Lecture 11: Goodness-of-Fit and Contingency Tables

Applied Statistics - STAN - 5.37 & 5.38
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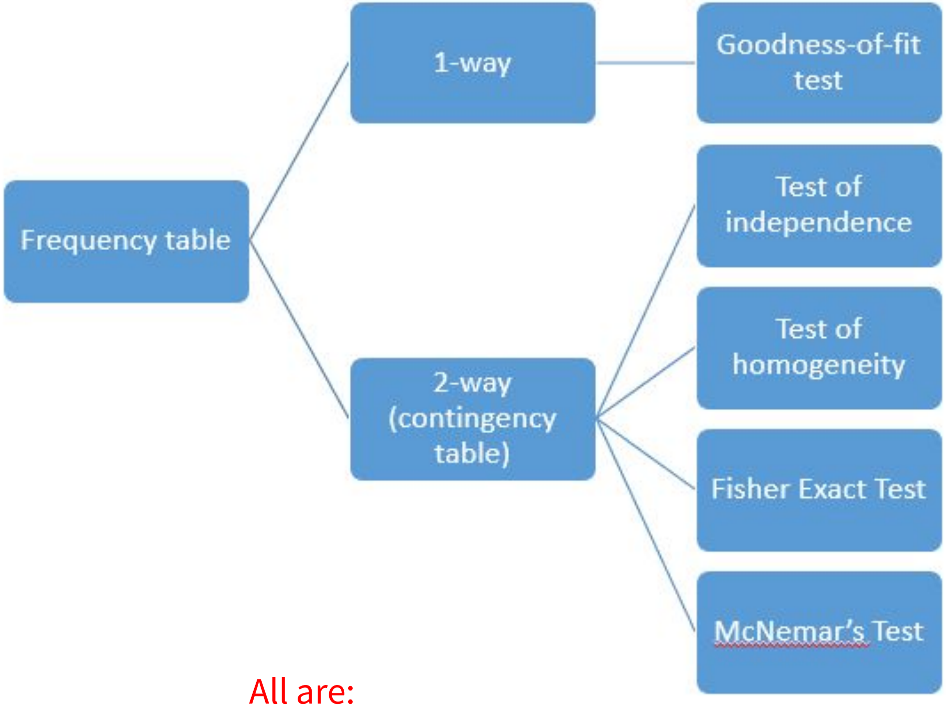
Test on Frequency Table



H0: The frequency counts agree (fit) with the claimed distribution
H1: The frequency counts do not agree (fit) with the claimed distribution

H0: The row and column variables are independent
H1: The row and column variables are dependent

H0: proporsi populasi di tiap baris sama (homogen)
H1: proporsi populasi di tiap baris tidak sama

All are:

- Right-tailed hypothesis test
- Use χ^2 for test statistics
- All except for fisher exact test require that the E >= 5
- Reject H0 → p-vaue < α

- requires us to find E
- Works based on (O-E)

Frequency Tables

Table 11-2 Last Digits of Weights

Last Digit	Frequency
0	7
1	14
2	6
3	10
4	8
5	4
6	5
7	6
8	12
9	8

One-way frequency table

Table 11-6 Results from Experiment with Echinacea

	Treatment Group		
	Placebo	Echinacea: 20% extract	Echinacea: 60% extract
Infected	88	48	42
Not infected	15	4	10

Two-way frequency table

- 2 corresponding variables → 1 for column and another 1 for row

I. Goodness-of-Fit Test

- A test to identify whether an observed frequency distribution fits some claimed distribution.
  - Singkatnya: test untuk melihat apakah sampel yang kita pakai mengikuti distribusi tertentu berdasarkan frekuensinya.
- How to test? → hypothesis test
- Ingredients:
  - Data consists of frequency counts for several different categories → example
  - The hypothesis
    H0: The frequency counts **agree/fit** with the claimed distribution
    H1: The frequency counts **do not agree/fit** with the claimed distribution
  - Test statistics: χ^2

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

O = observed frequency → **observation**
E = expected frequency → **theoritically**
k = number of different categories
df = k - 1

  - **E = n/k** → if it's assumed that the expected frequency are **all equal**.
  - **E = np** → if it's assumed that the expected frequency are **not all equal**.
  - For each category, **E >= 5**
  - Reject H0 if **p-value ≤ α**

Table 11-2 Last Digits of Weights

Last Digit	Frequency
0	7
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8	12
9	8

Example for E

a. **Equally likely:**
Sebuat dadu dilempar 45 kali dengan hasil seperti yang tampak pada tabel. Tentukan expected frequency (E)-nya.

