Exercise 1

Problem 1 a)

Children	0	1	2	3	4	5
Probability	0,15	0,49	0,27	0,06	0,02	0,01

C = total nr of siblings

$$P(C \leq 2) = 0,15 + 0,49 + 0,27 = \underline{0,91}$$

b)

$$P(C \geq 3 \mid C \geq 1)$$

$$= \frac{P(C \geq 3) \cdot P(C \geq 1 \mid C \geq 3)}{P(C \geq 1)}$$

$$= \frac{0,06 + 0,02 + 0,01}{1 - 0,15} \approx \underline{0,11}$$

c) Possibilities

	A	B	C	
1.)	1	1	1	□
2.)	2	1	0	○
3)	2	0	1	○
4)	3	0	0	△
5)	1	2	0	○
6)	0	2	1	○
7)	0	3	0	△
8)	1	0	2	○
9)	0	1	2	○
10)	0	0	3	△

$P(\square) = (0,49)^3 \approx 0,118$

$P(\bigcirc) = (0,27)\cdot(0,49)(0.15) = 0,020$

$P(\triangle) = (0,06)\cdot(0,15)^2 \approx 0,001$

$P(3 \text{ siblings}) = \square + 6\bigcirc + 3\triangle = \underline{0.241}$

d)

	Emma	Jacob	
1)	3	○	□
2)	2	1	△
3)	1	2	△
4)	0	3	*

$P(\square) = (0,06)(0,15) \simeq 0,009$

$P(\triangle) = (0,27)(0,49) \simeq 0,132$

$P(*) = P(\square) \simeq 0,009$

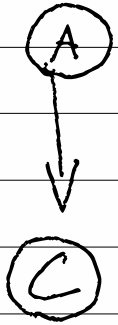
$P(3\ sib) = \square + * + 2\triangle = 0,282$

$$P(*\,|\,3\ sib) = \frac{P(*)}{P(3\ sib)} \simeq \underline{0,032}$$

Problem 2

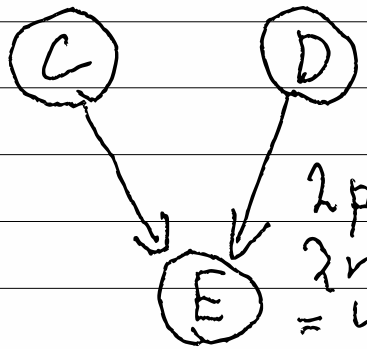
A has no parent and is a bernoulli distribution ⇒ only need 1 param. (has to equal 1.0)

A	P(C|A)
T	O
F	X



parent node + x = 2 param

C	D	P(E|C,D)
T	T	X
T	F	O
F	T	X
F	F	O



2 parent + 2 node = 4 param

Total = A + B + C + D + E + F + G + H

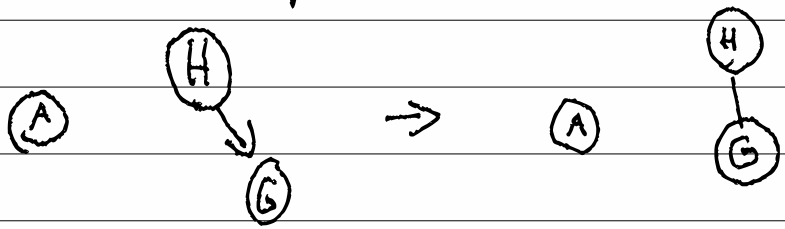
= 1 + 2 + 2 + 2 + 4 + 4 + 2 + 1

= 18 ⇒ True

"Each variable is conditionally independent of its non-descendants given its parents."

