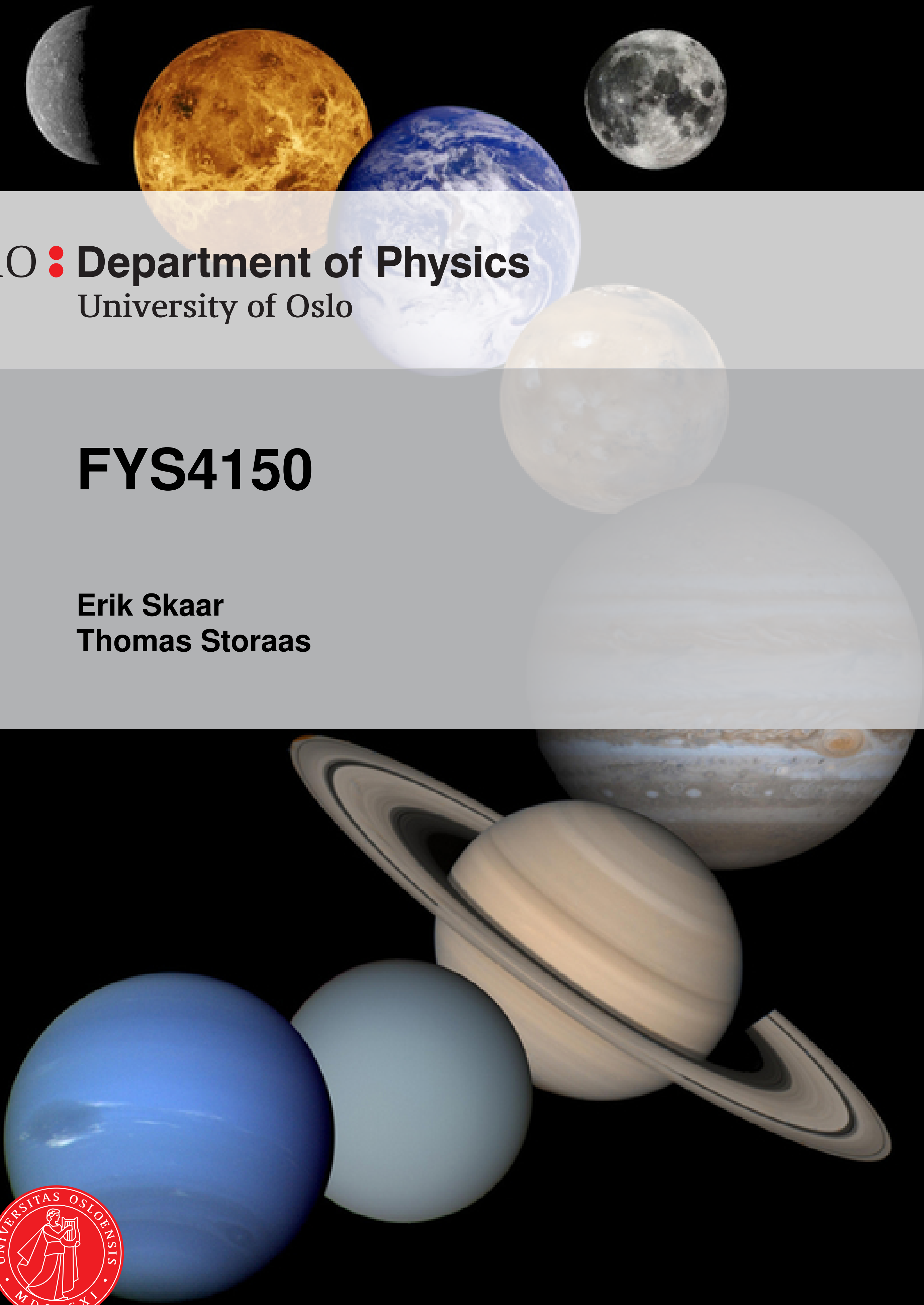

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Abstract

1 Introduction

For thousand of years humankind have looked up to the beyond and wondered. Specifically our race has wondered about the motion of our solar system. Finally after Newton the mystery was solved. Newton developed the gravitational law, which made it possible to predict the motion of the planets. After a couple of centuries Einstein came with the thoery of general relativity and made a small refinement to the law of motion that Newton proposed.

These laws are not enough to solve the motion of the planets. The laws makes differential equation which are not trivial or even impossible to solve. This is where a our computational physics course comes in . With the tools developed in this course we can make a prediction to the motion of the planets in our solarsystem.

In this project we will make an object oriented code to solve the solar system with Forward Eulers method and Velocity Verlet. Both method will be derived, discussed, implemented and benchmarked for the Earth-Sun system.

Finally, a relativistic correction was made.......

REF KURSETS HJEMMESIDE

WRITE ABOUT THEIR PERFORMANCE

NEED TO IMPLEMENT THIS AND GET FACTS

2 Theory

2.1 Gravitation

We will simulate the solar system with only the gravitational force affecting the planets. Newton's gravitational law is stated in equation (1). Where G is the gravitational constant ($6.67 \cdot 10^{-11}\text{Nm}^2/\text{s}^2$), r is the distance between the planets, m is the mass of the object and M is the mass of the other object.

$$\vec{F_G} = \frac{GmM}{|\vec{r}|^3}\vec{r} \tag{1}$$

Thankfully for us the force is an attractive force. It is worth noting that if the sun is at origo distance is simply the norm position vector of the planet. From now on we will denoted the different masses with $M_{planetname}$ except for the sun that has the special symbol $M_\odot$.

