FYS3120 Classical mechanics and electrodynamics
Final exam 2020

15112

September 8, 2021

Question 1

a)

The set-up looks something like the sketch shown in Figure 1.

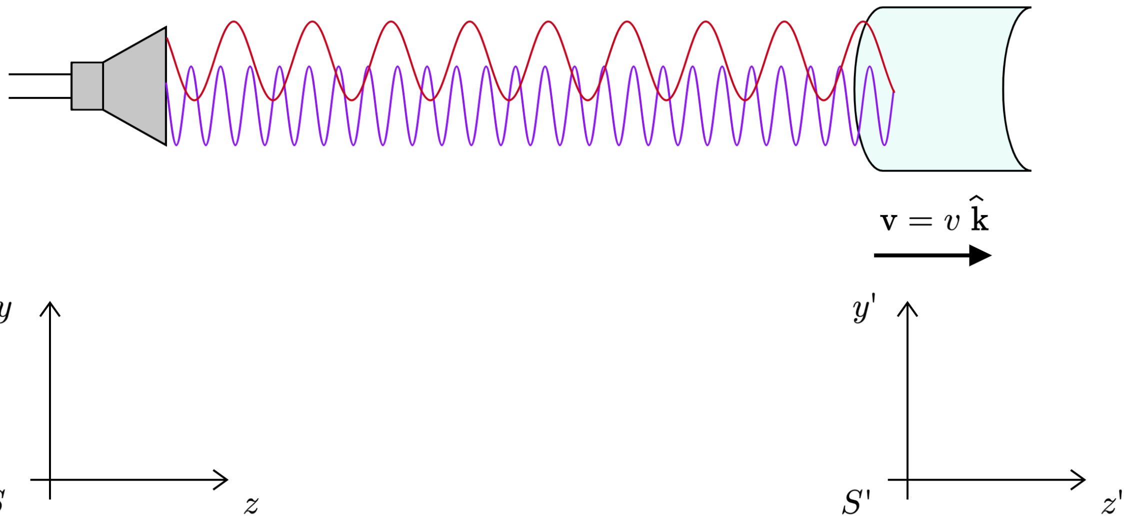

Figure 1

The reference frame S' is the rest frame of the mirror. The emitted light is represented by purple waves while reflected light is represented by red waves. The reflected light has a longer wavelength and a lower frequency than the emitted light.

b)

In the following I'd like to use the lower index e for the emitted light and the lower index r for the reflected light. I'll keep the lower index m for the mirror. Primed quantities means that they're measured in S' and unprimed quantities means that they're measured in S.

The Lorentz transformation for a boost in the z-direction is

$$x^{0'} = \gamma\left(x^0 - \beta x^3\right),\ x^{'1} = x^1,\ x^{'2} = x^2,\ x^{'3} = \gamma\left(x^3 - \beta x^0\right)$$

The components of the four-momentum in S of an emitted photon traveling in the z-direction are

$$\underline{\mathbf{P}}_e = \left(\frac{E_e}{c}, 0, 0, p_e\right)$$

The momentum of the photon measured in S' is thus

$$p'_e = \gamma\left(p_e - \beta\frac{E_e}{c}\right) = \gamma(1-\beta)p_e$$

where I've used the relationship $p = E/c$ which is valid for photons. Since $E = h\nu$ the momentum in terms of frequency is $p = h\nu/c$, so the relationship between the frequencies of an emitted photon in S and S' is

$$\nu'_e = \gamma(1-\beta)\nu_e \tag{1}$$

In S' the frequency of the reflected photon is the same as the frequency of the emitted photon, so $\nu'_r = \gamma(1-\beta)\nu_e$. To find the frequency of the reflected photon in S we can use Eq. (1) by substituting $\nu'_e \to \nu_r$ and $\nu_e \to \nu'_r$, since from the viewpoint of the mirror the mirror acts like a light source while the actual light source moves away from the mirror with velocity v. Thus

$$\nu_r = \gamma^2(1-\beta)^2\nu_e$$

$$= \frac{(1-\beta)^2}{1-\beta^2}\nu_e = \frac{(1-\beta)^2}{(1+\beta)(1-\beta)}\nu_e$$

$$= \frac{1-\beta}{1+\beta}\nu_e$$

c)

