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Problem description

Background

Multi-target tracking is a key ingredient in collision avoidance system for autonomous vehicles. Multi-frame tracking methods are commonly acknowledged as gold standards for multi-target tracking. The purpose of this master thesis is to develop a complete multi-frame system for autonomous ships, based on sensor inputs from radar and the Automatic Identification System (AIS).

Proposed tasks

The following task are proposed for this thesis:

- Extend an integer-linear-programming (ILP) based tracking method with suitable algorithms for track initiation and track management
- Develop a framework for fusion between radar tracks and AIS tracks
- Develop alternatives to N-scan pruning in order to enhance the computational efficiency of the tracking method
- Implement the tracking system in Python and/or C++
- Test the tracking system on simulated data
- Test the tracking system with real data recorded with the Navico 4G broadband radar mounted on Telemetron

Autosea

This thesis is associated with the AUTOSEA project, which is collaborative research project between NTNU, DNV GL, Kongsberg Maritime and Maritime Robotics, focused on achieving world-leading competence and knowledge in the design and verification of methods and systems for sensor fusion and collision avoidance for ASVs. The project has access to supervision and physical test platforms through our industry partners.

Preface

The work presented in this thesis

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Trondheim, 2017-06-05

Abstract

The answer is 42

Sammendrag

Løysinga er 42.

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Glossary

VHF The frequency range between 30 and 300 MHz.

AUTOSEA A collaborative research and development project between NTNU AMOS and the Norwegian maritime industry with aim to attain world leading knowledge in design and verification of control systems for ASVs.

Cluster Set of targets which share measurements.

Clutter Noise in the form of false measurements where the amount is assumed Poisson distributed.

COLREGS Convention on the International Regulations for Preventing Collisions at Sea.

Gross tonnage A measurement of a ship's overall internal volume.

Measurement A point in the measurement space where something is detected.

Nautical Mile Length used in maritime navigation. Equals 1 minute of latitude (1852 meters).

NMEA Communication protocol used between electronic maritime equipment, based on RS-422.

Radar Acronym for Radio Detection And Ranging. A device that uses radio waves to measure distance and bearing to other objects.

RS-232 Serial single ended communication standard.

RS-422 Serial differential communication standard.

SOLAS The International Convention for the Safety of Life at Sea.

Solver A program that solves optimization problems.

Target An actual object which the system is trying to track.

Track forest A forest of track hypothesis trees.

Track hypothesis A new measurement inside the gate for an existing track.

Acronyms

P_D Probability of detection

AIS Automatic Identification System

ASV Autonomous Surface Vessel

BFS Breath First Search

CAS Collision Avoidance System

CPA Closest point of Approach

CSTDMA Carrier Sense Time Division Multiple Access

DFS Depth First Search

DNV GL Det Norske Veritas Germanischer Lloyd

HOMHT Hypothesis Oriented Multi Hypothesis Tracker

ILP Integer Linear Programming

IMO International Maritime Organization

JPDFAF Joint Probabilistic Data Association Filter

MHT Multi Hypothesis Tracking

MILP Mixed Integer Linear Programming

MMSI Mobile Maritime Safety Identity

MUNIN Maritime Unmanned Navigation through Intelligence in Networks

NIS Normalized Innovation Squared

NM Nautical Mile

NTNU Norwegian University of Science and Technology

PDAF Probabilistic Data Association Filter

PDF Probability Density Function

RADAR RAdio Detection And Ranging

RFS Random Finite Set

SAR Synthetic Aperture Radar

SOTDMA Self Organized Time Division Multiple Access

TDMA Time Division Multiple Access

TOMHT Track Oriented Multi Hypothesis Tracker

VHF Very High Frequency

VTs Vessel Traffic Service

Nomenclature

$\bar{\mathbf{x}}$	Predicted state
$\hat{\mathbf{x}}$	Filtered state
λ_ν	Poisson spatial density of the number of new measurements
λ_ϕ	Poisson spatial density of the number of false measurements
λ_{ex}	Total spatial density of the number of “extraneous” measurements
cNLLR	Cumulative Negative Log Likelihood Ratio
$\text{NLLR}_{i,j}(k)$	Negative Log Likelihood Ratio for target j at time k for measurement i
\bar{P}	Predicted state covariance
Γ	Disturbance matrix
\hat{P}	Filtered state covariance
Φ	State transition matrix
H	State observation matrix
K	Kalman gain
Q	Process noise covariance matrix
R	Measurement covariance matrix

S	Residual covariance
$\hat{\mathbf{z}}_k$	Predicted measurement at time step k
ν_k	Weighted measurement innovation for time step k
τ	Binary vector where the selected track hypotheses are 1
$\tilde{\mathbf{y}}_k^i$	Measurement innovation for measurement i at time step k
\mathbf{v}	Observation noise
\mathbf{w}	Process noise
\mathbf{x}	State vector
\mathbf{z}_k	Measurement at time step k
i	Measurement index
j	Target index
k	Time index
l	Hypothesis index
M	Number of leaf nodes (track hypotheses in the track forest)
m_k	Number of measurements in scan k
ms	Millisecond
N	Number of scans to keep in track tree
N_1	Number of real measurements in cluster
N_2	Number of targets in cluster
t	Time
t_k	Time at time index

Chapter 1

Introduction

1.1 Motivation

Automation- and control technology have throughout the history been a crucial part of reliving humans from for instance dangerous, exhaustive, repetitive or boring work. Examples of this is automation and robotics in production facilities, remotely operated vehicles for working and exploring the deep sea and disarming explosives. The level of self control varies from remotely controlled to self sensing and planning without human interaction.

The early motivation for automation was probably, and in many situations still are, to improve speed, quality and consistency, which all tends to lead to better economics. With a still decreasing threshold for automating processes, more focus is applied on easing the burden on people, either by combining robotics and humans in the same operation, or fully automate the task. These jobs are typically repetitive, dangerous or both.

Although humans are capable of both self improving and easily adapting to new tasks, they will always have good and bad days, performing the same task slightly different or be bored and unfocused. These are all aspects that leads to inconsistency and errors, which may not be a problem in a production environment with quality inspections, though inconvenient, but can be fatal in critical applications.

There also exists several places where humans and automated system work together to exploit both strengths, for instance in aviation where the pilots are always present in the cockpit, but the autopilot are flying the plane most of the time. This gives the pilots freedom from a very static and repetitive task where a human error could have

fatal consequences. This symbiosis is somewhat similar to the workload on the bridge of commercial vessels, where the autopilot steering the ship most of the time, while the crew is setting the course.

For vessels that do very repetitive routes and jobs, like ferries and short domestic cargo transport, the mental fatigue on the crew can be an issue. Because of the need for crew in emergency situations, customer service and ship maintenance, larger ferries would still need crew. They could however be steered by an automated system, which is never tired, bored, intoxicated or distracted in any other way. This is one of many applications for Autonomous Surface Vessels (ASVs).

The sensor and control system needed for safe automation of any vessel is large and complex, and requires several layers of fault barriers to prevent system errors from spreading and the ability to self monitor its own performance. The control system would know its own position and desired position, it would have access to maps to make a route, a Collision Avoidance System (CAS) to deviate from its planned route to act in accordance with the rules at sea (COLREGS) based on real-time situation information from the sensors on the vessel.

For ASVs to be a viable alternative to human guided ships, the potential savings must be more than marginal, and the control system must be at least as safe as a human operated vessel. The state-of-the-art is not at this point yet, but recent initiatives by large corporations in development in ASVs and the regulation of a dedicated test area for ASVs in Trondheimsfjorden in Norway are just two examples on the direction this technology is headed.

The world's first autonomous ferry might be between Ravnkloa and Vestre kanalkai in Trondheim. The Norwegian University of Science and Technology (NTNU) and Det Norske Veritas Germanischer Lloyd (DNV GL) are working on a collaborate project to develop a small autonomous battery powered passenger and bike-cycle ferry, as an alternative to a bridge over a canal.

An indicator of the momentum autonomous surface vessels have is the Maritime Unmanned Navigation through Intelligence in Networks (MUNIN) project, which is a collaborate project between several European companies and research institutes, partially funded by the European Commission. The project aims at developing and verifying concept of autonomous vessels with remote control from onshore control stations.

This work is focused on the sensor fusion which generates a real time data stream into the control system, enabling situational awareness and the foundation for predictive CAS like [1], which also was a part of the AUTOSEA project.

1.2 Previous work

This work is based on a pre-master project executed autumn 2016 [2, url=true]. In this project, it was shown that several off-the-shelf Integer Linear Programming (ILP) solvers was capable of solving the data association optimization problem in a single sensor Track Oriented Multi Hypothesis Tracker (TOMHT). It also showed that under good to moderate conditions, the performance return when increasing multi-scan window more than a relative low threshold, was very low.

