EECS 142

Integrated Circuits for Communication

Lecture 27: Non-Linear Power Amplifiers

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Efficiency of Class A

- Consider the power dissipated by the transistor

$$p_t(t) = i(t) \times v(t) = (I_Q + i_o \cos \omega_0 t) \times (V_{CC} - v_o \cos \omega_0 t)$$

$$p_t(t) = I_Q V_{CC} - i_o v_o \cos^2 \omega_0 t + (i_o V_{CC} - I_Q v_o) \cos \omega_0 t$$

- The average power over a cycle is given by

$$\overline{p_t(t)} = I_Q V_{CC} - \frac{i_o v_o}{2}$$

- The above result is obvious from conservation of energy. It simply states that the difference between DC power and the power to the load must be the power lost to heat.

Efficiency Cont

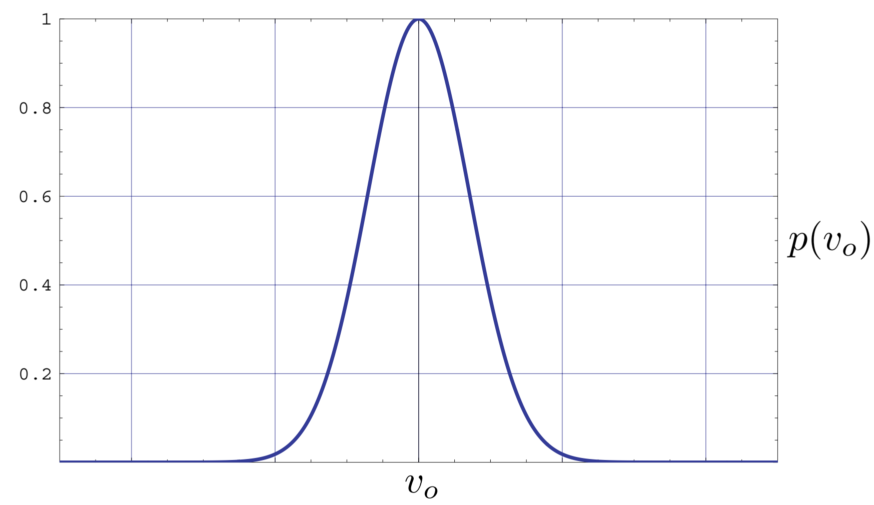
- Class A amplifiers have the undesirable property that the power dissipation is maximum for the absence of an input signal, a common condition. In fact, the efficiency is a linear function of the output power.
- The minimum power dissipation occurs for the maximum output power, where half the power is delivered to the load and the other half is converted to heat

$$\overline{p_t(t)}\Big|_{min} = \frac{I_Q V_{CC}}{2}$$

Average Efficiency

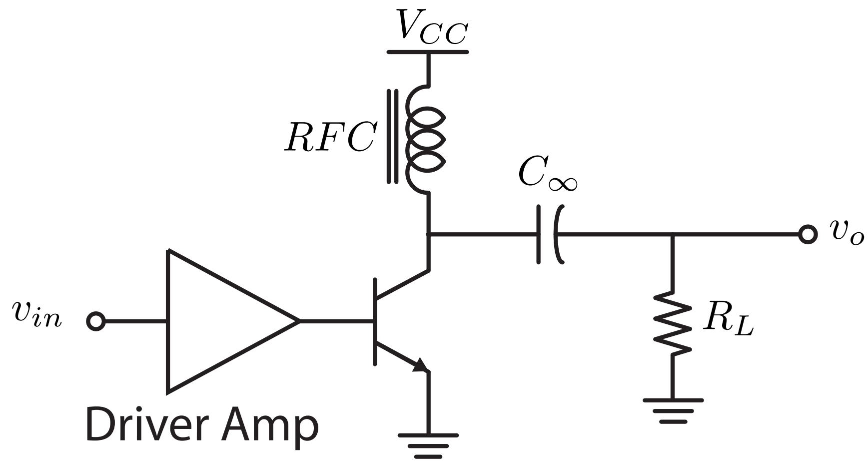
- Since the efficiency drops like V_2, the average efficiency for a signal distribution such as a Gaussian profile is particularly low

$$\eta_{av} = \int_{-\infty}^{\infty} \eta(v) p(v) dv$$



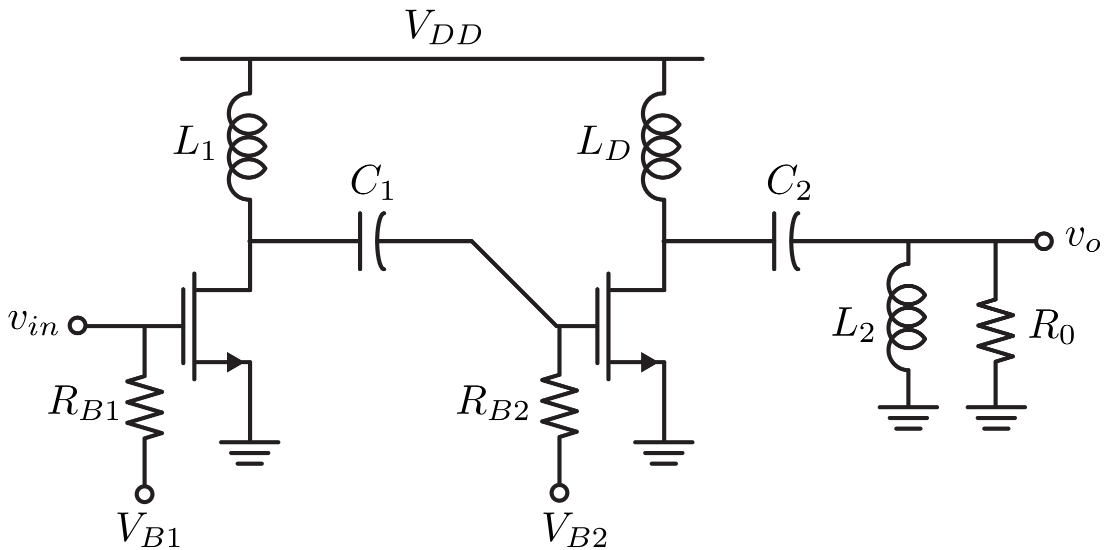
- A typical situation with Class A is that the peak efficiency is about 30%, but the average efficiency is as low as 6%.