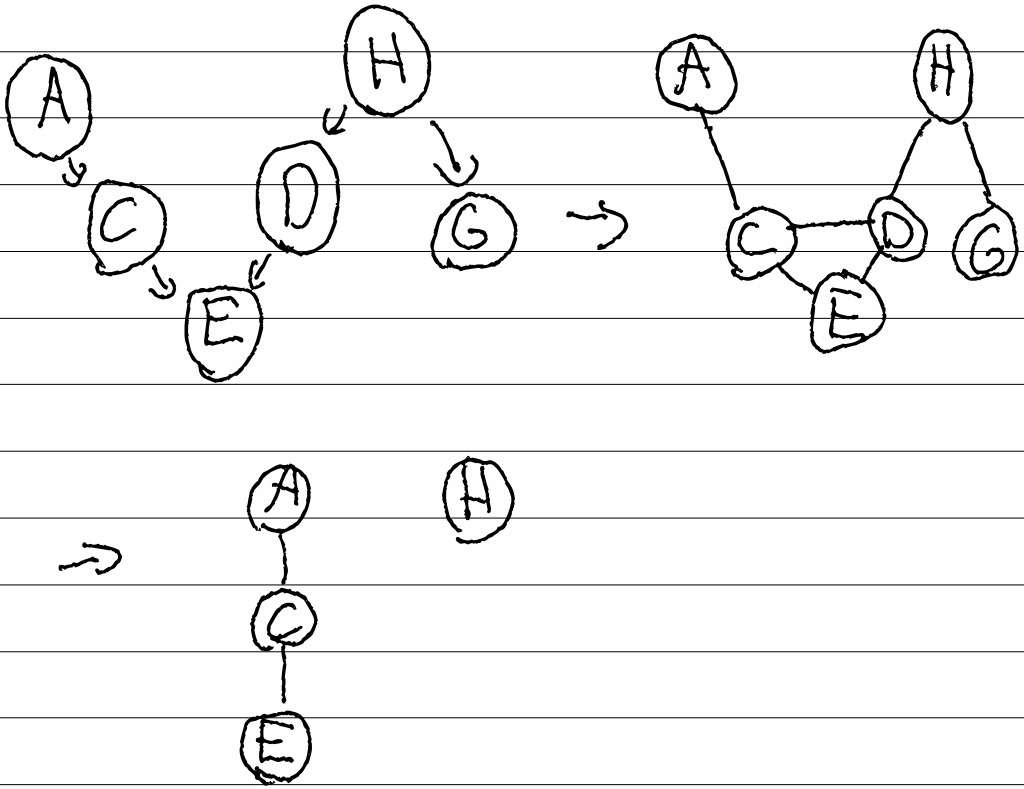
b) $G \perp\!\!\!\perp A$ using d-seperation

Ancestral Graph



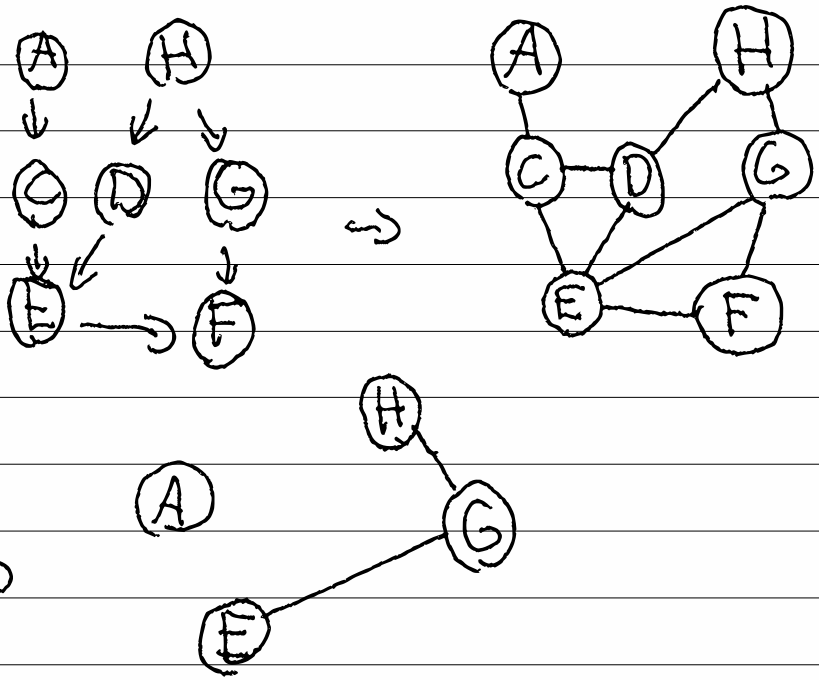
True, since G and A are not connected

c) $E \perp\!\!\!\perp H \mid \{D, G\}$



True, since E and H are not connected

d) $E \perp\!\!\!\perp H \{C, D, F\}$



False, since E and H are connected

Problem 3 a)

$$P(b) = P(b|a) \cdot P(a) + P(b|\neg a) P(\neg a)$$
$$= 0,5 \cdot 0,8 + 0,2 \cdot 0,2 = \underline{0,44}$$

b)