1.3 Outline of the thesis

Chapter 2 provides an introduction to the sensor systems used in this work, as well as some of the different Multi Hypothesis Tracking (MHT) variants that exist. Chapter 4 presents an overview of the complete measurement-to-guidance system and an in-depth explanation of the fused radar and Automatic Identification System (AIS) TOMHT tracking system. Chapter 5 presents the different scenarios that are used in performance evaluation of the tracker and the results of the simulated scenarios. A discussion of the results and evaluation of the performance with respect to safety at sea is presented in Chapter 6. Suggestion for future work is presented in Chapter 7, followed by a conclusion in Chapter 8.

Theoretical Background

2.1 Radar

2.1.1 Overview

Radio Detection And Ranging (RADAR) is a detection technology that uses radio waves to observe stationary and moving objects. A transmitter sends out radio waves and a receiver is waiting for reflected echo's from objects, the time the echo is delayed determines the distance to the object. The transmitter and receiver will in many situations be in the same location, can be both stationary and mobile and fixed or rotating orientation. Depending on frequency, a radar can observe solid objects like air-crafts, ships, terrain, road vehicles and less solid objects like people and weather formations.

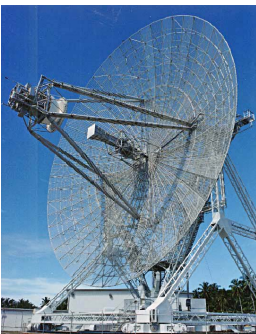


Figure 2.1: Fixed radar antenna



Figure 2.2: Rotating radar antenna



Figure 2.3: Maritime radar antenna

2.1.2 History

The first implementation of an instrument that were able to detect the presence of distance metallic objects by radio waves was done by Christian Hülsmeyer in 1904. His invention did not measure the distance to objects, but whether there was an object in the direction of the instrument. The radar as we know it today was introduced in the mid to late 1930's, with world war two triggering a research to improve the still immature technology to be used in military applications. After the war, the technology matured and where put in use in several civil applications, where air traffic control, maritime safety and weather monitoring is the most common.

2.1.3 Principles

The electromagnetic waves that a radar emits travels at the speed of light in air and vacuum. It reflects back when there is a change in the density of the medium it is travelling through, which is what happens when radio waves hit targets. Electrically conductive materials tends to be good reflectors, since they have a very different atomic density than air. On the other hand, materials with poor conductivity, and also some magnetic materials, then to absorb radio waves good. Like light, there is many ways an incoming radio wave can be reflected, primarily dependent on the geometry of the target. A corner with angles less than 180° will reflect the incoming radio waves directly back to the sender, and is a good thing on targets that want to be visible on a radar. This principle are the basis for radar reflectors commonly used to boost the radar signature on smaller vessels, see Figure 2.4. The opposite is used on targets that try to minimize their radar signature, and is the reason why stealth vessels and aircraft are tiled by flat areas.

2.2 AIS

The Automatic Identification System (AIS) is a maritime safety and information system primarily designed for collision avoidance. AIS works by broadcasting messages



Figure 2.5: USS Zumwalt



Figure 2.6: KNM Gnist



Figure 2.7: F177 Nighthawk

on the Very High Frequency (VHF) band at irregular intervals with information on the vessels. AIS transceivers are required on international voyaging vessels over 300 gross tonnage, and on all passenger vessels. AIS signals are received at both vessels and shore stations for use in Vessel Traffic Service (VTS) stations, open tracking databases like `www.marinetraffic.com`, fleet-monitoring and search and rescue. Since the AIS messages contain position, course and speed, AIS tracks can be overlaid on a map in a chart plotter or on top of a radar image, giving the operator two sensors to verify each other.

2.2.1 History

AIS was designed and developed by technical committees in the International Maritime Organization (IMO). Its objective was to enhance vessels' safety and efficiency by increasing their ability to see and identify other vessels. The main motivation for adopting AIS was its independence of humans in operation, since it automatically identifies other vessels and displays the information on the navigational system on the bridge. It also enables automatic calculation of Closest point

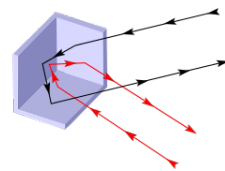


Figure 2.4: Corner reflector

of Approach (CPA) and time until CPA, in which the navigation system could alarm the bridge on incoming traffic on dangerous course. This gives the navigator on the bridge more and better information for making decisions, but with the caveat that not all vessels have AIS. In the 2002 IMO SOLAS Agreement, it is required that vessels over 300 gross tonnage and all passenger vessels must be equipped with class A AIS transceivers. A simpler and cheaper AIS version named class B aimed at smaller vessels and yachts was published in 2006, followed by a large increase in the amount of non-commercial vessels equipped with AIS.

Need citation

2.2.2 Messages

AIS broadcasts both static, dynamic and voyage information with varying intervals based on the vessels' speed, status and on request from shore stations. Static, dynamic and voyage messages are listed in Table tables 2.1 to 2.3. When the AIS standard was developed, the peak traffic situations in the two most densely trafficked waterways, Singapore and Dover Straits, were used to calculate the update frequency for the AIS system. Based on

these two locations and a desire to keep the number of reports per minute below 2000, the dynamic information report intervals for class A and B was set as in Table 2.4 and 2.5 respectively [3]. Static information is transmitted every 6 minutes, and on request from VTS stations. AIS transceivers are utilizing two reserved VHF channels; AIS 1 — 87B (161.975MHz) and AIS 2 — 88B (162.025MHz) to improve robustness against interference. An important note is that AIS transceivers are alternating which channel they are transmitting on, which means that if a receiver is only listening on one channel, the effective update rate halves.

2.2.3 Class A

Class A AIS transceivers are designed for Self Organized Time Division Multiple Access (SOTDMA) transmission, which is a way of reserving transmission time slot for the next broadcast. SOTDMA is based on Time Division Multiple Access (TDMA), with an extension allowing for self organizing of time slots compared to TDMA's dedicated timing manager. This effectively gives class A AIS transmissions priority over Class B equipment which may not have SOTDMA. Class A transceivers are also required to have build-in display, minimum transmission power of 12.5W, ability to filter targets and communication interfaces like RS-232 and NMEA.

2.2.4 Class B

Class B AIS transceivers are designed to be simpler and cheaper than Class A transceivers, which is accomplished through less strict requirements for hardware and operation. Class B AIS transmits at lower power, usually 2W and transmits at larger time intervals than Class A, see Table 2.5. It is not required to have a build-in display and can use both Carrier Sense Time Division Multiple Access (CSTDMA) and SOTDMA for transmission. CSTDMA is a simpler approach to time division than SOTDMA since it only listens for a single time slot to be unused before it transmits.

2.3 Tracking

2.3.1 Overview

Tracking, in this context, is to follow stationary and moving targets that are observed by a system without included association data. The problem is to know which measurements belong together over time, often referred to as the data association problem.

Static AIS information	
MMSI	Maritime Mobile Service Identity
Call sign	Maritime radio (VHF) call sign
Name	Name of vessel
IMO Number	Vessel IMO number
Length and beam	
Location of positioning fixing antenna	
Height over keel	

Table 2.1: Static AIS information

Dynamic AIS information	
Position	In WGS84 frame
Position accuracy	Better or worse than 10 meter
Position time stamp	UTC in whole seconds
Course over ground (COG)	
Speed over ground (SOG)	
Heading	
Navigational status	
Rate of turn (ROT)	

Table 2.2: Dynamic AIS information

Voyage AIS information	
Draught	Depth in water
Hazardous cargo	Type
Destination	Name of place
Estimated time of arrival (ETA)	
Route plan / waypoints	
Number of persons on board	

Table 2.3: Voyage AIS information

Vessels status	Reporting Interval
Ship at anchor or moored and not moving faster than 3 knots	3 minutes
Ship at anchor or moored and moving faster than 3 knots	10 seconds
Ship 0–14 knots	10 seconds
Ship 0–14 knots and changing course	3.3 seconds
Ship 14–23 knots	6 seconds
Ship 14–23 knots and changing course	2 seconds
Ship > 23 knots	2 seconds
Ship > 23 knots and changing course	2 seconds

Table 2.4: Class A Reporting Intervals

Vessels status	Reporting Interval
Ship < 2 knots	3 minutes
Ship 2–14 knots	30 seconds
Ship 14–23 knots	15 seconds
Ship > 23 knots	5 seconds
Search and Rescue aircraft	10 seconds
Aids to navigation	3 minutes
AIS base station	10 seconds

Table 2.5: Class B Reporting Intervals

2.3.2 History

Tracking is a relative new field of study, originating from the military technology race post 1945, enabled by the development of microprocessors and computers from the 60's. The applications ranged from sonar tracking on both submarines and navy vessels, to air control and missile guidance. This historical background is likely the reason for most published papers using these types of applications as background for testing. In recent years, tracking people and vehicles from visual- and Synthetic Aperture Radar (SAR) imagery have also become a topic in the research community [4]–[6]. There have also been published some research work on usage of tracking for ASVs [7], [8], although both are focused on interaction and response to external events.

