PROJECT 2 AST5220

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ABSTRACT

In this project we will estimate the fractional electron density, the cosmological optical depth and the visibility function. We will show that at a redshift around z ϵ [1800 < z < 650] both the fractional electron density and the optical depth decreases rapidly, indicating recombination. We will also show that the visibility function indicates that the last scattering surface occurred at a redshift of z $\approx$ 1100.

1. INTRODUCTION

The CMB photons observed today were decoupled during recombination, and to a large extent their fluctuations correspond to the fluctuations in the universe at that time. This project aim to calculate the optical depth, τ, and the visibility function, g, and their evolution through time. These values will give us a better insight in when recombination happened and how rapidly it happened. These variables are also needed later to integrate the Boltzmann-Einstein equations. In addition we will need to calculate the first and second derivatives of τ and g and the fractional electron density X_e.
The equations needed for this project were given with c = k_B = $\hbar$ = 1, therefore the equations stated in this report have been re-written for SI-units.
We will assume the same background geometry as in Project 1 (FRW metric). We will also neglect elements heavier than hydrogen, such as helium or lithium in our calculations, as well as excluding higher excitation states in the atoms. These approximations will result in the number density of hydrogen being equal to the number density of baryons (n_H = n_b).

2. METHOD

2.1. *Particles in thermal equilibrium*

A particle is in thermal equilibrium as long as

$$\Gamma(T) >> H(T), \tag{1}$$

where $\Gamma(T)$ is the scattering rate and H(T) is the expansion rate of the universe. In the case of photon decoupling the corresponding scattering rate due to Thomson scattering is given by

$$\Gamma_{\gamma e^- \to \gamma e^-} = X_e n_b \sigma_T \tag{2}$$

where X_e is the fractional electron density, $X_e = \frac{n_e}{n_b + n_H}$; n_e, n_b and n_H are the electron, baryon and helium densities respectively and σ_T is the Thomson cross section. As stated in the introduction we will neglect helium in our calculation reducing the X_e equation to

$$X_e = \frac{n_e}{n_H} \tag{3}$$

The hydrogen density, n_H, is given by

$$n_H = n_b \approx \frac{\rho_b}{mH} = \frac{\Omega_b \rho_c}{m_H a^3} \tag{4}$$

where Ω_b is the density fraction of baryons, ρ_c is the critical density of the universe, m_H is the hydrogen mass and a is the scale factor. Equation 3 can be re-written to the form

$$\frac{X_e^2}{1 - X_e} = \frac{1}{n_b}\Big(\frac{m_e k_B T_b}{2\pi\hbar^2}\Big)^{3/2} e^{-\epsilon_0/k_b T_b} \tag{5}$$

where k_B is the Boltzmann constant, T_b is the baryonic temperature ($T_b = \frac{T_0}{a}$, where T_0 is the temperature of photons today) and ϵ_0 is the ionization energy of hydrogen. Equation 5 is known as the Saha equation which is a good approximation for X_e as long as the system is in thermal equilibrium. However,ss the temperature and density fall the system leaves thermal equilibrium and an other equation is needed to estimate X_e. Thus if we can calculate X_e we can use equation 3 to calculate the electron density n_e

2.2. *Particles not in thermal equilibrium*

When the inequality equation $\Gamma(T) >> H(T)$ no longer hold we need to apply Peeble's equation:

$$\frac{dX_e}{dx} = \frac{C_r(T_b)}{H}\Big[\beta(T_b)(1 - X_e) - n_b \alpha^{(2)}(T_b) X_e^2\Big] \tag{6}$$

where

$$C_r(T_b) = \frac{\Lambda_{2s\to 1s} + \Lambda_\alpha}{\Lambda_{2s\to 1s} + \Lambda_\alpha + \beta^{(2)}(T_b)} \tag{7}$$

$$\Lambda_{2s\to 1s} = 8.227 s^{-1} \tag{8}$$

$$\Lambda_\alpha = H\frac{(3\epsilon_0)^3}{(8\pi)^2 n_{1s} (c\hbar)^3} \tag{9}$$

$$n_{1s} = (1 - Xe) n_b \tag{10}$$

$$\beta^{(2)}(T_b) = \beta(T_b) e^{3\epsilon_0/4k_B T_b} \tag{11}$$

$$\beta(T_b) = \alpha^{(2)}(T_b)\Big(\frac{m_e k_B T_b}{2\pi\hbar^2}\Big)^{3/2} e^{-\epsilon_0/k_b T_b} \tag{12}$$

$$\alpha^{(2)}(T_b) = \frac{64\pi}{\sqrt{27\pi}}\Big(\frac{\alpha}{m_e}\Big)^2 \sqrt{\frac{\epsilon_0}{k_B T_b}} \phi_2 \frac{\hbar^2}{c} \tag{13}$$

$$\phi_2(T_b) = 0.448 \ln \epsilon_0 / k_B T_b \tag{14}$$

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The limit at which we chose to activate Peeble's equation is set to when $X_e < 0.99$. This limit marks the time at which we deem thermal equilibrium to be no longer valid.

