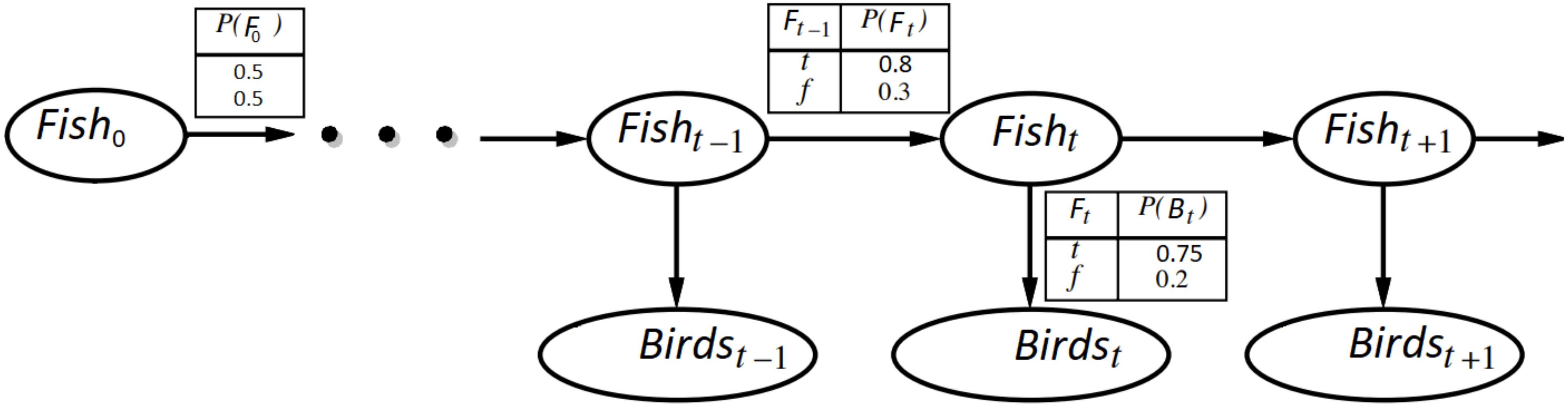
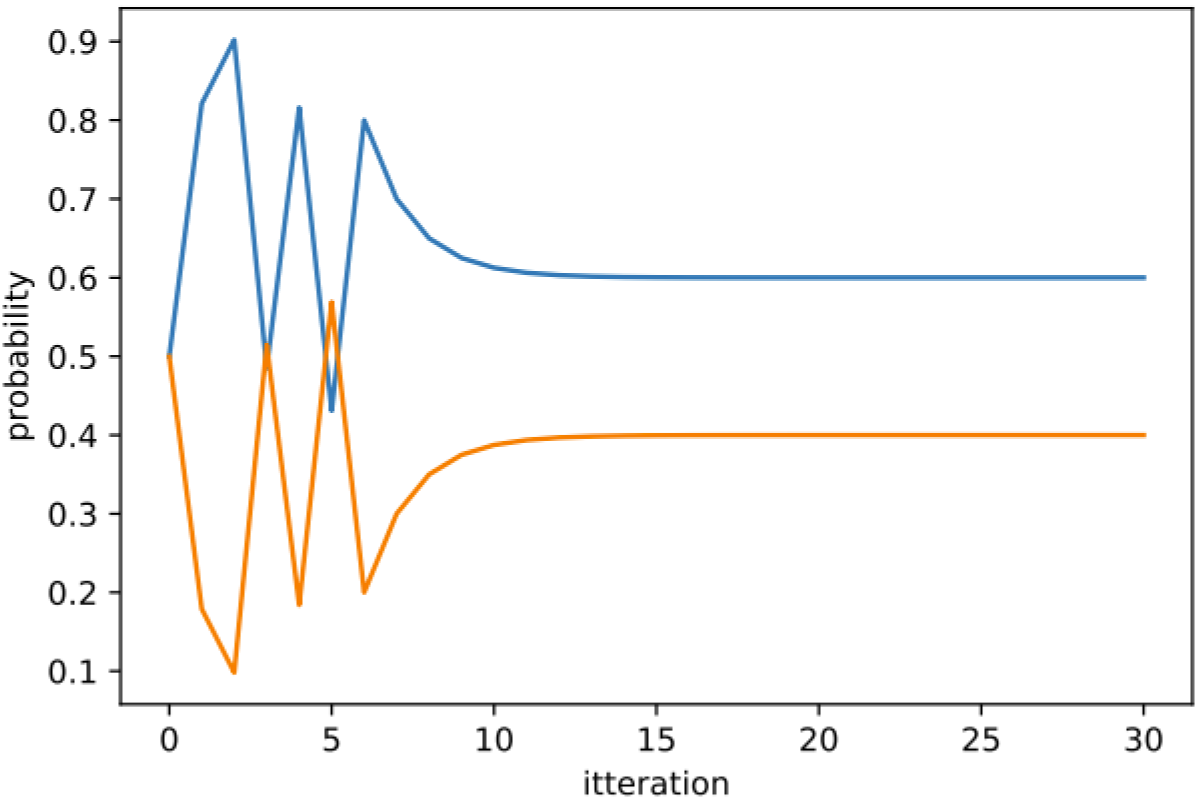
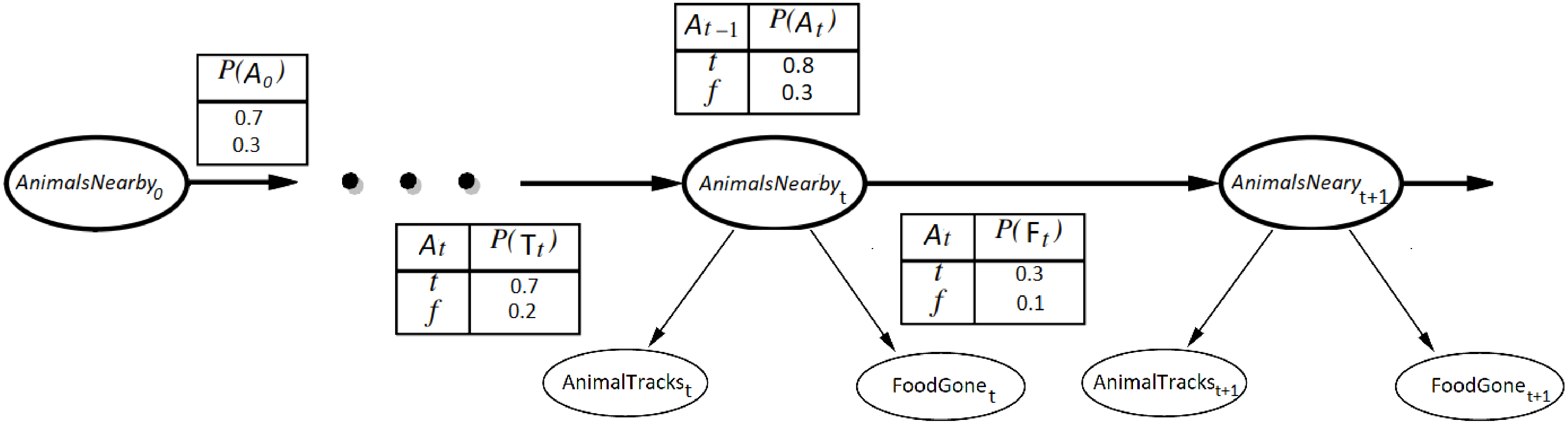

