differ they clearly overlap in the best-fit areas. With a significance of 99% from figures 1 and 2 further studies would have to be made to ensure the validity of these findings. We therefore recommend a more detailed investigation over wider frequencies from the PLANCK data released in a few year in order to achieve higher significance and better approximations. However, considering the findings compared to modern day inflation theory the results does indeed seem to be consistent.

5. SCRIPTS AND ORIGINAL DATA

`https://github.com/erikalev/AST2210/tree/master/LAB3`

REFERENCES

Eriksen, Hans K. K. and Lilje, Per B; Lecture notes
Drews, Ainar; Reports in AST2210