EECS 142

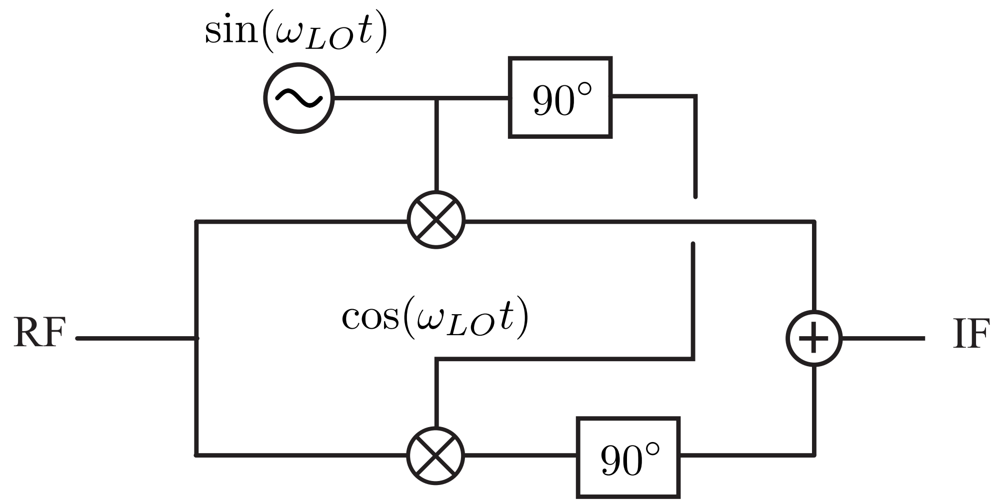
Integrated Circuits for Communication

Lecture 16: I/Q Mixers; BJT Mixers

Prof. Ali M. Niknejad

University of California, Berkeley

I/Q Hartley Mixer



- An I/Q mixer implemented as shown above is known as a Hartley Mixer.
- We shall show that such a mixer can be designed to select either the upper or lower sideband. For this reason, it is sometimes called a single-sideband mixer.
- We will also show that such a mixer can perform image rejection.

Delay Operation

- Consider the action of a 90° delay on an arbitrary signal. Clearley $\sin(x + 90^\circ) = \cos(x)$. Even though this is obvious, consider the effect on the complex exponentials

$$\sin(x - \frac{\pi}{2}) = \frac{e^{jx - j\pi/2} - e^{-jx + j\pi/2}}{2j}$$

$$= \frac{e^{jx}e^{-j\pi/2} - e^{-jx}e^{j\pi/2}}{2j} = \frac{e^{jx}(-j) - e^{-jx}(j)}{2j}$$

$$= -\frac{e^{jx} + e^{-jx}}{2} = -\cos(x)$$

- Notice that positive frequencies get multiplied by $-j$ and negative frequencies by $+j$. This is true for any waveform when it is delayed by 90°.

Complex Modulation

- Consider multiplying a waveform $f(t)$ by $e^{j\omega t}$ and taking the Fourier transform

$$\mathcal{F}\left\{e^{j\omega_0 t} f(t)\right\} = \int_{-\infty}^{\infty} f(t) e^{j\omega_0 t} e^{-j\omega t} dt$$

- Grouping terms we have

$$= \int_{-\infty}^{\infty} f(t) e^{-j(\omega-\omega_0)t} dt = F(\omega - \omega_0)$$

- It is clear that the action of multiplication by the complex exponential is a frequency shift.