Outcome	1	2	3	4	5	6
Observed freq. (O)	13	6	12	9	3	2
Expected freq (E)	??	??	??	??	??	??

→ **categories**

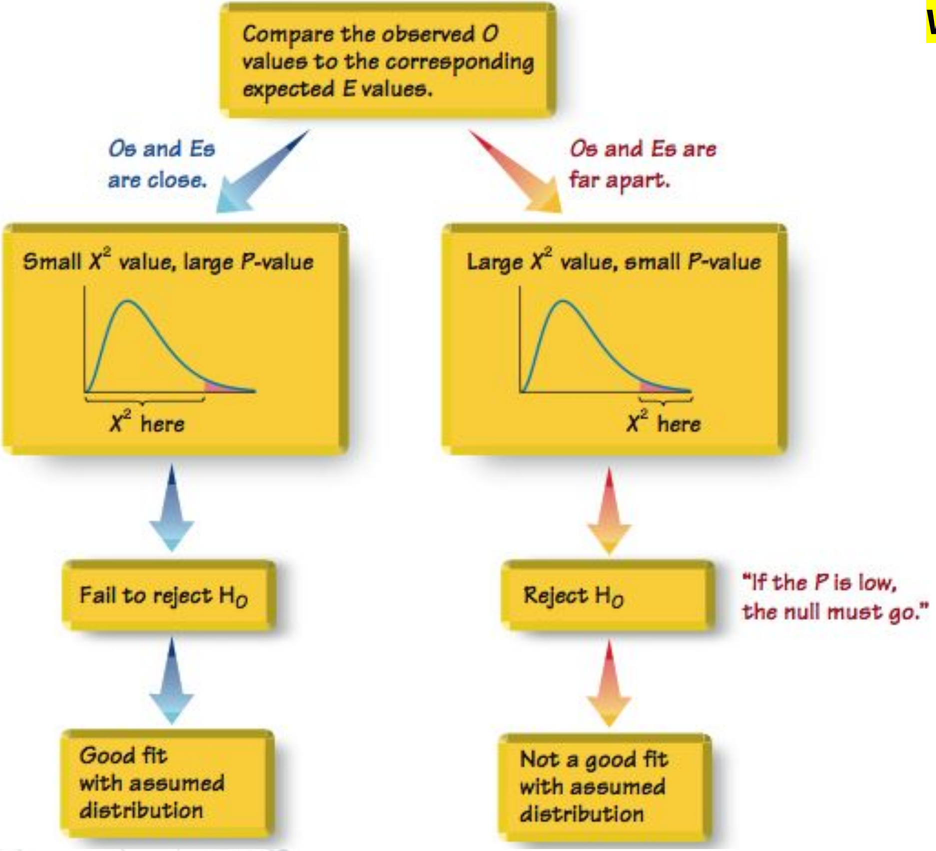
E = n/k

b. **Not equally likely**:
Let's say dadunya dibuat tidak seimbang, sehingga angka 1 memiliki peluang muncul 50%, sedangkan angka lainnya berpeluang muncul 10%.

Outcome	1	2	3	4	5	6
Probability	0.5	0.1	0.1	0.1	0.1	0.1
Observed freq. (O)	13	6	12	9	3	2
Expected freq (E)	??	??	??	??	??	??

E = np

Relationships Among the 2 Test Statistic, P-Value, and Goodness-of-Fit



Why is it a right-tailed test?

- The test statistics χ^2 is the difference between O and E.
- **Goodness-of-fit hypothesis** tests are always **right-tailed** → critical values will always be on the right side of the curve.
- The same thing applies to other test related to frequency table.