There are several factors contributing to the challenge of good tracking; clutter, lower than unity Probability of detection (P_D), multiple detections of the same vessel and wakes. Clutter is a term for unwanted measurements or noise, which is inherent in every observation system. For a maritime radar, this can be caused by waves, rain, snow, birds or shore echo. A common assumption on clutter is to assume the amount being Poisson P_D is a measure of how persistent the target is in the measurements, and will vary much between different types of targets.

2.3.3 Single-target Tracking

The simplest approach to tracking is single-target tracking, where it is assumed that there are only one target in the measurement area, and any other measurement is regarded as either extra measurements of the target or false measurements, often referred to as clutter.

2.3.4 Multitarget tracking

A subset of tracking is multitarget tracking, where the problem expands to jointly estimate both the number of targets and their trajectories [9]. While a large number of tracking techniques have been developed, the three most used are Joint Probabilistic Data Association Filter (JPDAF), MHT and Random Finite Set (RFS) [9].

2.3.5 JPDAF

JPDAF is a multitarget expansion of Probabilistic Data Association Filter (PDAF) which is a single-target tracking technique. The essence of them both is consider

2.3.6 RFS

2.3.7 MHT

Radar and AIS preprocessing

3.1 Radar preprocessing

3.2 AIS preprocessing

All AIS measurements are buffered from when they are received to the next radar scan. This is a design choice, which in some sense synchronize the AIS to the radar. Since the radar period is much smaller than (or in the best case for the AIS, equal to) the AIS transmit period, this will seldom lead to unused AIS measurements. On the other hand, since the radar period is relative short, the amount of time any AIS position will be predicted forward is typically small enough to not cause uncertainty larger than manageable for the algorithm.

As will be obvious in Section 4.5.3, this prediction is only used for the gating sequence.

3.2.1 Out-of-order filtering

3.2.2 ID swap filtering

MHT Module

To create a complete tracking *system*, rather than a tracking *algorithm*, it is often necessary to complement the main algorithm with other support modules. The system, or module if it is a part of a bigger system, presented here is an extension of the pre master project [2]. The aim of this chapter is to provide a complete walkthrough of the track oriented MHT system presented in this thesis. The motion model which is used throughout the entire tracking system when predicting and filtering target behaviour is explained first. Next follows an overview of the algorithm used to initiate new tracks into the MHT algorithm. Followed by the entire MHT tracking algorithm, with all its sub-routines and bookkeeping.

4.1 Motion Model

A local Cartesian NED-frame will be used throughout this thesis, with the assumption than all input sensors are transformed to this local frame. In real life, when working on an approximated local Cartesian frame is doable as long as the area in question is reasonable small. A global geodetic frame, like WGS84, would be preferable, but will yield non-linear motion equations. The state of the targets are modelled with four states

$$\mathbf{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T \quad (4.1)$$

where the x -axis is pointing east and the y -axis is pointing north. The two latest states are the velocity in their respective direction.

Since modelling the behaviour of any ship under unknown command is next to impossible, a common assumption in tracking theory is to assume that every target will continue on as usual. Although simple, this model captures the essence of most vessels at sea, and when looking at maritime training, regulation and best-practice, they all dictates that vessels should hold steady course and change course in clear decisive turns. To give room in our model for manoeuvring, the process noise covariance is increased according to the assumed manoeuvring capabilities of the vessels. This could be set as a fixed value for all targets, as done in this work, or estimated based on the history of the track.

This give raise for the linear dynamic model (4.2)

$$\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \mathbf{w}(k) \quad \mathbf{w} \sim \mathcal{N}(0; \mathbf{Q}) \quad (4.2)$$

where

Φ = the state transition matrix

\mathbf{w} = the process noise

k = time index

\mathbf{Q} = the system covariance matrix

and the matrices describing our constant-velocity model is (4.3).

$$\Phi = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{Q} = \sigma_v^2 \begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & T & 0 \\ 0 & \frac{T^2}{2} & 0 & T \end{bmatrix} \quad (4.3)$$

4.2 Track Initiation

In comparison with Hypothesis Oriented Multi Hypothesis Tracker (HOMHT) who treats every measurement as a potential new track since its hypotheses are essentially different ways of organizing its measurements into tracks, which are repeated for every iteration. TOMHT does not have any built-in initialization of tracks since it essentially only maintains an already existing track with track splitting and measurement-to-track association for every iteration. To remedy this lack, we need an algorithm that can find consistent and predictable patterns in an assumed uniformly distributed measurement space of clutter.

In this work, new tracks are initiated with 2/2 & m/n logic [9] on the unused measurements after each MHT iteration. As the name of the method indicates, this is a two step verification, where the first act as a rough filter and the second as a fine filter. As one of the main assumptions in most tracking systems, the measured radar clutter is assumed uniformly distributed in the measurement area. This assumption is quite rough in the radial measurement space, and even worse approximation in Cartesian measurement space, as illustrated in Figures 4.1 and 4.2. Although far from perfect, tests shows that we still get satisfactory performance from the 2/2&m/n method. The flow of the method is illustrated in Figure 4.3, and for better clarity, the algorithm is explained from the last step to the first step, since this is the sequence a newly started initiator will perform its operations.

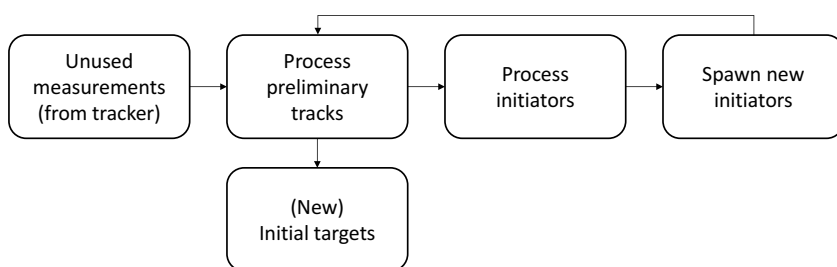


Figure 4.3: 2/2&m/n flowchart

4.2.1 Spawn new initiators

All measurement unused by the ‘Process preliminary tracks’ and ‘Process preliminary tracks’ steps will be the basis for new *initiators*. An initiator is a measurement that that awaits its match in the next scan. The idea is that uniformly distributed clutter will not (often) reappear at approximately the same location two times in a row, effectively filtering out most of the clutter.

4.2.2 Process initiators

When the next scan arrives, all the unused measurements from the ‘Process preliminary tracks’ step will be used as candidates in this step. Since an initiator is only a position and not a full state with velocity, all directions are equally likely, and the only design parameter in this step is maximum speed of targets to be tracked. This parameter sets an outer limit on the circle acting as a gate for the second and confirming measurement.

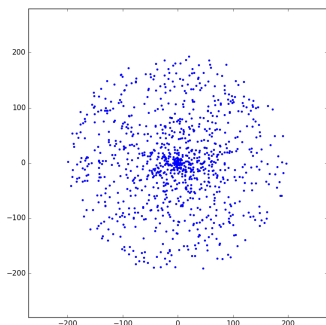


Figure 4.1: Uniform radial clutter

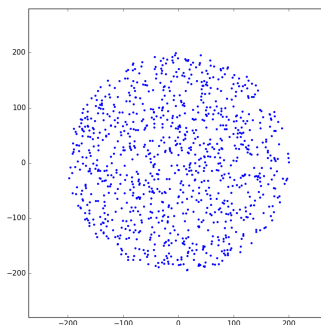


Figure 4.2: Uniform Cartesian clutter

When matching initiators with a second measurement, we want to select the closest measurement, making the assumption that the two consecutive measurements are the most likely to belong together. In a single target scenario, where this would be to calculate the distance to all the alternative measurements and select the lowest, the association is already done. While in a multitarget scenario, we *could* select the closest measurement to any initiator, but we would have to do this one initiator at a time. This would lead to different results depending on the arrangement of the initiators in the programming of this method. A different approach would be to calculate all the different distances for any possible combination of initiators and measurements, sort the list, and assign the distances from the shortest to the longest possible distances. This approach would not be influenced by randomness like the arrangement of the initiators in a programming language, but would not necessarily give the global optimal association regarding the how many initiators that are assigned measurements and their respective distances.