The components of the four-momenta in S of the emitted photon, the reflected photon and the mirror respectively are

$$\underline{\mathbf{P}}_e = \left(\frac{E_e}{c}, 0, 0, p_e\right) = p_e(1, 0, 0, 1)$$

$$\underline{\mathbf{P}}_r = \left(\frac{E_r}{c}, 0, 0, -p_r\right) = p_r(1, 0, 0, -1)$$

$$\underline{\mathbf{P}}_m = \left(\frac{E_m}{c}, 0, 0, p_m\right) = \left(\sqrt{p_m^2 + m^2c^2}, 0, 0, p_m\right)$$

and in S' they are

$$\underline{\mathbf{P}}_e = \left(\frac{E'_e}{c}, 0, 0, p'_e\right) = p'_e(1,0,0,1)$$

$$\underline{\mathbf{P}}_r = \left(\frac{E'_r}{c}, 0, 0, -p'_r\right) = p'_r(1,0,0,-1)$$

$$\underline{\mathbf{P}}_m = \left(\frac{E'_m}{c}, 0, 0, p'_m\right) = (mc,0,0,0)$$

where $p_i = |\mathbf{p}_i|$ and $p'_i = |\mathbf{p}'_i|$ for $i = e, r, m$. Here I've used $E = pc$ for the photons and $E^2 = p^2c^2 + m^2c^4$ for the mirror. Since S' is the rest frame of the mirror the relativistic momentum of the mirror in S' is zero and the energy of the mirror in S' is the rest energy mc^2.

In the following problems I will use an extra index to distinguish between quantities of the mirror before and after the collision with the emitted photon, e.g $p_{m,0}$ and p_m for the momentum of the mirror before and after the collision in S, respectively.

d)

Using the $(-,+,+,+)$ metric we have in S:

$$\underline{\mathbf{P}}_e + \underline{\mathbf{P}}_{m,0} = (p_e, 0, 0, p_e) + \left(\sqrt{p_{m,0}^2 + m^2c^2}, 0, 0, p_{m,0}\right) = \left(p_e + \sqrt{p_{m,0}^2 + m^2c^2}, 0, 0, p_e + p_{m,0}\right)$$

$$\underline{\mathbf{P}}_e \cdot \underline{\mathbf{P}}_{m,0} = -p_e\sqrt{p_{m,0}^2 + m^2c^2} + p_e p_{m,0} \tag{2}$$

and in S':

$$\underline{\mathbf{P}}_e + \underline{\mathbf{P}}_{m,0} = (p'_e, 0, 0, p'_e) + (mc, 0, 0, 0) = (p'_e + mc, 0, 0, p'_e)$$

$$\underline{\mathbf{P}}_e \cdot \underline{\mathbf{P}}_{m,0} = -p'_e mc + p'_e \cdot 0 = -p'_e mc$$

e)

The right side of Eq. (1) in the problem text is just Eq. (2). Using quantities in S the left side of Eq. (1) in the problem text can be expressed as

$$(\underline{\mathbf{P}}_e + \underline{\mathbf{P}}_{m,0}) \cdot \underline{\mathbf{P}}_r = \left(p_e + \sqrt{p_{m,0}^2 + m^2c^2}, 0, 0, p_e + p_{m,0}\right) \cdot (p_r, 0, 0, -p_r)$$

$$= -p_r\left(p_e + \sqrt{p_{m,0}^2 + m^2c^2}\right) - p_r(p_e + p_{m,0}) = -p_r\left(p_{m,0} + 2p_e + \sqrt{p_{m,0}^2 + m^2c^2}\right) \tag{3}$$

From conservation of four-momentum we have

$$\underline{\mathbf{P}}_e + \underline{\mathbf{P}}_{m,0} = \underline{\mathbf{P}}_r + \underline{\mathbf{P}}_m$$

In terms of quantities in S this equation is

$$\left(p_e + \sqrt{p_{m,0}^2 + m^2c^2}, 0, 0, p_e + p_{m,0}\right) = \left(p_r + \sqrt{p_m^2 + m^2c^2}, 0, 0, p_m - p_r\right) \tag{4}$$