Fish0
P(F0)
0.5
0.5
Fish t−1
Ft−1
P(Ft)
t
f
0.8
0.3
Fisht
Fish t+1
Ft
P(Bt)
t
f
0.75
0.2
Birds t−1
Birdst
Birds t+1

```
Problem 1b)

P(X_0|e_1:0) = [0.5, 0.5]
P(X_1|e_1:1) = [0.821, 0.179]
P(X_2|e_1:2) = [0.902, 0.098]
P(X_3|e_1:3) = [0.485, 0.515]
P(X_4|e_1:4) = [0.816, 0.184]
P(X_5|e_1:5) = [0.431, 0.569]
P(X_6|e_1:6) = [0.8, 0.2]

Problem 1c)

P(X_7|e_1:7) = [0.7000000000000002, 0.30000000000000004]
P(X_8|e_1:8) = [0.6500000000000001, 0.3500000000000001]
P(X_9|e_1:9) = [0.6250000000000001, 0.3750000000000001]
P(X_10|e_1:10) = [0.6125000000000002, 0.38750000000000007]
P(X_11|e_1:11) = [0.6062500000000002, 0.3937500000000001]
P(X_12|e_1:12) = [0.6031250000000001, 0.3968750000000001]
P(X_13|e_1:13) = [0.6015625000000002, 0.39843750000000006]
P(X_14|e_1:14) = [0.6007812500000002, 0.39921875000000007]
P(X_15|e_1:15) = [0.6003906250000002, 0.39960937500000004]
P(X_16|e_1:16) = [0.6001953125000001, 0.39980468750000003]
P(X_17|e_1:17) = [0.6000976562500001, 0.39990234375]
P(X_18|e_1:18) = [0.6000488281250002, 0.399951171875]
P(X_19|e_1:19) = [0.6000244140625002, 0.39997558593750004]
P(X_20|e_1:20) = [0.6000122070312501, 0.39998779296875003]
P(X_21|e_1:21) = [0.6000061035156251, 0.39999389648437506]
P(X_22|e_1:22) = [0.6000030517578125, 0.39999694824218757]
P(X_23|e_1:23) = [0.6000015258789063, 0.3999984741210938]
P(X_24|e_1:24) = [0.6000007629394533, 0.3999992370605469]
P(X_25|e_1:25) = [0.6000003814697267, 0.3999996185302735]
P(X_26|e_1:26) = [0.6000001907348634, 0.39999980926513673]
P(X_27|e_1:27) = [0.6000000953674318, 0.3999999046325684]
P(X_28|e_1:28) = [0.600000047683716, 0.3999999523162842 7]
P(X_29|e_1:29) = [0.600000023841858, 0.3999999761581422]
P(X_30|e_1:30) = [0.6000000119209291, 0.3999999880790711]

Problem 1d)

P(X_0|e_1:6) = [0.665, 0.335]
P(X_1|e_1:6) = [0.876, 0.124]
P(X_2|e_1:6) = [0.866, 0.134]
P(X_3|e_1:6) = [0.598, 0.402]
P(X_4|e_1:6) = [0.766, 0.234]
P(X_5|e_1:6) = [0.57, 0.43]

Problem 1e)

The most likely sequence is: [True, True, True, True, True, True]
```