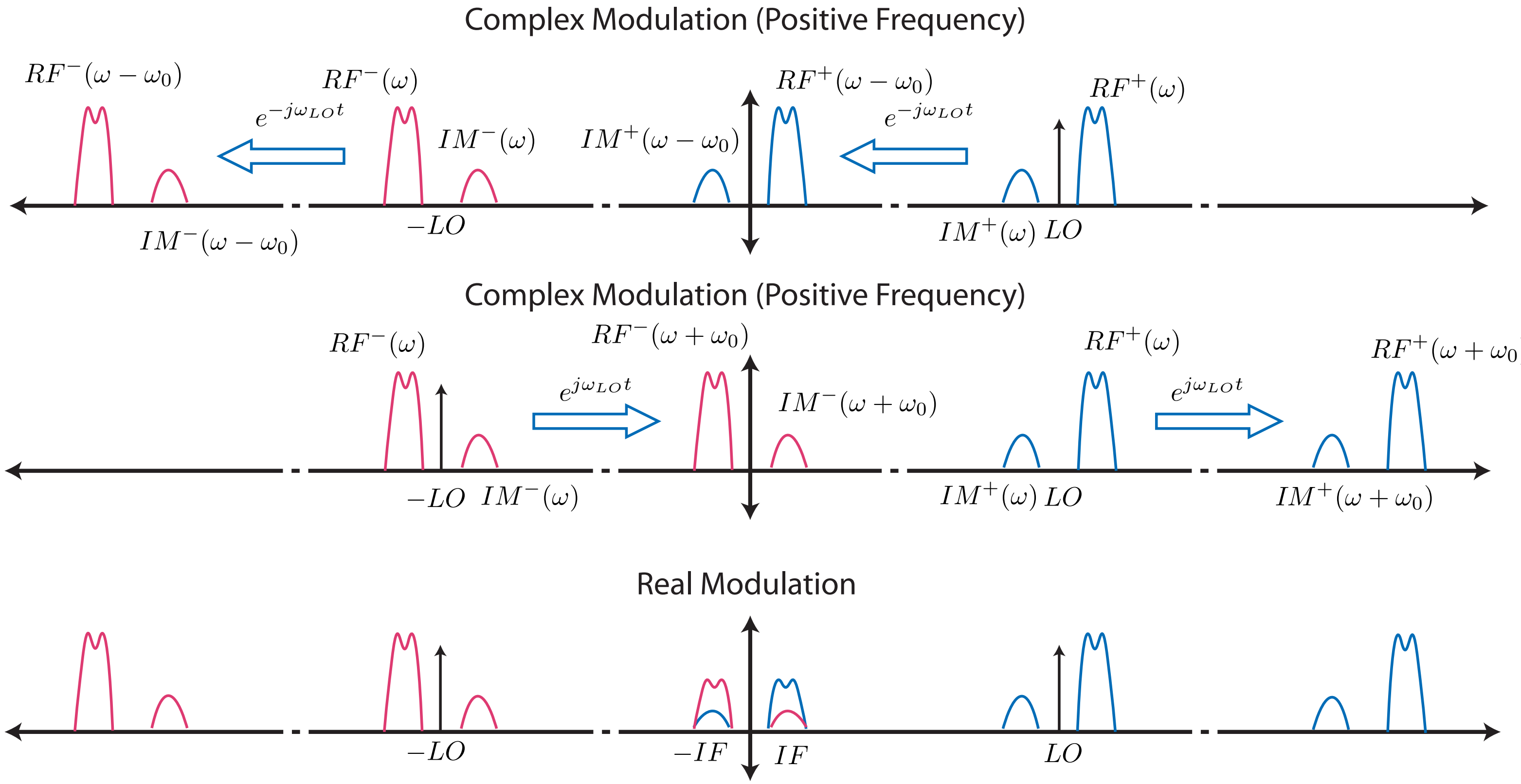
Real Modulation

- Now since $\cos(x) = (e^{jx} + e^{-jx})/2$, we see that the action of time domain multiplication is to produce two frequency shifts

$$\mathcal{F}\left\{\cos(\omega_0 t) f(t)\right\} = \frac{1}{2}F(\omega - \omega_0) + \frac{1}{2}F(\omega + \omega_0)$$

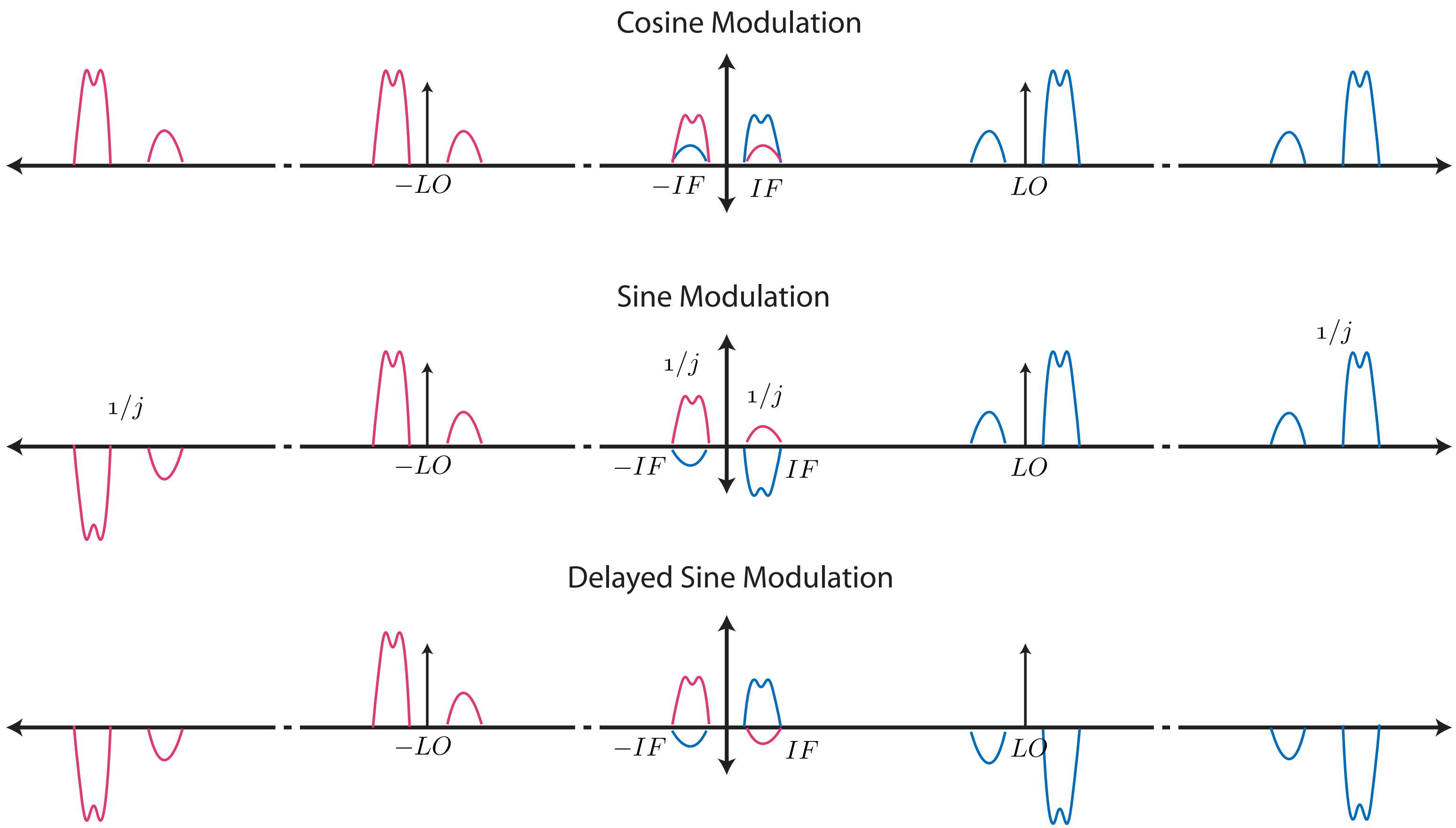
- These are the sum and difference (beat) frequency components.

