

1

(a)

$n \in \mathbb{N}$ then $2^n - (-1)^n$ is divisible by three.

Let's check the first steps:

$$\begin{array}{ll} n = 1 & 2^1 - (-1)^1 = 3 \\ n = 2 & 2^2 - (-1)^2 = 3 \\ n = 3 & 2^3 - (-1)^3 = 9 \\ n = 4 & 2^4 - (-1)^4 = 15 \end{array}$$

All of the first n 's are divisible by three. We go to general form:

$$\begin{array}{ll} n & 2^n - (-1)^n \\ n + 1 & 2^{n+1} - (-1)^{n+1} \\ & = 2(2^n) - (-1)^{n+1} \\ & = 2(2^n + (-1)^n + (-1)^{n+1}) - (-1)^{n+1} \\ & = 2(2^n + (-1)^n - (-1)^n) - (-1)^{n+1} \\ & = 2(2^n - (-1)^n) + 2(-1)^n + (-1)^n \\ & = 2(2^n - (-1)^n) + 3(-1)^n \end{array}$$

Notice that the expression for $n+1$ is made up by a two part. The first is 2 times the n expression and the second part is 3 times either -1 or +1. Both part are divisible by 3 and there by is the product divisible by 3.