0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
probability
0
5
10
15
20
25
30
itteration



P(A0)
0.7
0.3
At-1
P(At)
t
0.8
f
0.3
At
P(Tt)
t
0.7
f
0.2
At
P(Ft)
t
0.3
f
0.1
AnimalsNearby0
AnimalsNearbyt
AnimalsNearyt+1
AnimalTrackst
FoodGonet
AnimalTrackst+1
FoodGonet+1

Exercise 2

1\. b) This kind of opperation is called filtering. It computes the belief state given all evidence to date.

c) This is a prediction. It computes the future state given all evidence to date. When t increases the state converges to [0.6, 0.4].

d) This is called smoothing. It copmutes the posterior distribution over a past state, given all evidence up to the present.

e) This is called "Most likely explanation". It computes the most likely sequence of states to have generated the observations.

2b) $P(A_t \mid e_{1:t})$, for t 1, 2, 3, 4

$$P(A_1) = \sum_{a_0} P(A_1 \mid a_0) \cdot P(a_0)$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.7 + \langle 0.3, 0.7 \rangle \cdot 0.3$$
$$= \langle 0.65, 0.35 \rangle$$

$$P(A_1 \mid e_{T_1}, e_{F_1}) = \alpha \cdot P(e_{T_1}, e_{F_1} \mid A_1) \cdot P(A_1)$$

$$= \alpha \cdot \langle 0.65, 0.35 \rangle \cdot \langle 0.7 \cdot 0.3, 0.2 \cdot 0.1 \rangle$$

$$= \alpha \cdot \langle 0.1365, 0.007 \rangle \approx \underline{\underline{\langle 0.95, 0.05 \rangle}}$$

$$P(A_2) = \sum_{a_1} P(A_2 \mid a_1) \cdot P(a_1)$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.95 + \langle 0.3, 0.7 \rangle \cdot 0.05$$
$$= \langle 0.775, 0.225 \rangle$$

$$P(A_2 \mid e_{T2}, e_{F2}) = \alpha P(e_{T2}, e_{F2} \mid A_2) P(A_2)$$

$$= \alpha \cdot \langle 0.775, 0.225 \rangle \cdot \langle 0.3 \cdot 0.3, 0.8 \cdot 0.1 \rangle$$

