MF772 Project Report

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Abstract

In our project, we study the comparative analysis of Merton, KMV Merton, and CreditGrades models to assess credit risk and determine implied credit risk premiums within the Bonds and CDS markets. By harnessing historical financial data and applying structural models, we aim to validate the models' effectiveness and explore arbitrage opportunities. We also evaluate the models' performance in predicting market spreads, employ the Dickey-Fuller test for stationarity in pricing errors, and discuss the implementation of trading strategies based on model outcomes.

1. Introduction

In recent years, the dynamics of the credit market have increasingly captured the interest of financial analysts and investors, particularly in the context of Bonds and Credit Default Swaps (CDS). Motivated by the need for deeper insights into market behaviors and the search for profitable trading strategies, our analysis is set to focus on these financial instruments. Our approach involves using models like Merton, KMV, and Creditgrades, which establish links between the equity market and the implied probability of default. Our goal is to determine the implied credit risk premiums, which can be interpreted as the implied spreads for Bonds or CDS. The main objective of our study is to conduct an in-depth analysis and discussion of the results, focusing on aspects such as correlation and stationarity. This will involve verifying the effectiveness of our chosen models and conducting comparative assessments across different models. Additionally, we plan to uncover pricing anomalies between the implied CDS spreads we calculated and the market CDS spread. Utilizing these findings, we aim to develop trading strategies that capitalize on these arbitrage opportunities as revealed by our models.

2. Data

In this project, we utilized Bloomberg for historical financial analysis data and historical option implied volatility data, providing a comprehensive view of risk assessments and components that would be used in our credit risk models. Data Stream is the platform allowing us to obtain historical CDS spread data, crucial for understanding market dynamics over time. Refinitiv is the database source and we filtered the database with our focus on the U.S. market, concentrating on senior unsecured, non-restructured CDS. (The initial CDS universe is around 224 after the first filtration, each with 5 different maturities). Later we filtered out unavailable or abnormal data) This selective approach ensured that our analysis was tailored to a specific, relevant subset of the financial market, enhancing the accuracy and relevance of our findings.

3. Models

We tend to use the Merton Model, KMV Merton Model, and Credit Grades Model to identify and exploit arbitrage opportunities with Credit Default Swaps (CDS) and compare the results. Using structural models, we shall obtain more insight into the relationship between companies' equity value and credit risks hence assisting us to create trading strategies that enhance risk-adjusted returns.

Merton Model

The Merton Model is a structural credit risk model that uses a firm's equity value and volatility to estimate the probability of default. It's based on the Black-Scholes option pricing theory, where a company's equity is viewed as a call option on its assets. Figure 1 shows the key formulas for the Merton model we used in our project: Et represents the equity value, which is modeled as a call option on the firm's assets, At, with a strike price equal to the debt's face value, L. The d1 and d2 terms are functions of asset value, asset volatility sigma A, risk-free rate r, and time to maturity T. The 'distance to default' (DD) is a measure derived from these variables, reflecting the number of standard deviations the firm's assets are above the default point. The default probability (DP) is then inferred from the cumulative distribution function of the standard normal distribution, N(DD).

$$E_T = A_T N(d_1) - L_T e^{-rT} N(d_2)$$

$$d_1 = \frac{1}{\sigma_A \sqrt{T}} (ln \frac{A_T}{E_T} + \left(r + \frac{\sigma^2{}_A}{2}\right) T)$$

$$d_2 = d_1 - \sigma_A \sqrt{T}$$

$$\sigma_A = \sigma_e N(d_1) \frac{E}{A}$$

$$DD = \frac{\log\left(\frac{A_T}{L}\right) + \left(r + \frac{1}{2}\sigma^2 T\right)}{\sigma_A \sqrt{T}}$$

$$\mathrm{Dflt}_i(0, T) = 1 - N(DD)$$

Figure 1

KMV Merton Model

The KMV Merton Model, is an extension of the original Merton Model. The Default Point is set as a firm-specific combination of Long term and Short term debt. It relies more heavily on market data to estimate a firm's asset value and asset volatility.

