

# Assignment 3, TTK4190

Shiv Jeet Rai  
Arne Selle  
Erik Liland

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# 1 Autopilot design

## 1.1 Heading autopilot

### Analysis of ship characteristics

To be able to model the ship in a good way we ran many simulations with different rudder angle and measured the steady-state yaw rate. We then made a  $\delta - r$  plot of the result. Since the ship was turning port while giving a positive rudder command, this plot the rest of the assignment is made with a fixed gain of -1 on  $\delta_c$ .

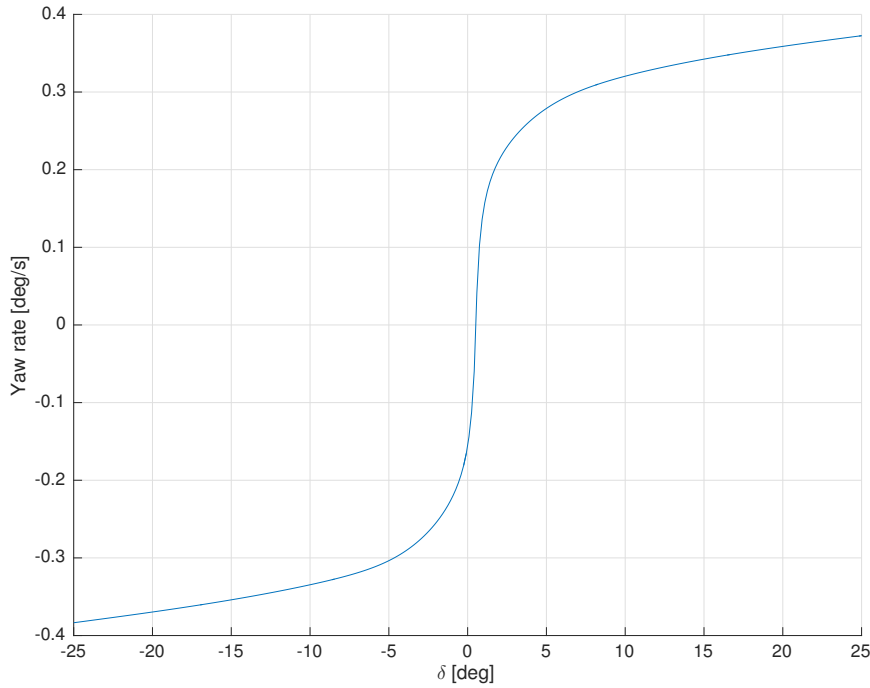


Figure 1:  $\delta - r$  plot

From figure 1 we clearly see the non-linear behavior of the ship. This motivates a 1-DOF heading model i.e. first- or second order Monoto model with non-linear extensions. To further investigate the effect of the non-linear characteristics of this ship, we compare the actual response with different models at different rudder angles. It should also be noticed that the ship has a constant drift to starboard with  $\delta_c = 0$ , as seen by the curve not passing through the origin. We compensate for this through the rest of the modeling part by adding a fixed rudder angle of  $0.52^\circ$  to the rudder input. We only need this correction while estimating the model parameters. In a closed loop, the integral effect will cancel both this drift and drift caused by wind, current and waves.

## 2. order linear Nomoto

$$\frac{r}{\delta}(s) = \frac{K_\nu(1 + T_3s)}{(1 + T_1s)(1 + T_2s)} \quad (1)$$

$$T_1T_2\ddot{\psi} + (T_1 + T_2)\dot{\psi} + \psi = K(\delta + T_3\dot{\delta})$$

The second order Nomoto model follows the ships overshoot some, but not enough.

