

Summary of TTK4215: System Identification and Adaptive Control

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Contents

1	Todo	1
2	Preliminaries	2
2.1	Norms	2
2.2	Transfer function properties	2
2.3	(Strictly) positive real transfer functions	3
3	Models for dynamic systems	3
3.1	SISO, LTI system	3
4	Parametric models	4
4.1	Linear	4
4.2	Bilinear	4
5	Parameter estimation	4
5.1	SPR Lyapunov method	4
5.2	Gradient method	4
5.2.1	Instantaneous cost	4
5.2.2	Integral cost	5
5.3	With projection	5
5.4	Least squares	5
5.4.1	Pure least squares	5
5.4.2	With covariance resetting	5
5.4.3	With forgetting	5
6	Model reference adaptive control (MRAC)	5
7	Adaptive pole placement control (APPC)	6
7.1	Indirect APPC	6

1 Todo

- Canonical forms
- PR and SPR

- Unbounded input (necessary shit for proofs or whatever)
- List of abbreviations (SPR - strict pos. real)
- Lemma 3.5.2 - 3.5.4
- PE (p177)

2 Preliminaries

2.1 Norms

General p -norm

$$\|x\|_p = \left(\int_0^\infty |x(\tau)|^p dt \right)^{1/p} \quad (1)$$

\mathcal{L}_∞ -norm

$$\|x\|_\infty = \sup_{t \geq 0} |x(t)| \quad (2)$$

and we say $x \in \mathcal{L}_\infty$ when $\|x\|_\infty$ exists.

2.2 Transfer function properties

Consider the polynomial

$$X(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \dots + \alpha_0 \quad (3)$$

and the transfer function

$$G(s) = \frac{Z(s)}{R(s)}. \quad (4)$$

Monic: $X(s)$ is *monic* iff $\alpha_n = 1$.

Hurwitz: $X(s)$ is *Hurwitz* if all roots of $X(s) = 0$ are in the left half plane.

Minimum phase: A system defined by the t.f. $G(s)$ is *minimum phase* iff $Z(s)$ is Hurwitz.

Stability: A system defined by the t.f. $G(s)$ is *stable* if $R(s)$ is Hurwitz.

Coprime: Two polynomials are *coprime* if they have no common factors other than a constant.

2.3 (Strictly) positive real transfer functions

KYP Lemma Given a square matrix A with eigenvalues $\Re(\lambda) \leq 0$, a vector B such that (A, B) controllable, a vector C , and scalar $d \geq 0$, then the t.f.

$$G(s) = d + C^T(sI - A)^{-1}B \quad (5)$$

is PR iff \exists a symmetric pos. def. matrix P and a vector q such that

$$A^T P + PA = -qq^T \quad (6)$$

$$PB - C = \pm\sqrt{2d} \cdot q. \quad (7)$$

LKY Lemma Given a stable matrix A , a vector B such that (A, B) controllable, a vector C and a scalar $d \geq 0$, then the t.f.

$$G(s) = d + C^T(sI - A)^{-1}B \quad (8)$$

is SPR iff for any pos. def. matrix L , \exists a symmetric pos. def. matrix P , a scalar $\nu > 0$ and a vector q such that

$$A^T P + PA = -qq^T - \nu L \quad (9)$$

$$PB - C = \pm q\sqrt{2d}. \quad (10)$$

MKY Lemma Given a stable matrix A , vectors B, C , and a scalar $d \geq 0$, we have: If

$$G(s) = d + C^T(sI - A)^{-1}B \quad (11)$$

is SPR, then for any $L = L^T > 0$, \exists a scalar $\nu > 0$, a vector q and a $P = P^T > 0$ such that

$$A^T P + PA = -qq^T - \nu L \quad (12)$$

$$PB - C = \pm q\sqrt{2d}. \quad (13)$$

3 Models for dynamic systems

3.1 SISO, LTI system

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \quad (14)$$

$$\mathbf{y} = C^T \mathbf{x} \quad (15)$$

Controllability

$$P_c \triangleq \begin{bmatrix} B \\ AB \\ \vdots \\ A^{n-1}B \end{bmatrix} \quad (16)$$

If P_c is nonsingular, the system is controllable, and can be transformed to the *controllability canonical form* by

$$\mathbf{x}_c = P_c^{-1} \mathbf{x} \quad (17)$$

Properness A transfer function $G(s) = \frac{N(s)}{D(s)}$ is

- *proper* if $\deg(N) \leq \deg(D)$,
- *biproper* if $\deg(N) = \deg(D)$,
- *strictly proper* if $\deg(N) < \deg(D)$.