We note for future reference that

$$p_e - p_r + \sqrt{p_{m,0}^2 + m^2c^2} = \sqrt{p_m^2 + m^2c^2} \tag{5}$$

$$p_e + p_r + p_{m,0} = p_m \tag{6}$$

"Squaring" both sides of Eq. (4) and simplifying:

$$-\left(p_e + \sqrt{p_{m,0}^2 + m^2c^2}\right)^2 + (p_e + p_{m,0})^2 = -\left(p_r + \sqrt{p_m^2 + m^2c^2}\right)^2 + (p_m - p_r)^2$$

$$-\left(2p_e\sqrt{p_{m,0}^2 + m^2c^2} + m^2c^2\right) + 2p_e p_{m,0} = -\left(2p_r\sqrt{p_m^2 + m^2c^2} + m^2c^2\right) - 2p_m p_r$$

$$-2p_e\sqrt{p_{m,0}^2 + m^2c^2} - m^2c^2 + 2p_e p_{m,0} = -2p_r\sqrt{p_m^2 + m^2c^2} - m^2c^2 - 2p_m p_r$$

$$-p_e\sqrt{p_{m,0}^2 + m^2c^2} + p_e p_{m,0} = -p_r\sqrt{p_m^2 + m^2c^2} - p_m p_r \tag{7}$$

Comparing the left side of Eq. (7) with Eq. (2) we see that this is $\underline{\mathbf{P}}_e \cdot \underline{\mathbf{P}}_{m,0}$. Using Eq. (5) and (6) we can write the right side of Eq. (7) as

$$-p_r\sqrt{p_m^2 + m^2c^2} - p_m p_r = -p_r\left(p_m + \sqrt{p_m^2 + m^2c^2}\right)$$

$$= -p_r\left[(p_e + p_r + p_{m,0}) + \left(p_e - p_r + \sqrt{p_{m,0}^2 + m^2c^2}\right)\right] = -p_r\left(p_{m,0} + 2p_e + \sqrt{p_{m,0}^2 + m^2c^2}\right)$$

which is Eq. (3). So it has been shown that

$$(\underline{\mathbf{P}}_e + \underline{\mathbf{P}}_{m,0}) \cdot \underline{\mathbf{P}}_r = \underline{\mathbf{P}}_e \cdot \underline{\mathbf{P}}_{m,0}$$

f)

It has been shown that the following equality is true:

$$-p_e\sqrt{p_{m,0}^2 + m^2c^2} + p_e p_{m,0} = -p_r\left(p_{m,0} + 2p_e + \sqrt{p_{m,0}^2 + m^2c^2}\right)$$

Solving for p_r:

$$p_r = \frac{p_e\sqrt{p_{m,0}^2 + m^2c^2} - p_e p_{m,0}}{p_{m,0} + 2p_e + \sqrt{p_{m,0}^2 + m^2c^2}} \tag{8}$$

g)

If $p_{m,0} \gg p_e$ we can neglect the $2p_e$ term in the denominator of Eq. (8):

$$p_r \approx \frac{\sqrt{p_{m,0}^2 + m^2c^2} - p_{m,0}}{\sqrt{p_{m,0}^2 + m^2c^2} + p_{m,0}} p_e$$

$$= \frac{\frac{E_{m,0}}{c} - p_{m,0}}{\frac{E_{m,0}}{c} + p_{m,0}} p_e = \frac{E_{m,0} - cp_{m,0}}{E_{m,0} + cp_{m,0}} p_e$$

$$= \frac{\gamma mc^2 - c\gamma mv}{\gamma mc^2 + c\gamma mv} p_e = \frac{c - v}{c + v} p_e$$

$$= \frac{1 - \frac{v}{c}}{1 + \frac{v}{c}} p_e = \frac{1 - \beta}{1 + \beta} p_e$$

In Question b) we found $\nu_r = \dfrac{1-\beta}{1+\beta}\nu_e$. Multiplying both sides by h/c gives

$$p_r = \frac{1 - \beta}{1 + \beta} p_e$$

So Eq. (8) agrees with the result in Question b) when $p_{m,0} \gg p_e$.

