1

(a)

$n \in \mathbb{N}$ then $2^n - (-1)^n$ is divisible by three.
Let's check the first steps:

$$\begin{aligned} n = 1 \qquad & 2^1 - (-1)^1 = 3 \\ n = 2 \qquad & 2^2 - (-1)^2 = 3 \\ n = 3 \qquad & 2^3 - (-1)^3 = 9 \\ n = 4 \qquad & 2^4 - (-1)^4 = 15 \end{aligned}$$

All of the first n's are divisible by three. We go to general form:

$$\begin{aligned} n & \qquad 2^n - (-1)^n \\ n+1 & \qquad 2^{n+1} - (-1)^{n+1} \\ & = 2(2^n) - (-1)^{n+1} \\ & = 2(2^n + (-1)^n + (-1)^{n+1}) - (-1)^{n+1} \\ & = 2(2^n + (-1)^n - (-1)^n) - (-1)^{n+1} \\ & = 2(2^n - (-1)^n) + 2(-1)^n + (-1)^n \\ & = 2(2^n - (-1)^n) + 3(-1)^n \end{aligned}$$

Notice that the expression for n+1 is made up by a two part. The first is 2 times the n expression and the second part is 3 times either -1 or +1. Both part are divisible by 3 and there by is the product divisible by 3.

n	General	Specific	LU	fastest	slowest	$\frac{slowest}{fastest}$
10	6.5e-05	5e-06	4e-05	Specific	General	13.0
100	7.5e-05	8e-06	0.0023	Specific	LU	287.5
1000	0.00014	4e-05	0.26	Specific	LU	6500
10000	0.0007	0.0005	142.5	Specific	LU	285000