Image Problem (Again)



- We see that the image problem is due to to multiplication by the sinusoid and not a complex exponential. If we could synthesize a complex exponential, we would not have the image problem.

Sine/Cosine Modulation



- Using the same approach, we can find the result of multipling by $\sin$ and $\cos$ as shown above. If we delay the $\sin$ portion, we have a very desirable situation! The image is inverted with respect to the $\cos$ and can be cancelled.

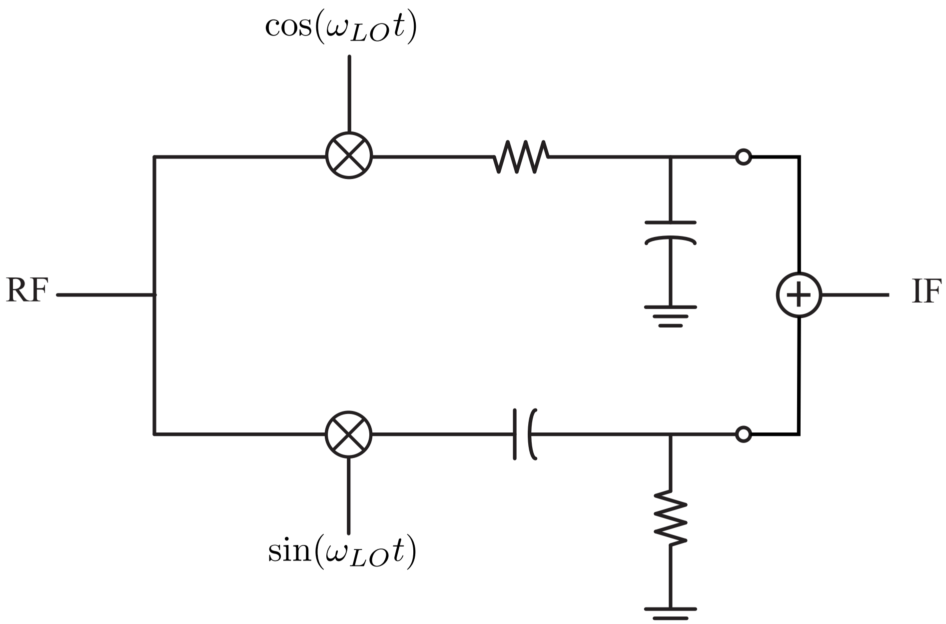
Image Rejection

- The image rejection scheme just described is very sensitive to phase and gain match in the I/Q paths. Any mismatch will produce only finite image rejection.
- The image rejection for a given gain/phase match is approximately given by

$$IRR(\mathrm{dB}) = 10 \cdot \log \frac{1}{4}\left(\left(\frac{\delta A}{A}\right)2 + (\delta\theta)^2\right)$$

- For typical gain mismatch of $0.2 - 0.5\,\mathrm{dB}$ and phase mismtach of $1^\circ - 4^\circ$, the image rejection is about $30\,\mathrm{dB}$ - $40\,\mathrm{dB}$. We usually need about $60 - 70\,\mathrm{dB}$ of total image rejection.

$\pm 45°$ Delay Element



- The passive R/C and C/R lowpass and highpass filters are a nice way to implement the delay. Note that their relative phase difference is always $90°$.

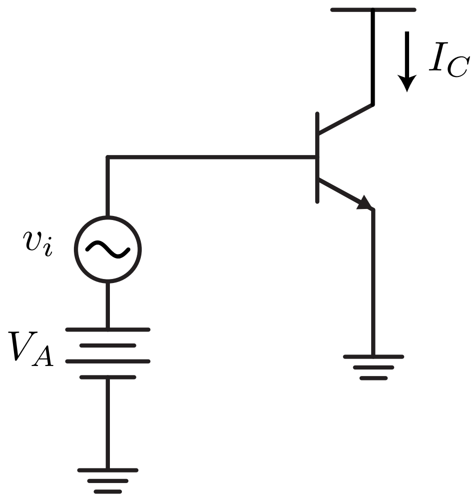
$$\angle H_{lp} = \angle \frac{1}{1 + j\omega RC} = -\arctan \omega RC$$

$$\angle H_{hp} = \angle \frac{j\omega RC}{1 + j\omega RC} = \frac{\pi}{2} - \arctan \omega RC$$

Gain Match / Quadrature LO Gen

- But to have equal gain, the circuit must operate at the $1/RC$ frequency. This restricts the circuit to relatively narrowband systems. Multi-stage polyphase circuits remedy the situation but add insertion loss to the circuit.
- The I/Q LO signal is usually generated directly rather than through an high-pass and low-pass network.
- Two ways to generate the I/Q LO is through a divide-by-two circuit (requires $2 \times LO$) or a quadrature oscillator (requires two tanks).