Question 2

a)

The standard expression for the energy current density (Poynting's vector) is

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

where the magnetic wave is related to the electric wave by $\mathbf{B} = \hat{\mathbf{n}} \times \mathbf{E}/c$ (see Eq. (10.18) on p. 263 in the course book). Using the triple product relation $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ we have

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \left(\frac{1}{c} \hat{\mathbf{n}} \times \mathbf{E} \right)$$

$$= \frac{\mathbf{E} \cdot \mathbf{E}}{\mu_0 c}\ \hat{\mathbf{n}} - \frac{\mathbf{E} \cdot \hat{\mathbf{n}}}{\mu_0 c}\ \mathbf{E} = c\epsilon_0 E^2\ \hat{\mathbf{n}}$$

where I've used that $\mathbf{E} \perp \hat{\mathbf{n}}$ and $1/\mu_0 c = c\epsilon_0$.

b)

The Lorentz force acting on the electron is

$$\mathbf{F} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Taking the absolute value of the Lorentz force and using the triangle inequality:

$$F = e|\mathbf{E} + \mathbf{v} \times \mathbf{B}|$$

$$\leq eE + e|\mathbf{v} \times \mathbf{B}|$$

Since the both the electric and magnetic contributions to the Lorentz force accelerates the electron in a direction perpendicular to the magnetic field, the velocity of the electron is always perpendicular to the magnetic field. So we have

$$F \leq eE + ev|\mathbf{B}|$$

$$= eE + e\frac{v}{c}|\hat{\mathbf{n}} \times \mathbf{E}| = eE + eE\frac{v}{c}$$

$$= eE\left(1 + \frac{v}{c}\right) \approx eE$$

if we assume $v \ll c \rightarrow v/c \ll 1$. So in the following, as long as the velocity is much less than c, we can ignore the magnetic contribution to the Lorentz force and write

$$\mathbf{F} = -e\mathbf{E}$$

Since there are no constraints, the electron has $d = 3$ degrees of freedom and we can choose the Cartesian coordinates x, y, z as the generalized coordinates. We can set the z-axis along $\hat{\mathbf{n}}$ and the x-axis in the direction of the electric field at $z = 0$ and $t = 0$. Then the force acting on the particle is

$$\mathbf{F} = -eE_0 \cos(kz - \omega t)\ \hat{\mathbf{i}}$$

but since the force is acting exclusively along the x-axis, the z-coordinate of the electron doesn't change (The magnetic contribution to the Lorentz force can act along the z-axis, but we are neglecting this). So we might as well place the origin where the electron is initially at rest. Then the force is $\mathbf{F} = -eE_0 \cos(\omega t)\ \hat{\mathbf{i}}$ which is the gradient of the potential

$$V = eE_0 x \cos(\omega t)$$

The Lagrangian of the system is thus

$$L = T - V$$

$$= \frac{1}{2}m_e\dot{x}^2 - eE_0 x \cos(\omega t)$$

Here I've set $\dot{y} = \dot{z} = 0$ since the electron is initially at rest and is not accelerated in the y-or z-directions by the force. The equation of motion is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$m_e \ddot{x} + eE_0 \cos(\omega t) = 0 \tag{9}$$

We can integrate the equation of motion over time to find the velocity and the position:

$$\dot{x} = -\frac{eE_0}{m_e \omega} \sin(\omega t)$$

$$x = \frac{eE_0}{m_e \omega^2} \cos(\omega t) - \frac{eE_0}{m_e \omega^2}$$

$$= -\frac{eE_0}{m_e \omega^2}[1 - \cos(\omega t)] = -\frac{2eE_0}{m_e \omega^2} \sin^2\left(\frac{\omega t}{2}\right)$$

So the position of the electron if it starts at rest in the origin and always has a velocity $v \ll c$ is

$$\mathbf{r}(t) = -\frac{2eE_0}{m_e \omega^2} \sin^2\left(\frac{\omega t}{2}\right)\ \hat{\mathbf{i}}$$

c)

The amplitude of the electron's velocity is $v_{\max} = eE_0/m_e\omega$, so for $v \ll c$ to be true the amplitude and the frequency of the electric field has to satisfy

$$\frac{E_0}{\omega} \ll \frac{m_e c}{e}$$

d)

We can put the acceleration given in Eq. (9) into Larmor's radiation formula to find

$$P = \frac{\mu_0 q^2}{6\pi c} a^2$$

$$= \frac{\mu_0 e^2}{6\pi c} \cdot \frac{e^2 E_0^2}{m_e^2} \cos^2(\omega t) = \frac{\mu_0 e^4 E_0^2}{6\pi c m_e^2} \cos^2(\omega t)$$

