EECS 142

Integrated Circuits for Communication

Lecture 22: Oscillator Steady-State Analysis

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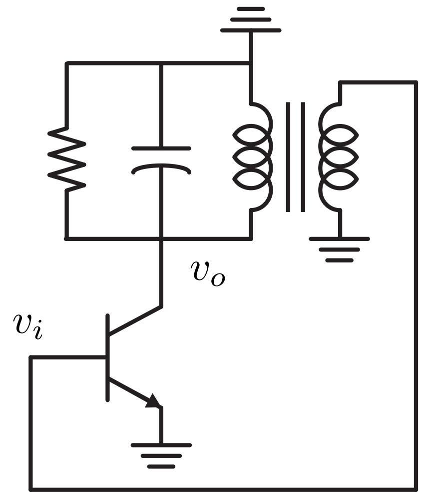
Summary of Last Lecture



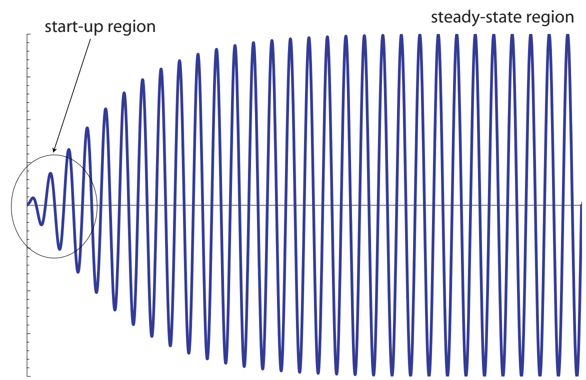
- Last lecture we analyzed the small-signal behavior of the above circuit. We found that the closed-loop gain is given by

$$H(s) = \frac{g_m R s \frac{L}{R}}{1 + s\frac{L}{R}(1 - A_\ell) + s^2 LC}$$

Review: Role of Loop Gain

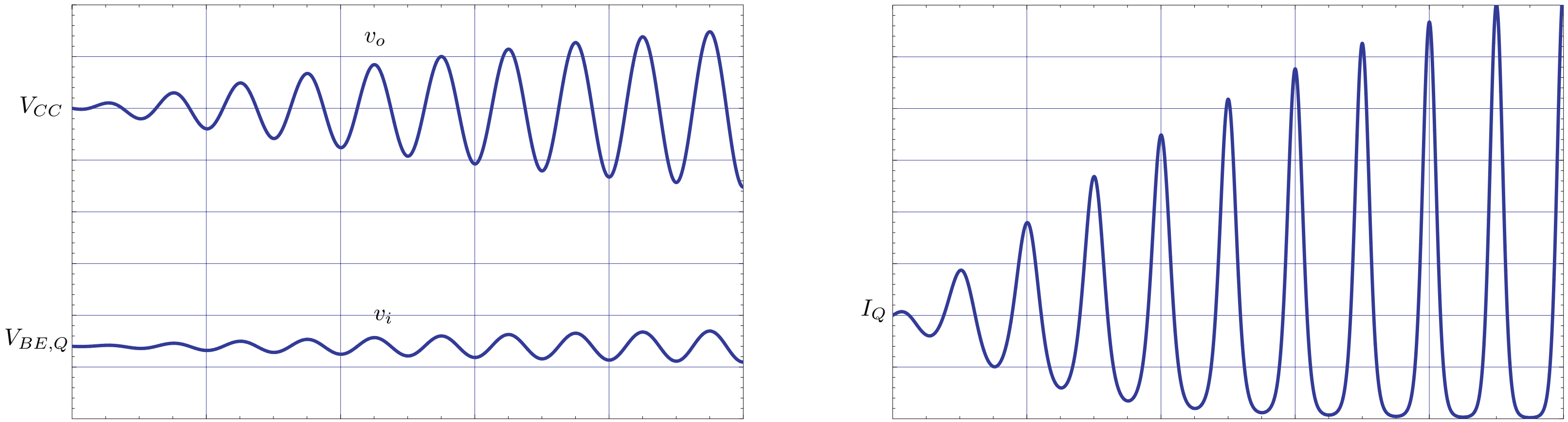
- The behavior of the circuit is determined largely by A_ℓ, the loop gain at DC and resonance. When $A_\ell = 1$, the poles of the system are on the $j\omega$ axis, corresponding to constant amplitude oscillation.
- When $A_\ell < 1$, the circuit oscillates but decays to a quiescent steady-state.
- When $A_\ell > 1$, the circuit begins oscillating with an amplitude which grows exponentially. Eventually, we find that the steady state amplitude is fixed.