4 Parametric models

4.1 Linear

$$z = \theta^{*\top} \phi \quad (18)$$

$$y = \theta_\lambda^{*\top} \phi \quad (19)$$

4.2 Bilinear

$$y = k_0(\theta^{*\top} \phi + z_0) \quad (20)$$

5 Parameter estimation

5.1 SPR Lyapunov method

Based on choosing an adaptive law so that a *Lyapunov-like* function guarantees $\tilde{\theta} \rightarrow 0$. The parametric model $z = W(s)\theta^{*\top} \phi$ is rewritten $z = W(s)L(s)\theta^{*\top} \phi$, with $L(s)$ a proper stable t.f., and $W(s)L(s)$ a proper SPR t.f.

$$z = W(s)L(s)\theta^{*\top} \phi \quad (21)$$

$$\hat{z} = W(s)L(s)\theta^\top \phi \quad (22)$$

$$\epsilon = z - \hat{z} - W(s)L(s)\epsilon n_s^2 \quad (23)$$

$$\dot{\theta} = \Gamma \epsilon \phi \quad (24)$$

5.2 Gradient method

$$z = \theta^{*\top} \phi \quad (25)$$

$$\hat{z} = \theta^\top \phi \quad (26)$$

$$\epsilon = \frac{z - \hat{z}}{m^2} \quad (27)$$

5.2.1 Instantaneous cost

$$\dot{\theta} = \Gamma \epsilon \phi \quad (28)$$

5.2.2 Integral cost

$$\dot{\theta} = -\Gamma(R\theta + Q) \quad (29)$$

$$\dot{R} = -\beta R + \frac{\phi\phi^T}{m^2} \quad (30)$$

$$\dot{Q} = -\beta Q - \frac{z\phi}{m^2} \quad (31)$$

5.3 With projection

$$\dot{\theta} = \begin{cases} \Gamma\epsilon\phi & \text{if } \theta \in \mathcal{S}^0 \\ \Gamma\epsilon\phi - \Gamma \frac{\nabla g \nabla g^T}{\nabla g^T \Gamma \nabla g} \Gamma\epsilon\phi & \text{otherwise} \end{cases} \quad (32)$$

5.4 Least squares

$$z = \theta^{*T} \phi \quad (33)$$

$$\hat{z} = \theta^T \phi \quad (34)$$

$$\epsilon = \frac{z - \hat{z}}{m^2} \quad (35)$$

5.4.1 Pure least squares

$$\dot{\theta} = P\epsilon\phi \quad (36)$$

$$\dot{P} = -P \frac{\phi\phi^T}{m^2} P \quad (37)$$

5.4.2 With covariance resetting

$$\dot{\theta} = P\epsilon\phi \quad (38)$$

$$\dot{P} = -P \frac{\phi\phi^T}{m^2} P, \quad P(t_r^+) = P_0 = \rho_0 I \quad (39)$$

5.4.3 With forgetting

$$\dot{\theta} = P\epsilon\phi \quad (40)$$

$$\dot{P} = \begin{cases} \beta P - P \frac{\phi\phi^T}{m^2} P & \text{if } \|P(t)\| \leq R_0 \\ 0 & \text{otherwise} \end{cases} \quad (41)$$

6 Model reference adaptive control (MRAC)

MRAC requires a plant and a reference model. A controller is made so that the controller and plant together behave similar to the reference model. An adaptive algorithm estimates the controller parameters θ . There are two main categories:

- *Direct*, where θ is equal to the controller gains.

- *Indirect*, where the controller gains are a function of θ .

Huge drawback: Requires plant of minimum phase.

7 Adaptive pole placement control (APPC)

7.1 Indirect APPC

Objective: Choose u_p so that the closed-loop poles are the roots of $A^*(s) = 0$.