2.3. *Cosmological optical depth*

The main source of the absorption of photons is due to Thomson scattering of free electrons. The optical depth due to this scattering is given by

$$\tau(\eta) = \int_{\eta}^{\eta_0} n_e \sigma_T a d\eta \tag{15}$$

where η is the conformal time as described in Project 1. Written on a differential form τ is given by

$$\tau' = -\frac{n_e \sigma_T a}{\mathcal{H}} \tag{16}$$

where $\mathcal{H}$ is the scaled Hubble parameter as described in Project 1. As an initial parameter for the differential form we can use the fact that the universe is transparent today, τ(z=0) = 0.

2.4. *Visibility function*

The visibility function, g, describes the probability of a photon to scatter at a time η

$$g = -\dot{\tau} e^{-\tau(\eta)} \tag{17}$$

which can be scaled and re-written in terms of x as

$$\tilde{g}(x) = -\tau'(x) e^{-\tau(x)} = \frac{g(x)}{\mathcal{H}} \tag{18}$$

3. RESULTS

First we present the fractional electron density X_e. Both the optical depth and the visibility function depends this density and it is therefore vital that it is calculated correctly.

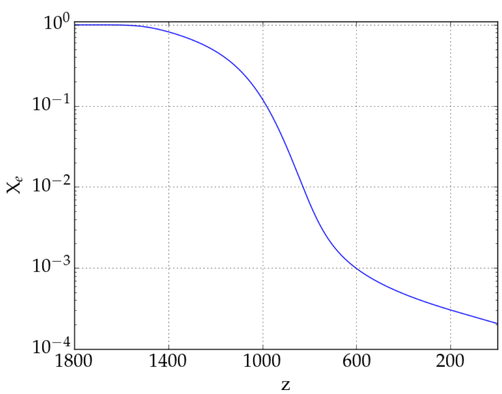

FIG. 1.—: Fractional electron density X_e as a function of x = ln(a).

We also present the optical depth, the visibility function and their first and second derivatives as a function of x.

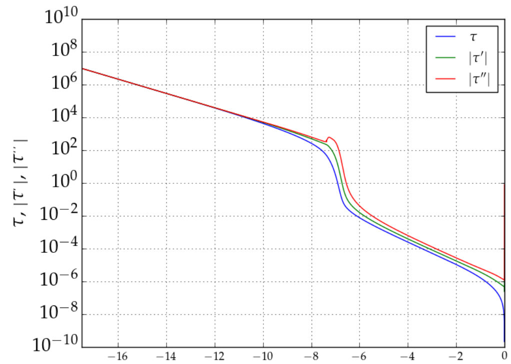

FIG. 2.—: The cosmological optical depth with it's first and second derivatives as a function of x = ln(a).

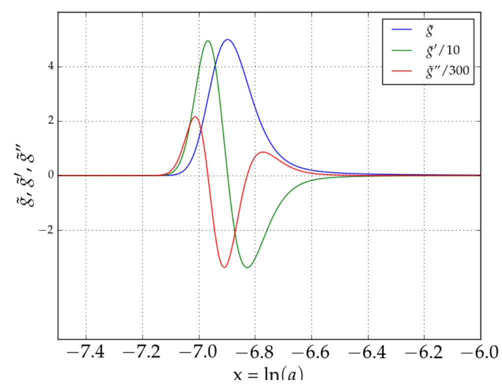

FIG. 3.—: The interpolated visibility function with it's first and second derivatives as a function of x = ln(a). g' has been divided by 10 and g" has been divided by 300 to make the curves fit into the same figure.

4. DISCUSSION AND CONCLUSION

As seen from figure 1, at high redshifts the fractional electron density is approximately 1. At redshift z $\approx$ 1587 Peeble's equation activates as $X_e < 0.99$ and the system leaves thermal equilibrium. We also note that recombination occurs at redshift around z = 1100, which on figure 1 is where the X_e values starts to drop fast.

From figure 2 we note that the optical depth at low x-values (high redshift) corresponds to the relatively low decrease in X_e for high redshift. We also note that the optical depth drops rapidly in the area x ϵ [-7.5 < x < -6.5] which approximately corresponds to redshifts in the area z ϵ [1800 < z < 650] which from figure 1 relates to the drop in X_e. According to figure 2 the early universe is opaque ($\tau >> 1$) and transparent in later times ($\tau << 1$). We also note that the absolute value of τ' and τ'' follow the same shape as τ. These values will be needed later as we calculate multipoles.

Combining the results from figure 1 and 2 we would expect the majority of photons being scattered through Thomson scattering to be scattered at x ϵ [-7.5 $<$ x $<$ -6.5] since this is where both the X_e and τ values drop.

Figure 3 shows that this indeed is the case. The peak of the visibility function indicates that the last scattering surface occurred at x $\approx$ -7 which would correspond to a redshift of z $\approx$ 1100, which is in agreement with prior calculations of the time of recombination.