Example for Goodness-of-Fit Test

last_digit	obs_freq	exp_freq
0	46	10
1	1	10
2	2	10
3	3	10
4	3	10
5	30	10
6	4	10
7	0	10
8	8	10
9	3	10

Dikumpulkan data **berat badan 100** orang secara random. Researchers ingin menguji apakah data ini dikumpulkan dengan benar-benar mengukur berat 100 orang tersebut, atau hanya bertanya ke ybs saja, dimana **orang cenderung membulatkan beratnya ke digit berakhiran 0 atau 5**. So, with $\alpha = 5\%$, we'd like to test the claim that the digits do not occur with the same frequency. → H0: all digits have the same probability of being occured → **uniform distribution**

H0: $p0 = p1 = p2 = p3 = p4 = p5 = p6 = p7 = p8 = p9$
(the last digits occur with same relative frequency)

H1: At least one of the p is different from others.
(the last digits do not occur with same relative frequency)

H0: The frequency counts agree (fit) with the claimed distribution
H1: The frequency counts do not agree (fit) with the claimed distribution

The Manual Way

last_digit	obs_freq	exp_freq	oe	oe2	oe2e
0	46	10	36	1296	129.6
1	1	10	-9	81	8.1
2	2	10	-8	64	6.4
3	3	10	-7	49	4.9
4	3	10	-7	49	4.9
5	30	10	20	400	40
6	4	10	-6	36	3.6
7	0	10	-10	100	10
8	8	10	-2	4	0.4
9	3	10	-7	49	4.9
Jumlah	100	100	0	2128	**212.8**

Test statistics: $\chi^2 = 212.8$ → df = ??

P-value = …?

Critical value = …?

Decision= reject H0?

Interpretation:

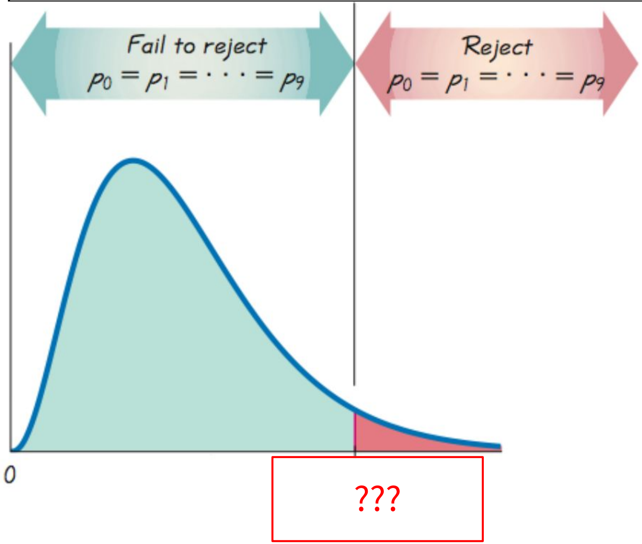
Try it using R

Using R: *chisq.test(obs_data)*

```
> weight_gof <- chisq.test(weight$obs_freq)
> weight_gof

        Chi-squared test for given probabilities

data:  weight$obs_freq
X-squared = 212.8, df = 9, p-value < 2.2e-16
```



Contingency Tables: Independency Test

- 2-way frequency table → tabel frekuensi 2 arah.
  - have **at least 2 rows** and **at least 2 columns**
  - frequencies correspond to two variables
  - **1st** variable → categorize **rows**
  - **2nd** variable → categorize **columns**
  - For every cell in the contingency table, the expected frequency E is at least 5

Table 11-6 Results from Experiment with Echinacea

	Treatment Group		
	Placebo	Echinacea: 20% extract	Echinacea: 60% extract
Infected	88	48	42
Not infected	15	4	10

- Hypothesis

  H0: The row and column variables are independent

  H1: The row and column variables are dependent

Contingency Tables (2)

- **Hypothesis:**
  H0: The row and column variables are independent

  H1: The row and column variables are dependent
- Test Statistics

$$\boxed{\chi^2 = \sum \frac{(O-E)^2}{E}}$$

df = (r-1)(c-1)

$$\boxed{E = \frac{(\text{row total})\,(\text{column total})}{(\text{grand total})}}$$

Derived from the ***n*p*** formula below

$$E = \cancel{\text{grand total}} \cdot \frac{\text{row total}}{\cancel{\text{grand total}}} \cdot \frac{\text{column total}}{\text{grand total}}$$

$$\boldsymbol{n} \cdot \boldsymbol{p}$$

(probability of a cell)

Table 11-6 Results from Experiment with Echinacea

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- Tests of Independence are always **right-tailed**.