Credit Grades Model

The Credit Grades Model is also a structural model. It combines market information like stock prices and debt levels with asset volatility to estimate the risk of default. This model is often used for pricing and managing credit derivatives and structured credit products. In Figure 2, the recovery rate L upon default is considered random, rather than fixed. The expected recovery rate is denoted as L bar, which is the mean of the probability distribution of L. The variability of the recovery rate is captured by lambda's square , which is the variance of the natural logarithm of L. These parameters are critical in the model as they influence the pricing of credit derivatives, reflecting the uncertainty and risk associated with the recovery rate post-default.

$$\bar{L} = E(L);\ L = \text{ the (random) recovery rate given default}$$

$$\lambda^2 = var(\ln(L))$$

Figure 2

The survival probability formula within our project quantifies the likelihood that a firm will continue to operate without defaulting over a given period. It is computed using a combination of the firm's debt structure and market data. In Figure 3, the parameter d is a ratio indicating the firm's debt per share over the equity value per share, exponentialized by the equity volatility. D represents the total debt per share, considering both short and long-term liabilities. The asset variance At^2 is an aggregate of equity volatility and the variance of the recovery rate, reflecting the underlying asset risk. The formula captures the dynamic nature of the firm's credit state through market variables, determining the probability of default and thus the firm's creditworthiness over time.

$$q(t) = \Phi\left(\frac{-A_t}{2} + \frac{\ln(d)}{A_t}\right) - d * \Phi\left(-\frac{A_t}{2} - \frac{\ln(d)}{A_t}\right),$$

Where:

Φ: *cumulative normal distribution*

$$\mathrm{d} = \frac{V_0}{\bar{L}D} e^{\lambda^2}$$

D = debt per share

$$= \frac{\text{(short term debt + long term debt) + 0.5(short term libaility+long term liabiltiy)}}{\text{share outstanding}}$$

$${A_t}^2 = \sigma^2 t + \lambda^2$$

${A_t}^2$ = variance of asset

V = value per share = $S + \bar{L}D$; uses a linear approximation of V

$\sigma = \sigma_S \frac{S}{S + \bar{L}D}$; σ_S = equity volatility

Figure 3

CDS

Since the probability of default is not directly observable, the model's robustness is tested by applying the calculated default probabilities to the CDS pricing equation. This process yields a theoretical CDS spread. By comparing these model-derived spreads with actual market spreads, we gain actionable insights into the model's predictive accuracy. This comparative analysis not only serves as a validation tool for our model but also sets the stage for its practical application in subsequent phases of our project.

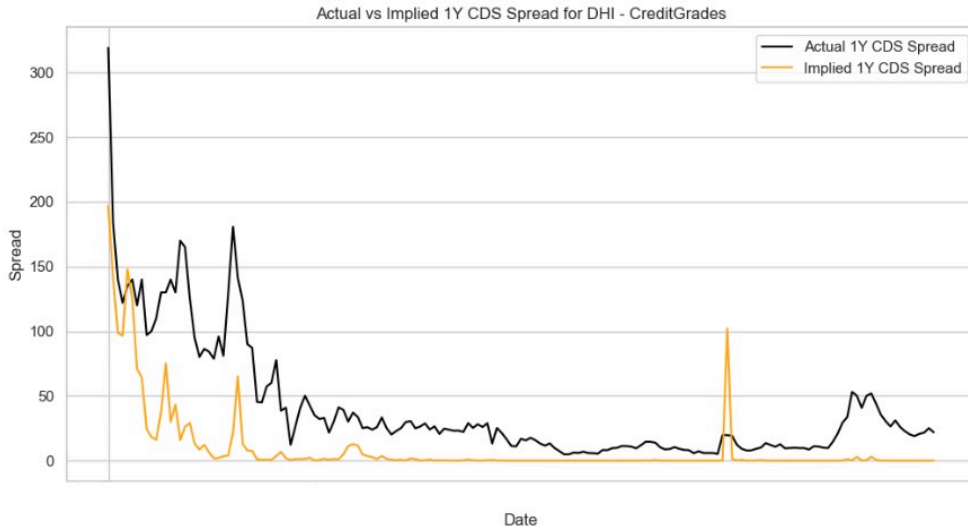

Figure 4

Dicky-Fuller Unit Root Test

The Dickey-Fuller Unit Root Test is used to determine if a time series is stationary. Specifically, it tests the null hypothesis that a unit root is present in an autoregressive model. The CRV (credit spread valuation) is the difference between the actual and theoretical credit spreads for a given financial instrument at time t. The equation in Figure 5 describes the autoregressive process of order 1. If delta < 0, the CRV is mean reverting, which is a characteristic of stationary time series. In this context, k represents the number of months. The test aims to find if the delta is significantly different from zero, indicating that the series does revert to its mean over time.