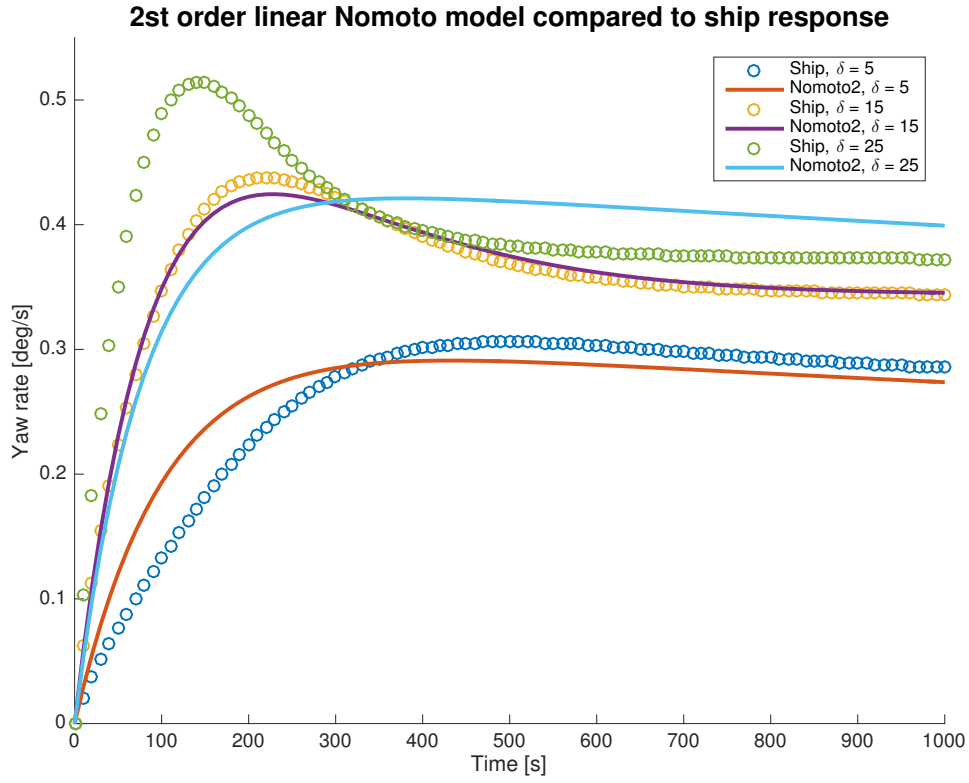


Figure 2: 2.order linear Nomoto model

### 1. order linear Nomoto

$$\frac{r}{\delta}(s) = \frac{K}{(1 + Ts)} \quad (2)$$

$$T\ddot{\psi} + \dot{\psi} = K\delta$$

We also tried the first order version Nomoto, and as expected the model will only be accurate for small rudder angles, and is therefore not very good for modeling the non-linearities.

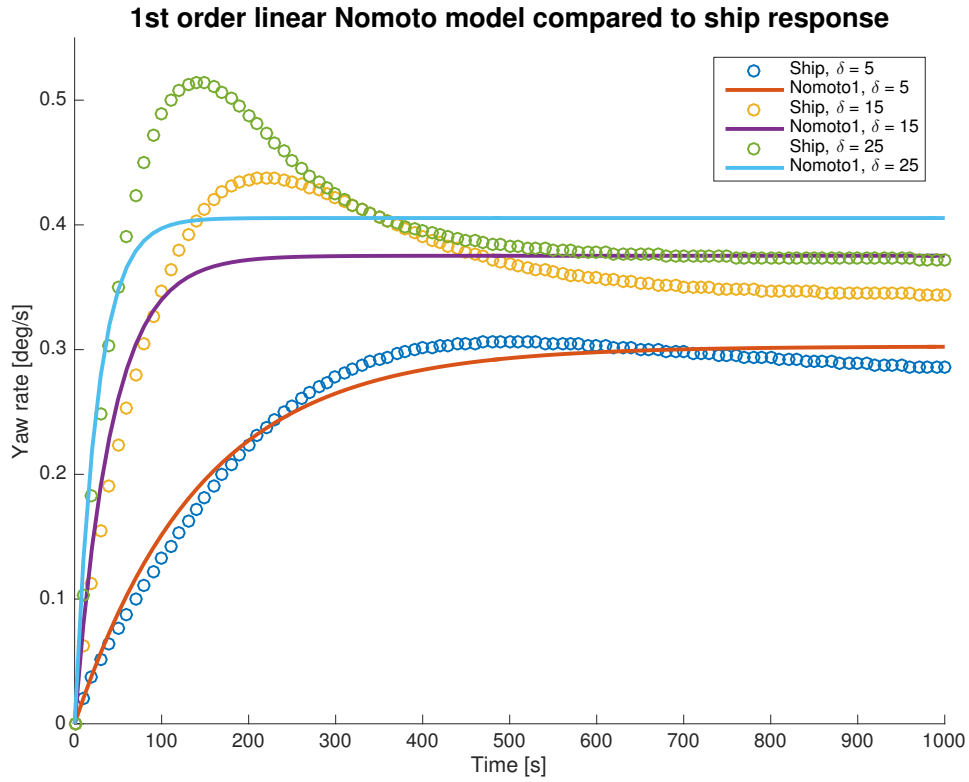


Figure 3: 1.order linear Nomoto model

## 2. order non-linear Nomoto

$$T_1 T_2 \ddot{r} + (T_1 + T_2) \dot{r} + K H_b(r) = K(\delta + T_3 \dot{\delta}) \quad (3)$$