Example

Table 11-6 Results from Experiment with Echinacea

	Treatment Group			
	Placebo	Echinacea: 20% extract	Echinacea: 60% extract	
Infected	88 88.57	48 44.71	42 44.71	178
Not infected	15 14.43	4 7.28	10 7.28	29
	103	52	52	**207**

$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

In a test of the effectiveness of echinacea to fight rhinovirus, some test subjects were treated with echinacea extracted with **20% ethanol**, some were treated with echinacea extracted with **60% ethanol**, and others were given a **placebo**. All of the test subjects were then exposed to **rhinovirus**. Use a 0.05 significance level to test the claim that **getting an infection (cold) is independent of the treatment group**. What does the result indicated about the effectiveness of echinacea as a treatment for colds?

Answer:

H0: Getting an infection is independent of the treatment

H1: Getting an infection and the treatment are dependent

Test statistics

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$= \frac{(88 - 88.570)^2}{88.570} + \ldots + \frac{(10 - 7.285)^2}{7.285}$$

$$= 2.925$$

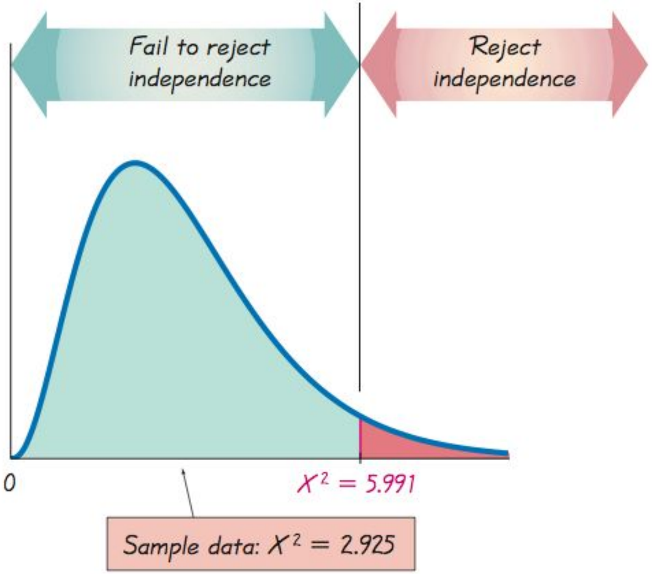
df = (r-1)(c-1) = (2-1)(3-1) = 2

For the **1st cell (88)** → **E = 178*103/207 = 88.57**

Interpretation:

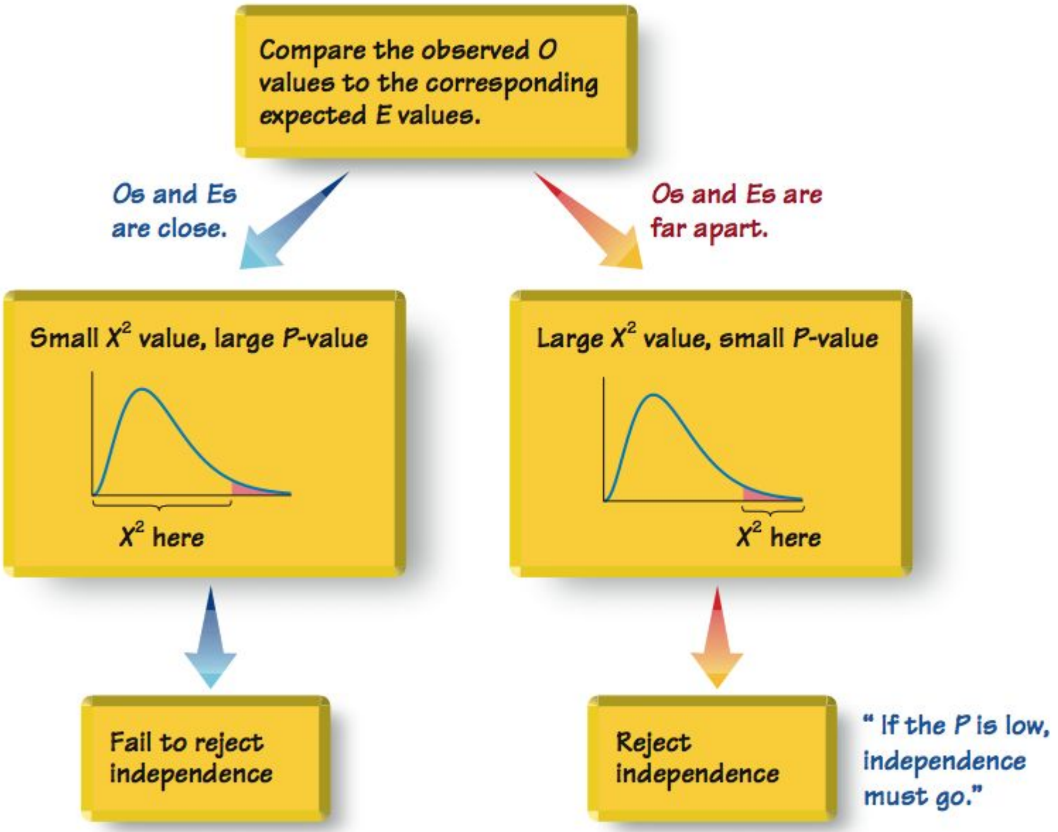
if we assume that getting an infection is independent of the treatment, then we **expect to find that 88.57** of the subjects would be given a placebo and would get an infection.