$$CRV_{i,t+k} - CRV_{i,t+k-1} = \alpha_i + \delta_i CRV_{i,t} + \mu_{i,t}$$

Figure 5

Ticker DHI CreditGrades Model

Maturity	Pearson Cor.	Spearman Cor.	P-value of Beta	Beta of Implied Spread	Dicky Fuller test p-value
0.5Y	0.655	0.862	0.008164	2.936	0.000
1Y	0.609	0.890	0.000468	1.590	0.000
2Y	0.627	0.854	0.007935	0.820	0.001
3Y	0.627	0.812	0.006419	0.765	0.158
5Y	0.604	0.734	0.008897	0.565	0.288
10Y	0.547	0.633	0.004169	0.289	0.220

Figure 6

Figure 6 shows the p value for the 6-month to 10-year maturity. According to the chart, the p-values of 0.5 year, 1 year and 2 years are less than 0.05. It suggests that the null hypothesis can be rejected, indicating that the time series is stationary, which agrees with the mean-reverting series. For 3 years, 5 years, and 10 years, the p values are higher than 0.05. It suggests that the null hypothesis will not be rejected, which means that the time series is not stationary.

4. Model Results

From the Figure 7 below we can look into the results generated from all the three models. As for the Pearson correlation, we took the median values of the fifty firms' data. We can see that with the CreditGrades model, we obtained comparably larger numbers than the other two models. In this way, the CreditGrades model performs much better than the other two models.

Also, figure 8 shows the corresponding beta values and median p-values. We can observe that the CreditGrades model generates a much higher beta and lower p-value than the other two models.

To sum up, the CreditGrades model performs better than the Merton model and KMV Merton model in our study.

Maturity	Merton	KMV	Credit Grades
0.5Y	0.024946	0.017080	0.199914
1Y	0.038185	0.047944	0.205441
2Y	0.102003	0.073864	0.289732
3Y	0.146755	0.103504	0.341095
5Y	0.209076	0.162106	0.386167
10Y	0.161238	0.126677	0.296992

Figure 7

Maturity	Merton		KMV		Credit Grades	
	Corresponding Beta	Median of p-value	Corresponding Beta	Median of p-value	Corresponding Beta	Median of p-value
0.5Y	0.003907	0.162963	0.000944	0.552280	0.417179	0.013814
1Y	0.032854	0.114618	0.041196	0.377689	0.389649	0.015655
2Y	0.051865	0.088171	0.117032	0.239121	0.247322	0.000796
3Y	0.067846	0.072224	0.160285	0.162288	0.213461	0.000063
5Y	0.079690	0.016294	0.127202	0.046073	0.140876	0.000005
10Y	0.031642	0.001476	0.051147	0.023905	0.062289	0.000586

Figure 8

5. Trading Strategy

For trading strategy, we assume as arbitrageurs, have a time series of observed CDS market spreads, ct = c(t, t+T), also available is a time series of observed equity prices, St. And this allows us to calculate a theoretical CDS spread based on a structural model. This implied CDS is denoted ct' (prime) and the difference between the two-time series is denoted et = ct - ct'. Specifically, suppose we examine the behavior of et and find that it has a mean of Expectation of e and a standard deviation of sigma e, As mentioned, we should not expect the pricing error to be unbiased, but suppose a point comes at which the deviation is unusually large. In this case, implied volatility sigma (imp) may be too high and will decline to a lower level, sigma. The correct strategy in that case is to sell CDS and sell equity as a hedge because they are mean reverting which is akin to selling overpriced stock options and use delta hedging to neutralize the effect of a changing stock price. And when et < E(e) - 2*sigma (e) we long CDS and long equity as a hedge. And for each kind of trading strategy, we traded the same amount of value.

$$\pi(t,T) = \left[c(t,T) - c(0,T)\right] \int_t^T P(t,s)\, q_t(s)\, ds, \qquad \delta(t,T) = \frac{\partial \pi(t,T)}{\partial S_t},$$

Figure 9

To summarize, the risk involved in capital structure arbitrage can be understood in terms of the subsequent movements of the CDS spread and the equity price. For example, after we have sold credit protection and shorted equity, four scenarios can happen:

When ct is decreasing and St is decreasing, both CDS and equity price decline, this is the case of convergence, which allows us to profit from both positions.