Multi-Stage PAs



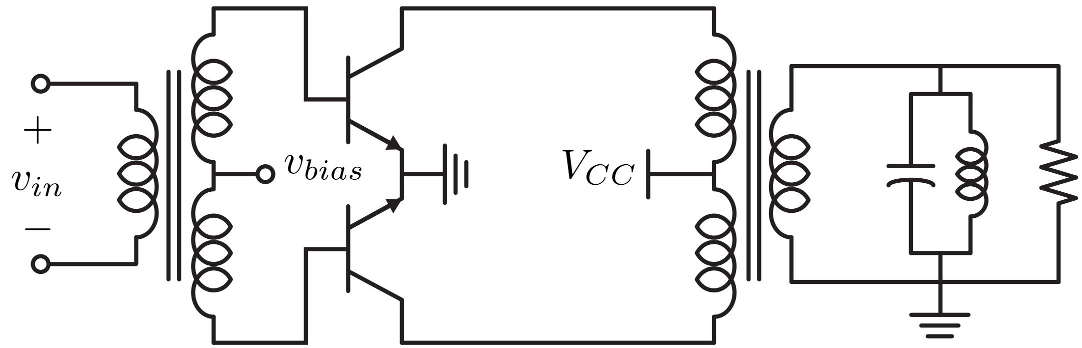
- Often 2-3 stages of power amplification are integrated into a PA to achieve $\sim 30\,\mathrm{dB}$ of gain.
- In a single-ended design, more gain is dangerous, due to parasitic feedback paths which limit stability.
- In a high power design the *driver* amplifier must deliver a considerable amount of power to the load.

Two Stage Example



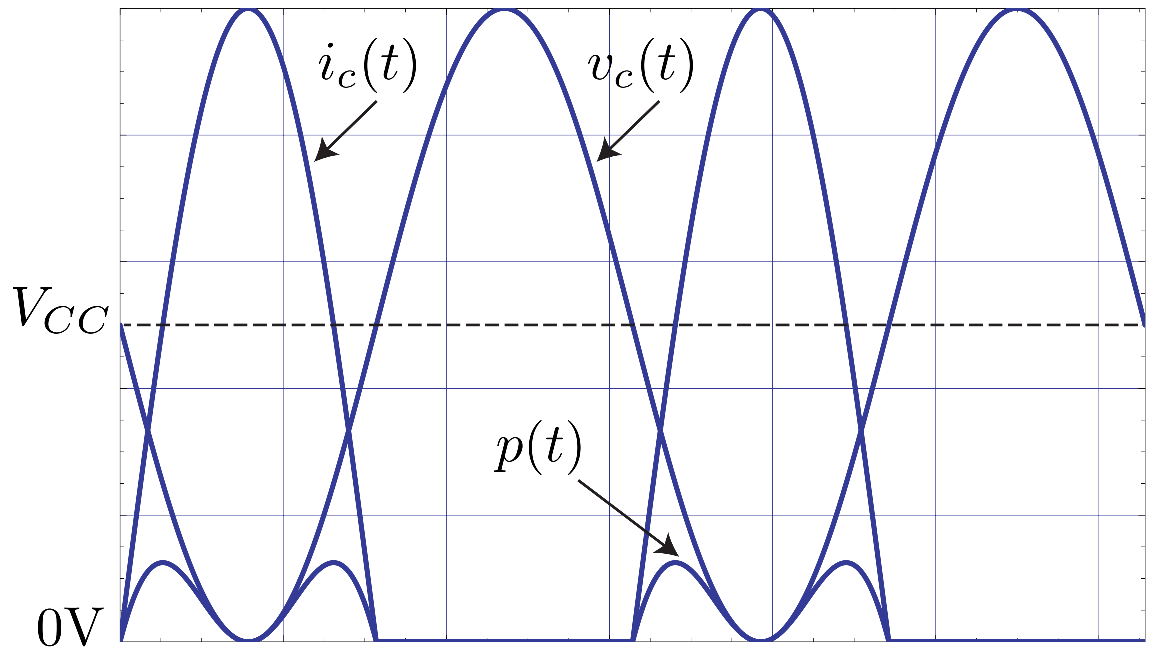
- If the gain of the output stage is not too great, then the power of the driver is not negligible and the overall efficiency must include a careful driver design.
- Here, an L matching network is used to convert the input impedance of the output stage (usually low) to a higher value close to the R_{opt} of the driver stage.

Class B PA



- The above circuit utilizes two transistors. Each device only delivers a half sinusoid pulse and the full sinusoid is recovered by phase inversion through the transformer.
- The base and collector bias voltages come from the transformer center tap. The base (or gate) is biased at the edge of conduction (threshold).

Class B Operation



- Since the voltage at the load is ideally a perfect sinusoid, the voltage on the collectors is likewise sinusoidal.
- The power dissipated by each transistor is thus the product of a sine and a half sine as shown above.