$$H_B(r) = b_3 r^3 + b_2 r^2 + b_1 r + b_0$$

Where the steady state of  $H_B(r) = \delta$ .  $b_0$  have already been taken care of in the fixed rudder offset, and by symmetry in the hull leads to  $b_2 = 0$ . We then only need the first- and third-order term to describe the maneuvering characteristics. By curve fitting  $H_B(r) = b_3 r^3 + b_1 r = \delta$  to the obtained delta-r curve, we estimate the parameters  $b_3$  and  $b_1$ .

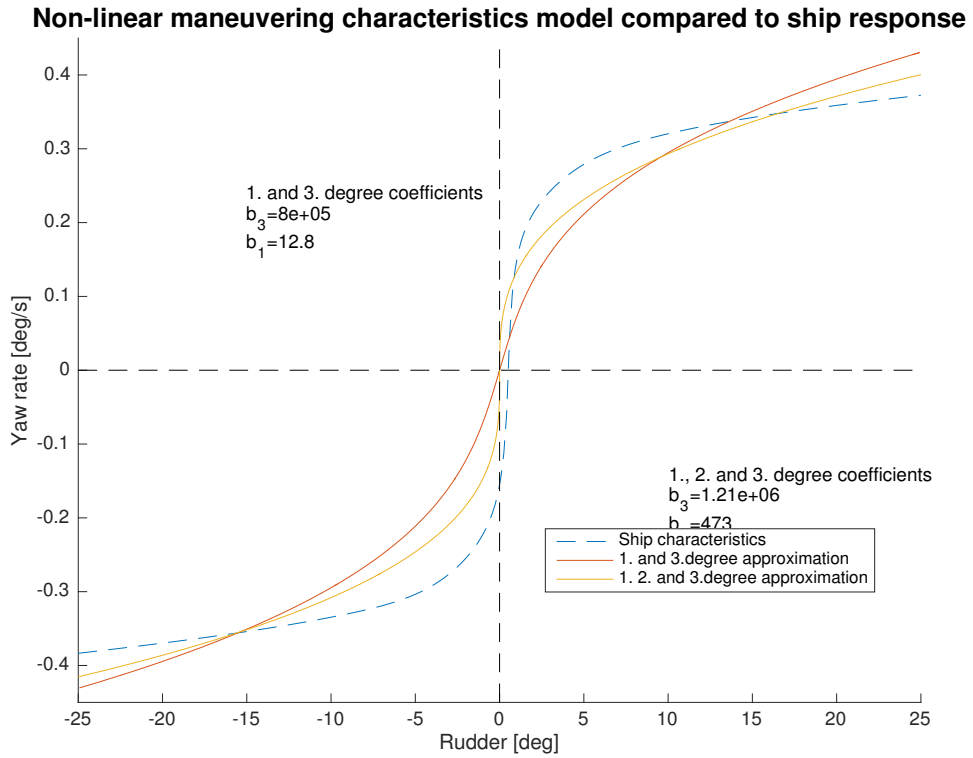


Figure 4: Third degree approximation of ship characteristics

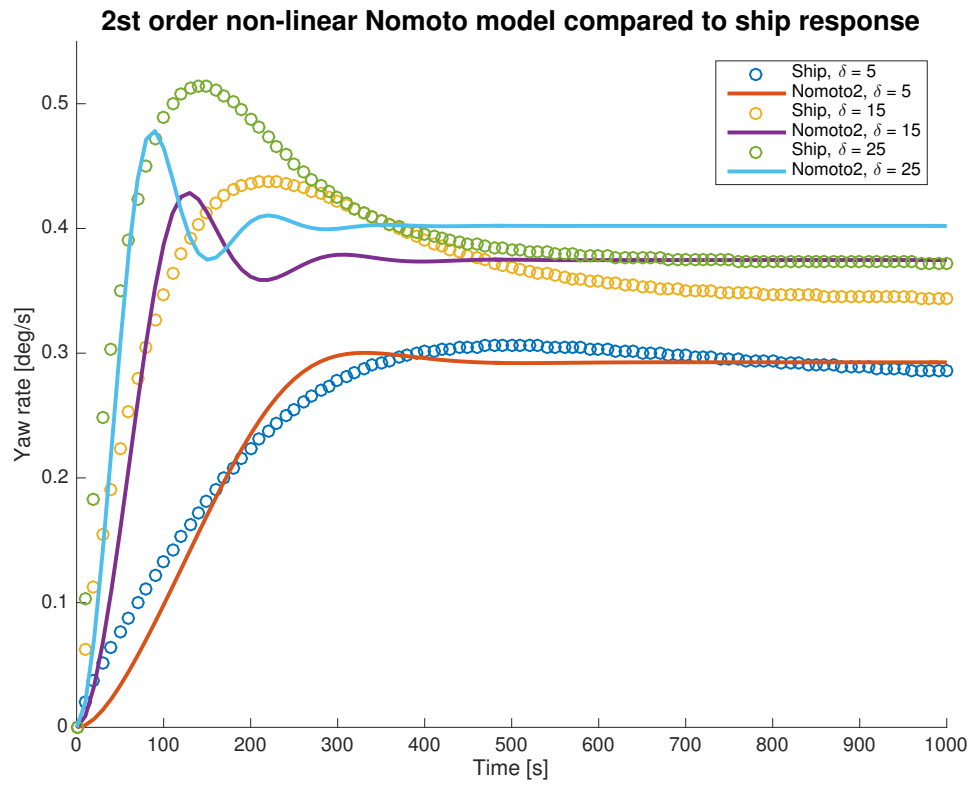


Figure 5: 2.order non-linear Nomoto model

### 1. order non-linear Nomoto

Norbins' extension of the linear first order model:

$$\begin{aligned} T\dot{r} + H_N(r) &= K\delta \\ H_N(r) &= n_3r^3 + n_2r^2 + n_1r + n_0 \end{aligned} \quad (4)$$

Where the steady state of  $H_N(r) = K\delta$ . We know that  $n_i = \frac{b_i}{|b_1|}$ , and since our ship is stable we know that  $n_1 = 1$ , and  $n_3 = \text{sign}(b_3) = 1$  thus resulting in following model.

$$T\dot{r} + r^3 + r = K\delta \quad (5)$$

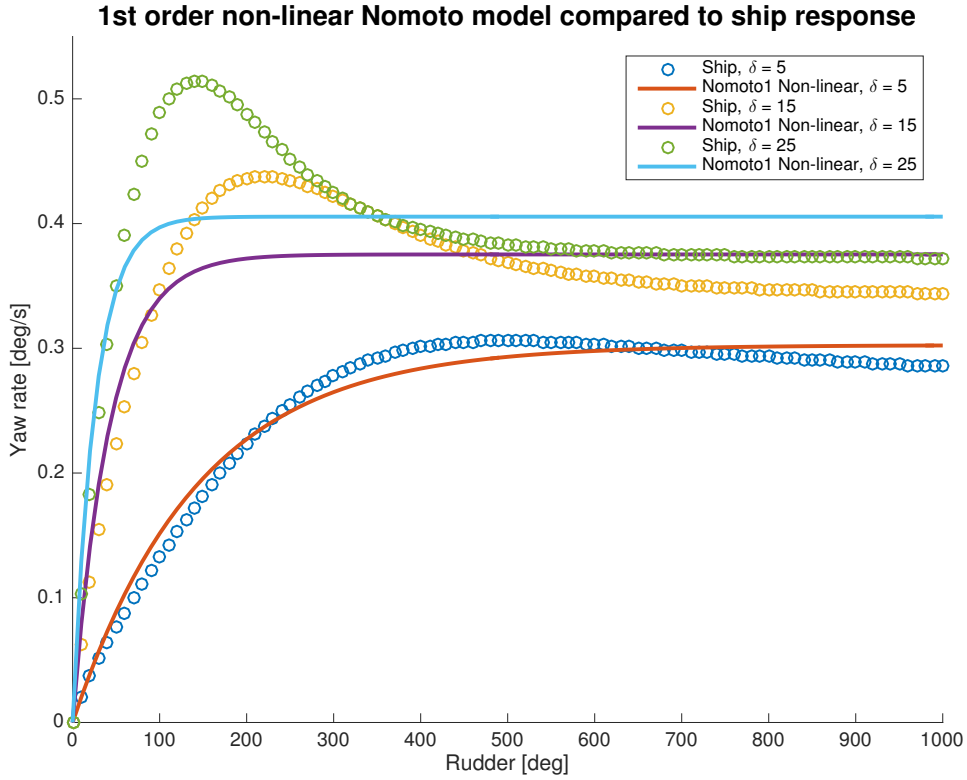


Figure 6: 1.order non-linear Nomoto model

## 1.2 Speed autopilot

To control the surge speed of MS Fartøystyring we suggest using a linearized model, where the surge speed is decoupled from the rest of the system. We are assuming

