# Exercise 1

## Problem 1 a)

| Children | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Probability | 0,15 | 0,49 | 0,27 | 0,06 | 0,02 | 0,01 |

$C$ = total nr of siblings

$$P(C \leq 2) = 0{,}15 + 0{,}49 + 0{,}27$$
$$= \underline{\underline{0{,}91}}$$

b)

$$P(C \geq 3 \mid C \geq 1)$$

$$= \frac{P(C \geq 3) \cdot \overset{1}{\overset{\shortparallel}{P(C \geq 1 \mid C \geq 3)}}}{P(C \geq 1)},$$

$$= \frac{0{,}06 + 0{,}02 + 0{,}01}{1 - 0{,}15} \approx \underline{\underline{0{,}11}}$$

c) Possibilities

| | A | B | C | |
|---|---|---|---|---|
| 1.) | 1 | 1 | 1 | $\square$ |
| 2.) | 2 | 1 | 0 | 0 |
| 3) | 2 | 0 | 1 | 0 |
| 4) | 3 | 0 | 0 | $\Delta$ |
| 5) | 1 | 2 | 0 | 0 |
| 6) | 0 | 2 | 1 | 0 |
| 7) | 0 | 3 | 0 | $\Delta$ |
| 8) | 1 | 0 | 2 | 0 |
| 9) | 0 | 1 | 2 | 0 |
| 10) | 0 | 0 | 3 | $\Delta$ |

$P(\square) = (0,49)^3 \approx 0,118$

$P(0) = (0,27)\cdot(0,49)(0.15) \approx 0.020$

$P(\Delta) = (0,06)\cdot(0,15)^2 \approx 0,001$

$P(3 \text{ siblings}) = \square + 6 0 + 3\Delta = \underline{\underline{0,241}}$

d)

| | Emma | Jacob | |
|---|---|---|---|
| 1.) | 3 | 0 | □ |
| 2.) | 2 | 1 | Δ |
| 3) | 1 | 2 | Δ |
| 4) | 0 | 3 | * |

$P(\square) = (0{,}06)(0{,}15) \simeq 0{,}009$

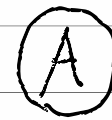
$P(\Delta) = (0{,}27)(0{,}49) \simeq 0{,}132$

$P(*) = P(\square) \simeq 0{,}009$

$P(3\,sib) = \square + * + 2\Delta = 0{,}282$

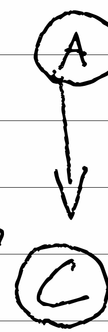
$$P(* \mid 3\,sib) = \frac{P(*)}{P(3\,sib)} \simeq \underline{\underline{0{,}032}}$$

## Problem 2



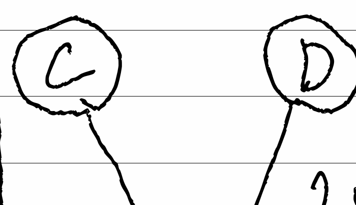

A has no parent and is a <u>bernoulli distribution</u> $\Rightarrow$ only need 1. param. (has to equal 1.0!)

| A | P(C\|A) |
|---|---|
| T | 0 |
| F | x |




parent node + x = 2 param

| C | D | P(E\|C,D) |
|---|---|---|
| T | T | x |
| T | F | o |
| F | T | x |
| F | F | o |



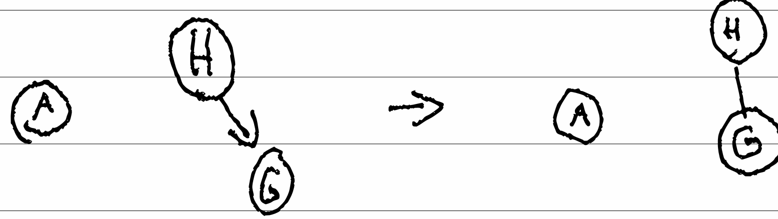

2 parent + 2 node = 4 param

$$\text{Total} = A + B + C + D + E + F + G + H$$
$$= 1 + 2 + 2 + 2 + 4 + 4 + 2 + 1$$
$$= 18 \Rightarrow \underline{\text{True}}$$

"Each variable is conditionally independent of its non-descendants given its parents."