BJT with Large Sine Drive



$$v_i = \hat{V}_i \cos \omega t$$

$$I_C = I_S e^{\frac{v_{BE}}{V_t}}$$

$$v_{BE} = V_A + \hat{V}_i \cos \omega t$$

- Consider a bipolar device driven with a large sine signal.
- This occurs in many types of non-linear circuits, such as oscillators, frequency multipliers, mixers and class C amplifiers.

BJT Collector Current

- The collector current can be factored into a DC bias term and a periodic signal

$$I_C = I_S e^{V_A} V_t e^{\frac{\hat{V}_i}{V_t}\cos\omega t}$$

$$I_C = I_S e^a e^{b\cos\omega t}$$

- Where the normalized bias is $a = V_a/V_t$ and the normalized drive signal is $b = \hat{V}_i/V_t$.
- Since I_C is a periodic function, we can expand it into a Fourier Series. Note that the Fourier coefficients of $e^{b\cos\omega t}$ are modified Bessel functions $I_n(b)$

$$e^{b\cos\omega t} = I_0(b) + 2I_1(b)\cos\omega t + 2I_2(b)\cos 2\omega t + \cdots$$

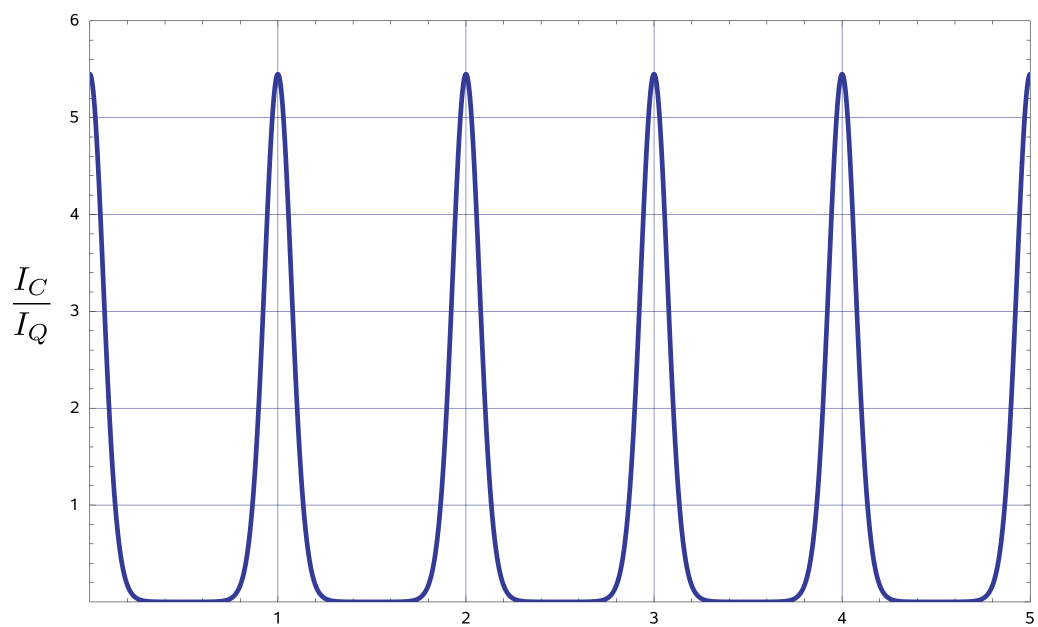
BJT DC Current

- Assume that the bias current of the amplifier is stabilized. Then

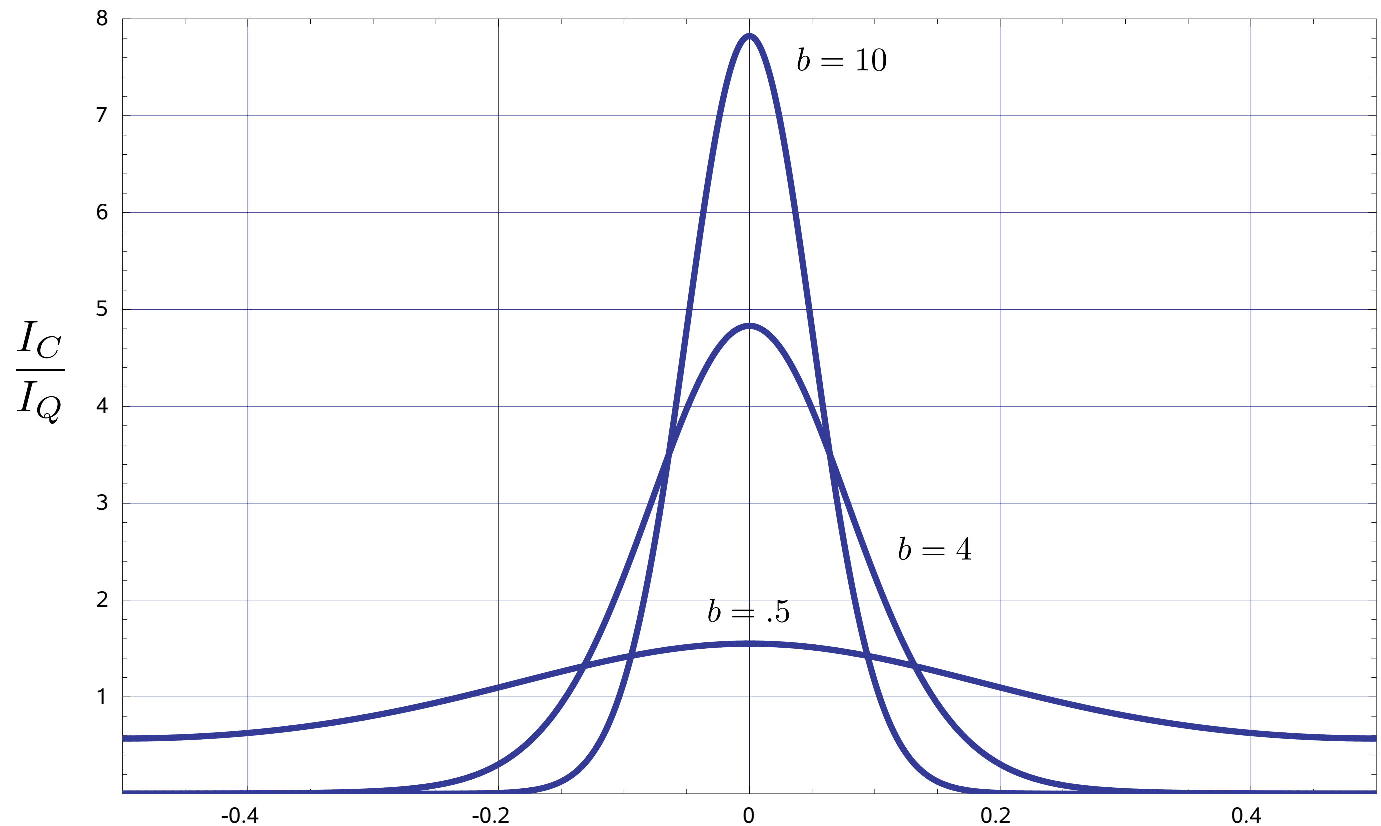
$$I_C = \underbrace{I_S e^a I_0(b)}_{I_Q} \left(1 + \frac{2I_1(b)}{I_0(b)} \cos \omega t + \frac{2I_2(b)}{I_0(b)} \cos 2\omega t + \cdots \right)$$