If you have n object attracting each other the total gravitational force, F_k, for each object is:

$$F_k = \sum_{i=1}^{N} \vec{F_i} = \frac{Gm_k m_i}{|\vec{r_k - r_i}|^3}(\vec{r_k} - \vec{r_i})(1 - \delta_{k,i}) \tag{2}$$

Where $\delta_{k,i}$ is the function:

$$\delta_{k,i} = \begin{cases} 1 & \text{if} \quad k = i \\ 0 & \text{if} \quad k \neq i \end{cases}$$

We can use Newton's second law to determine the accelration of the planet. Newton's second law state $F = m\vec{a}$. Which translate to:

$$a_k = \sum_{i=1}^{N} \frac{Gm_i}{|\vec{r_k} - \vec{r_i}|^3}(\vec{r_k} - \vec{r_i})(1 - \delta_{k,i}) \tag{3}$$

2.2 Units

As a famous person once said, Ïf you use seconds and meters you will not finish within the deadline ̈. I think our professor, Morten Hjorth-Jensen, is on to something. Seconds and meters are impractical in this project. Therefore we use more suitable units. The distance between the earth and sun, 149 597 870 691 meter, is defined as one astronomical unit (au). For time one year seems reasonable. These are the units that will be used in this project.

We can now express the gravitational constant with these units. To do this we use the formula for the acceleration for a circular orbit is $\frac{v^2}{r}$. Combine this with the gravitational law, we get:

$$\frac{v^2}{r} = \frac{GM_\odot}{|\vec{r}|^3}\vec{r} \implies G = \frac{v^2 r}{M_\odot} = \frac{\mathrm{au}(2\pi \cdot \mathrm{au/year})^2}{M_\odot} = \frac{4\pi^2}{M_\odot}\mathrm{au}^3/\mathrm{year}^2$$

Applying this to equation (3), we get:

$$a_k = \sum_{i=1}^{N} 4\pi \frac{m_i}{M_\odot} \frac{(\vec{r_k} - \vec{r_i})}{|\vec{r_k} - \vec{r_i}|^3}(1 - \delta_{k,i})\mathrm{au}^3/\mathrm{year}^2 \tag{4}$$

2.3 prehelion

since this is not implemented in the code yet nothing is writen

2.4 Numerical methods

Equation (4) initial conditions for velocity and position determines the orbit of the planets. These equations are for x,y,z direction and when writen out you get a coupled set of differential equations. This set is near impossible to solve analytically. It might be impossible. We will use numerical methods to solve this set. More specifically we will use the Forward Euler method and the Velocity-Verlet method. A given t_i is equal to $t_0 + ih$, where $h = (t_0 + t_n)/n$.

2.4.1 Forward Euler

Both method use a Taylor polynomial approximation to solve the set of differential equation. The Forward Euler use the first order Taylor polynomial. With $r'(t) = v(t)$ and $v'(t) = a(t)$ the Forward Euler method result in:

$$\vec{r_i}(t+h) \approx \vec{r_i}(t) + h\vec{v_i}(t) \tag{5}$$
$$\vec{v_i}(t+h) \approx \vec{v_i}(t) + h\vec{a}(t) \tag{6}$$

The error for a first order taylor polynomial goes as $\mathcal{O}(h^2)$. This is the error for each step. The error is accumulated for each step and will thereby be proportional to h.

REF TO LECTURE NOTES

2.4.2 Velocity-Verlet

You guessed it, this is the second order Taylor polynomial. . As I said the Velocity-Verlet method is based on a Taylor polynomial approximation. And this is the second order.

$$\vec{r_i}(t+h) \approx \vec{r_i}(t) + h\vec{v_i}(t) + \frac{1}{2}h^2 a(t) \tag{7}$$

$$\vec{v_i}(t+h) \approx \vec{v_i}(t) + h\vec{a}(t) + \frac{1}{2}h^2 \vec{a}'(t) \tag{8}$$

Since we have no formula for the derivative of a, we use the approximation:

ER DET DELT PÅ 2 EL-LER h?

$$\vec{a}'(t) \approx \frac{\vec{a}(t+h) - \vec{a}(t)}{h}$$

We simply update the position first. We keep the old acceleration and calculate the acceleration at the new position. Using this equation (**??**) we get:

$$\vec{v_i}(t+h) \approx \vec{v_i}(t) + h\vec{a}(t) + \frac{1}{2}h(\vec{a}(t+h) - \vec{a}(t))$$
$$\vec{v_i}(t+h) \approx \vec{v_i}(t) + \frac{1}{2}h(\vec{a}(t+h) + \vec{a}(t))$$

The error for a first order taylor polynomial goes as $\mathcal{O}(h^2)$. This is the error for each step. The error is accumulated for each step and will thereby be proportional to h^2. This is one order of magnitude better, then the Forward Euler method.

REF TO LECTU-RE NO-TES

3 Method

$2458045.500000000 = A.D. 2017{-}Oct{-}19 00:00:00.0000 TDB X = 2.213296131976958E{-}03 Y = 5.740795718142255E{-}03 Z = -1.300333836064062E{-}04 VX = -5.236918819978495E{-}06 VY = 5.487345385589584E-06 VZ = 1.229796132639033E-07$

4 Result

4.1 Earth-Sun system

4.1.1 Stability

4.1.2 Conserved quantities

4.1.3 Escape velocity

4.2 Three body system

4.2.1 Fixed mass for jupitur

4.2.2 Varying mass for jupitur

4.3 Solar system

4.3.1 Three planets and all moving

4.3.2 Solar system all moving

4.4 The perihelion precession of Mercury

4.4.1 missing this part

5 Discussion

6 Conclusion

7 Appendix