Efficiency of Class B

- The average current drawn by each transistor is given by

$$I_Q = \frac{1}{T}\int_0^T i_c(t)dt = \frac{I_p}{T}\int_0^{T/2}\sin\omega t dt = \frac{I_p}{2\pi}\int_0^{\pi}\sin\theta d\theta = \frac{I_p}{\pi}$$

- Where I_p is the peak voltage drawn from the supply.
- The peak current drawn from the supply is just the load current swing reflected to the collector, $I_p = i_o \times n$.

$$\eta = \frac{1}{2}\left(\frac{I_p}{2I_Q}\right)\left(\frac{v_o}{V_{CC}}\right)$$

- Note that the total DC current draw is twice I_Q since both devices draw current from the supply.

Efficiency (cont)

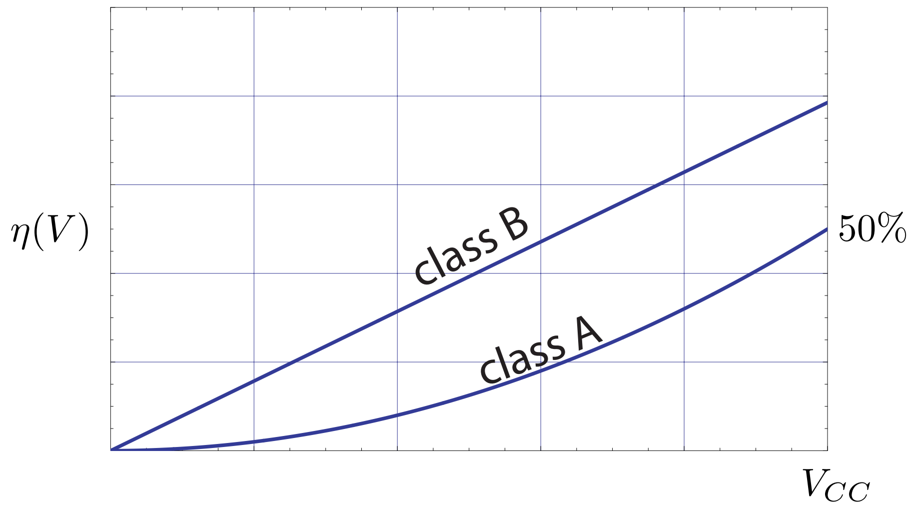
- Since the collector voltage swing can be as large as V_{CC} (similar to an inductively loaded Class A), the efficiency is bounded by

$$\eta \leq \frac{1}{2}\left(\frac{I_p}{2I_Q}\right) = \frac{1}{4}\left(\frac{I_p}{I_Q}\right)$$

$$\eta \leq \frac{\pi}{4} \approx 78\%$$

- This is a big improvement over the peak efficiency of Class A.
- Note that the average current naturally scales with output power, and so efficiency drops more gracefully as we back-off from peak power.

Class B Efficiency versus Voltage

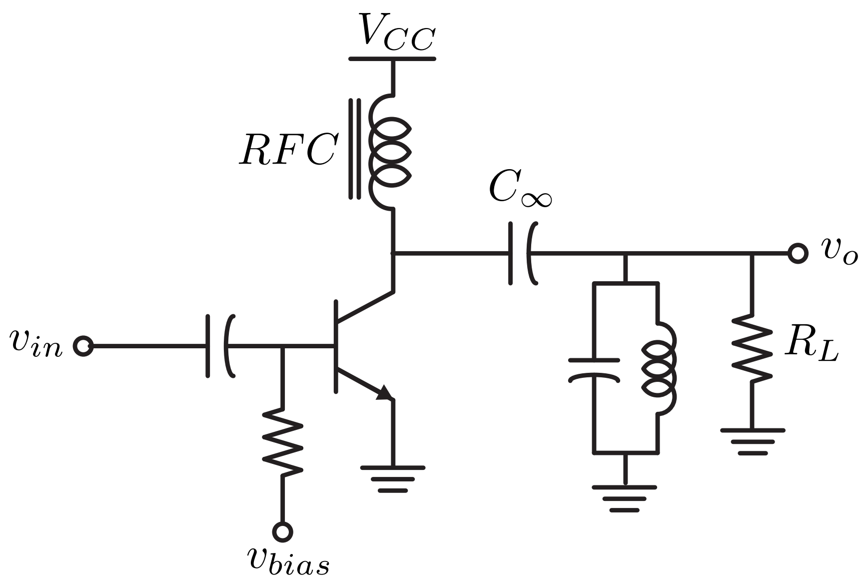


- The efficiency drops linearly as we back-off from the peak output voltage

$$\eta(v) = \frac{\pi}{4}\left(\frac{v_c}{V_{CC}}\right)$$

- where v_c is the collector voltage swing, which is just n times smaller than the load voltage, $v_c = v_o/n$.

Transformerless Class B



- A tuned Class B amplifier works with a single devices by sending half sinusoid current pulses to the load. The device is biased at the edge of conduction.
- The load voltage is sinusoidal because a high Q RLC tank shunts harmonics to ground.