Steady-State Analysis



- To find the steady-state behavior of the circuit, we will make several simplifying assumptions. The most important assumption is the high tank Q assumption (say $Q > 10$), which implies the output waveform v_o is sinusoidal.
- Since the feedback network is linear, the input waveform $v_i = v_o/n$ is also sinusoidal.
- We may therefore apply the large-signal periodic steady-state analysis of the BJT to the oscillator.

Steady-State Waveforms



- The collector current is not sinusoidal, due to the large signal drive.
- The output voltage,though, is sinusoidal and given by

$$v_o \approx I_{\omega_1} Z_T(\omega_1) = G_m Z_T v_i$$

Steady State Equations

- But the input waveform is a scaled version of the output

$$v_o = G_m Z_T \frac{v_o}{n} = \frac{G_m Z_T}{n} v_o$$

- The above equation implies that

$$\frac{G_m Z_T}{n} \equiv 1$$

- Or that the loop gain in steady-state is unity and the phase of the loop gain is zero degrees (an exact multiple of 2π)

$$\left| \frac{G_m Z_T}{n} \right| \equiv 1 \qquad \angle \frac{G_m Z_T}{n} \equiv 0^\circ$$

Large Signal G_m

- Recall that the small-signal loop gain is given by

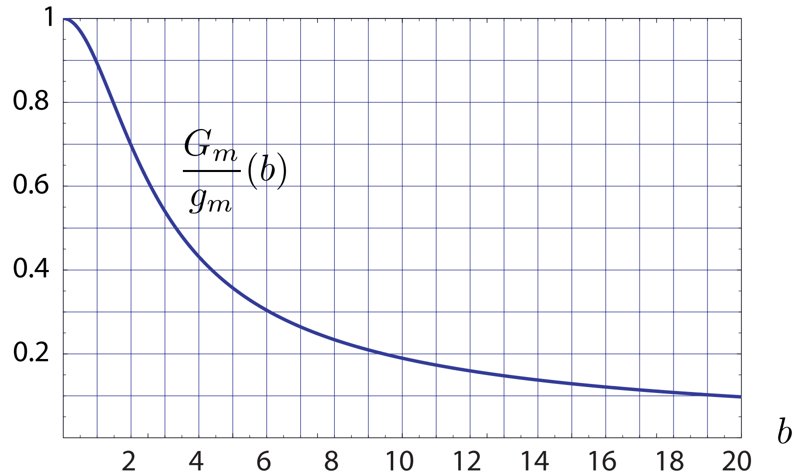
$$|A_\ell| = \left|\frac{g_m Z_T}{n}\right|$$

- Which implies a relation between the small-signal start-up transconductance and the steady-state large-signal transconductance

$$\left|\frac{g_m}{G_m}\right| = A_\ell$$

- Notice that g_m and A_ℓ are design parameters under our control, set by the choice of bias current and tank Q. The steady state G_m is therefore also fixed by initial start-up conditions.

Large Signal G_m (II)



- To find the oscillation amplitude we need to find the input voltage drive to produce G_m.
- For a BJT, we found that under the constraint that the bias current is fixed