$$I_C = I_S e^a e^{b \cos \omega t}$$

$$= \frac{I_Q}{I_0(b)} e^{b \cos \omega t}$$

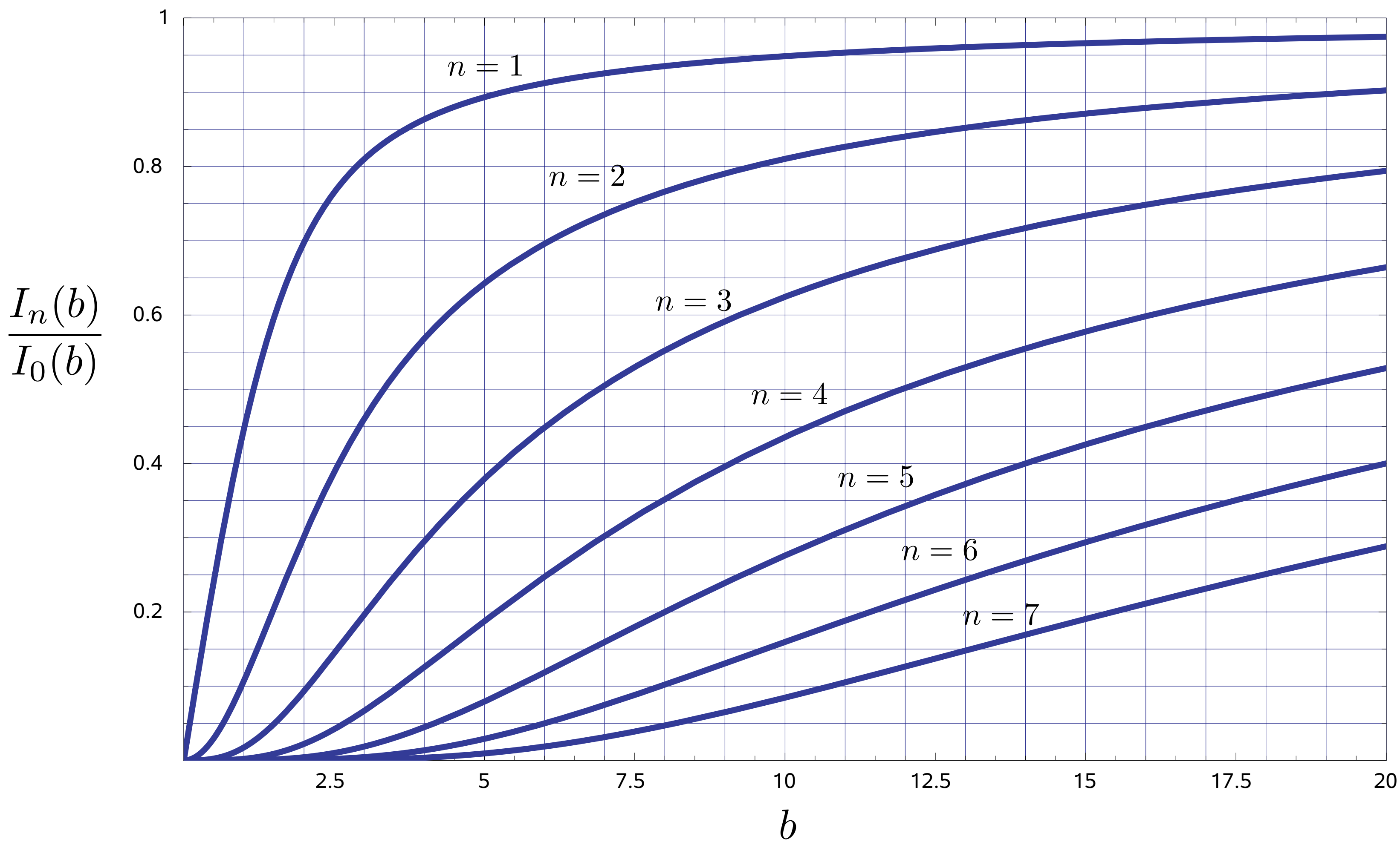


Collector Current Waveform



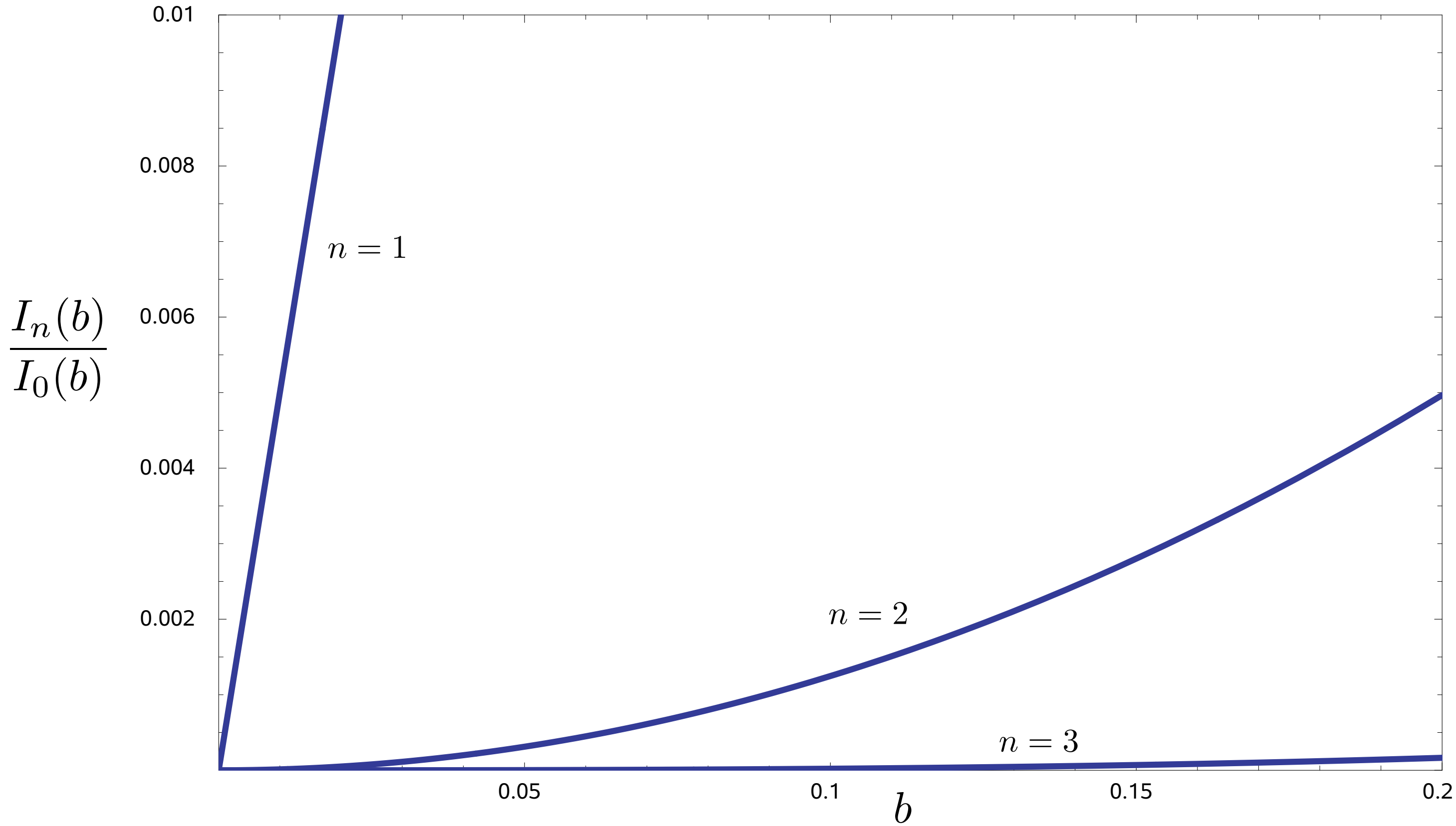
- With increasing input drive, the current waveform becomes “peaky”. The peak value can exceed the DC bias by a large factor.