Since we can have situations like exemplified in Figure 4.4, where two initiators have the same measurement inside their gates, and one of them have a second measurement inside its gate, we need to take the global consequence of any assignment into consideration.

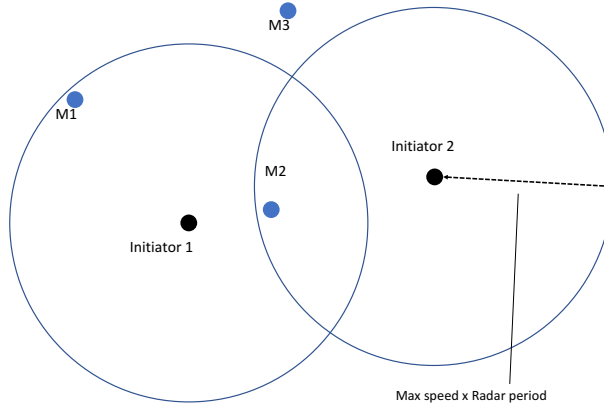


Figure 4.4: Initiator gating example

If using method 1; to sequentially select the best, we have two possible outcomes. When starting with initiator 1, this initiator would be associated with measurement 2, and initiator 2 would not be associated with any measurements. On the other hand, starting with initiator 2 would lead to this initiator being associated with measurement 2, and initiator 1 would be associated with measurement 1. This randomness in outcome based on which initiator the algorithm starts with is clearly not a desired property. If using using method 2; to sequentially select the globally shortest distance, we would first associate initiator 1 with measurement 2, and there would not be any measurements left for initiator 2, leaving this empty.

A third option is to formulate the problem as a global combinatorial problem, and use an ‘off-the-shelf’ solution to solve the problem. We have essentially a matrix with initiators along one axis and measurements along the second axis and the distance between them in their intersections, as in (4.4) for our example.

$$\begin{matrix} & M_1 & M_2 & M_3 \\ \begin{matrix} I_1 \\ I_2 \end{matrix} & \begin{bmatrix} 3 & 1 & 5 \\ 7 & 2 & 6 \end{bmatrix} \end{matrix} \quad (4.4)$$

The values above the threshold set by the maximum speed multiplied with the time period between the radar scans can be set to infinity to symbolise that this combination is not

possible, see (4.5) where the gate threshold is 4.

$$\begin{matrix} & M_1 & M_2 & M_3 \\ I_1 & \begin{bmatrix} 3 & 1 & \infty \end{bmatrix} \\ I_2 & \begin{bmatrix} \infty & 2 & \infty \end{bmatrix} \end{matrix} \quad (4.5)$$

If we remove the columns with only infinity, we are removing measurements that cannot be associated under any circumstances, thus reducing the size of the problem, see (4.6). With this pre processing, we want to assign each row to a column so that the sum of the selected intersections are minimal.

$$\begin{matrix} & M_1 & M_2 \\ I_1 & \begin{bmatrix} 3 & 1 \end{bmatrix} \\ I_2 & \begin{bmatrix} \infty & 2 \end{bmatrix} \end{matrix} \quad (4.6)$$

We now have formulated our problem in a way that it can be solved by the ‘Hungarian’ algorithm, which will give us the association $I_1 \rightarrow M_1$ and $I_2 \rightarrow M_2$

4.2.3 Process preliminary tracks

4.3 MHT Overview

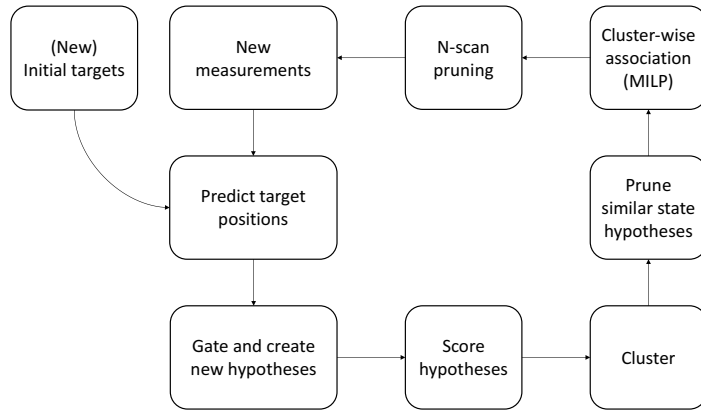


Figure 4.5: Algorithm flowchart

4.4 Predict target position

$$\begin{aligned}\bar{\mathbf{x}}(k+1) &= \Phi \hat{\mathbf{x}}(k) \\ \bar{\mathbf{P}}(k+1) &= \Phi \hat{\mathbf{P}}(k) \Phi^T + \mathbf{Q}\end{aligned}\tag{4.7}$$

4.5 Gate and Create new hypotheses

$$\begin{aligned}\mathbf{S} &= \mathbf{H} \bar{\mathbf{P}} \mathbf{H}^T + \mathbf{R} \\ \tilde{\mathbf{z}} &= \mathbf{z} - \mathbf{H} \bar{\mathbf{x}} \\ \tilde{\mathbf{z}}^T \mathbf{S}^{-1} \tilde{\mathbf{z}} &\leq \eta^2\end{aligned}\tag{4.8}$$

4.5.1 Zero hypothesis

To account for the possibility that the target is not present in this scan, a *zero* hypothesis, or *dummy* hypothesis as it is sometimes called, is generated with the predicted state and covariance.

4.5.2 Pure radar hypotheses

For every radar measurement inside the gate in (4.8), a new track hypothesis is generated with filtered state and covariance according to regular Kalman measurement update equations (4.9).

$$\begin{aligned}\tilde{\mathbf{y}} &= \mathbf{z} - \mathbf{H} \bar{\mathbf{x}} \\ \mathbf{S} &= \mathbf{H} \bar{\mathbf{P}} \mathbf{H}^T + \mathbf{R} \\ \mathbf{K} &= \bar{\mathbf{P}} \mathbf{H}^T \mathbf{S}^{-1} \\ \hat{\mathbf{x}}(k) &= \bar{\mathbf{x}} + \mathbf{K} \tilde{\mathbf{y}} \\ \hat{\mathbf{P}}(k) &= (\mathbf{I} - \mathbf{K} \mathbf{H}) \bar{\mathbf{P}}\end{aligned}\tag{4.9}$$

4.5.3 Combined radar and AIS hypotheses

As elaborated in Section 3.2 all AIS measurements are preprocessed to remove out-of-order messages and ID-swap errors. And for each radar scan, only the latest AIS update from each target (Mobile Maritime Safety Identity (MMSI) number) are passed through to the MHT tracking loop.

For every (predicted) AIS measurement, there are generated a new complete set of hypotheses consisting of the AIS measurement and all the radar measurements inside

the gate. This will lead to a large number of new hypotheses, but since the number of AIS measurements inside any gate at any time will seldom be larger than one, and in most cases zero, this will not cause any substantially larger explosion in the number of track hypotheses that the already exponentially nature of any MHT.

Since the AIS measurements originates before the radar measurement, the fusion is carried out in two steps as a sequential update [10].

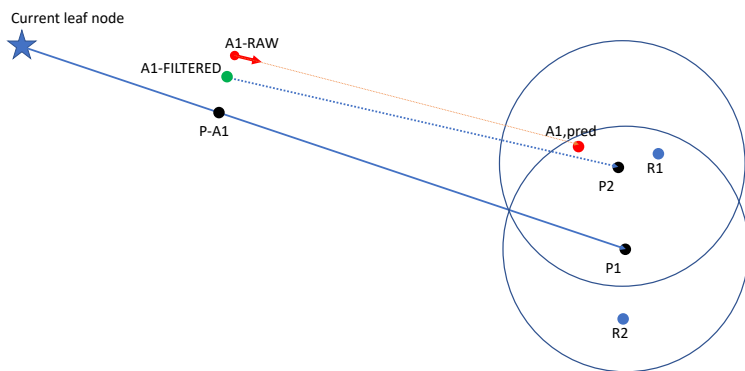


Figure 4.6: AIS gating

To get a more accurate score when summing two measurements, we are first predicting the origin hypothesis to the time of the AIS measurement. This predicted state is then filtered with the AIS measurement, giving rise for a new state, covariance and Normalized Innovation Squared (NIS). The AIS score is then calculated based on how good this intermittent prediction matches the AIS measurement. The new state is then predicted to the time of the radar measurement, which is most likely not at the same location as the original prediction. This prediction is then filtered with the radar measurement, and a radar score is calculated. The new hypothesis are given the accumulative score for the radar and AIS. This process is repeated for each new radar measurement in the gate.