$$I_{\omega_1} = \frac{2I_1(b)}{I_0(b)} I_Q = G_m v_i = G_m b \frac{kT}{q}$$

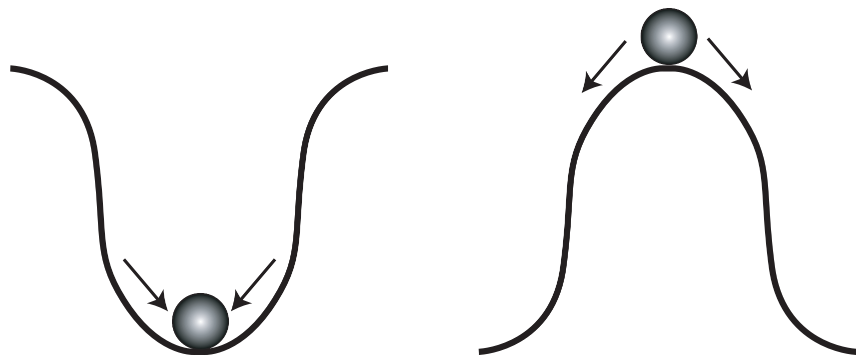
Large Signal G_m (III)

- Thus the large-signal G_m is given by

$$G_m = \frac{2I_1(b)}{bI_0(b)} \frac{qI_Q}{kT} = \frac{2I_1(b)}{bI_0(b)} g_m$$

$$\frac{G_m}{g_m} = \frac{2I_1(b)}{bI_0(b)}$$

Stability (Intuition)



- Here's an intuitive argument for how the oscillator reaches a stable oscillation amplitude. Assume that initially $A_l > 1$ and oscillations grow. As the amplitude of oscillation increases, though, the G_m of the transistor drops, and so effectively the loop gain drops.
- As the loop gain drops, the poles move closer to the $j\omega$ axis. This process continues until the poles hit the $j\omega$ axis, after which the oscillation ensues at a constant amplitude and $A_\ell = 1$.

Intuition (cont)

- To see how this is a stable point, consider what happens if somehow the loop gain changes. If the loop gain changes to $A_\ell + |\epsilon|$, then we already see that the system will roll back. If the loop gain drops below unity, $A_\ell - |\epsilon|$, then the poles move into the LHP and amplitude of oscillation will begin to decay.
- As the oscillation amplitude decays, the G_m increases and this causes the loop gain to grow. Thus the system also rolls back to the point where $A_\ell = 1$.

BJT Oscillator Design

- Say we desire an oscillation amplitude of $v_0 = 100\mathrm{mV}$ at a certain oscillation frequency ω_0.
- We begin by selecting a loop gain $A_\ell > 1$ with sufficient margin. Say $A_\ell = 3$.
- We also tune the LC tank to ω_0, being careful to include the loaded effects of the transistor (r_o, C_o, C_{in}, R_{in})
- We can estimate the required first harmonic current from

$$I_{\omega_0} = \frac{v_o}{R'_T}$$

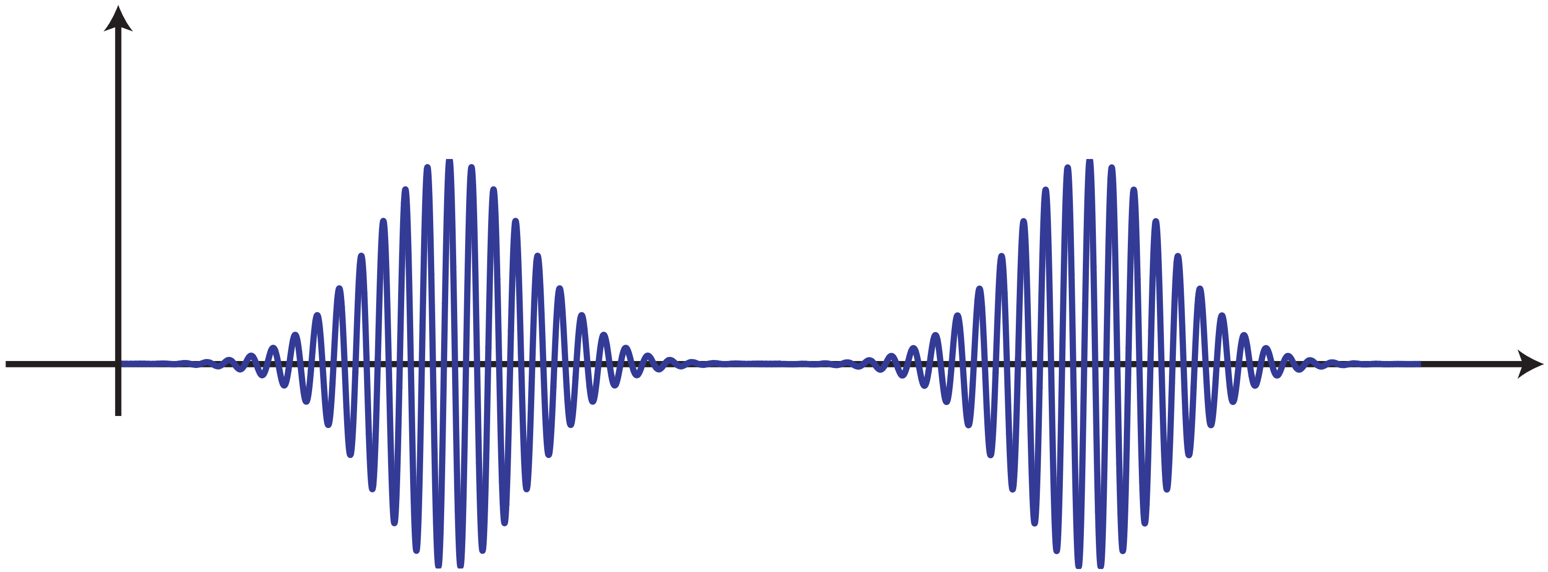
Design (cont)

