Lemma 0.1. Let $\phi : A \to B$, and suppose that M is a B-module, and that $b_1, b_2, \ldots, b_r$ is an M-sequence. Then $\phi^{-1}(b_1), \phi^{-1}(b_2), \ldots, \phi^{-1}(b_r)$ is also an M-sequence.

Proof. □

Lemma 0.2. Suppose that A is a Cohen-Macaulay ring. Then $A[x]$ is Cohen-Macaulay as well.

Proof. Let $i : A \to A[x]$ be the injection, and $\mathfrak{m}$ be a maximal ideal of $A[x]$. Then let $\mathfrak{m}' = i^{-1}(\mathfrak{m})$. Then $\mathfrak{m}'$ is maximal in A as the kernel of the composition

$$A \xrightarrow{i} A[x] \xrightarrow{\pi} A[x]/\mathfrak{m}$$

is given by $\mathfrak{m}'$, whence $A/\mathfrak{m}' \cong A[x]/\mathfrak{m}$ are both fields.

As $\dim(A[x]) = \dim(A) + 1$, and the length of any regular sequence is bounded by the Krull dimension, it will suffice that there exist a regular sequence in $A_{\mathfrak{m}'}$ of length $\dim(A_{\mathfrak{m}'}) = \dim(A[x]_{\mathfrak{m}}) - 1$.

Let $a_1, a_2, \ldots, a_r$ be a maximal regular sequence in $A[x]_{\mathfrak{m}}$

□