The trial wave function for N particles is given by

$$\Psi_T(\mathbf{r}) = \Psi_T(\mathbf{r}_1, \mathbf{r}_2, \dots \mathbf{r}_N, \alpha, \beta) = \left[\prod_i g(\alpha, \beta, \mathbf{r}_i)\right] \left[\prod_{j<k} f(a, |\mathbf{r}_j - \mathbf{r}_k|)\right], \quad (1)$$

where

$$g(\alpha, \beta, \mathbf{r}_i) = \exp\left[-\alpha(x_i^2 + y_i^2 + \beta z_i^2)\right] = \phi(\mathbf{r}_i) \quad (2)$$

and

$$f(a, |\mathbf{r}_i - \mathbf{r}_j|) = \begin{cases} 0 & |\mathbf{r}_i - \mathbf{r}_j| \le a \\ (1 - \frac{a}{|\mathbf{r}_i - \mathbf{r}_j|}) & |\mathbf{r}_i - \mathbf{r}_j| > a. \end{cases} \quad (3)$$

Defining $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ and $u(r_{ij}) = \ln f(r_{ij})$, this can be rewritten as

$$\Psi_T(\mathbf{r}) = \left[\prod_i \phi(\mathbf{r}_i)\right] \exp\left(\sum_{j<m} u(r_{jm})\right)$$

The gradient with respect to particle k is then:

$$\begin{aligned}
\nabla_k \Psi_T(\mathbf{r}) &= \nabla_k \left(\left[\prod_i \phi(\mathbf{r}_i)\right] \exp\left(\sum_{j<m} u(r_{jm})\right)\right) \\
&= \left(\nabla_k \left[\prod_i \phi(\mathbf{r}_i)\right]\right) \exp\left(\sum_{j<m} u(r_{jm})\right) + \left[\prod_i \phi(\mathbf{r}_i)\right] \left(\nabla_k \exp\left(\sum_{j<m} u(r_{jm})\right)\right) \\
&= \nabla_k \phi(\mathbf{r}_k) \left[\prod_{i\neq k} \phi(\mathbf{r}_i)\right] \exp\left(\sum_{j<m} u(r_{jm})\right) \\
&\quad + \left[\prod_i \phi(\mathbf{r}_i)\right] \exp\left(\sum_{j<m} u(r_{jm})\right) \sum_{l\neq k} \nabla_k u(r_{kl})
\end{aligned}$$

where we used the chain rule for gradients and the fact that only $\phi(\mathbf{r}_k)$ and $\exp(\sum_{l\neq k} u(r_{kl}))$ are functions of $\mathbf{r}_k$.

The second derivative divided by the trial wave function is then given by

$$
\begin{aligned}
\frac{1}{\Psi_T(\mathbf{r})}\nabla_k^2\Psi_T(\mathbf{r}) &= \frac{1}{\Psi_T(\mathbf{r})}\nabla_k\cdot(\nabla_k\Psi_T(\mathbf{r})) \\
&= \frac{1}{\Psi_T(\mathbf{r})}\nabla_k\cdot\left(\nabla_k\phi(\mathbf{r}_k)\left[\prod_{i\neq k}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right)\right) \\
&\quad + \frac{1}{\Psi_T(\mathbf{r})}\nabla_k\cdot\left(\left[\prod_{i}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right)\sum_{l\neq k}\nabla_k u(r_{kl})\right) \\
&= \frac{1}{\Psi_T(\mathbf{r})}\left(\nabla_k^2\phi(\mathbf{r}_k)\right)\left[\prod_{i\neq k}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right) \\
&\quad + \frac{1}{\Psi_T(\mathbf{r})}\left(\left[\prod_{i\neq k}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right)\nabla_k\phi(\mathbf{r}_k)\cdot\sum_{l\neq k}\nabla_k u(r_{kl})\right) \\
&\quad + \frac{1}{\Psi_T(\mathbf{r})}\left(\left[\prod_{i\neq k}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right)\nabla_k\phi(\mathbf{r}_k)\cdot\sum_{l\neq k}\nabla_k u(r_{kl})\right) \\
&\quad + \frac{1}{\Psi_T(\mathbf{r})}\left[\prod_{i}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right)\left(\sum_{l\neq k}\nabla_k u(r_{kl})\right)^2 \\
&\quad + \frac{1}{\Psi_T(\mathbf{r})}\left[\prod_{i}\phi(\mathbf{r}_i)\right]\exp\left(\sum_{j<m}u(r_{jm})\right)\sum_{l\neq k}\nabla_k^2 u(r_{kl}) \\
&= \frac{\nabla_k^2\phi(\mathbf{r}_k)}{\phi(\mathbf{r}_k)} + 2\frac{\nabla_k\phi(\mathbf{r}_k)}{\phi(\mathbf{r}_k)}\cdot\sum_{l\neq k}\nabla_k u(r_{kl}) \\
&\quad + \sum_{l\neq k}\nabla_k u(r_{kl})\cdot\sum_{i\neq k}\nabla_k u(r_{ki}) \\
&\quad + \sum_{l\neq k}\nabla_k^2 u(r_{kl})
\end{aligned}
$$

where we used the chain rule for divergences and gradients. The expressions for the gradient of $u(r_{ik})$ can be reexpressed using the chain rule. We have that, by the chain rule

$$
\frac{\partial u(r_{ki})}{\partial x_k}\hat{\mathbf{x}}_{\mathbf{k}} = \frac{\partial r_{ki}}{\partial x_k}\frac{\partial u(r_{ki})}{\partial r_{ki}}\hat{\mathbf{x}}_{\mathbf{k}} = u'(r_{ki})\frac{x_k - x_i}{r_{ki}}\hat{\mathbf{x}}_{\mathbf{k}}
$$

and we get the equivalent expressions for the partial derivatives with respect to y_k and z_k due to symmetry. Combining these, we get

$$
\nabla_k u(r_{ki}) = \frac{(\mathbf{r}_k - \mathbf{r}_i)}{r_{ki}}u'(r_{ki})
$$

Similarly, we get for the second partial derivative that

$$\frac{\partial^2 u(r_{ki})}{\partial x_k^2} = \frac{\partial}{\partial x_k}\left(u'(r_{ki})\frac{x_k - x_i}{r_{ki}}\right) = \frac{\partial u'(r_{ki})}{\partial x_k}\frac{x_k - x_i}{r_{ki}} + u'(r_{ki})\frac{\partial}{\partial x_k}\left(\frac{x_k - x_i}{r_{ki}}\right)$$
$$= u''(r_{ki})\frac{(x_k - x_i)^2}{r_{ki}^2} + u'(r_{ki})\left(\frac{1}{r_{ki}} - \frac{(x_k - x_i)^2}{r_{ki}^3}\right)$$

and again, we get equivalent expressions for the second partial derivatives with respect to y_k and z_k. Because $(x_k - x_i)^2 + (y_k - y_i)^2 + (z_k - z_i)^2 = r_{k_i}^2$, the Laplacian becomes

$$\nabla_k^2 u(r_{kl}) = u''(r_{ki})\frac{r_{ki}^2}{r_{ki}^2} + u'(r_{ki})\left(\frac{3}{r_{ki}} - \frac{r_{ki}^2}{r_{ki}^3}\right) = u''(r_{kj}) + \frac{2}{r_{kj}}u'(r_{kj})$$

Inserting this in the expression for the second derivative, we get Morten's result.