Exercise 1

Problem 1 a)

Children	0	1	2	3	4	5
Probability	0,15	0,49	0,27	0,06	0,02	0,01

C = total nr of siblings

$$P(C \leq 2) = 0,15 + 0,49 + 0,27$$
$$= \underline{\underline{0,91}}$$

b)

$$P(C \geq 3 \mid C \geq 1)$$

$$= \frac{P(C \geq 3) \cdot \overset{\overset{1}{\shortparallel}}{P(C \geq 1 \mid C \geq 3)}}{P(C \geq 1)},$$

$$= \frac{0,06 + 0,02 + 0,01}{1 - 0.15} \simeq \underline{\underline{0,11}}$$

c) Possibilities

	A	B	C	
1.)	1	1	1	$\square$
2.)	2	1	0	0
3)	2	0	1	0
4)	3	0	0	Δ
5)	1	2	0	0
6)	0	2	1	0
7)	0	3	0	Δ
8)	1	0	2	0
9)	0	1	2	0
10)	0	0	3	Δ

$P(\square) = (0,49)^3 \approx 0,118$

$P(0) = (0,27)\cdot(0,49)(0.15) \approx 0,020$

$P(\Delta) = (0,06)\cdot(0,15)^2 \approx 0,001$

$P(3 \text{ siblings}) = \square + 6 0 + 3\Delta = \underline{\underline{0,241}}$

d) Emma Jacob

1.)	3	0	$\square$
2.)	2	1	Δ
3)	1	2	Δ
4)	0	3	$*$

$$P(\square) = (0{,}06)(0{,}15) \simeq 0{,}009$$

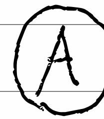
$$P(\Delta) = (0{,}27)(0{,}49) \simeq 0{,}132$$

$$P(*) = P(\square) \simeq 0{,}009$$

$$P(3\,sib) = \square + * + 2\Delta = 0{,}282$$

$$P(* \mid 3\,sib) = \frac{P(*)}{P(3\,sib)} \simeq \underline{\underline{0{,}032}}$$

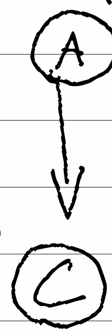
Problem 2



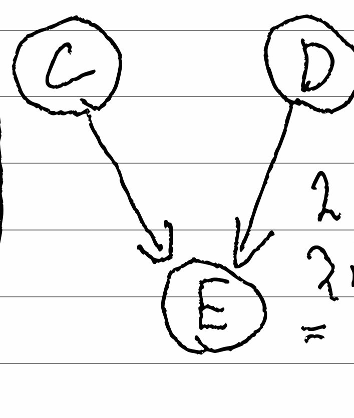
A has no parent and is a <u>bernoulli distribution</u> $\Rightarrow$ only need 1. param. (has to equal 1.0!)

A	P(C|A)
T	0
F	x

parent node + x = 2 param



C	D	P(E|C,D)
T	T	0
T	F	x
F	T	x
F	F	x



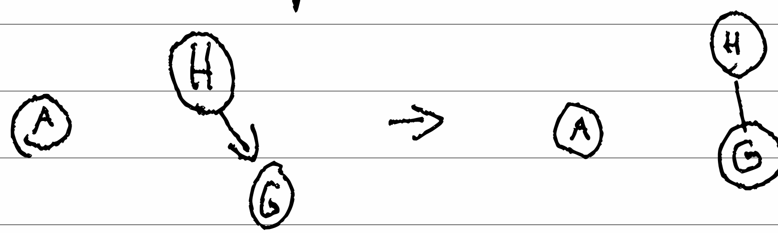
2 parent + 2 node = 4 param

$$\text{Total} = A + B + C + D + E + F + G + H$$
$$= 1 + 2 + 2 + 2 + 4 + 4 + 2 + 1$$
$$= 18 \Rightarrow \underline{\text{True}}$$

"Each variable is conditionally independent of its non-descendants given its parents."