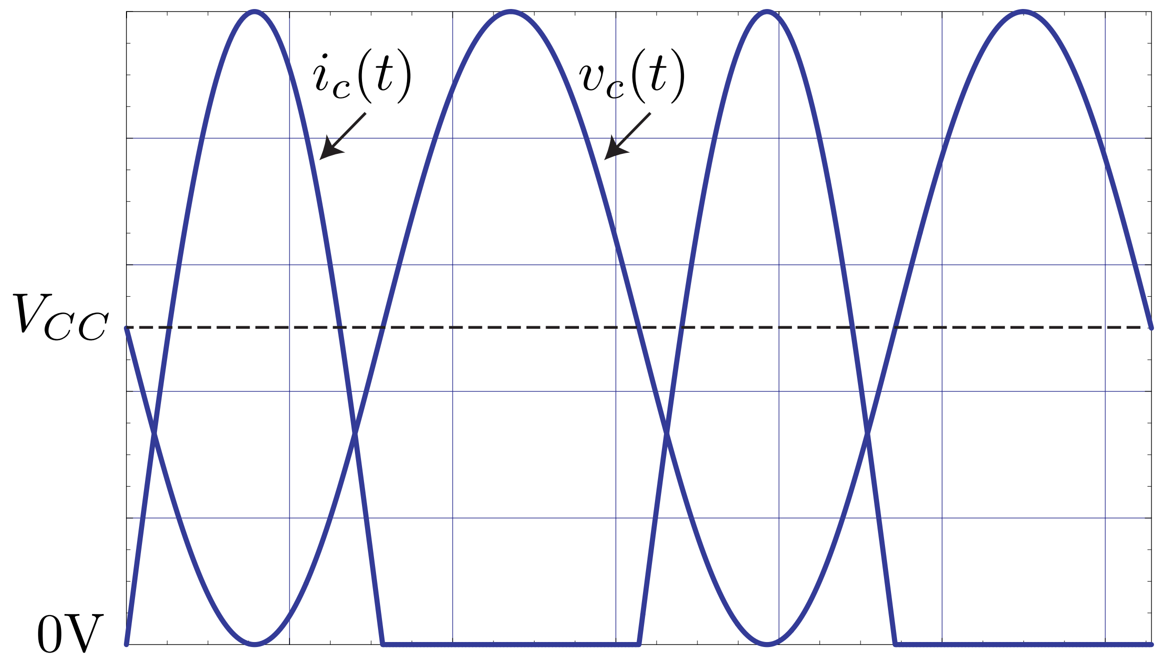
Class B Tank

- In a single transistor version, the "minus" pulse is in fact delivered by the RLC tank. The Q factor of the tank needs to be large enough to do this. This is analogous to pushing someone on a swing. You only need to push in one direction, and the reactive energy stored will swing the person back in the reverse direction.
- The average current drawn from the supply is the same as before, $I_Q = I_p/\pi$. The harmonic current delivered to the load is given by Fourier analysis of the half pulse

$$I_{\omega_1} = \frac{2}{2\pi} I_p \int_0^{\pi} \sin\theta sin\theta d\theta = \frac{1}{\pi} I_p \int_0^{\pi} \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \frac{1}{\pi} \frac{\pi}{2} I_p = \frac{I_p}{2}$$

Class B Waveforms



- We see that the transistor is cutoff when the collector voltage swings above V_{CC}. Thus, the power dissipated during this first half cycle is zero.
- During the second cycle, the peak current occurs when the collector voltage reaches zero.

Class B Power Dissipation

- The efficiency is therefore the same

$$\eta = \frac{1}{2}\frac{I_{\omega_1}}{I_Q}\frac{v_c}{V_{CC}} =\leq \frac{1}{2}\frac{\pi}{2} = \frac{\pi}{4}$$

- The DC power drawn from the supply is proportional to the output voltage

$$P_{dc} = I_Q V_{CC} = \frac{V_{CC} I_p}{\pi} \qquad I_p = \frac{v_c}{R_{opt}} = \frac{n v_o}{R_{opt}}$$

- The power loss in the transistor is given by

$$p_t(t) = \frac{1}{2\pi}\int_0^{\pi} I_p \sin\theta (V_{CC} - v_c \sin\theta) d\theta$$

Power Loss (cont)

- Integrating the above expression we have

$$p_t(t) = \frac{V_{CC} I_p}{2\pi} \left(-\cos\theta\big|_0^{\pi} - \frac{v_c}{I_p} 2\pi\right.$$

$$= \frac{1}{2\pi}\left(2V_{CC}I_p - \frac{v_c I_p}{2}\pi\right)$$

$$= \frac{I_p}{\pi}V_{CC} - \frac{v_c I_p}{4}$$

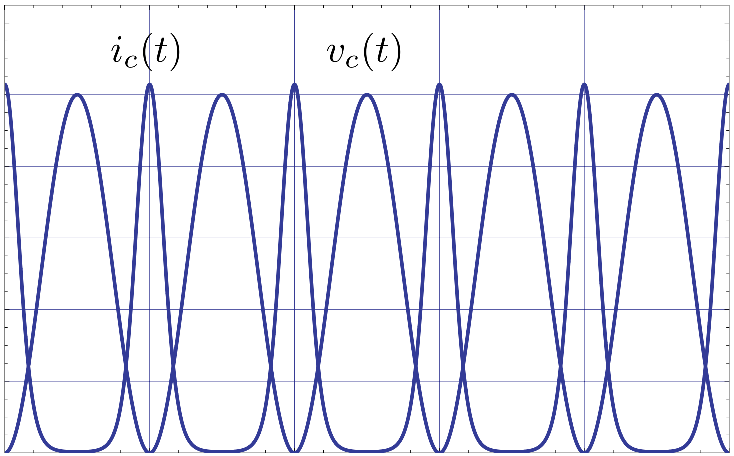
$$= I_Q \cdot V_{CC} - \frac{v_c I_{\omega_1}}{2}$$