- This is an estimate because the exact value of R_T is not known until we specify the operating point of the transistor. But a good first order estimate is to neglect the loading and use R'_T
- We can now solve for the bias point from

$$I_{\omega_1} = \frac{2I_1(b)}{I_0(b)} I_Q$$

- b is known since it's the oscillation amplitude normalized to kT/q and divided by n. The above equation can be solved graphically or numerically.
- Once I_Q is known, we can now calculate R''_T and iterate until the bias current converges to the final value.

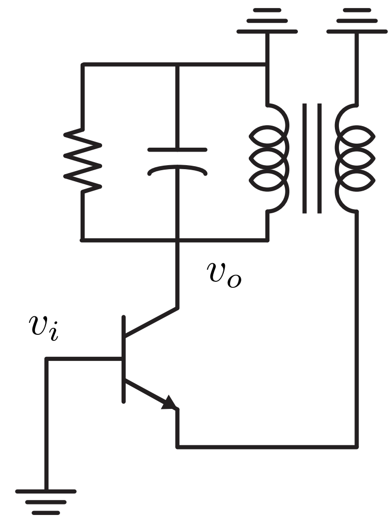
Squegging



- Squegging is a parasitic oscillation in the bias circuitry of the amplifier.
- It can be avoided by properly sizing the emitter bypass capacitance

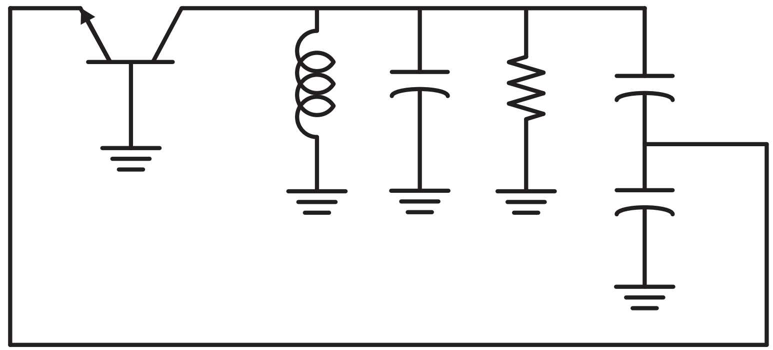
$$C_E \leq nC_T$$

Common Base Oscillator



- Another BJT oscillator uses the common-base transistor. Since there is no phase inversion in the amplifier, the transformer feedback is in phase.
- Since we don't need phase inversion, we can use a simpler feedback consisting of a capacitor divider.