$$u \gg v$$

which leads to

$$U = u$$

. We then use a quadratic speed model

$$K_1 \dot{u} - X_u u_r - X_{|u|u} |u_r| u_r = \tau \quad (6)$$

which leads to

$$\dot{u} = \frac{\tau + X_{|u|u} |u_r| u_r + X_u u_r}{m - X_{\dot{u}}} = \frac{X_{|u|u} |u_r| u_r + X_u u_r}{m - X_{\dot{u}}} + \tau_{nl} \quad (7)$$

where

$$\tau_{nl} = \frac{\tau}{m - X_{\dot{u}}} \Rightarrow \tau = \tau_{nl} (m - X_{\dot{u}}) \quad (8)$$

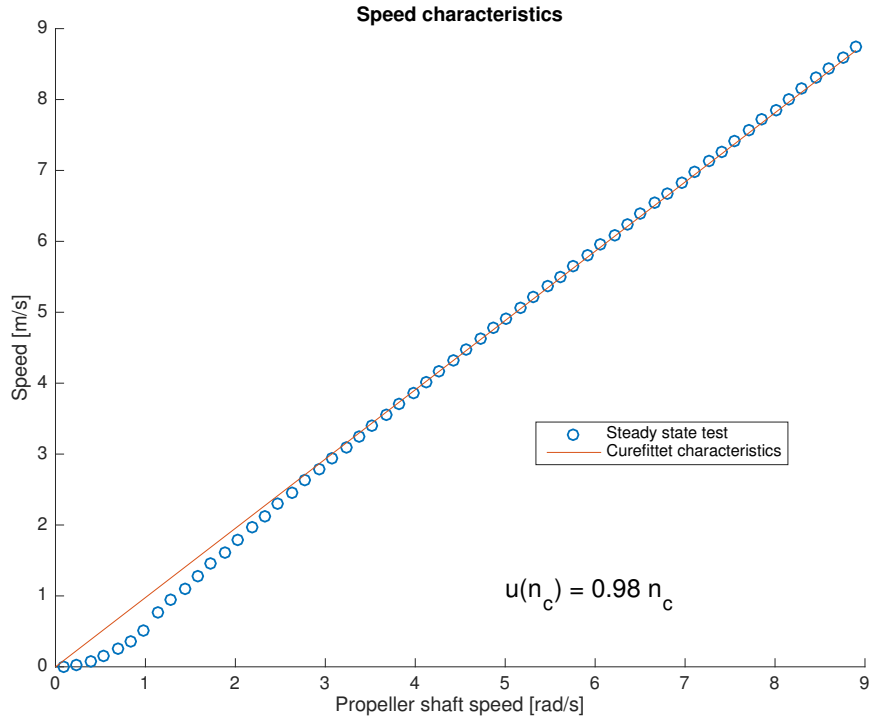


Figure 7: Speed characteristics



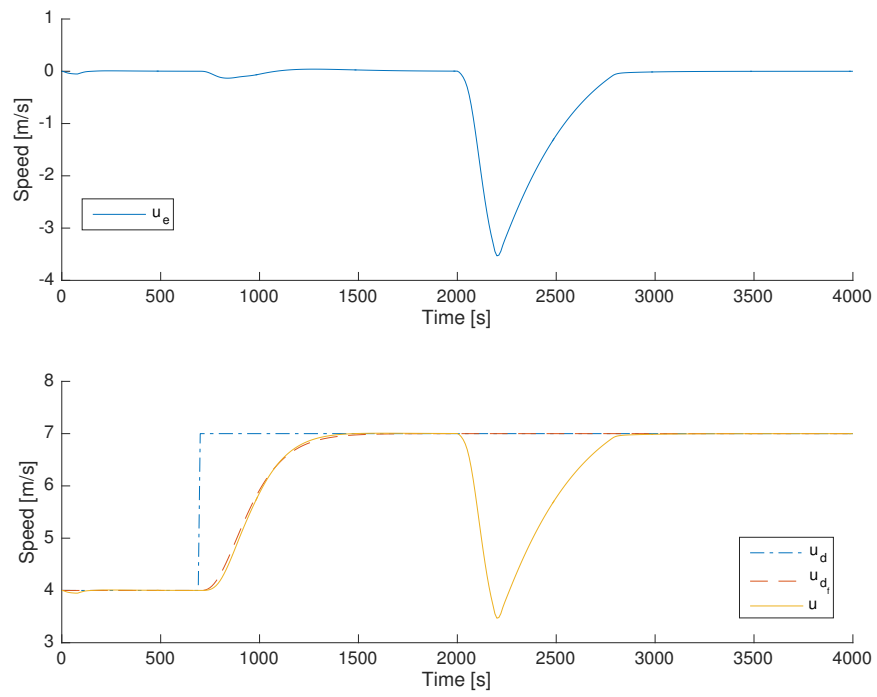


Figure 8: Speed step response

## 2 Path following and Path tracking

### 2.1 Path Generation

We can see from figure 9 that both methods based on continuous interpolation ("Piecewise Cubic Hermite Interpolating Polynomial" and "Cubic spline data interpolation") are very smooth, but they are more off-track than on-track. The piece-wise continuous interpolation are the crudest method of them all, since it in no way takes in to consideration the dynamics of the ship. The combination of circles and straight lines may look like the obvious best solution, but it does have a step in yaw rate ( $r$ ), which means that the ship will not be able to follow the circle exactly while turning. Beside that, the lines and circles makes an excellent path for a ship to follow since it reduces the amount of time the ships actively uses