Since $\mu_0/c = 1/\epsilon_0 c^3$ we can also write

$$P = \frac{e^4 E_0^2}{6\pi \epsilon_0 c^3 m_e^2} \cos^2(\omega t)$$

$$= \left(\frac{e^2}{4\pi\epsilon_0 m_e c^2}\right)^2 \cdot \frac{8}{3}\pi c \epsilon_0 E_0^2 \cos^2(\omega t) = \frac{8}{3}\pi c \epsilon_0 r_0^2 E_0^2 \cos^2(\omega t)$$

e)

$$\langle S \rangle \sigma = \langle P \rangle$$

The dimension of the magnitude of Poynting's vector is energy per area per time. Since the dimension of the power is energy per time the dimension of the scattering cross section σ has to be area, i.e square-meters m^2 in SI-units.

f)

The average magnitude of Poynting's vector is

$$\langle S \rangle = \left\langle c\epsilon_0 E^2 \right\rangle = c\epsilon_0 E_0^2 \left\langle \cos^2(\omega t) \right\rangle = \frac{1}{2} c\epsilon_0 E_0^2$$

and the average power that the free electron radiates is

$$\langle P \rangle = \left\langle \frac{8}{3}\pi c\epsilon_0 r_0^2 E_0^2 \cos^2(\omega t) \right\rangle = \frac{8}{3}\pi c\epsilon_0 r_0^2 E_0^2 \left\langle \cos^2(\omega t) \right\rangle = \frac{4}{3}\pi c\epsilon_0 r_0^2 E_0^2$$

So the scattering cross section of the free electron is

$$\sigma = \frac{\langle P \rangle}{\langle S \rangle} = \frac{\frac{4}{3}\pi c\epsilon_0 r_0^2 E_0^2}{\frac{1}{2} c\epsilon_0 E_0^2} = \frac{8}{3}\pi r_0^2$$

$$= \frac{8}{3} \cdot \pi \cdot (2.8179\ \mathrm{fm})^2 = 66.523\ \mathrm{fm}^2$$

g)

From Newton's second law we have

$$m_e \ddot{\mathbf{r}} = -m_e \omega_0^2 \mathbf{r} - m_e \Gamma \dot{\mathbf{r}} - e\mathbf{E}$$

which consists of the three coupled differential equations:

$$\ddot{x} + \Gamma \dot{x} + \omega_0^2 x = -\frac{eE_0}{m_e} \cos(kz - \omega t)$$

$$\ddot{y} + \Gamma \dot{y} + \omega_0^2 y = 0$$

$$\ddot{z} + \Gamma \dot{z} + \omega_0^2 z = 0$$

The y-and z-equations are just second order ODE's. Their characteristic polynomials are

$$\lambda^2 + 2\gamma\lambda + \omega_0^2 = 0$$

where $2\gamma \equiv \Gamma$. The solutions are

$$\lambda = \frac{-2\gamma \pm \sqrt{4\gamma^2 - 4\omega_0^2}}{2} = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

$$= -\gamma \pm i\sqrt{\omega_0^2 - \gamma^2} \equiv -\gamma \pm i\omega'$$

So the solution to both the y- and z-equations are on the form

$$e^{-\gamma t}[A\cos(\omega' t) + B\sin(\omega' t)]$$

which vanishes after some time no matter what the initial conditions are. So the motion of the electron will after some time be purely along the x-axis. Then the equation of motion is

$$\ddot{x} + \Gamma\dot{x} + \omega_0^2 x = -\frac{eE_0}{m_e}\cos(\omega t)$$

where the homogeneous solution is on the same form as the solution to the y- and z-equations so it vanishes after some time as well. So if we consider the motion of the electron only after some time we only need to find a particular solution of the differential equation. To do this we can consider the ODE

$$\ddot{\chi} + \Gamma\dot{\chi} + \omega_0^2\chi = -\frac{eE_0}{m_e}e^{i\omega t} \tag{10}$$

where $x(t) = \mathrm{Re}\{\chi(t)\}$. We can try a solution on the form $\chi(t) = Ae^{i\omega t}$ where A may be a complex number. Putting this into Eq. (10)

$$-\omega^2 Ae^{i\omega t} + i\omega\Gamma Ae^{i\omega t} + \omega_0^2 Ae^{i\omega t} = -\frac{eE_0}{m_e}e^{i\omega t}$$

$$-\omega^2 A + i\omega\Gamma A + \omega_0^2 A = -\frac{eE_0}{m_e}$$

$$A = \frac{1}{\omega^2 - \omega_0^2 - i\omega\Gamma}\frac{eE_0}{m_e} = \frac{1}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{eE_0}{m_e}\cdot\left(\omega^2 - \omega_0^2 + i\omega\Gamma\right)$$

$$= \frac{\omega^2 - \omega_0^2}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{eE_0}{m_e} + i\frac{\omega\Gamma}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{eE_0}{m_e}$$