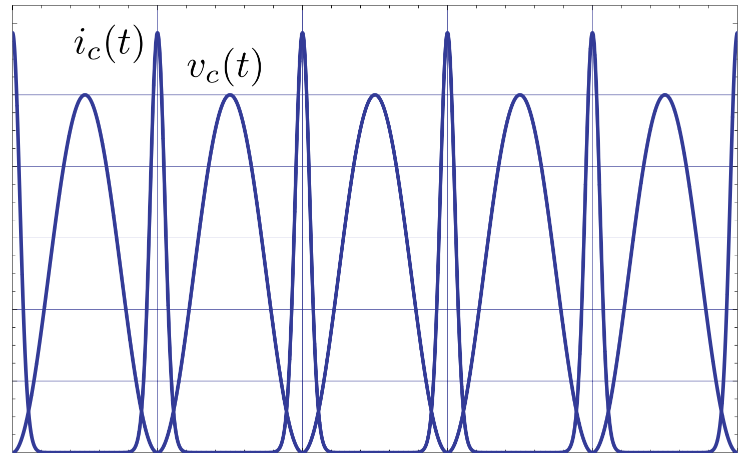
$$= P_{dc} - P_L$$

Conduction Angle

- Often amplifiers are characterized by their conduction angle, or the amount of time the collector current flows during a cycle.
- Class A amplifiers have 360° conduction angle, since the DC current is always flowing through the device.
- Class B amplifiers, though, have 180° conduction angle, since they conduct half sinusoidal pulses.
- In practice most Class B amplifiers are implemented as Class AB amplifiers, as a trickle current is allowed to flow through the main device to avoid cutting off the device during the amplifier operation.

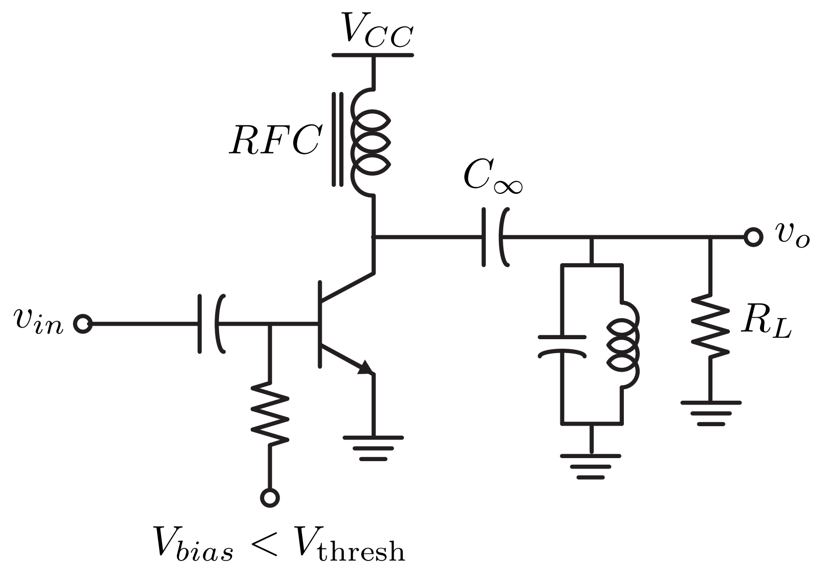
Improving The Efficiency





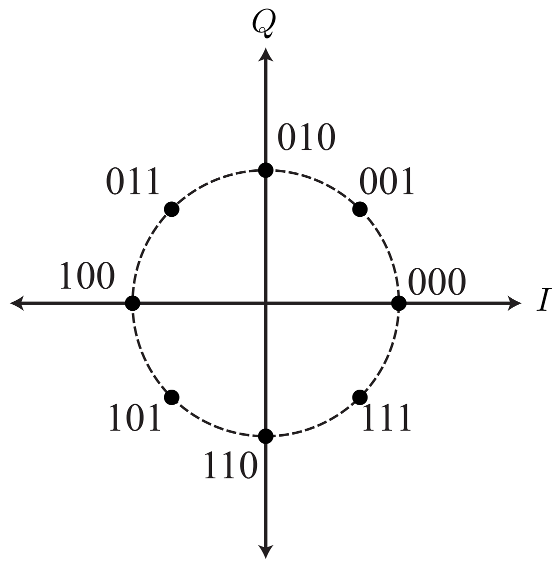
- The most optimal waveform is shown above, where a current pulse is delivered to the load during the collector voltage minimum (ideally zero)
- As the pulse is made sharper and sharper, the efficiency improves. To deliver the same power, though, the pulse must be taller and taller as it's made more narrow. In fact, in the limit the current spike approaches a delta function.

Class C



- Class C amplifiers are a wide family of amplifiers with conduction angle less than $180°$. One way to achieve this is to bias a transistor below threshold and allow the input voltage to turn on the device for a small fraction of the cycle.

