MATIN1105 - INDUKSJON

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(a)

$n \in \mathbb{N}$ then $2^n - (-1)^n$ is divisible by three.
Let's check the first steps:

$$
\begin{aligned}
n = 1 \qquad & 2^1 - (-1)^1 = 3 \\
n = 2 \qquad & 2^2 - (-1)^2 = 3 \\
n = 3 \qquad & 2^3 - (-1)^3 = 9 \\
n = 4 \qquad & 2^4 - (-1)^4 = 15
\end{aligned}
$$

All of the first n's are divisible by three. We go to general form:

$$
\begin{aligned}
n \qquad\qquad & 2^n - (-1)^n \\
n + 1 \qquad\qquad & 2^{n+1} - (-1)^{n+1} \\
& = 2(2^n) - (-1)^{n+1} \\
& = 2(2^n + (-1)^n + (-1)^{n+1}) - (-1)^{n+1} \\
& = 2(2^n + (-1)^n - (-1)^n) - (-1)^{n+1} \\
& = 2(2^n - (-1)^n) + 2(-1)^n + (-1)^n \\
& = 2(2^n - (-1)^n) + 3(-1)^n
\end{aligned}
$$

Notice that the expression for n+1 is made up by a two part. The first is 2 times the n expression and the second part is 3 times either -1 or +1. Both part are divisible by 3 and there by is the product divisible by 3.