Harmonic Current Amplitudes



- The BJT output spectrum is rich in harmonics.

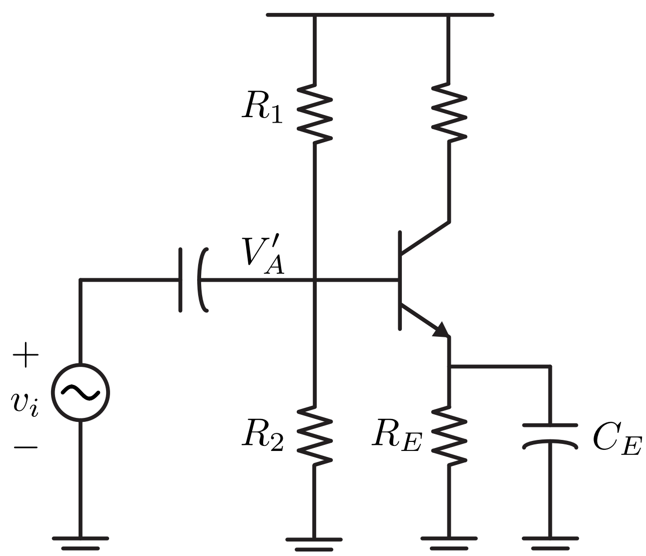
Small-Signal Region



- If we zoom in on the curves to small b values, we enter the small-signal regime, and the weakly non-linear behavior is predicted by our power series analysis.