$$= \alpha \cdot \langle 0.06975, 0.018 \rangle \approx \underline{\langle 0.79, 0.21 \rangle}$$

$$P(A_3) = \sum_{a_2} P(A_3 \mid a_2) \cdot P(a_2)$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.79 + \langle 0.3, 0.7 \rangle \cdot 0.21$$
$$= \langle 0.695, 0.305 \rangle$$

$$= P(A_3 \mid e_{T3}, e_{F3}) = \alpha P(e_{T3}, e_{F3} \mid A_3) P(A_3)$$

$$= \alpha \cdot \langle 0.695, 0.305 \rangle \cdot \langle 0.3 \cdot 0.7, 0.8 \cdot 0.9 \rangle$$

$$\alpha \cdot \langle 0.14595, 0.2196 \rangle \approx \underline{\langle 0.4, 0.6 \rangle}$$

$$P(A_4) = \sum_{a_3} P(A_4 \mid a_3) \cdot P(a_3)$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.4 + \langle 0.3, 0.7 \rangle \cdot 0.6$$
$$= \langle 0.5, 0.5 \rangle$$

$$P(A_4 \mid e_{T4}, e_{F4}) = \alpha P(e_{T4}, e_{F4} \mid A_4) P(A_4)$$

$$= \alpha \cdot \langle 0.5, 0.5 \rangle \cdot \langle 0.7 \cdot 0.7, 0.2 \cdot 0.9 \rangle$$

$$= \alpha \cdot \langle 0.245, 0.09 \rangle \approx \underline{\underline{\langle 0.73, 0.27 \rangle}}$$

2 c) $P(X_t \mid e_{1:4})$, for $t = 5, 6, 7, 8$

$$P(A_5 \mid \overbrace{e_{T1:4}, e_{F1:4}}^{e_{TF}}) = \sum P(A_5, a_4) \cdot P(a_4 \mid e_{TF})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.73 + \langle 0.3, 0.7 \rangle \cdot 0.27$$
$$= \underline{\underline{\langle 0.665, 0.335 \rangle}}$$

$$P(A_6 \mid e_{TF}) = \sum P(A_6, a_5) P(a_5 \mid e_{TF})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.665 + \langle 0.3, 0.7 \rangle \cdot 0.335$$
$$\approx \underline{\underline{\langle 0.633, 0.367 \rangle}}$$

$$P(A_7 \mid e_{TF}) = \sum P(A_7, a_6) P(a_6 \mid e_{TF})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.633 + \langle 0.3, 0.7 \rangle \cdot 0.367$$
$$= \underline{\langle 0.617, 0.383 \rangle}$$

$$P(A_8 \mid e_{TF}) = \sum P(A_8, a_7) P(a_7 \mid e_{TF})$$

$$= \langle 0.8, 0.2 \rangle \cdot 0.617 + \langle 0.3, 0.7 \rangle \cdot 0.383$$
$$= \underline{\underline{\langle 0.608, 0.392 \rangle}}$$

d) $\lim_{t \to \infty} P(A_t | e_{1:4})$:

$$\begin{pmatrix} 0.8 , 0.3 \\ 0.2 , 0.7 \end{pmatrix} \cdot \begin{pmatrix} p \\ 1-p \end{pmatrix}$$

$$0.8p + 0.3(1-p) = p$$

$$\Rightarrow p = 0.6$$

$$\Rightarrow \lim_{t \to \infty} P(A_t | e_{1:4}) = \begin{pmatrix} 0.6 \\ 1-0.6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0.6 \\ 0.4 \end{pmatrix}}}$$

e) $P(X_t \mid e_{1:4})$, for $t = 0, 1, 2, 3$

Lets start by defining observation matrices for e:

$$O_1 = \begin{pmatrix} 0.7 \cdot 0.3, & 0 \\ 0, & 0.2 \cdot 0.1 \end{pmatrix} = \begin{pmatrix} 0.21, & 0 \\ 0, & 0.02 \end{pmatrix}$$

$$O_2 = \begin{pmatrix} 0.3 \cdot 0.3, & 0 \\ 0, & 0.8 \cdot 0.1 \end{pmatrix} = \begin{pmatrix} 0.09, & 0 \\ 0, & 0.08 \end{pmatrix}$$

$$O_3 = \begin{pmatrix} 0.3 \cdot 0.7, & 0 \\ 0, & 0.8 \cdot 0.9 \end{pmatrix} = \begin{pmatrix} 0.21, & 0 \\ 0, & 0.72 \end{pmatrix}$$

$$O_4 = \begin{pmatrix} 0.7 \cdot 0.7, & 0 \\ 0, & 0.2 \cdot 0.9 \end{pmatrix} = \begin{pmatrix} 0.49, & 0 \\ 0, & 0.18 \end{pmatrix}$$