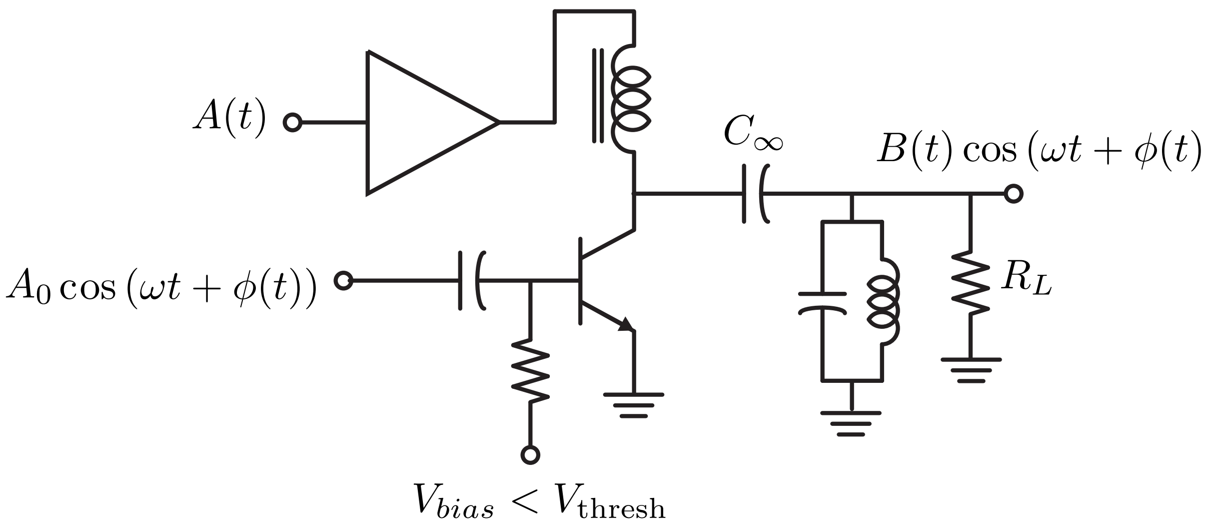
Class C Linearity



$$I^2 + Q^2 = A^2$$

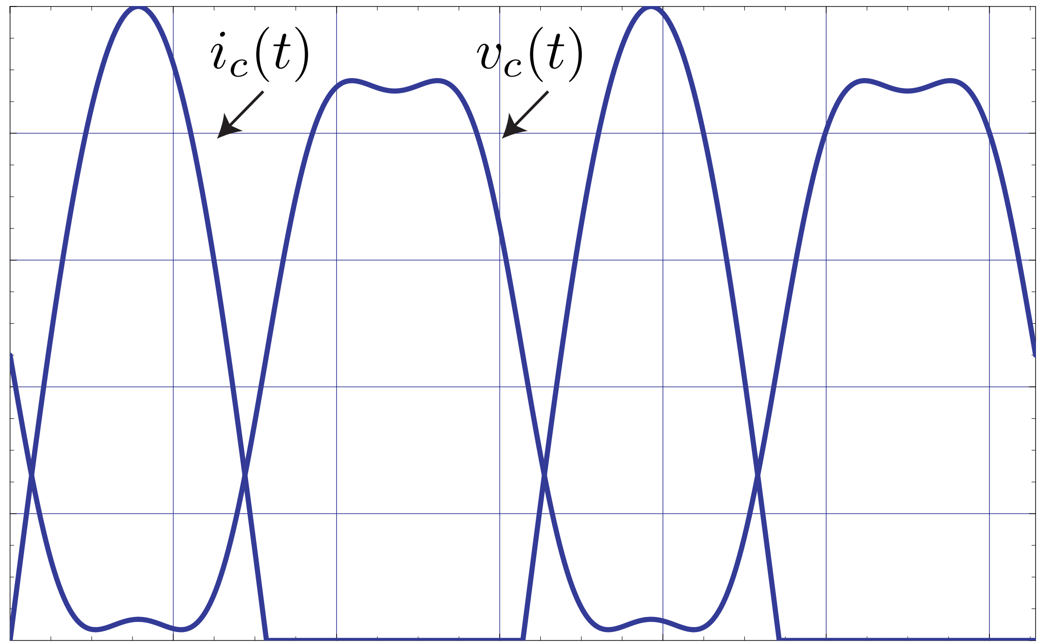
- The Class C amplifier is very non-linear, and it is only appropriate for applications where the modulation is constant envelope. For instance, FM uses a constant amplitude carrier and only modulates the frequency to convey information. Likewise, any digital modulation scheme with a constellation in a circle is constant envelope

Collector Modulation



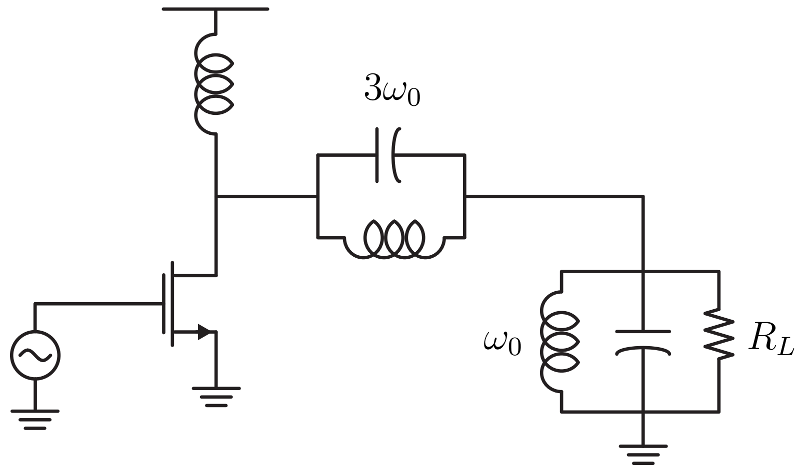
- While the amplifier is a non-linear function of the input amplitude, the Class C amplifier can be made to act fairly linearly to the collector voltage.
- By driving the amplifier into "saturation" in each cycle, e.g. with a large enough swing to rail the supplies, then the output power is related to the voltage supply
- Collector modulation then uses the power supply to introduce amplitude modulation into the carrier

Class F



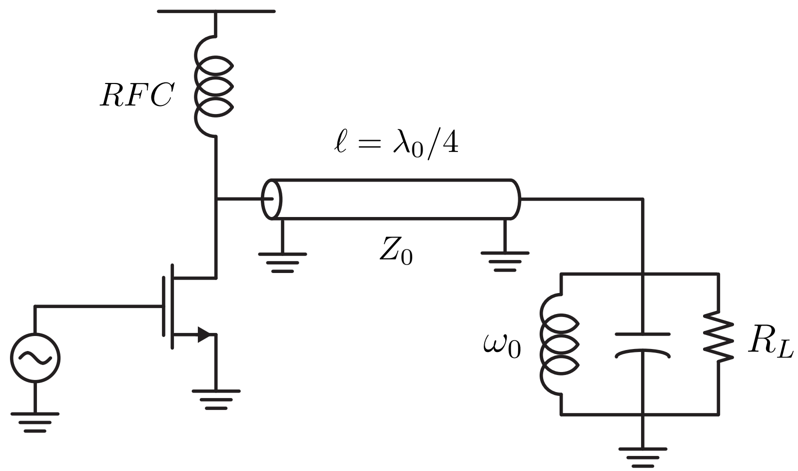
- Since it's difficult to create extremely narrow pulses at high frequency, we can take a different approach and attempt to square up the drain voltage.