So the solution $x(t) = \mathrm{Re}\left\{Ae^{i\omega t}\right\}$ is

$$x(t) = \frac{\omega^2 - \omega_0^2}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{eE_0}{m_e}\cos(\omega t) - \frac{\omega\Gamma}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{eE_0}{m_e}\sin(\omega t)$$

The acceleration of the electron is

$$\ddot{x}(t) = -\frac{\omega^2 - \omega_0^2}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{e\omega^2 E_0}{m_e}\cos(\omega t) + \frac{\omega\Gamma}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{e\omega^2 E_0}{m_e}\sin(\omega t)$$

Taking the square:

$$\ddot{x}^2(t) = \left[\frac{\omega^2 - \omega_0^2}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{e\omega^2 E_0}{m_e}\cos(\omega t)\right]^2 + \left[\frac{\omega\Gamma}{\left(\omega^2 - \omega_0^2\right)^2 + \omega^2\Gamma^2}\frac{e\omega^2 E_0}{m_e}\sin(\omega t)\right]^2 + \text{cross term}$$

and the average:

$$\langle \ddot{x}^2 \rangle = \frac{\left(\omega^2-\omega_0^2\right)^2}{\left[\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2\right]^2}\frac{e^2\omega^4E_0^2}{2m_e^2} + \frac{\omega^2\Gamma^2}{\left[\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2\right]^2}\frac{e^2\omega^4E_0^2}{2m_e^2}$$

$$= \frac{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}{\left[\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2\right]^2}\frac{e^2\omega^4E_0^2}{2m_e^2} = \frac{\omega^4}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{e^2E_0^2}{2m_e^2}$$

where the average of the cross term vanishes since it is proportional to $\cos(\omega t)\sin(\omega t)$ which has average zero. Thus the average power that the electron radiates is

$$\langle P \rangle = \frac{\mu_0 e^2}{6\pi c}\langle \ddot{x}^2 \rangle = \frac{\omega^4}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{e^4}{6\pi\epsilon_0c^3}\frac{E_0^2}{2m_e^2}$$

where I've again used $\mu_0/c = 1/\epsilon_0 c^3$. The equation for the cross section of the bound electron is

$$\langle S \rangle \sigma = \langle P \rangle$$

$$\frac{1}{2}c\epsilon_0E_0^2\sigma = \frac{\omega^4}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{e^4}{6\pi\epsilon_0c^3}\frac{E_0^2}{2m_e^2}$$

$$\sigma = \frac{\omega^4}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{e^4}{6\pi\epsilon_0^2c^4m_e^2}$$

$$= \frac{\omega^4}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\frac{16\pi^2}{6\pi}\cdot\frac{e^4}{4^2\pi^2\epsilon_0^2c^4m_e^2} = \frac{\omega^4}{\left(\omega^2-\omega_0^2\right)^2+\omega^2\Gamma^2}\cdot\frac{8\pi}{3}r_0^2$$

h)

The cross sections for both the free and bound electron are plotted in Figure 2 as functions of the frequency $f = \omega/2\pi$ of the incoming wave. The visible spectrum which consists of frequencies roughly in the range of $[400,\ 800]$ THz is shown for reference.

i)

Since $\langle S \rangle$ is independent of frequency, the average radiated power of the electron is $\langle P \rangle \propto \sigma$. The cross section can thus be interpreted as a measure of how much radiation the electron emits. Since $x(t) = \mathrm{Re}\left\{Ae^{i\omega t}\right\}$ the electron oscillates with the same frequency as that of the incoming electromagnetic wave. The radiation field that the electron produces is proportional to the acceleration of the electron (see Eq. (49) in the formula collection), so the radiation field has the same frequency as the incoming electromagnetic wave. Light is thus being scattered by the electron and $\langle P \rangle \propto \sigma$ tells us how much.