4.5.4 Pure AIS hypotheses

If no radar measurements are present in the gate while there are AIS measurements present inside the gate, pure AIS hypotheses are created. This can be the situation when a target is broadcasting an AIS message, but is either in radar shadow or is not detected by the radar for any reason. These hypotheses are not created when one or more radar mea-

measurements are available, based on the assumption that if a radar measurement is present, the difference between a fused hypothesis and a pure AIS hypothesis is quite small since the AIS measurement covariance typically will be much smaller than the radar measurement covariance, leading to a fused state very close to the AIS measurement.

4.6 Scoring

The scoring used in this tracking system is based on a dimensionless score function by Bar-Shalom [11]. His paper discusses the issue of scoring measurement-to-track associations and comparing scores based on different numbers of measurement and measurement dimensions. He proposes a dimensionless *likelihood ratio*, which is the Probability Density Function (PDF) of a measurement having originating from the track, to the PDF of it not originating from the track.

Fill in the main steps in the derivation

Each track hypothesis is scored according to (4.10). Where the cumulative score being the sum through time since we are using the logarithm of the likelihood ration. For the fused hypotheses, their score is the cumulative score of both the measurements, giving them a better score reflecting that they are more likely hypotheses than pure radar or AIS hypotheses.

$$\text{NLLR} = \frac{1}{2} \left[\tilde{z}^T S^{-1} \tilde{z} \right] + \ln \frac{\lambda_{ex} |2\pi S|^{1/2}}{P_D} \quad (4.10)$$

$$\tilde{z} = z - \hat{z}$$

$$\text{cNLLR}_k^j \triangleq \sum_{l=0}^k \text{NLLR}_{i,j}(l) \quad (4.11)$$

4.7 Clustering

The problem of finding the globally optimal set of track hypotheses increases exponentially with the number of hypotheses in the problem. To reduce the size of the problem, it is desirable to split it into smaller independent problems. Both because it enables parallel computation and it reduces the total cost of solving the problem. Track trees that have common measurements must be solved together, since they can have mutual exclusive leaf nodes. (Remember that each target can maximum produce on radar measurement at

each scan.) The clustering can be done efficiently through Breath First Search (BFS) or Depth First Search (DFS) on a graph made from the hypothesis tree.

By constructing a 0–1 adjacency matrix describing the connection between all the nodes in the track forest, the clustering problem is equivalent to the *connected components* problem in graph theory [12].

4.8 Optimal data association

When the targets are divided into independent clusters, each of them can be treated as a global problem where we want to minimize the cost or maximize the score of the selected track hypotheses (leaf nodes). The selected track hypotheses must also fulfil the constraints, that each measurement can only be a part of one track, and that minimum and maximum one track hypothesis can be selected from each target. Since only binary values, selected or not selected, is possible for selection of hypotheses, the problem becomes an ILP. In the case where a cluster is only containing one target tree, the best hypothesis can be selected by running a search among the leaf nodes after the highest score, since none of the leaf nodes are excluding other leaf nodes in other target trees. This will often be the case for targets that are largely spaced out, and their gates are not and have not overlapped in a while. For any other case, where there are two or more targets in a cluster, the procedure in Section 4.8.1 must be carried out.

4.8.1 Integer Linear programming

The essence of any optimization problem is a cost function and a set of constraints. In our problem, we want to select the combination of hypotheses (leaf nodes) that gives the highest score / lowest cost, while not selecting any measurement more than one time and ensure that we select minimum and maximum one hypothesis from each target.

Our cost vector \mathbf{c} is

them all is (4.12), where \mathbf{c} is a vector of costs (minimize) or scores (maximize) and $\boldsymbol{\tau}$ is a selection vector, where each row in \mathbf{c} and $\boldsymbol{\tau}$ represents one branch in the track hypothesis tree.

$$\min_{\boldsymbol{\tau}} \quad \mathbf{c}^T \boldsymbol{\tau} \quad (4.12)$$

There are two sets of constraints (4.13), one equality and one inequality. The inequality constraints $\mathbf{A}_1 \boldsymbol{\tau} \leq \mathbf{b}_1$ ensures that each measurement are maximum (but not minimum) used one time. The equality constraints $\mathbf{A}_2 \boldsymbol{\tau} = \mathbf{b}_2$ ensures that minimum

and maximum one track from each track tree is selected. The complete ILP formulation becomes (4.13), where τ is a binary vector with dimension equal the number of leaf nodes in the track forest.

$$\begin{aligned}
 & \max_{\tau} \quad \mathbf{c}^T \tau \\
 & \text{s.t.} \quad \mathbf{A}_1 \tau \leq \mathbf{b}_1 \\
 & \quad \quad \mathbf{A}_2 \tau = \mathbf{b}_2 \\
 & \quad \quad \tau \in \{0, 1\}^M
 \end{aligned} \tag{4.13}$$

\mathbf{A}_1 is a $N_1 \times M$ binary matrix with N_1 real measurements and M track hypotheses (all leaf nodes), where $\mathbf{A}_1(l, i) = 1$ if hypothesis l are utilizing measurement i , 0 otherwise. The measurements and hypothesis are indexed by the order they are visited by DFS. \mathbf{A}_2 is an $N_2 \times M$ binary matrix where N_2 is the number of targets in the cluster and $\mathbf{A}_2(l, j) = 1$ if hypothesis l belongs to target j . \mathbf{b}_1 is a N_1 long vector with ones and \mathbf{b}_2 is a N_2 long vector with ones. \mathbf{c} is a M long vector with a measure of the goodness of the track hypotheses. For example, in Figure 4.7 at time step 2, the \mathbf{A} matrices and \mathbf{C} vector would be (4.14).

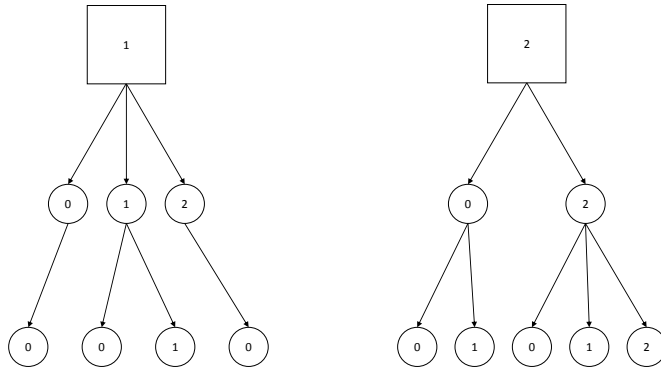


Figure 4.7: Track hypothesis tree

$$\begin{aligned}
\mathbf{A}_1 &= \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \\
\mathbf{A}_2 &= \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\
\mathbf{c} &= [\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6 \quad \lambda_7 \quad \lambda_8 \quad \lambda_9]^T
\end{aligned} \tag{4.14}$$

The explicit enumeration that becomes necessary when creating these \mathbf{A} matrices is exhaustive since the dimension of τ , which is equal to the number of leaf nodes in the track forest, can be very large. Both \mathbf{A}_1 and \mathbf{A}_2 grows quadratically with the number of track hypotheses and real measurements or number of targets respectively.

4.8.2 Solvers

There are a lot of off-the-shelf ILP and Mixed Integer Linear Programming (MILP) solvers on the market, both free open source and commercial. Since the problem in this report is formulated on standard form, it can easily be executed on several solvers, and we can compare runtime and performance. The performance difference of some of these were tested in [2], where the difference was found marginal, probably because of the nature of the problem.

4.9 Dynamic window

4.10 N-Scan pruning

4.11 Track termination

Results

5.1 Testing scheme

The performance evaluation of the MHT tracking system is tedious in that it is necessary to test very many different situations to get a good understanding of how the system is performing. The two largest factors contributing to the difficulty is the random nature of the clutter and lost detections. It is also desirable to evaluate the initialization and tracking performance under both varying environmental (external) conditions and tuning (internal) setting. We want good tracking of targets with low probability of detections in cluttered environment, and secondly it must be able to do this within the time frame of the radar rotation period. The initialization module must be able to detect targets with probability of detection lower than unity without initializing too many false tracks into the MHT algorithm. The testing is separated into two parts; initialization and tracking.