When ct is decreasing and St is increasing, we lose on the equity but profit from the CDS. We will profit overall if the CDS spread falls more rapidly than the equity price rises, allowing partial convergence.

When ct is increasing and St is decreasing, we lose on the CDS bet, but the equity position acts as a hedge against this loss. We will achieve an overall profit if the equity price falls more rapidly than the CDS spread rises.

When ct is increasing and St is increasing, this is a sure case of divergence. We suffer losses from both positions regardless of the size of the equity hedge.

Below are the results of the trading strategy. The Figure 10 shows the cumulative PNL using the Merton model. It is clear to see that the strategy based on shorter maturities outperforms the strategy based on longer maturities. Figure 11, it also shows that the same pattern holds for the KMV model as well. But if we look carefully at the PNL, there are a lot of big jumps during the same period using the KMV model. There are some periods where portfolio value did not change at all because there is no signal detected by our trading strategy for those periods. For the CreditGrades model (Figure 12), the strategy is making money as well. However, the extent of the fluctuation is much less than in the KMV model.

In conclusion, the same strategy based on 3 different models is making money. Because the trading strategy is based on convergence, if we assume the convergence phenomenon holds, these increased portfolios could give some insights into the accuracy of those 3 models.

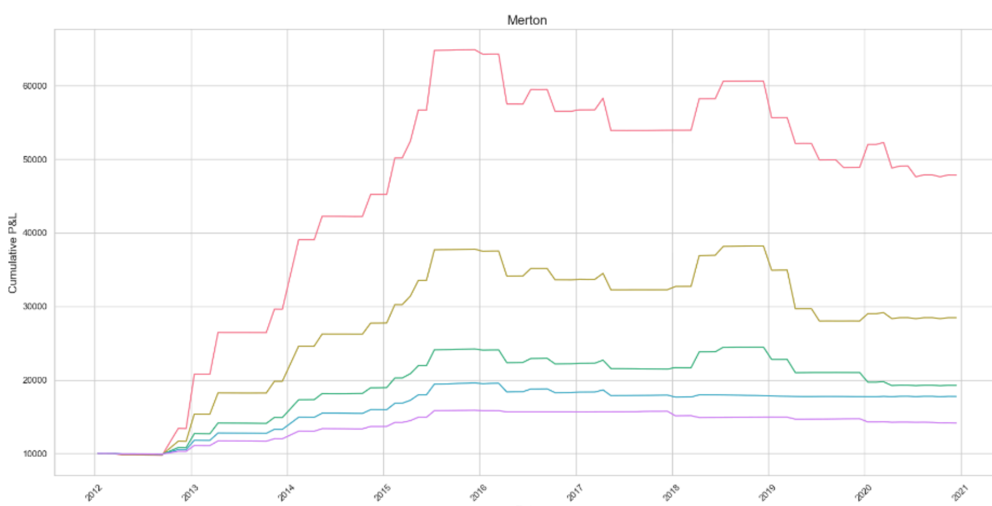

Figure 10

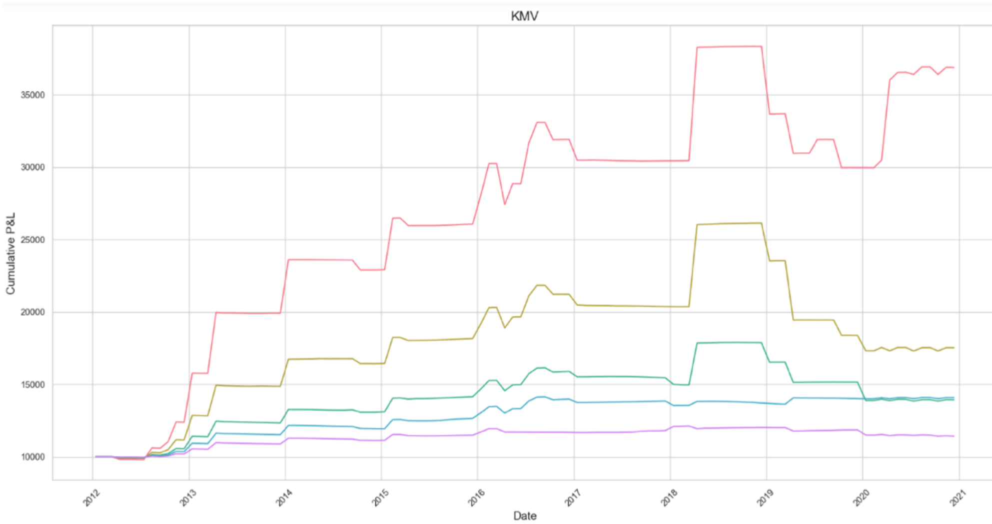

Figure 11

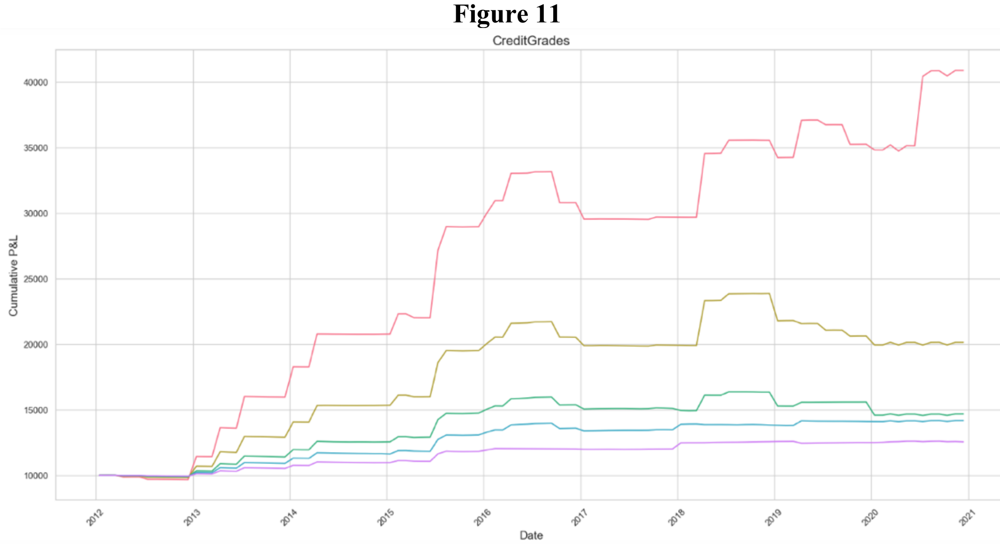

Figure 12

Secondly, Figure 13 is the risk evaluation metrics for the 6-month and 3-year maturity strategy. For 6 months' results, the creditgrades model performed the best, with the largest Sharpe ratio and least max drawdown. But for the 3-year risk metric. The Merton model performs the best, which it might be explained by the simplicity of the Merton model's structure, as it is not that sensitive to the noise in the market, especially for longer matured CDS.