BJT with Stable Bias

- Neglecting base current ($\beta \gg 1$), the voltage at the base is given by



$$V'_A = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$V_E = V'_A - V_{BE}$$

$$I_Q = \frac{V_E}{R_E} = \frac{V'_A - V_{BE}}{R_E}$$

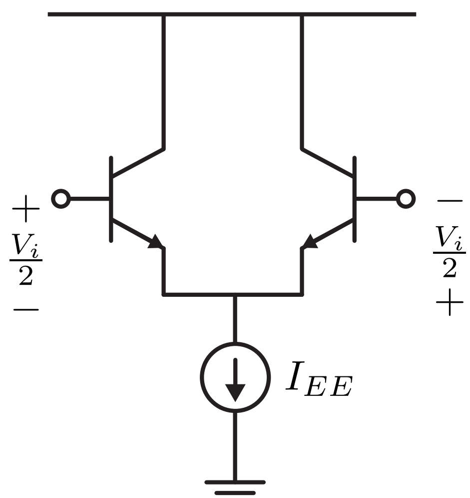
- We see that the bias is fixed since V_{BE} does not vary too much. Typically V'_A is a few volts.
- In this circuit C_E is an emitter bypass capacitor used to short R_E at high frequency.

Differential Pair with Sine Drive

- The large signal equation for I_{C1} is given by

$$I_{C1} + I_{C2} = I_{EE}$$

$$V_{BE1} - V_{BE2} = V_i = V_t \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$