Class F Circuit



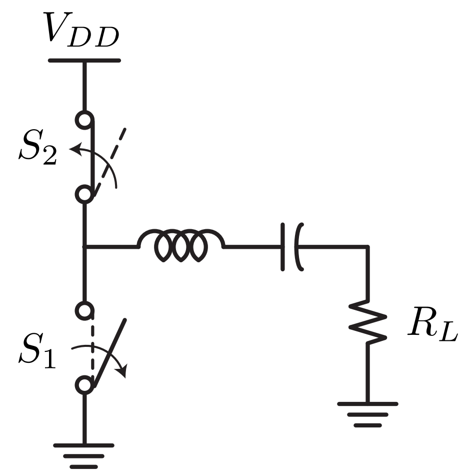
- The above circuit isolates the fundamental resonant load from the drain at the third harmonic, allowing the drain voltage to contain enough third harmonic to create a more square like waveform.
- It can be shown that just adding a third harmonic boost the efficiency from 78% (Class B) to 88%.

Quarter Wave Class F



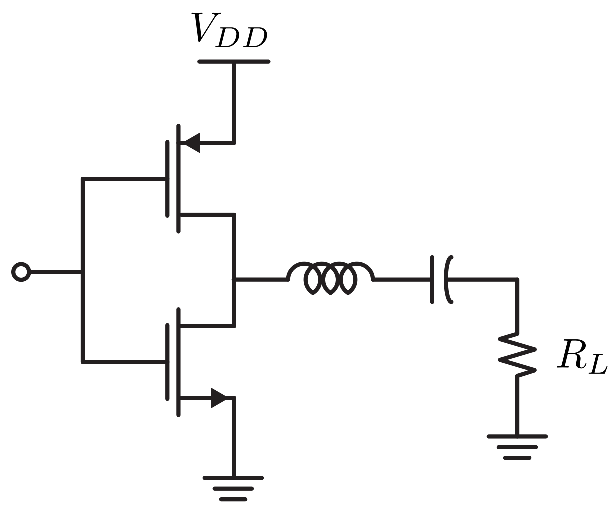
- In this circuit a quarter wave transformer converts the low impedance at the load (due to the capacitance) at harmonics of the fundamental to a high impedance for all odd harmonics. Even harmonics are unaltered as they see a $\lambda/2$ line.
- In theory then we can create a perfect square wave at the drain of the transistor and thus achieve 100% efficiency.

Class D Switching Amplifier



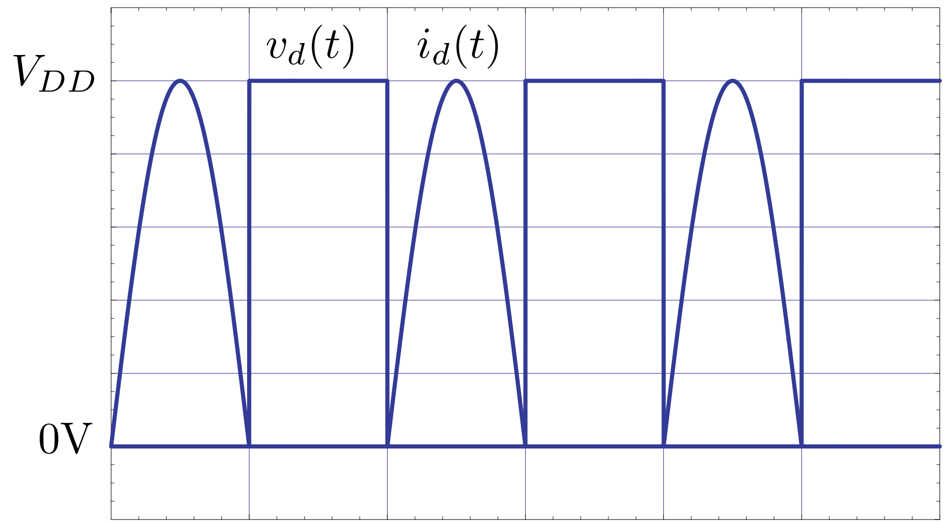
- If give up linearity, then we can create some intrinsically efficient amplifiers using switches. An ideal switch does not dissipate any power since either the voltage or current is zero.
- By varying the switching rate, we can impart frequency modulation onto the load.
- A real switch has on-resistance and parasitic off capacitance and conductance.

MOS Class D Inverter



- Switching amplifiers are realized with transistors operating as switches. MOS transistors make particularly good switches.
- The input voltage is large enough to quickly move the operating point from cut-off to triode region.

Class D Waveforms



- The MOS drain voltages switches from V_{SS} to V_{DD} at the rate of the input signal.
- A series LCR filter only allows the first harmonic of voltage to flow into the load. Since current only flows through a device when it's fully on (ideally $V_{ds} = 0$), little power dissipation occurs in the devices.