O - E → 88 - 88.57 → key component of the test statistics.



- Test statistics = 2.925
- P-value =
  > pchisq(2.925, 2, lower.tail = FALSE)
  [1] **0.2316564**
- Critical value = with α=5% & df = 2
  > qchisq(0.05, 2, lower.tail = FALSE)
  [1] **5.991465**
- P-value > α → fail to reject H0.
- Interpretation:
  Getting an infection is **independent** of the treatment group. This suggests that **echinacea is not an effective treatment for colds.**

Relationships Among Key Components in Test of Independence



Test of Homogeneity

- Claim: **different populations** have the **same proportions** of some characteristics
- **Hypothesis:**
  H0: proporsi populasi di tiap baris/kolom sama (homogen)
  H1: proporsi populasi di tiap baris/kolom tidak sama
- E and test statistics == test of independence.

$$\boxed{\chi^2 = \sum \frac{(O-E)^2}{E}}$$

df = (r-1)(c-1)

$$\boxed{E = \frac{(\text{row total})\,(\text{column total})}{(\text{grand total})}}$$

- Independency Test vs Homogeneity Test:
  - **Sample size**: pada **homogeneity test**, jumlah sampel untuk tiap proporsi (kolom/baris) sudah **ditentukan dari awal**, sementara pada **independency test** kita menarik 1 set sampel besar, kemudian **secara random dihitung** berapa yang masuk ke masing-masing kolom/baris.
  - **Tujuan:**
    - **Homogeneity** test: **apakah proporsi** populasi di tiap baris **sama**?
    - **Independence** test: **apakah ada keterkaitan** antara column variable dengan row variable?

Example

H0: The proportions of agree/disagree responses are the same for the subjects interviewed by men and the subjects interviewed by women.
H1: The proportions are different.

A group of surveyed men were asked if they agreed with this statement: "*Abortion is a private matter that should be left to the woman to decide without government intervention.*" Assume that the survey was designed so that **male interviewers** were instructed to obtain **800 responses** from male subjects, and **female interviewers** were instructed to obtain **400 responses** from male subjects. Using a **0.05** significance level, test the claim that the **proportions of agree/disagree** responses are the **same** for the subjects **interviewed by men** and the subjects **interviewed by women**.

Table 11-6 Gender and Survey Responses

	Gender of Interviewer	
	Man	Woman
Men who agree	560	308
Men who disagree	240	92

```
> abortion_chisq

	Pearson's Chi-squared test with Yates' continuity correction

data:  abortion
X-squared = 6.1842, df = 1, p-value = 0.01289
```

P-value < α → reject H0
Meaning: **no homogeneity**

Fisher Exact Test

- for a 2 x 2 contingency table.
- Having 1 or more **expected frequencies < 5**
- calculations are quite **complex**, it's a good idea to use **technologies** (R, excel, minitab, spss, etc).
- With R:
  fisher.exact(data) → *fisher.exact(abortion)*

McNemar's Test for Matched Pair

- Checking on the discordance
- testing the null hypothesis that the **frequencies from the discordant (different) categories occur in the same proportion**.
- Data are place in a 2 x 2 table where each observation is classified in two ways.
- The test only compares categories that are **different** (**discordant** pairs).