Now lets calculate the backwards probabilities:

$$b_4 = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$$

$$b_3 = \alpha \overset{\text{transition}}{\begin{pmatrix} 0.8, & 0.2 \\ 0.3, & 0.7 \end{pmatrix}} \overset{O_4}{\begin{pmatrix} 0.49, & 0 \\ 0, & 0.18 \end{pmatrix}} \overset{b_4}{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}$$

$$= \alpha \begin{pmatrix} 0.428 \\ 0.273 \end{pmatrix} \approx \begin{pmatrix} 0.611 \\ 0.389 \end{pmatrix}$$

$$b_2 = \alpha \begin{pmatrix} 0.8, & 0.2 \\ 0.3, & 0.7 \end{pmatrix} \begin{pmatrix} 0.21, & 0 \\ 0, & 0.72 \end{pmatrix} \begin{pmatrix} 0.611 \\ 0.389 \end{pmatrix}$$

$$= \alpha \begin{pmatrix} 0.159 \\ 0.235 \end{pmatrix} = \begin{pmatrix} 0.404 \\ 0.596 \end{pmatrix}$$

$$b_1 = \alpha \begin{pmatrix} 0.8, 0.2 \\ 0.3, 0.7 \end{pmatrix} \begin{pmatrix} 0.09, & 0 \\ 0 & , 0.08 \end{pmatrix} \begin{pmatrix} 0.404 \\ 0.596 \end{pmatrix}$$

$$= \alpha \begin{pmatrix} 0.0386 \\ 0.0443 \end{pmatrix} \approx \begin{pmatrix} 0.466 \\ 0.534 \end{pmatrix}$$

$$b_0 = \alpha \begin{pmatrix} 0.8, 0.2 \\ 0.3, 0.7 \end{pmatrix} \begin{pmatrix} 0.21, & 0 \\ 0 & , 0.02 \end{pmatrix} \begin{pmatrix} 0.466 \\ 0.534 \end{pmatrix}$$

$$= \alpha \begin{pmatrix} 0.0804 \\ 0.0368 \end{pmatrix} \approx \begin{pmatrix} 0.686 \\ 0.314 \end{pmatrix}$$

Now we can finally find the smoothed values by multiplying the forward values with their backwards values.

$$\gamma_0 = \alpha \begin{pmatrix} 0.7 \\ 0.3 \end{pmatrix} \circ \begin{pmatrix} 0.686 \\ 0.314 \end{pmatrix} = \alpha \begin{pmatrix} 0.4802 \\ 0.0942 \end{pmatrix}$$

$$\approx \underline{\underline{\begin{pmatrix} 0.836 \\ 0.164 \end{pmatrix}}}$$

$$\gamma_1 = \alpha \begin{pmatrix} 0.95 \\ 0.05 \end{pmatrix} \circ \begin{pmatrix} 0.466 \\ 0.534 \end{pmatrix} = \alpha \begin{pmatrix} 0.4427 \\ 0.0267 \end{pmatrix}$$

$$\approx \underline{\underline{\begin{pmatrix} 0.943 \\ 0.057 \end{pmatrix}}}$$

$$r_2 = \alpha \begin{pmatrix} 0.79 \\ 0.21 \end{pmatrix} \cdot \begin{pmatrix} 0.404 \\ 0.596 \end{pmatrix} = \alpha \begin{pmatrix} 0.319 \\ 0.125 \end{pmatrix}$$

$$\approx \underline{\underline{\begin{pmatrix} 0.718 \\ 0.282 \end{pmatrix}}}$$

$$r_3 = \alpha \begin{pmatrix} 0.4 \\ 0.6 \end{pmatrix} \cdot \begin{pmatrix} 0.611 \\ 0.389 \end{pmatrix} = \alpha \begin{pmatrix} 0.244 \\ 0.233 \end{pmatrix}$$

$$\approx \underline{\underline{\begin{pmatrix} 0.511 \\ 0.488 \end{pmatrix}}}$$