Class D Efficiency

- We can see that the efficiency has to be 100% for an ideal switch. That's because there is no where for the DC power to flow except to the load.
- The collector voltage can be decomposed into a Fourier series

$$v_d = \frac{V_{DD}}{2}\left(1 + s(\omega t)\right)$$

$$s(\theta) = \textsf{sign}(\sin(\theta)) = \frac{4}{\pi}\left(\sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{5}\sin 5\theta + \cdots\right)$$

- The load current is therefore

$$i_L = \frac{4}{\pi}\frac{V_{DD}}{2R}\sin\theta = \frac{2V_{DD}}{\pi R_L}\sin\theta$$

Class D Efficiency (cont)

- The load power is thus

$$P_L = \frac{i_L^2 R_L}{2} = \frac{2}{\pi^2}\frac{V_{DD}^2}{R_L} \approx 0.2\frac{V_{DD}^2}{R_L}$$

- The drain current are half-sinusoid pulses. The average current drawn from the supply is the average PMOS current

$$I_Q = \frac{I_p}{\pi} = \frac{2}{\pi^2}\frac{V_{DD}}{R_L}$$

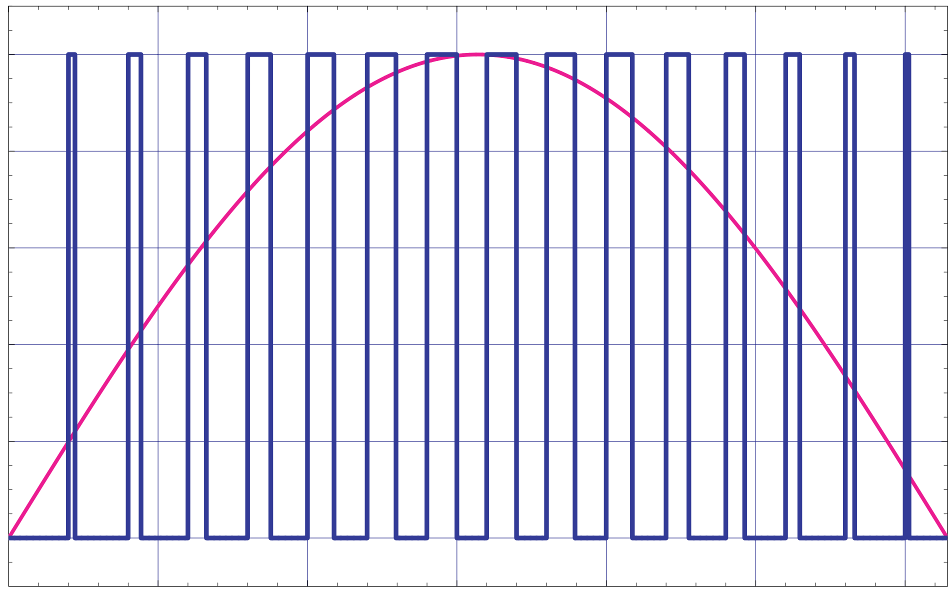
$$P_{DC} = I_Q \cdot V_{DD} = \frac{2V_{DD}^2}{\pi^2 R_L} = P_L$$

- As we expected, the ideal efficiency is $\eta = 100\%$

Class D Losses

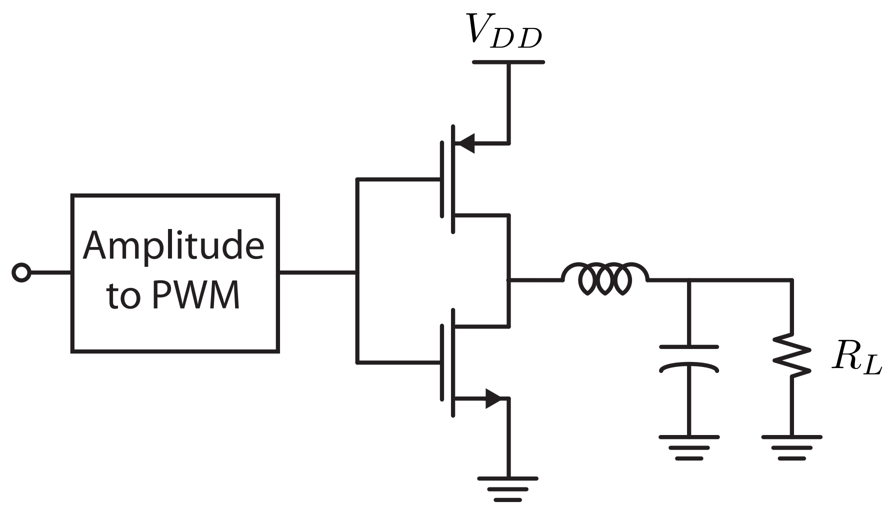
- As previously noted, a real Class D amplifier efficiency is lowered due to the switch on-resistance
- We can make our switches bigger to minimize the resistance, but this in turn increases the parasitic capacitance
- There are two forms of loss associated with the parasitic capacitance, the capacitor charging losses $CV^2 f$ and the parasitic substrate losses.
- The power required to drive the switches also increases proportional to C_{gs} since we have to burn $CV^2 f$ power to drive the inverter or. A resonant drive can lower the drive power by the Q of the resonator.
- In practice a careful balance dictates the maximum switch size

Class S



- If the switching rate is much higher than the fundamental, then amplitude modulation can be converted to pulse-width modulation.
- A low-pass filter at the output faithfully recreates the envelope of the signal.

Class S



- Class S amplifiers are commonly used at low frequencies (audio) since the transistors can be switched at a much higher frequency than the fundamental.
- This allows efficiencies approaching 100% with good linearity.