Colpitts Oscillator

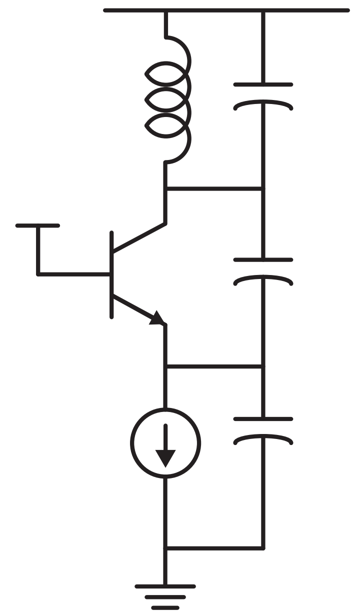


- The cap divider works at higher frequencies. Under the cap divider approximation $$f \approx \frac{C_1}{C_1 + C_2'} = \frac{1}{n}$$

$$n = 1 + \frac{C_2'}{C_1}$$

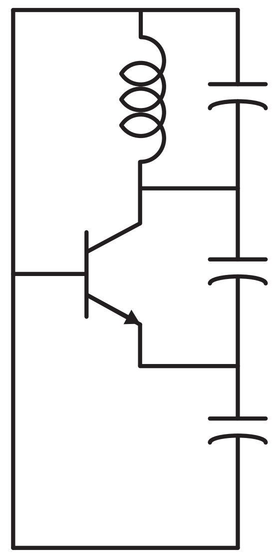
- C_2' includes the loading from the transistor and current source.

Colpitts Bias



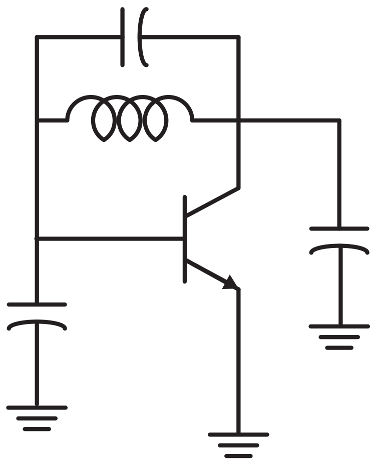
- Since the bias current is held constant by a current source I_Q or a large resistor, the analysis is identical to the BJT oscillator with transformer feedback. Note the output voltage is divided and applied across v_{BE} just as before.

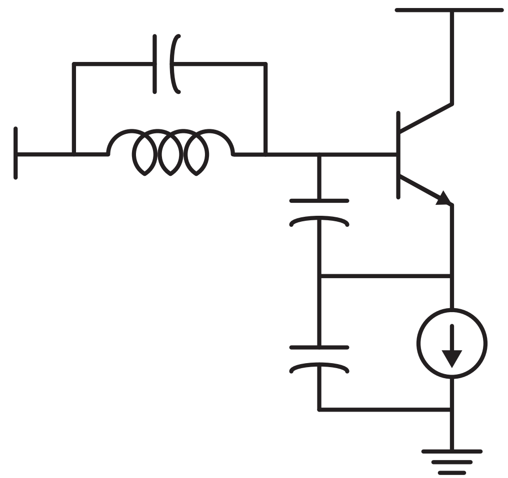
Colpitts Family



- If we remove the explicit ground connection on the oscillator, we have the template for a generic oscillator.
- It can be shown that the Colpitts family of oscillator never squegg.

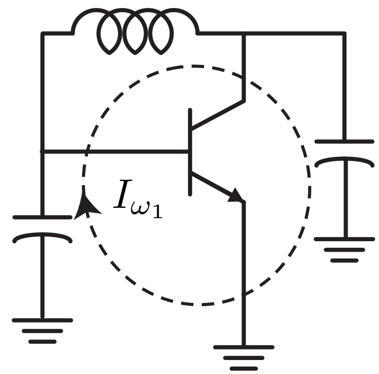
CE and CC Oscillators





- If we ground the emitter, we have a new oscillator topology, called the Pierce Oscillator. Note that the amplifier is in CE mode, but we don't need a transformer!
- Likewise, if we ground the collector, we have an emitter follower oscillator.
- A fraction of the tank resonant current flows through $C_{1,2}$. In fact, we can use $C_{1,2}$ as the tank capacitance.