Table 11-9 2 × 2 Table with Frequency Counts from Matched Pairs

		Treatment X	
		Cured	Not Cured
Treatment Y	Cured	*a*	$\boxed{b}$
	Not cured	$\boxed{c}$	*d*

use only the frequencies from the pairs of categories that are different

a, b, c, and d represent the frequency counts

- H0: The proportions of the frequencies b and c are the same.
  H1: The proportions of the frequencies b and c are different.
- Test statistics: $\chi^2 = \dfrac{(|b - c| - 1)^2}{b + c}$

Example

A randomized controlled trial was designed to **test the effectiveness of hip protectors** in preventing hip fractures in the **elderly**. Nursing home residents each **wore protection on one hip, but not the other (matched pair)**. Results are summarized in table. Using a **0.05** significance level, apply McNemar's test to test the null hypothesis that the following two proportions are the same:

		No Hip Protector Worn	
		No Hip Fracture	Hip Fracture
Hip Protector Worn	No Hip Fracture	309	$\boxed{10}$ b
	Hip Fracture	$\boxed{15}$ c	2

- The proportion of subjects with **no hip fracture on the protected hip** and **a hip fracture on the unprotected hip**.
- The proportion of subjects with a **hip fracture on the protected hip** and **no hip fracture on the unprotected hip**.

Based on the results, **do the hip protectors appear to be effective in preventing hip fractures**?

$$\chi^2 = \frac{(|b - c| - 1)^2}{b + c} = \frac{(|10 - 15| - 1)^2}{10 + 15} = 0.640$$

```
> mcnemar.test(hip)

	McNemar's Chi-squared test with continuity correction

data:  hip
McNemar's chi-squared = 0.64, df = 1, p-value = 0.4237
```

It appears that the **proportion of hip fractures with the protectors worn** is <u>**not significantly different**</u> from the proportion of **hip fractures without the protectors worn**. The hip protectors **do not appear to be effective** in preventing hip fractures.

Compute the Test Statistics (manual way)

	Before marketing video	
After marketing video	Do not support	support
Support	40	30
Do not support	18	12

	After: present	**After:** absent	Row total
Before: present	101	121	222
Before: absent	59	33	92
Column total	160	154	314

Exercise (Note: all tests use α = 5%)

1. Lakukan goodness-of-fit test pada data internet (internet.csv), definisikan hipotesisnya, hitung test statistiknya, buat keputusan dan interpretasi. → gof test
   a. Dengan cara manual
   b. Dengan R
2. Berdasarkan tabel berikut, ujilah independensi antara “the type of treatment for stress fracture” vs ‘its success’. → independency

	success	failure
surgery	54	12
Weight-bearing cast	41	51
non-weight-bearing cast for 6 weeks	70	3
non-weight-bearing cast for < 6 weeks	17	5

Exercise

3. Berdasarkan tabel berikut, lakukanlah uji homogenitas dengan $\alpha = 5\%$

UU Baru	Partai politik			Total
	Golkar	PDI-P	Demokrat	
Setuju	82	70	62	**214**
Menentang	93	62	67	**222**
Tanpa pendapat	25	18	21	**64**
Total	**200**	**150**	**150**	**500**

Exercise

4. Berdasarkan tabel berikut, lakukanlah uji independency-nya. Hitunglah E-nya. Apakah sebaiknya menggunakan uji independency biasa atau fisher exac test?

	Non smoker	smoker
athlete	7	2
Non athlete	0	5

5. Lakukan mcnemar test pada data berikut. Sertakan juga hasil penghitungan test statisticsnya dengan cara manual.

1st survey		2nd survey	
		approve	disapprove
1st survey	approve	794	**150**
	disapprove	**86**	570

Thanks!

Any questions?

You can find me at:

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