The performance metrics for the initialization module is how long time it takes to initialize the correct tracks, which is tested under a range of internal and external conditions, see (5.1). All combinations of these parameters were simulated on all scenarios, see Table 5.1, which are the same routes but with different AIS configurations. From these simulations, the time to initiate true targets and amount of false targets are calculated. A track is categorized as correct initialized if the state difference between the true track and the initial track is less than a threshold. All initial tracks that does not correspond to a true track is categorized as erroneous. To analyse the impact of the erroneous tracks, the lifespan of falsely initiated tracks is plotted to see whether they die out at the same

rate as they are initiated, or if they accumulate.

$$\begin{aligned}
\mathbf{P_D} &= \begin{bmatrix} 1.0 & 0.8 & 0.6 \end{bmatrix} \\
\mathbf{M/N} &= \begin{bmatrix} (1/1) & (1/2) & (1/3) & (1/4) \\ (2/2) & (2/3) & (2/4) & (2/5) \\ (3/3) & (3/4) & (3/5) & (3/6) \end{bmatrix} \\
\boldsymbol{\lambda_\phi} &= \begin{bmatrix} 0 & 2 \cdot 10^{-6} & 4 \cdot 10^{-6} \end{bmatrix}
\end{aligned} \tag{5.1}$$

When testing the tracking performance, it is desirable to remove the variable of initialization to better see difference in *tacking* rather than *initialization*. Therefore are all the simulations testing tracking performance carried out with all targets correctly initialized at initial time, and with the initiator set to $M = 2, N = 4$ such that the unused measurements from the tracking algorithm would be treated as normal. This would also give lost targets a change to get re-initialized, which is an important property for any safety critical system.

$$\begin{aligned}
\mathbf{P_D} &= \begin{bmatrix} 1.0 & 0.8 & 0.6 \end{bmatrix} \\
\mathbf{N} &= \begin{bmatrix} 1 & 3 & 6 & 9 \end{bmatrix} \\
\boldsymbol{\lambda_\phi} &= \begin{bmatrix} 0 & 2 \cdot 10^{-6} & 4 \cdot 10^{-6} \end{bmatrix}
\end{aligned} \tag{5.2}$$

Both initialization- and tracking performance is averaged over a set of 50 Monte Carlo simulations with differently seeded clutter- and detections points.

5.2 Scenario

All simulations in this work is based on a generated scenario, as shown in Figure 5.1, with black dots marking the initial time and position. The radar range is 5500 meter (3 Nautical Miles (NMs)), which gives an area of surveillance of approximately 95 square km. The scenario contains 16 targets, which all starts inside the observable area of the radar. The scenario contains a mixture of fast and slow moving vessels, some with sharp turns and some almost at stand still.

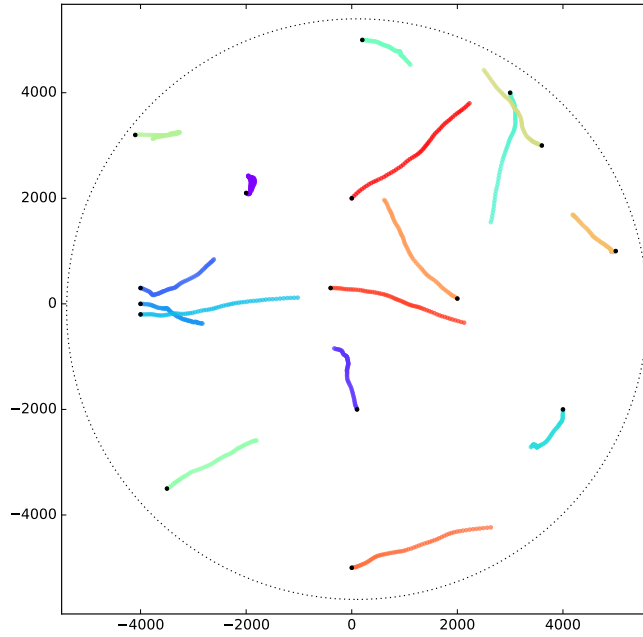


Figure 5.1: True tracks

From this base scenario, five scenarios were generated with different AIS configuration on the vessels, see Table 5.1. The first scenario represents the baseline with only radar information available, whereas the rest have some level of AIS information. Scenario 2 adds three class B AIS transmitters, and is representing a situation where all the targets are smaller vessels with some voluntarily installed AIS transceivers. In scenario 3, all vessels have AIS class B installed. This scenario represents a best case situation regarding yacht and leisure vessels from an autonomous anti collision perspective and is only realistic if AIS class B were to be mandatory for these vessel classes. Scenario 4 is the same as scenario 2, with the difference that the vessels have class A transmitters instead of class B. This gives them higher and smarter rate of transmission, which in theory should improve tracking under challenging conditions. This scenario can be viewed as a few commercial vessels travelling in between a large group of yachts. The last scenario, where all targets are equipped with class A transmitters is the ultimate situation for any fusion tracking system. This case would be realistic in a crowded professional working

area, for instance harbours, fishing areas and off-shore installations.

5.3 Simulation

For each simulation

AIS scenario configuration					
Target 0	Scenario				
	1	2	3	4	
0	—	—	B	—	A
1	—	—	B	—	A
2	—	—	B	—	A
3	—	B	B	A	A
4	—	—	B	—	A
5	—	—	B	—	A
6	—	—	B	—	A
6	—	—	B	—	A
6	—	—	B	—	A
7	—	—	B	—	A
8	—	—	B	—	A
9	—	B	B	A	A
10	—	—	B	—	A
11	—	—	B	—	A
12	—	B	B	A	A
13	—	—	B	—	A

Table 5.1: AIS scenario configuration

Chapter 6

Discussion

Discussion here.

6.1 Alternative design / open questions

What to do when a track previously associated with an MMSI no longer has new AIS measurements from that MMSI inside its gate? Make an extra hypothesis with a jump? An how to score this?

What to do with unused AIS measurements, i.e. ones that are not previously associated with any tracks and are not within any gate? Use them as measurements in the initialization procedure? Initiate them as new tracks immediately?

In what ways can AIS meta-data improve the radar tracking? From AIS length information, it might be possible to estimate the maximum turning rate for a vessel.

Chapter 7

Future work

Chapter 8

Conclusion

Conclusion here.

Appendices

Appendix **A**

Initialization time plot

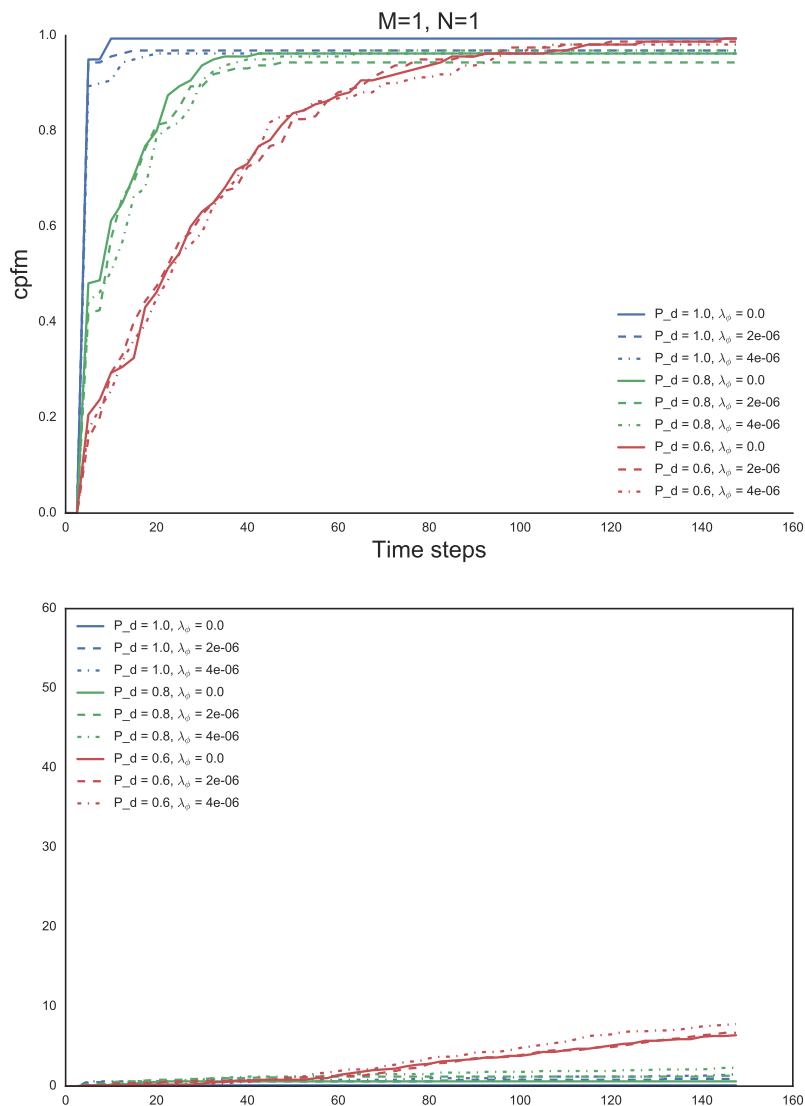


Figure A.1: Initialization time (1/1)