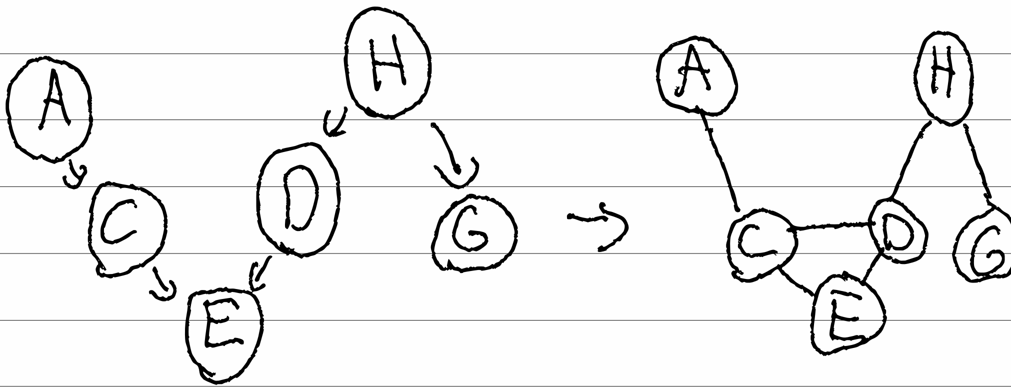
b) $G \perp\!\!\!\perp A$ using d-seperation

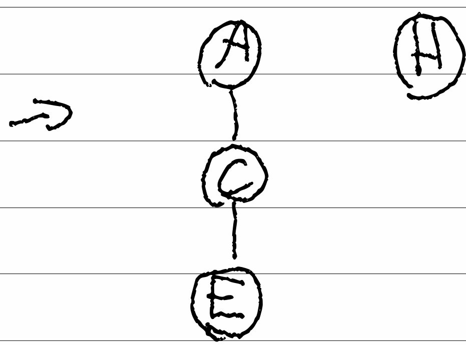
Ancestral Graph




True, since G and A are not connected

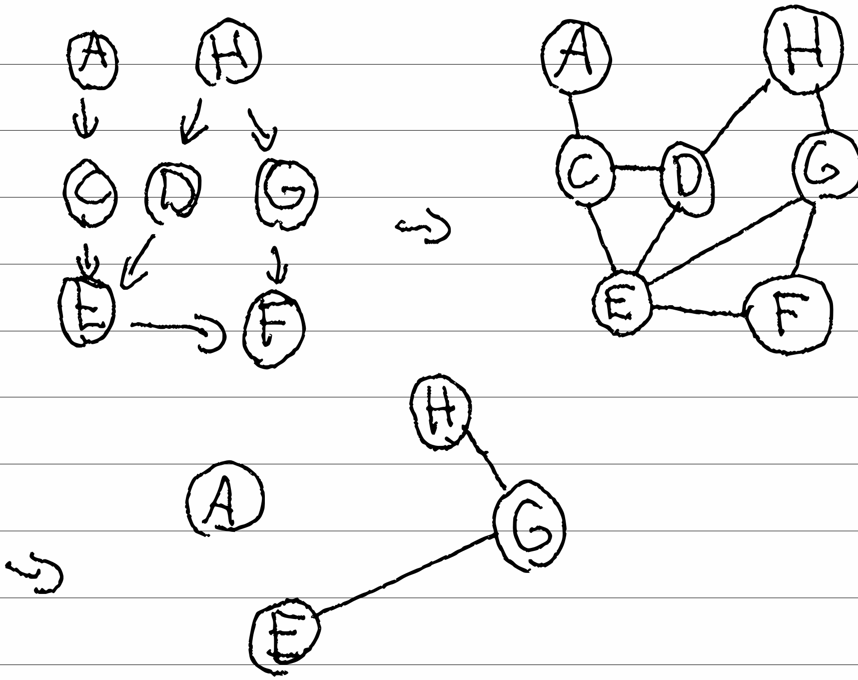
c) $E \perp\!\!\!\perp H \mid \{D, G\}$




True, since E and H are not connected

d) $E \perp\!\!\!\perp H \ \{C, D, F\}$




False, since E and H are connected

Problem 3 a)

$$P(b) = P(b|a)\cdot P(a) + P(b|\neg a)P(\neg a)$$
$$= 0{,}5\cdot 0{,}8 + 0{,}2\cdot 0{,}2 = \underline{\underline{0{,}44}}$$

b)