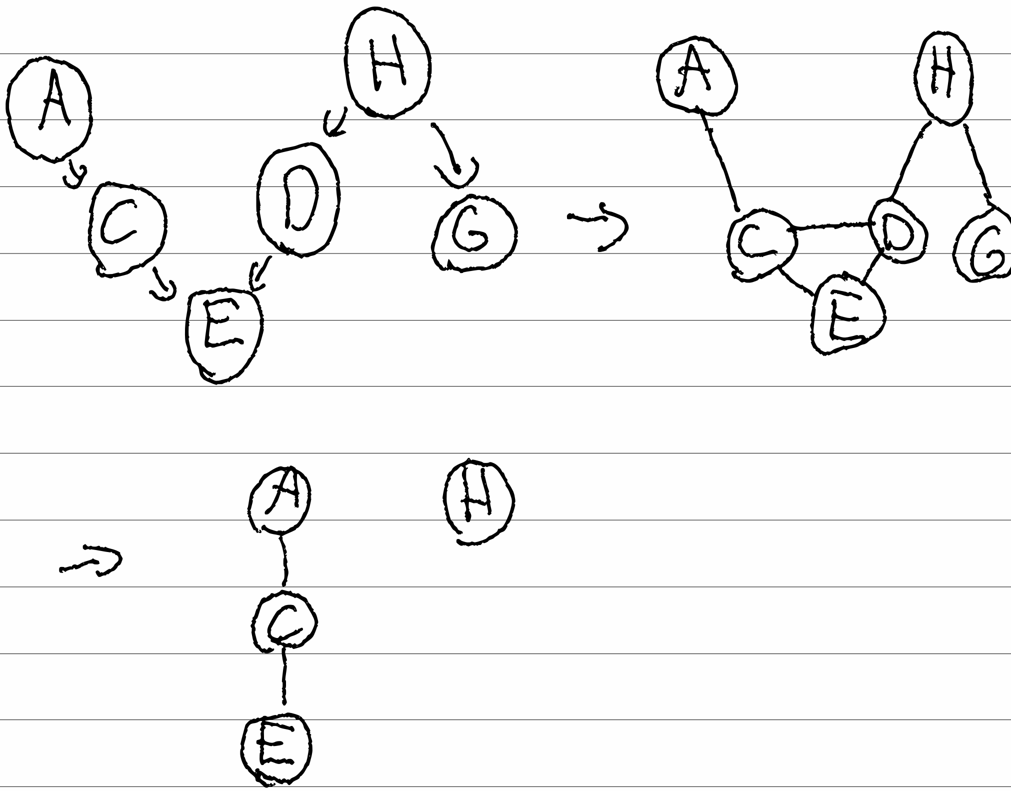
b) $G \perp\!\!\!\perp A$ using d-seperation

Ancestral Graph



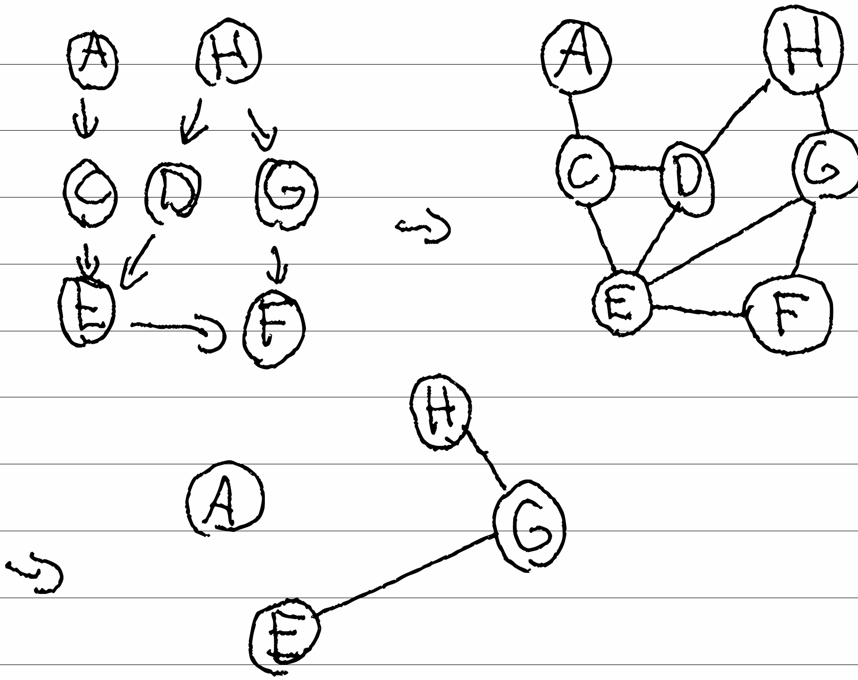
True, since G and A are not connected

c) $E \perp\!\!\!\perp H \mid \{D, G\}$



True, since E and H are not connected

d) $E \perp\!\!\!\perp H \{C, D, F\}$



False, since E and H are connected

Problem 3 a)

