The following question concerns the proof of Proposition 4.28. I have trouble following the argument "as $X : U \to X(U)$ is a diffeomorphism, it follows that every differentiable curve $\gamma : I \to X(U)$ with $\gamma(0) = p$ can be obtained this way". I interpret this as follows:

1. Let $\gamma : I \to U$ be a differentiable curve on U.
2. Then as $X : U \to X(U)$ is a diffeomorphism, so is $X^{-1} : X(U) \to U$, hence $\alpha = X^{-1} \circ \gamma$ is a differentiable curve on U such that $\alpha(0) = X^{-1}(\gamma(0)) = X^{-1}(p) = 0$.
3. Then $\gamma = X \circ \alpha$, and every curve arises this way.

I don't follow step (2). I agree that X^{-1} is differentiable as a map between regular surfaces, but this is not enough to guarantee that α is smooth as a map into $\mathbb{R}^3$? Or am I mistaken?

For example, let $M = \mathbb{R}^2 \times \{0\} \subset \mathbb{R}^3$, and let $X : \mathbb{R}^2 \to M$ be a local parameterization at $(0, 0, 0)$ given by $X : (x, y) \mapsto (x^3, y^3, 0)$. Then the corresponding local chart $X^{-1} : (x, y, z) \mapsto (x^{1/3}, y^{1/3})$ is not differentiable at $(0, 0, 0)$ as a map $\mathbb{R}^3 \to \mathbb{R}^2$. In particular, the differentiable curve $\gamma : \mathbb{R} \to M$ given by $\gamma : x \mapsto (x, 0, 0)$ can't be decomposed as a differentiable curve into the local coordinates postcomposed with X, since $X^{-1} \circ \gamma : x \mapsto (x^{1/3}, 0, 0)$ isn't a differentiable curve.

I tried to read Pressley instead to understand better, but they just make the same claim (Equation (4.2), Section 4.4), and say that it will be proved in Section 5.6, but then I can't find a proof in Section 5.6.

I've hade some troubles with differentiablity in general, and am wondering if I'm missunderstanding something fundamental here?