Pierce Oscillator



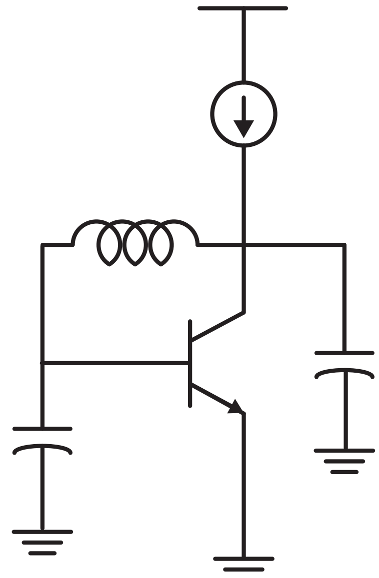
- If we assume that the current through $C_{1,2}$ is larger than the collector current (high Q), then we see that the same current flows through both capacitors. The voltage at the input and output is therefore

$$v_o = I_{\omega_1} \frac{1}{j\omega C_1} \qquad v_i = -I_{\omega_1} \frac{1}{j\omega C_2}$$

- or

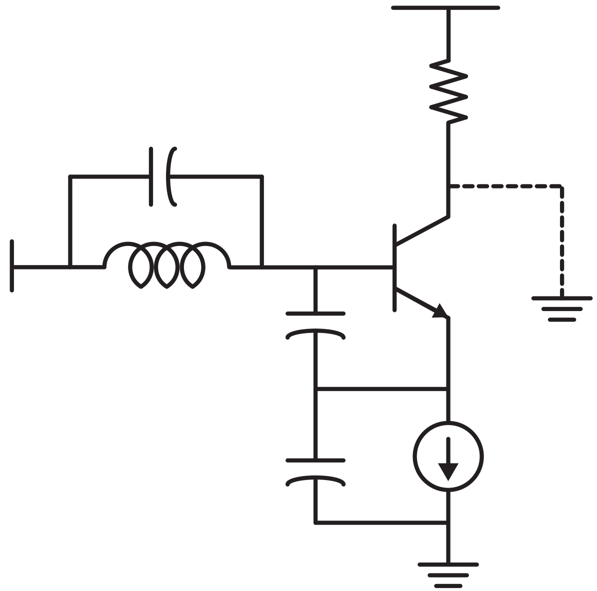
$$\frac{v_o}{v_i} = n = \frac{C_1}{C_2}$$

Pierce Bias



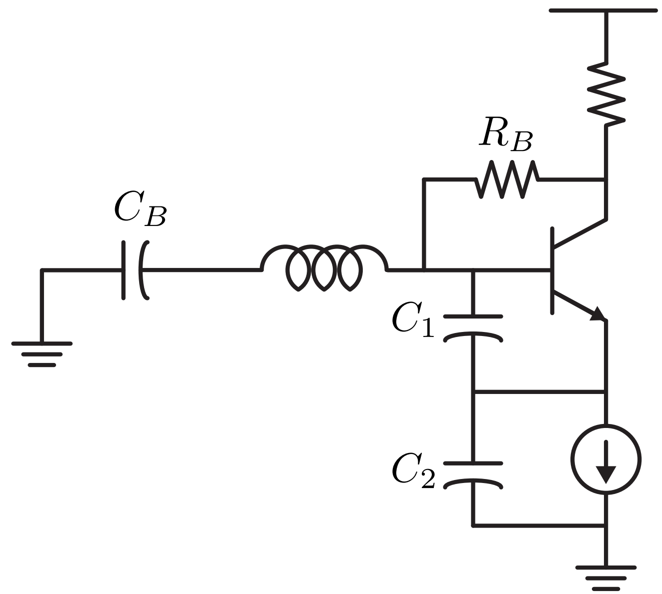
- A current source or large resistor can bias the Pierce oscillator.
- Since the bias current is fixed, the same large signal oscillator analysis applies.

Common-Collector Oscillator



- Note that the collector can be connected to a resistor without changing the oscillator characteristics. In fact, the transistor provides a buffered output for “free”.

Clapp Oscillator



- The common-collector oscillator shown above uses a large capacitor C_T to block the DC signal at the base. R_B is used to bias the transistor.
- If the shunt capacitor C_T is eliminated, then the capacitor C_B can be used to resonate with L and the series combination of C_1 and C_2. This is a series resonant circuit.