

**UiO** : **Department of Mathematics**  
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# **MATIN1105 - INDUKSJON**

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# 1

## (a)

$n \in \mathbb{N}$  then  $2^n - (-1)^n$  is divisible by three.

Let's check the first steps:

$$\begin{aligned}n = 1 & \quad 2^1 - (-1)^1 = 3 \\n = 2 & \quad 2^2 - (-1)^2 = 3 \\n = 3 & \quad 2^3 - (-1)^3 = 9 \\n = 4 & \quad 2^4 - (-1)^4 = 15\end{aligned}$$

All of the first  $n$ 's are divisible by three. We go to general form:

$$\begin{aligned}n & \quad 2^n - (-1)^n \\n + 1 & \quad 2^{n+1} - (-1)^{n+1} \\& = 2(2^n) - (-1)^{n+1} \\& = 2(2^n + (-1)^n + (-1)^{n+1}) - (-1)^{n+1} \\& = 2(2^n + (-1)^n - (-1)^n) - (-1)^{n+1} \\& = 2(2^n - (-1)^n) + 2(-1)^n + (-1)^n \\& = 2(2^n - (-1)^n) + 3(-1)^n\end{aligned}$$

Notice that the expression for  $n+1$  is made up by a two part. The first is 2 times the  $n$  expression and the second part is 3 times either -1 or +1. Both part are divisible by 3 and there by is the product divisible by 3.