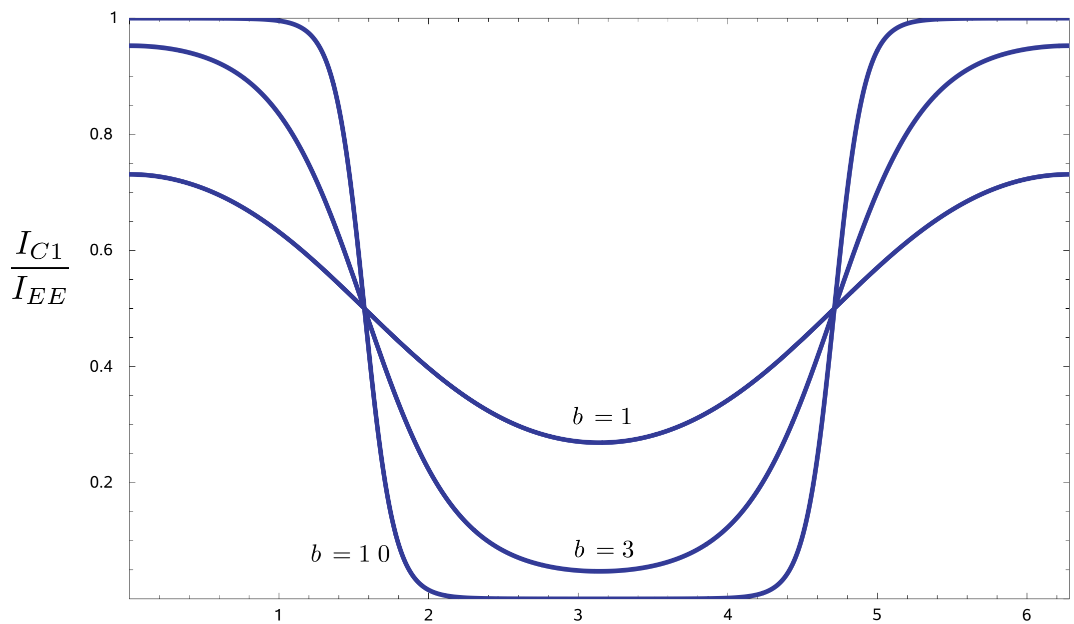


$$I_{C1} = \frac{I_{EE}}{1 + e^{-v_i/V_t}}$$

$$v_i = \hat{V}_i \cos \omega t$$

$$I_{C1} = \frac{I_{EE}}{1 + e^{-b \cos \omega t}}$$

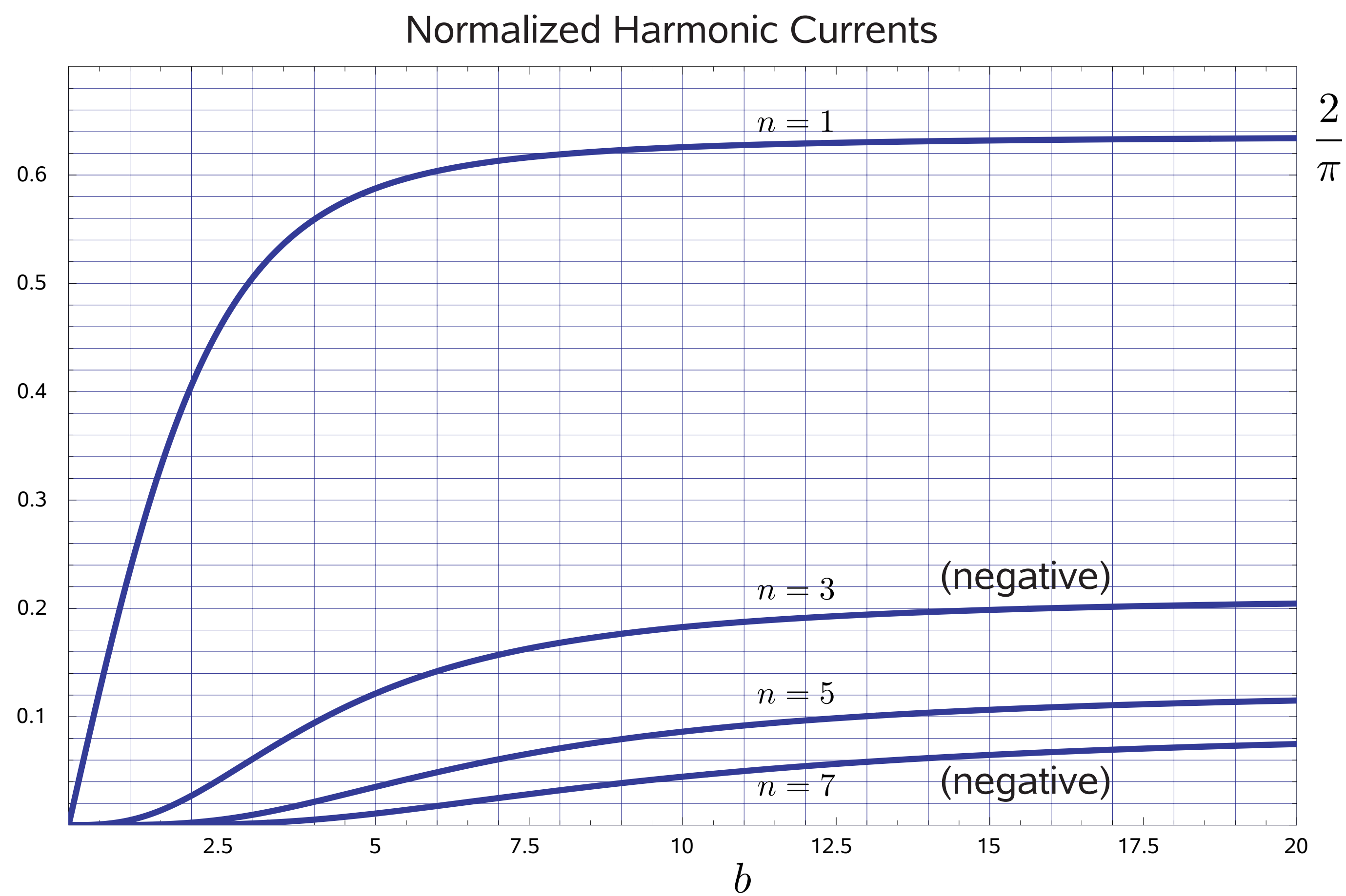
Diff Pair Waveforms



- For large b, the waveform approaches a square wave

$$\frac{I_C}{I_{EE}} = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega t - \frac{1}{3}\cos 3\omega t + \frac{1}{5}\cos 5\omega t + \cdots\right)$$

Diff Pair Harmonic Currents



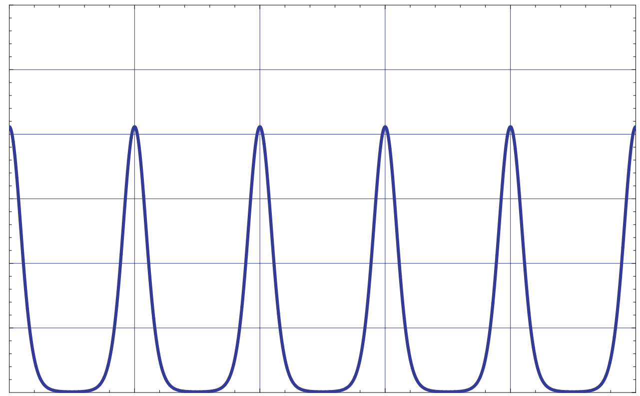
- As expected, the ideal differential pair does not produce any even harmonics.

Mixer Analysis

- As we have seen, a mixer has three ports, the LO, RF, and IF port.
- Assume that a circuit is “pumped” with a periodic large signal at the LO port with frequency ω_0.
- From the RF port, though, assume we apply a small signal at frequency ω_s.
- Since the RF input is small, the circuit response should be linear (or weakly non-linear). But since the LO port changes the operating point of the circuit periodically, we expect the overall response to the RF port to be a linear time-varying response

$$i_o(t) = g(t)v_{in}$$

Mixer Assumptions



- The transconductance varies periodically and can be expanded in a Fourier series

$$g(t) = g_0 + g_1 \cos \omega_0 t + g_2 \cos 2\omega_0 t + \cdots$$

- Applying the input $v_{in} = \hat{V}_1 \cos \omega_s t$

$$i_o(t) = (g_0 + g_1 \cos \omega_0 t + g_2 \cos 2\omega_0 t + \cdots) \times \hat{V}_1 \cos \omega_s t$$

Mixer Output Signal

- Expanding the product, we have

$$i_o(t) = g_0 \hat{V}_1 \cos \omega_s t + \frac{g_1}{2} \hat{V}_1 \cos(\omega_0 \pm \omega_s)t + \frac{g_2}{2} \hat{V}_1 \cos(2\omega_0 \pm \omega_s)t + \cdots$$

- The first term is just the input amplified. The other terms are all due to the mixing action of the linear time-varying periodic circuit.
- Let's say the desired output is the IF at $\omega_0 - \omega_s$. The conversion gain is therefore defined as

$$g_{conv} = \frac{|\text{IF output current}|}{|\text{RF input signal voltage}|} = \frac{g_1}{2}$$