$$P(\neg b) = 1 - P(b) = 0,56$$
$$P(d) = P(d|\neg b) \cdot P(\neg b) + P(d|b) P(b)$$
$$= 0,8 \cdot 0,56 + 0,6 \cdot 0,44$$
$$= \underline{0,712}$$

$$P(c) = P(c|\neg b) \cdot P(\neg b) + P(c|b) \cdot P(b)$$
$$= 0,3 \cdot 0,56 + 0,1 \cdot 0,44$$
$$= 0,212$$

c)

$$P(C \mid \neg d) = \frac{\text{I } P(C, \neg d)}{\text{II } P(\neg d)}$$

I $\Rightarrow P(C, \neg d)$

$$= \sum_a \sum_b P(a, b, C, d)$$

$$= \sum_a \sum_b P(a) \cdot P(b \mid a) \cdot P(d \mid b) \cdot P(C \mid b)$$

if $c = t, d = f$

$a = t, b = t = P(a) \cdot P(b \mid a) \cdot P(\neg d \mid b) \cdot P(C \mid b)$

$= 0,8 \cdot 0,5 \cdot 0,4 \cdot 0,1 = 0,016$

$a = f, b = t = P(\neg a) \cdot P(b \mid \neg a) \cdot P(\neg d \mid b) \cdot P(C \mid b)$

$= 0,2 \cdot 0,2 \cdot 0,4 \cdot 0,1 = 0,0016$

$a = t, b = f = P(a) \cdot P(\neg b \mid a) \cdot P(\neg d \mid \neg b) \cdot P(C \mid \neg b)$

$= 0,8 \cdot 0,6 \cdot 0,2 \cdot 0,3 = 0,024$

$c = f, b = f \Rightarrow P(\neg a) \cdot P(\neg b \mid \neg a) \cdot P(\neg d \mid \neg b) \cdot P(C \mid \neg b)$

$0,2 \cdot 0,8 \cdot 0,2 \cdot 0,3 = 0,0096$

$$P(C \mid \neg d) = \frac{0,016 + 0,0016 + 0,024 + 0,0096}{1 - P(d)}$$

$\approx 0,178$

d) $\underline{P(a \mid \neg c, d)}$

$$P(a, b, c, d)$$
$$= P(a) \cdot P(b \mid a) \cdot P(d \mid a, b) \cdot P(c \mid a, b)$$
$$= P(a) \cdot P(b \mid a) \cdot P(d \mid b) \cdot P(c \mid b)$$

$$P(a \mid \neg c, d) = \sum_b \frac{P(a, b, \neg c, d)}{P(\neg c, d)}$$

$$= \alpha \cdot P(a) \sum_b P(b \mid a) \cdot P(d \mid b) \cdot P(\neg c \mid b)$$

$$= \alpha \cdot 0,8 \Big(\underbrace{0,5 \cdot 0,6 \cdot 0,9}_{b} + \underbrace{0,5 \cdot 0,8 \cdot 0,7}_{\neg b} \Big)$$

$$= \alpha \cdot 0,44$$

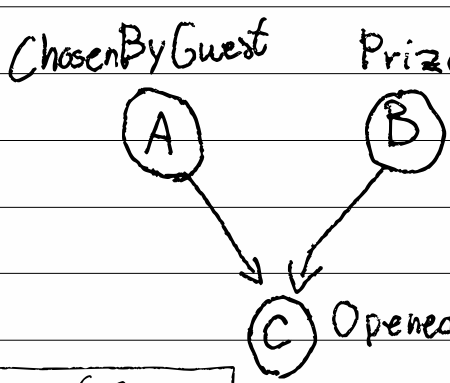
$$P(\neg c \wedge d) = (1 - 0,212) \cdot (0,712)$$
$$\approx 0,561$$

$$P(a \mid \neg c, d) = \frac{0,44}{0,561} = \underline{\underline{0,784}}$$

4 c)

$$P(\text{Prize} \mid \text{ChosenByGuest} = 1, \text{OpenedByHost} = 3)$$

A	P(A)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$



B	P(B)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$

		C =		
A	B	1	2	3
1	1	0	0,5	0,5
1	2	0	0	1,0
1	3	0	1,0	0
2	1	0	0	1,0
2	2	0,5	0	0,5
2	3	1,0	0	0
3	1	0	1,0	0
3	2	1,0	0	0
3	3	0,5	0,5	0