Summary of TTK4215: System Identification and Adaptive Control

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Contents

1 Todo

- Canoncal forms
- PR and SPR

- Unbounded input (necessary shit for proofs or whatever)
- List of abbreviations (SPR - strict pos. real)
- Lemma 3.5.2 - 3.5.4
- PE (p177)

2 Preliminaries

2.1 Norms

General p-norm

$$||x||_p = \left(\int_0^\infty |x(\tau)|^p \, \mathrm{d}t \right)^{1/p} \tag{1}$$

$\mathcal{L}_\infty$-norm

$$||x||_\infty = \sup_{t \geq 0} |x(t)| \tag{2}$$

and we say $x \in \mathcal{L}_\infty$ when $||x||_\infty$ exists.

2.2 Transfer function properties

Consider the polynomial

$$X(s) = \alpha_n s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_0 \tag{3}$$

and the transfer function

$$G(s) = \frac{Z(s)}{R(s)}. \tag{4}$$

Monic: $X(s)$ is *monic* iff $\alpha_n = 1$.

Hurwitz: $X(s)$ is *Hurwitz* if all roots of $X(s) = 0$ are in the left half plane.

Minimum phase: A system defined by the t.f. $G(s)$ is *minimum phase* iff $Z(s)$ is Hurwitz.

Stability: A system defined by the t.f. $G(s)$ is *stable* if $R(s)$ is Hurwitz.

Coprime: Two polynomials are *coprime* if they have no common factors other than a constant.

2.3 (Strictly) positive real transfer functions

KYP Lemma Given a square matrix A with eigenvalues $\Re(\lambda) \leq 0$, a vector B such that (A, B) controllable, a vector C, and scalar $d \geq 0$, then the t.f.

$$G(s) = d + C^{\mathrm{T}}(s\mathrm{I} - A)^{-1}B \tag{5}$$

is PR iff $\exists$ a symmetric pos. def. matrix P and a vector q such that

$$A^{\mathrm{T}}P + PA = -qq^{\mathrm{T}} \tag{6}$$

$$PB - C = \pm\sqrt{2d} \cdot q. \tag{7}$$

LKY Lemma Given a stable matrix A, a vector B such that (A, B) controllable, a vector C and a scalar $d \geq 0$, then the t.f.

$$G(s) = d + C^{\mathrm{T}}(s\mathrm{I} - A)^{-1}B \tag{8}$$

is SPR iff for any pos. def. matrix L, $\exists$ a symmetric pos. def. matrix P, a scalar $\nu > 0$ and a vector q such that

$$A^{\mathrm{T}}P + PA = -qq^{\mathrm{T}} - \nu L \tag{9}$$

$$PB - C = \pm q\sqrt{2d}. \tag{10}$$

MKY Lemma Given a stable matrix A, vectors B, C, and a scalar $d \geq 0$, we have: If

$$G(s) = d + C^{\mathrm{T}}(s\mathrm{I} - A)^{-1}B \tag{11}$$

is SPR, then for any $L = L^{\mathrm{T}} > 0$, $\exists$ a scalar $\nu > 0$, a vector q and a $P = P^{\mathrm{T}} > 0$ such that

$$A^{\mathrm{T}}P + PA = -qq^{\mathrm{T}} - \nu L \tag{12}$$

$$PB - C = \pm q\sqrt{2d}. \tag{13}$$

3 Models for dynamic systems

3.1 SISO, LTI system

$$\dot{\mathbf{x}} = \boldsymbol{A}\mathbf{x} + \boldsymbol{B}\mathbf{u} \tag{14}$$

$$\mathbf{y} = \boldsymbol{C}^{\mathrm{T}}\mathbf{x} \tag{15}$$

Controllability

$$\boldsymbol{P}_c \triangleq \begin{bmatrix} \boldsymbol{B} \\ \boldsymbol{AB} \\ \vdots \\ \boldsymbol{A}^{n-1}\boldsymbol{B} \end{bmatrix} \tag{16}$$

If $\boldsymbol{P}_c$ is nonsingular, the system is controllable, and can be transformed to the *controllability canonical form* by

$$\mathbf{x}_c = \boldsymbol{P}_c^{-1}\mathbf{x} \tag{17}$$

Properness A transfer function $\boldsymbol{G}(s) = \frac{\boldsymbol{N}(s)}{\boldsymbol{D}(s)}$ is

- *proper* if $\deg(\boldsymbol{N}) \leq \deg(\boldsymbol{D})$,
- *biproper* if $\deg(\boldsymbol{N}) = \deg(\boldsymbol{D})$,
- *stricly proper* if $\deg(\boldsymbol{N}) < \deg(\boldsymbol{D})$.