its rudder, and therefore minimizes the drag on the ship. The turning radius set in the last method would be selected equal or larger to the ships turning radius. Preferably set it large enough to not loose to much speed, and small enough to not collide with someone/something. The nice about these turning radius, is that they can be adapted to the situation.

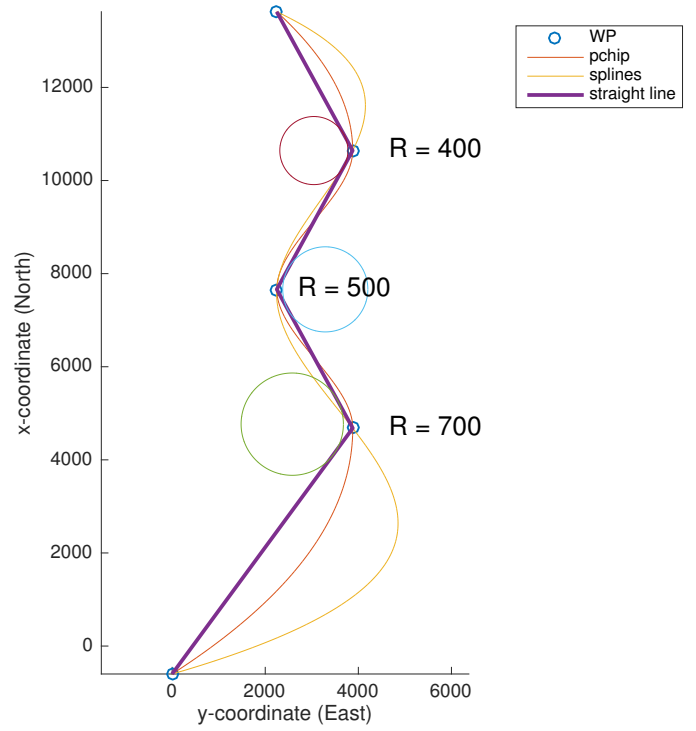


Figure 9: Different trajectories

## 2.2 Path following

We implemented a lookahead-based steering law, based on figure (10.10) in Fossen (2011). The desired course  $\chi_d$  is made up of two parts, a path-angle  $\chi_p$  and a cross-track error correction  $\chi_r$ . The cross-track error correction is essentially a PI-controller, normalized with an inverse tangent. The proportional gain is the inverse of the lookahead distance  $\Delta_s$  which is a design parameter.

$$K_p = \frac{1}{\Delta_s} \quad (9)$$

The integral effect of the controller is quite complex, since we only want the integrator to compensate for small slow-changing disturbances like wind and current. To achieve this, we made an integral structure that takes into consideration the yaw rate of the ship  $r$ , the cross-track error  $e$ , a soft maximum integral effect and an anti-wind-up corrector and a regular gain on the end result. The total control law is listed in equation 10.

