TPPE32 - Riskhantering

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1 Chapter 1: Volatility and Variance

Portfolio expected return: $\mu_p = w_1 * \mu_1 + w_2 * \mu_2$

Portfolio volatility: $\sigma_P = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2\rho w_1 w_2 \sigma_1 \sigma_2}$

Steiners sats: $V(X) = E(X^2) - E(X)^2$ Where X^2 is the outcome squared times the probability.

CAPM: $E(R) = R_F + \beta(E(R_M) - R_F)$ Where R_F is the risk free interest rate and β shows the exposure to systemic risk (market returns). $\beta = \rho\frac{\sigma}{\sigma_M}$
Lets say that the actual return on the portfolio is greater than the expected return. That means that the manager produced superior returns for the amount of systemic risk being taken. This is referred as the alpha created by the portfolio. (Excess return above expected)

2 Chapter 2: Volatility

$\sigma_{yr} = \sigma_{day}\sqrt{252}$ or $\sigma_{day} = \frac{\sigma_{yr}}{\sqrt{252}}$

EWMA: $\sigma_n^2 = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2$

GARCH: $\sigma_n^2 = \gamma V_L + \beta\sigma_{n-1}^2 + \alpha u_{n-1}^2$ Where $\gamma = (1 - \alpha - \beta)$

The expected volatility for day n+t

$$\sigma_{n+t}^2 = V_L + (\alpha + \beta)^t * (\sigma_n^2 + V_L) \qquad (1)$$

The "(1,1)" in GARCH(1,1) indicates that σ^2 n is based on the most recent observation of u2 and the most recent estimate of the variance rate. The more general GARCH(p, q) model calculates σ^2 n from the most recent p observations on u2 and the most recent q estimates of the variance rate.

3 Copulas and Correlation

Definition of correlation: $\rho = \frac{cov(V_1,V_2)}{SD(V_1)SD(V_2)}$
$cov(V_1, V_2) = E(V_1V_2) - E(V_1)E(V_2)$

4 VaR and Expected Shortfall

However, when VaR is used in an attempt to limit the risks taken by a trader, it can lead to undesirable results. Suppose that a bank tells a trader that the one-day 99VaR of the trader's portfolio must be limited to \$10 million. The trader can construct a portfolio where there is a 99.1% chance that the daily loss is less than \$10 million and a 0.9% chance that it is \$500 million. The trader is satisfying the risk limits imposed by the bank but is clearly taking unacceptable risks.

Whereas VaR asks the question: "How bad can things get?" ES asks: "If things do get bad, what is the expected loss?"

$$ES = \mu + \sigma \frac{e^{\frac{-\mathcal{N}(c)^2}{2}}}{\sqrt{2\pi} * (1-c)} \tag{2}$$

$$VaR = \mathcal{N}^{-}1(c) * \sigma * (\sqrt{T}) \tag{3}$$

5 Backtesting

$$Z = \frac{X_p - Tp}{\sqrt{(T_p(1-p))}}, \text{ where } p = 1 - c \tag{4}$$

Two sided test statistic of say $\alpha = 0.10$ gives us using $\mathcal{N}^{-1}(1-0.95)$ and $\mathcal{N}^{-1}(1-0.05)$

6 Extreme Value Theory

$$VaR_c = u + \frac{\beta}{\xi}((\frac{n}{n_u}(1-c))^{-\xi} - 1) \tag{5}$$