0.5Y maturity	Merton	KMV	Credit grades
Annual return	16.75	13.68	14.18
Annual volatility	23.17	18.74	15.68
Max drawdown	-26.62	-21.86	-10.97
Annual sharpe ratio	59.35	56.98	71.30

3Y maturity	Merton	KMV	Credit grades
Annual return	5.55	3.27	3.31
Annual volatility	6.99	4.50	3.99
Max drawdown	-9.80	-4.21	-4.16
Annual sharpe ratio	36.52	6.00	7.88

Figure 13

6. Discussion

There are certainly potential improvements in our projects. In terms of data, Datastream certainly has its limitations and we were not able to work with the most ideal CDS sets since some of the ticker information is not available on Bloomberg, causing obstacles for us to obtain valid financial analysis. We have to manually filter out CDS tickers given the limits of information available. Meanwhile, the time horizon does not necessarily match between CDS pricing date and the financial data date. We would have to choose the closest date. Improvements in our data accuracy implemented in the models will be helpful during the process of calculating our default probabilities. Also, we were not able to retrieve the data of default information of CDS/companies so we used the debt barrier to mimic the default signal. This process is not ideal and surely there could be other efficient methods to more accurately approximate the default.

For models, the Merton model and KMV Merton model heavily relied on the accuracy of the data source, where inaccuracies would be aggregated in the iteration process, leading to unrealistically high or low default probability. Thus, there are still a lot of improvements. For example, we can involve time-varying risk premia into the model and have more sophisticated ways of implied spread calculator.

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