$$P(\neg b) = 1 - P(b) = 0{,}56$$
$$P(d) = P(d|\neg b)\cdot P(\neg b) + P(d|b)P(b)$$
$$= 0{,}8\cdot 0{,}56 + 0{,}6\cdot 0{,}44$$
$$= \underline{\underline{0{,}712}}$$

$$P(c) = P(c|\neg b)\cdot P(\neg b) + P(c|b)\cdot P(b)$$
$$= 0{,}3\cdot 0{,}56 + 0{,}1\cdot 0{,}44$$
$$= 0{,}212$$

c)

$$P(C \mid \neg d) = \frac{\text{I} \; P(C, \neg d)}{\text{II} \; P(\neg d)}$$

$\text{I} \Rightarrow P(C, \neg d)$

$$= \sum_a \sum_b P(a, b, C, d)$$

$$= \sum_a \sum_b P(a) \cdot P(b \mid a) \cdot P(d \mid b) \cdot P(C \mid b)$$

if $c = t$, $d = f$

$a = t, b = t = P(a) \cdot P(b \mid a) \cdot P(\neg d \mid b) \cdot P(C \mid b)$

$= 0{,}8 \cdot 0{,}5 \cdot 0{,}4 \cdot 0{,}1 = 0{,}016$

$a = f, b = t = P(\neg a) \cdot P(b \mid \neg a) \cdot P(\neg d \mid b) \cdot P(C \mid b)$

$= 0{,}2 \cdot 0{,}2 \cdot 0{,}4 \cdot 0{,}1 = 0{,}0016$

$a = t, b = f = P(a) \cdot P(\neg b \mid a) \cdot P(\neg d \mid \neg b) \cdot P(C \mid \neg b)$

$= 0{,}8 \cdot 0{,}5 \cdot 0{,}2 \cdot 0{,}3 = 0{,}024$

$c = f, b = f \Rightarrow P(\neg a) \cdot P(\neg b \mid \neg a) \cdot P(\neg d \mid \neg b) \cdot P(C \mid \neg b)$

$0{,}2 \cdot 0{,}8 \cdot 0{,}2 \cdot 0{,}3 = 0{,}0096$

$$P(C \mid \neg d) = \frac{0{,}016 + 0{,}0016 + 0{,}024 + 0{,}0096}{1 - P(d)}$$

$\approx 0{,}178$

d) $\underline{P(a \mid \neg c, d)}$

$$P(a, b, c, d)$$
$$= P(a) \cdot P(b \mid a) \cdot P(d \mid a, b) \cdot P(c \mid a, b)$$
$$= P(a) \cdot P(b \mid a) \cdot P(d \mid b) \cdot P(c \mid b)$$

$$P(a \mid \neg c, d) = \sum_b \frac{P(a, b, \neg c, d)}{P(\neg c, d)}$$

$$= \alpha \cdot P(a) \sum_b P(b \mid a) \cdot P(d \mid b) \cdot P(\neg c \mid b)$$

$$= \alpha \cdot 0{,}8 (\underbrace{0{,}5 \cdot 0{,}6 \cdot 0{,}9}_{b} + \underbrace{0{,}5 \cdot 0{,}8 \cdot 0{,}7}_{\neg b})$$

$$= \alpha \cdot 0{,}44$$

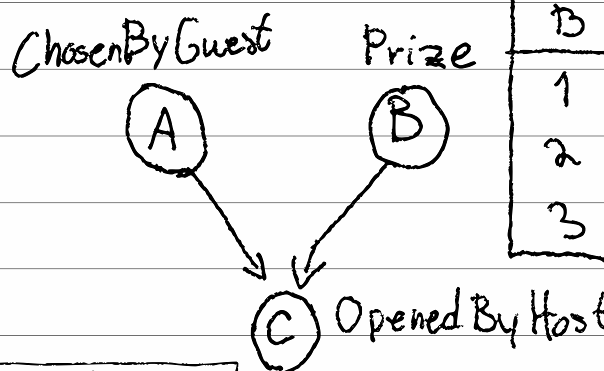
$$P(\neg c \wedge d) = (1 - 0{,}212) \cdot (0{,}712)$$
$$\approx 0{,}561$$

$$P(a \mid \neg c, d) = \frac{0{,}44}{0{,}561} = \underline{\underline{0{,}784}}$$