4 Parametric models

4.1 Linear

$$z = {\theta^*}^{\mathrm{T}} \phi \tag{18}$$

$$y = {\theta_\lambda^*}^{\mathrm{T}} \phi \tag{19}$$

4.2 Bilinear

$$y = k_0({\theta^*}^{\mathrm{T}} \phi + z_0) \tag{20}$$

5 Parameter estimation

5.1 SPR Lyapunov method

Based on choosing an adaptive law so that a *Lyapunov-like* function guarantees $\tilde{\theta} \to 0$. The parametric model $z = W(s){\theta^*}^{\mathrm{T}} \psi$ is rewritten $z = W(s)L(s){\theta^*}^{\mathrm{T}} \phi$, with $L(s)$ a proper stable t.f., and $W(s)L(s)$ a proper SPR t.f.

$$z = W(s)L(s){\theta^*}^{\mathrm{T}} \phi \tag{21}$$
$$\hat{z} = W(s)L(s)\theta^{\mathrm{T}} \phi \tag{22}$$
$$\epsilon = z - \hat{z} - W(s)L(s)\epsilon n_s^2 \tag{23}$$
$$\dot{\theta} = \Gamma \epsilon \phi \tag{24}$$

5.2 Gradient method

$$z = {\theta^*}^{\mathrm{T}} \phi \tag{25}$$
$$\hat{z} = \theta^{\mathrm{T}} \phi \tag{26}$$
$$\epsilon = \frac{z - \hat{z}}{m^2} \tag{27}$$

5.2.1 Instantaneous cost

$$\dot{\theta} = \Gamma \epsilon \phi \tag{28}$$

5.2.2 Integral cost

$$\dot{\theta} = -\Gamma(R\theta + Q) \tag{29}$$

$$\dot{R} = -\beta R + \frac{\phi\phi^{\mathrm{T}}}{m^2} \tag{30}$$

$$\dot{Q} = -\beta Q - \frac{z\phi}{m^2} \tag{31}$$

5.3 With projection

$$\dot{\theta} = \begin{cases} \Gamma\epsilon\phi & \text{if } \theta \in \mathcal{S}^0 \\ \Gamma\epsilon\phi - \Gamma\frac{\nabla g \nabla g^{\mathrm{T}}}{\nabla g^{\mathrm{T}}\Gamma\nabla g}\Gamma\epsilon\phi & \text{otherwise} \end{cases} \tag{32}$$

5.4 Least squares

$$z = {\theta^*}^{\mathrm{T}}\phi \tag{33}$$

$$\hat{z} = \theta^{\mathrm{T}}\phi \tag{34}$$

$$\epsilon = \frac{z - \hat{z}}{m^2} \tag{35}$$

5.4.1 Pure least squares

$$\dot{\theta} = P\epsilon\phi \tag{36}$$

$$\dot{P} = -P\frac{\phi\phi^{\mathrm{T}}}{m^2}P \tag{37}$$

5.4.2 With covariance resetting

$$\dot{\theta} = P\epsilon\phi \tag{38}$$

$$\dot{P} = -P\frac{\phi\phi^{\mathrm{T}}}{m^2}P, \quad P(t_r^+) = P_0 = \rho_0 I \tag{39}$$

5.4.3 With forgetting

$$\dot{\theta} = P\epsilon\phi \tag{40}$$

$$\dot{P} = \begin{cases} \beta P - P\frac{\phi\phi^{\mathrm{T}}}{m^2}P & \text{if } ||P(t)|| \leq R_0 \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

6 Model reference adaptive control (MRAC)

MRAC requires a plant and a reference model. A controller is made so that the controller and plant together behave similar to the reference model. An adaptive algorithm estimates the controller parameters θ. There are two main categories:

- *Direct*, where θ is equal to the controller gains.

- *Indirect*, where the controller gains are a function of θ.

Huge drawback: Requires plant of minimum phase.

7 Adaptive pole placement control (APPC)

7.1 Indirect APPC

Objective: Choose u_p so that the closed-loop poles are the roots of $A^*(s) = 0$.