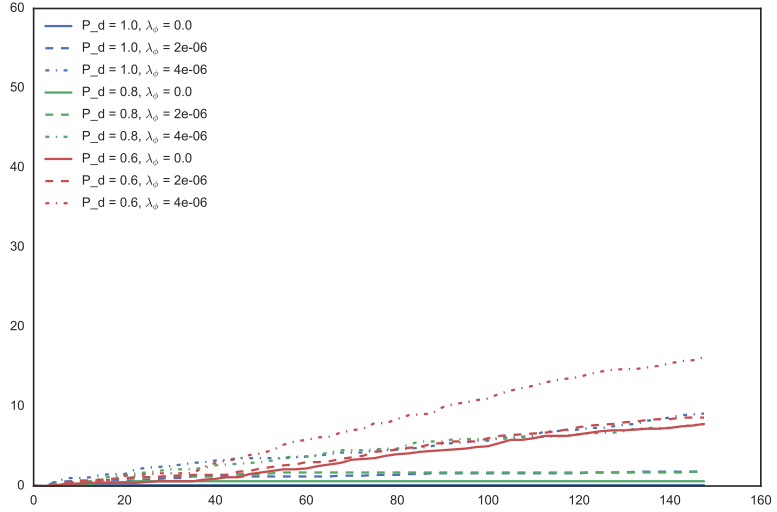
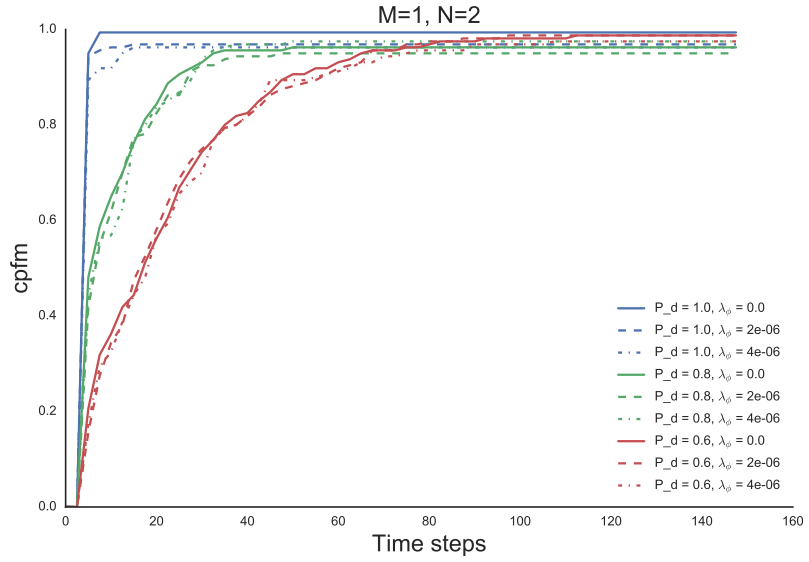


Figure A.2: Initialization time (1/2)

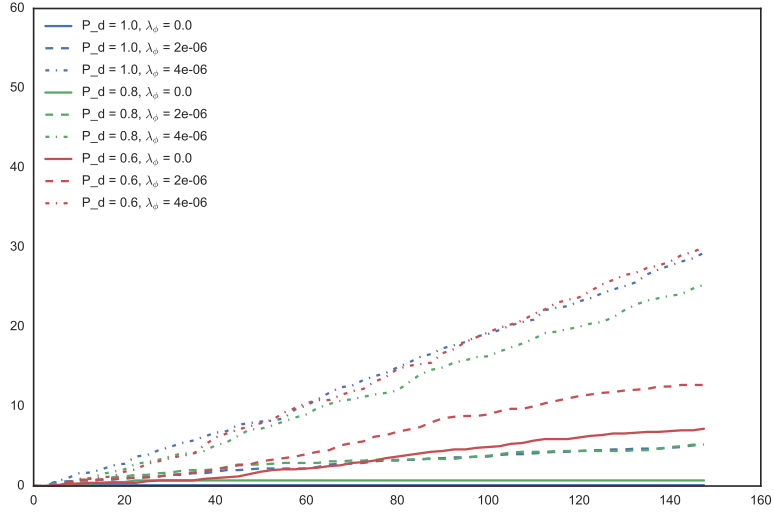
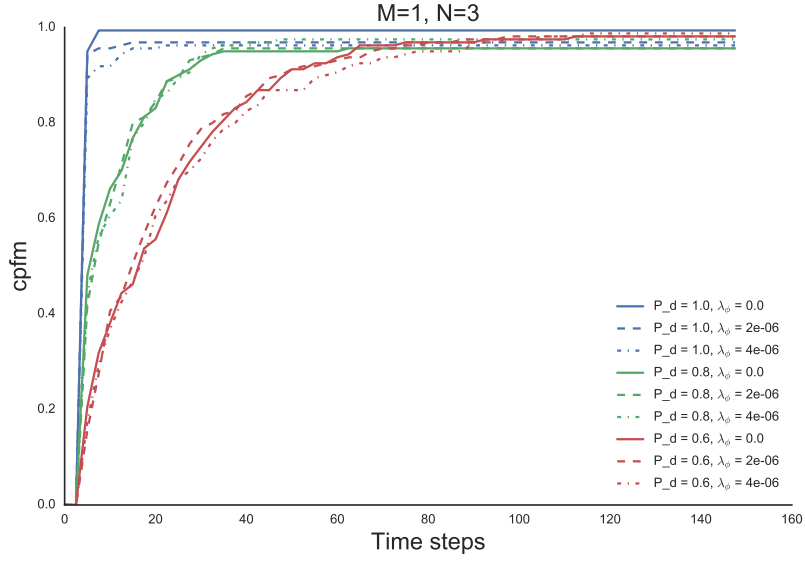


Figure A.3: Initialization time (1/3)

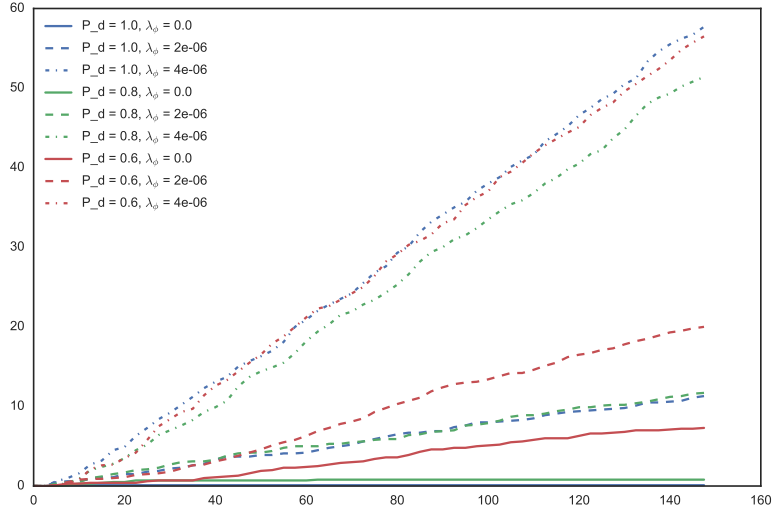
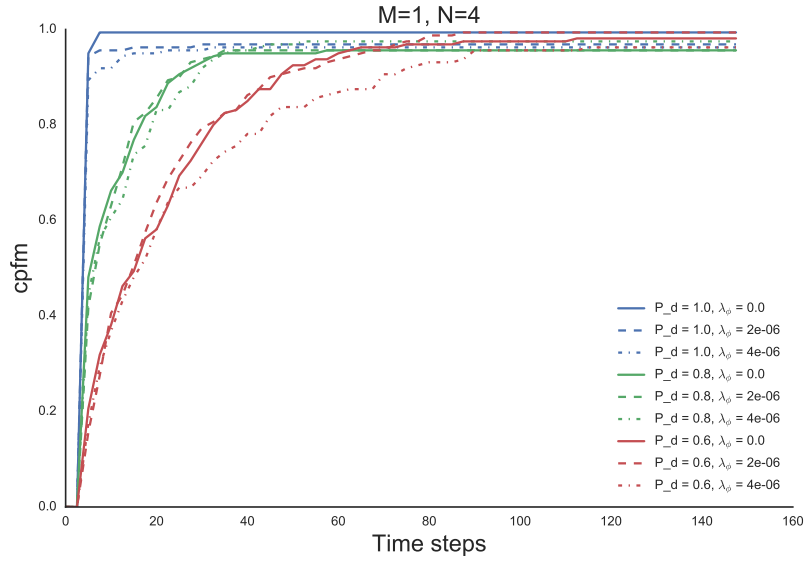


Figure A.4: Initialization time (1/4)