4 c)

$$P(\overset{B}{\text{Prize}} \mid \overset{A}{\text{ChosenByGuest}} = 1, \overset{C}{\text{OpenedByHost}} = 3)$$

| A | P(A) |
|---|---|
| 1 | $\frac{1}{3}$ |
| 2 | $\frac{1}{3}$ |
| 3 | $\frac{1}{3}$ |




| B | P(B) |
|---|---|
| 1 | $\frac{1}{3}$ |
| 2 | $\frac{1}{3}$ |
| 3 | $\frac{1}{3}$ |

| | | C = | | |
|---|---|---|---|---|
| A | B | 1 | 2 | 3 |
| 1 | 1 | 0 | 0,5 | 0,5 |
| 1 | 2 | 0 | 0 | 1,0 |
| 1 | 3 | 0 | 1,0 | 0 |
| 2 | 1 | 0 | 0 | 1,0 |
| 2 | 2 | 0,5 | 0 | 0,5 |
| 2 | 3 | 1,0 | 0 | 0 |
| 3 | 1 | 0 | 1,0 | 0 |
| 3 | 2 | 1,0 | 0 | 0 |
| 3 | 3 | 0,5 | 0,5 | 0 |

```
C:\Studie\NTNU\Metoder i kunstig intelligens\oving1>python Exercise-1_new.py
Problem3c:
Probability distribution, P(A)
+----------+----------+
|   A(0)   |  0.8000  |
+----------+----------+
|   A(1)   |  0.2000  |
+----------+----------+

Probability distribution, P(B | A)
+----------+----------+----------+
|    A     |   A(0)   |   A(1)   |
+----------+----------+----------+
|   B(0)   |  0.5000  |  0.2000  |
+----------+----------+----------+
|   B(1)   |  0.5000  |  0.8000  |
+----------+----------+----------+

Probability distribution, P(C | B)
+----------+----------+----------+
|    B     |   B(0)   |   B(1)   |
+----------+----------+----------+
|   C(0)   |  0.1000  |  0.3000  |
+----------+----------+----------+
|   C(1)   |  0.9000  |  0.7000  |
+----------+----------+----------+

Probability distribution, P(D | B)
+----------+----------+----------+
|    B     |   B(0)   |   B(1)   |
+----------+----------+----------+
|   D(0)   |  0.6000  |  0.8000  |
+----------+----------+----------+
|   D(1)   |  0.4000  |  0.2000  |
+----------+----------+----------+

Probability distribution, P(C | !D)
+----------+----------+
|   C(0)   |  0.1778  |
+----------+----------+
|   C(1)   |  0.8222  |
+----------+----------+

Monty Hall:
Probability distribution, P(A)
+----------+----------+
|   A(0)   |  0.3333  |
+----------+----------+
|   A(1)   |  0.3333  |
+----------+----------+
|   A(2)   |  0.3333  |
+----------+----------+

Probability distribution, P(B)
+----------+----------+
|   B(0)   |  0.3333  |
+----------+----------+
|   B(1)   |  0.3333  |
+----------+----------+
|   B(2)   |  0.3333  |
+----------+----------+

Probability distribution, P(C | A, B)
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|    A     |   A(0)   |   A(1)   |   A(2)   |   A(0)   |   A(1)   |   A(2)   |   A(0)   |   A(1)   |   A(2)   |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|    B     |   B(0)   |   B(0)   |   B(0)   |   B(1)   |   B(1)   |   B(1)   |   B(2)   |   B(2)   |   B(2)   |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|   C(0)   |  0.0000  |  0.0000  |  0.0000  |  0.0000  |  0.5000  |  1.0000  |  0.0000  |  1.0000  |  0.5000  |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|   C(1)   |  0.5000  |  0.0000  |  1.0000  |  0.0000  |  0.0000  |  0.0000  |  1.0000  |  0.0000  |  0.5000  |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+
|   C(2)   |  0.5000  |  1.0000  |  0.0000  |  1.0000  |  0.5000  |  0.0000  |  0.0000  |  0.0000  |  0.0000  |
+----------+----------+----------+----------+----------+----------+----------+----------+----------+----------+

Probability distribution, P(B | A, C)
+----------+----------+
|   B(0)   |  0.3333  |
+----------+----------+
|   B(1)   |  0.6667  |
+----------+----------+
|   B(2)   |  0.0000  |
+----------+----------+
```