$$\chi_d = \chi_p + \chi_r$$

$$\chi_r = \text{atan}(P - I)$$

$$P = K_p e$$

$$I = \int K_i e_i$$

$$e_i = \begin{cases} 0, & \text{if } |r| > r_{lim} \text{ or } |e| > e_{lim} \\ \frac{e}{k_1 + k_2 \frac{|I|}{I_{max}}}, & \text{if } (I > I_{max} \text{ and } e > 0) \text{ or } (I < -I_{max} \text{ and } e < 0) \end{cases} \quad (10)$$

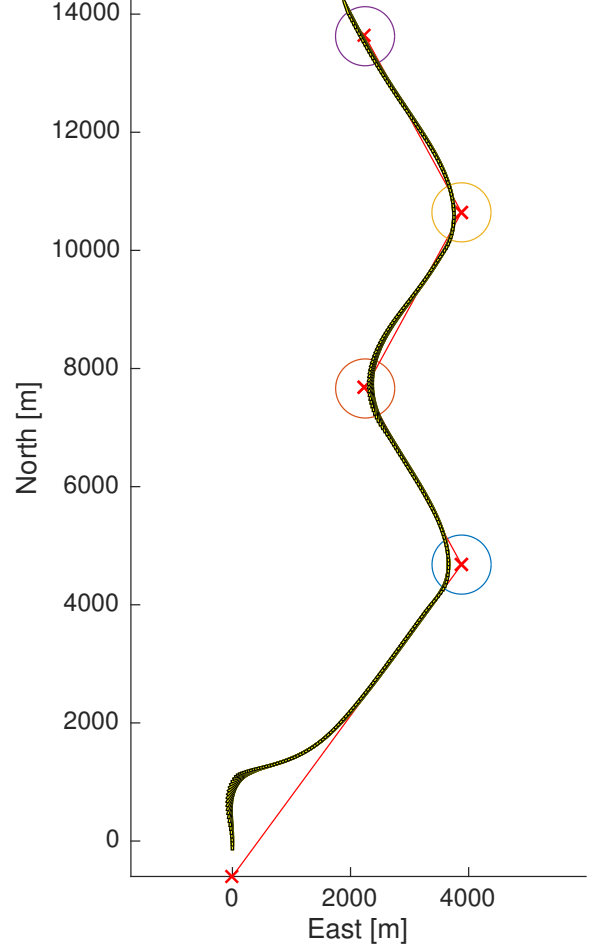


Figure 10: Path following

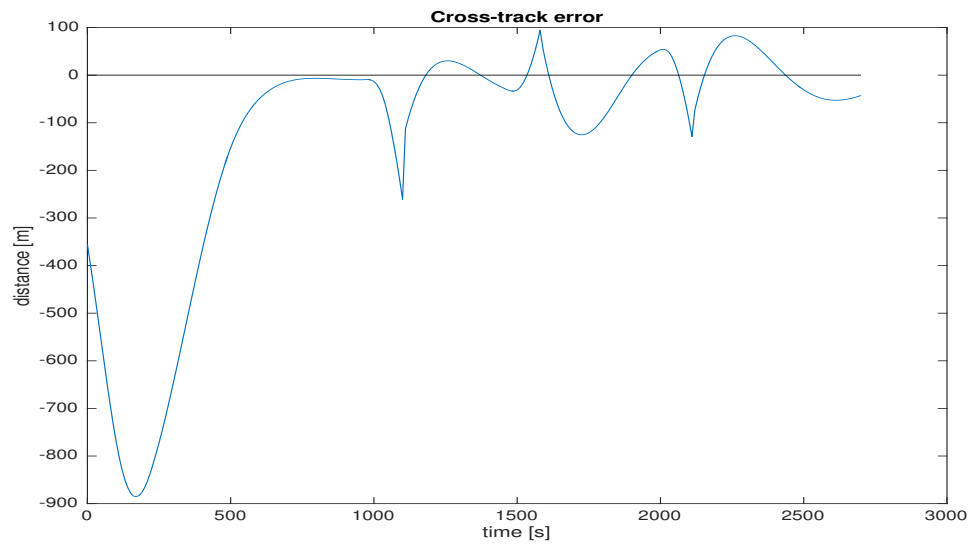


Figure 11: Cross-track error

## 2.3 Path Tracking