$$P(b) = P(b|a)\cdot P(a) + P(b|\neg a)P(\neg a)$$
$$= 0{,}5\cdot 0{,}8 + 0{,}2\cdot 0{,}2 = \underline{\underline{0{,}44}}$$

b)

$$P(\neg b) = 1 - P(b) = 0{,}56$$
$$P(d) = P(d|\neg b)\cdot P(\neg b) + P(d|b)P(b)$$
$$= 0{,}8\cdot 0{,}56 + 0{,}6\cdot 0{,}44$$
$$= \underline{\underline{0{,}712}}$$

c)
$$P(c|\neg d) = \frac{P(\neg d|c)\cdot P(c)}{P(\neg d)}$$

$$P(c) = P(c|b)\cdot P(b) + P(c|\neg b)\cdot P(\neg b)$$
$$= 0{,}1\cdot 0{,}44 + 0{,}3\cdot 0{,}56 = \underline{\underline{0{,}212}}$$

d) $\underline{P(a \mid \neg c, d)}$

$$P(a, b, c, d)$$
$$= P(a) \cdot P(b \mid a) \cdot P(d \mid a, b) \cdot P(c \mid a, b)$$
$$= P(a) \cdot P(b \mid a) \cdot P(d \mid b) \cdot P(c \mid b)$$

$$P(a \mid \neg c, d) = \sum_b \frac{P(a, b, \neg c, d)}{P(\neg c, d)}$$

$$= \alpha \cdot P(a) \sum_b P(b \mid a) \cdot P(d \mid b) \cdot P(\neg c \mid b)$$

$$= \alpha \cdot 0{,}8 (\underbrace{0{,}5 \cdot 0{,}6 \cdot 0{,}9}_{b} + \underbrace{0{,}5 \cdot 0{,}8 \cdot 0{,}7}_{\neg b})$$

$$= \alpha \cdot 0{,}44$$

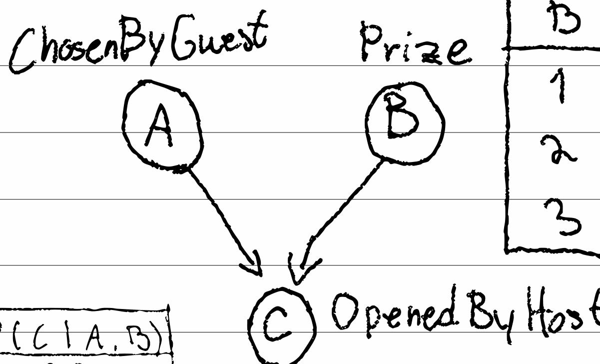
$$P(\neg c \wedge d) = (1 - 0{,}212) \cdot (0{,}712)$$
$$\approx 0{,}561$$

$$P(a \mid \neg c, d) = \frac{0{,}4}{0{,}561} = \underline{\underline{0{,}784}}$$

4 c)

$$P(\text{Prize} \mid \overbrace{\text{ChosenByGuest}}^{A} = 1, \overbrace{\text{OpenedByHost}}^{C} = 3)$$

A	P(A)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$



B	P(B)
1	$\frac{1}{3}$
2	$\frac{1}{3}$
3	$\frac{1}{3}$

A	B	C	P(C | A, B)
1	1	2	0,5
1	1	3	0,5
1	2	3	1,0
1	3	2	1,0
2	2	1	0.5
2	2	3	0.5
2	1	3	1,0
2	3	1	1,0
3	3	1	0,5
3	3	2	0,5
3	1	2	1,0
3	2	1	1,0

		C =		
A	B	1	2	3
1	1	0	0,5	0,5
1	2	0	0	1,0
1	3	0	1,0	0
2	1	0	0	1,0
2	2	0,5	0	0,5
2	3	1,0	0	0
3	1	0	1,0	0
3	2	1,0	0	0
3	3	0,5	0,5	0

C:

A	B	C	P(C | A, B)
1	1	2	0,5
1	1	3	0,5
1	2	3	1,0
1	3	2	1,0
2	2	1	1,0
2	2	3	1,0
2	1	3	1,0
2	3	1	1,0
3	3	1	0,5
3	3	2	0,5
3	1	2	1,0
3	2	1	1,0

A	B	C = 1	2	3
1	1	0	0,5	0,5
1	2	0	0	1,0
1	3	0	1,0	0
2	1	0	0	1,0
2	2	0,5	0	0,5
2	3	1,0	0	0
3	1	0	1,0	0
3	2	1,0	0	0
3	3	0,5	0,5	0