On a first sight it might look like any interesting part of Figure 2 is far away from the visible spectrum, but Figure 3 gives a closer look. We see that higher frequency visible light (the blue end of the visible spectrum) is scattered ~ 5 times more than lower frequency visible light (the red end of the visible spectrum). The sun emits mostly light with frequencies in the visible spectrum which all at once appears as white light. So the particles in the atmosphere separate blue light from the white light to a larger extent than red or yellow light. Blue light can get scattered several times before it by chance reaches my eye from all parts of the sky, while red light will to a larger extent pass directly through the atmosphere without being scattered as much.

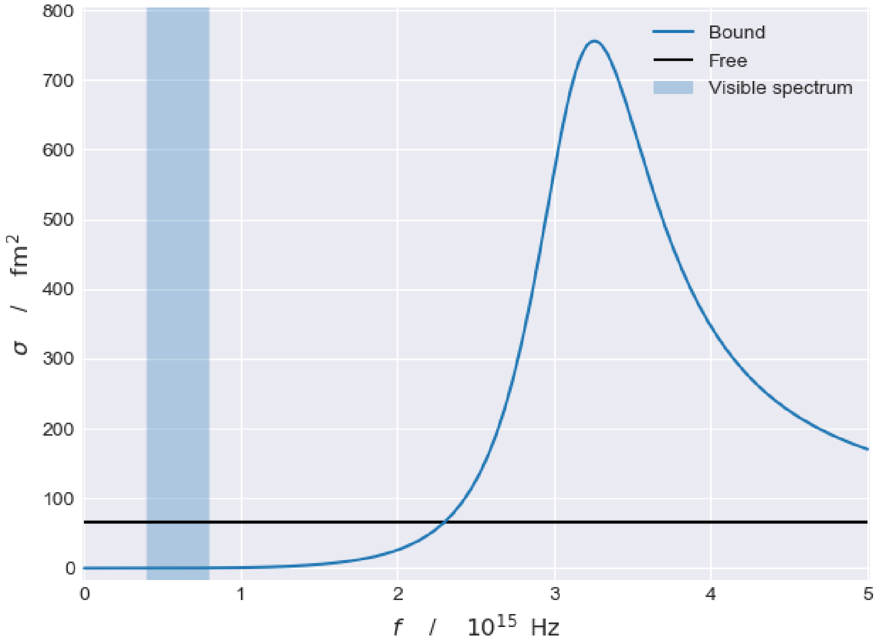

Figure 2

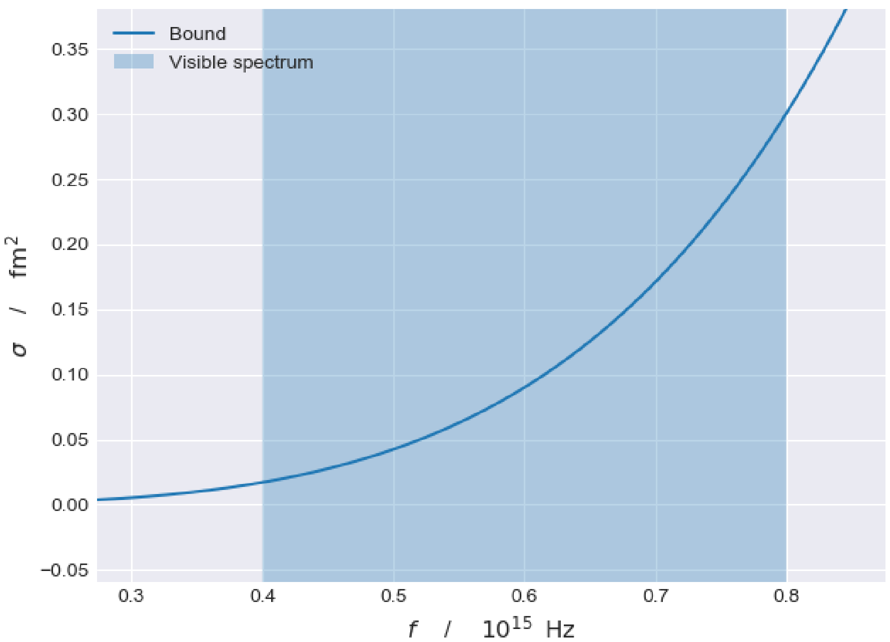

Figure 3