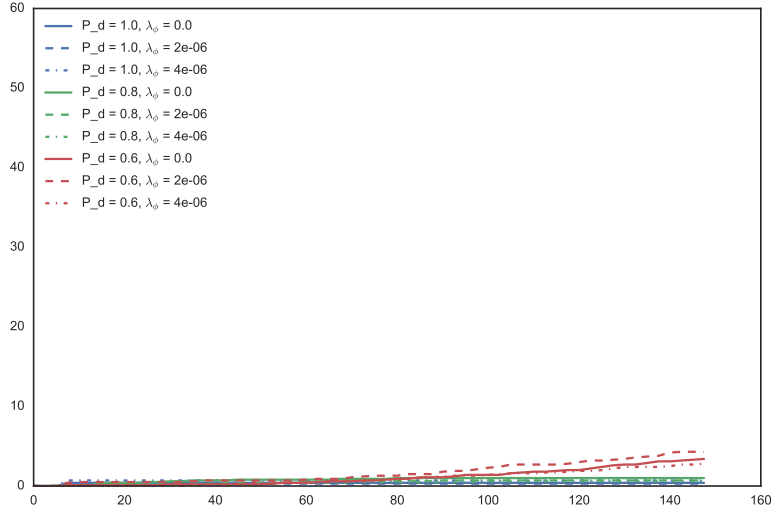
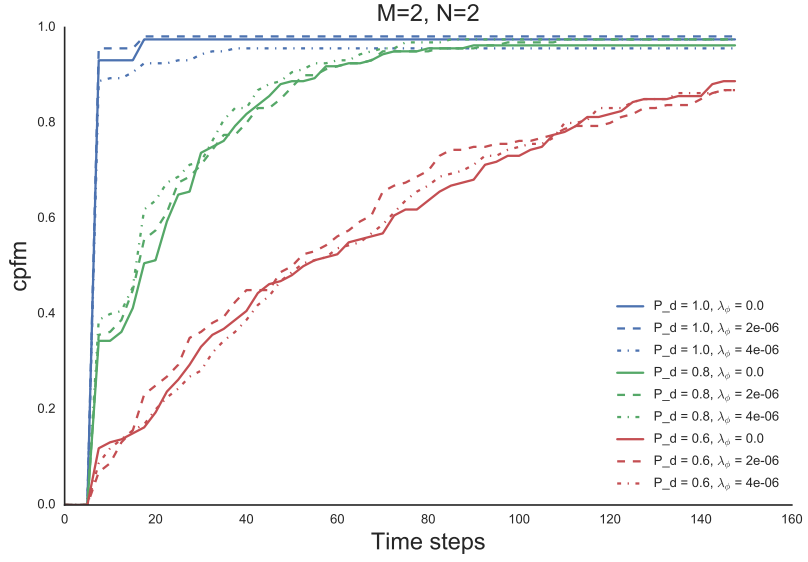


Figure A.5: Initialization time (2/2)

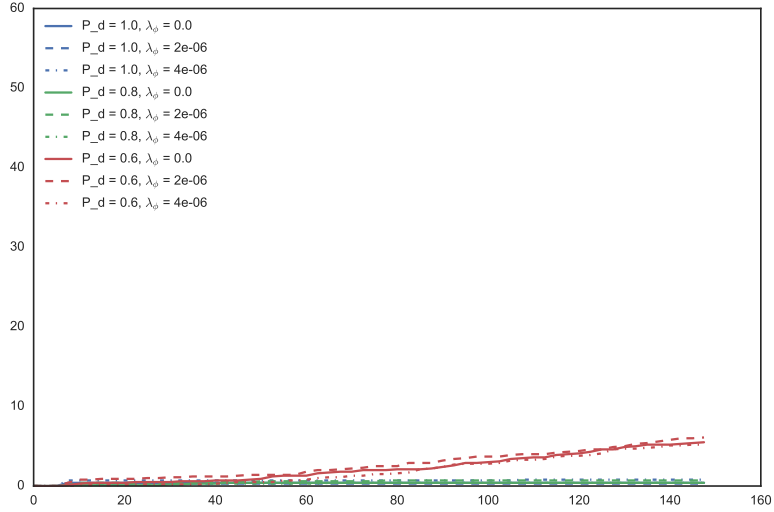
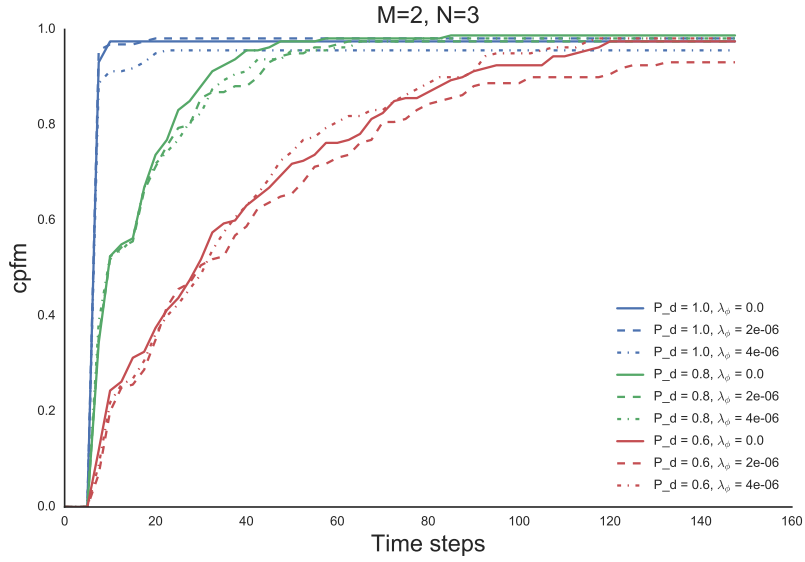


Figure A.6: Initialization time (2/3)

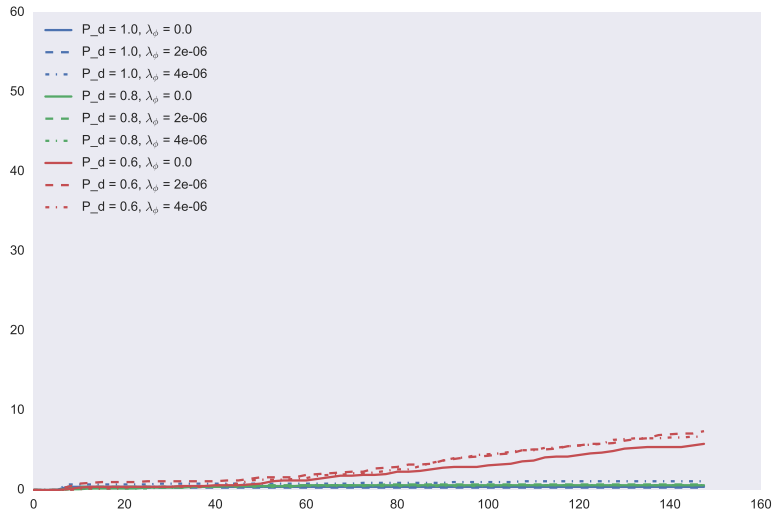
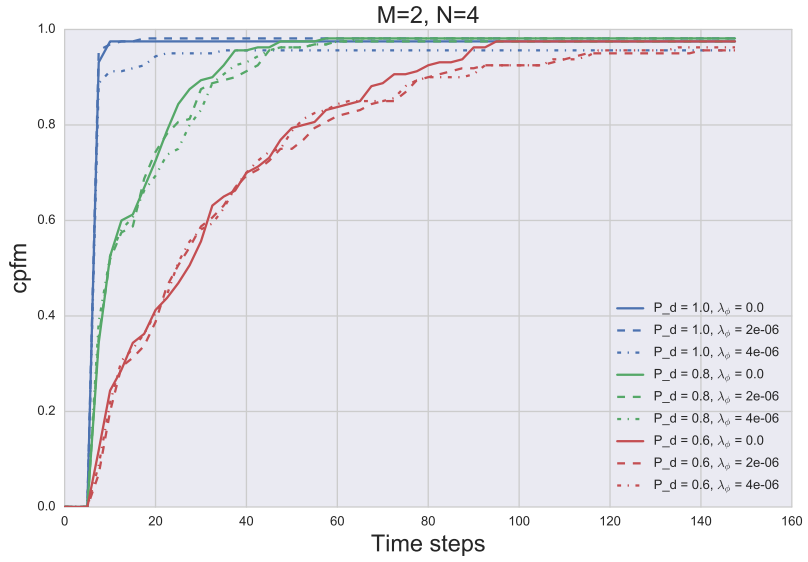


Figure A.7: Initialization time (2/4)

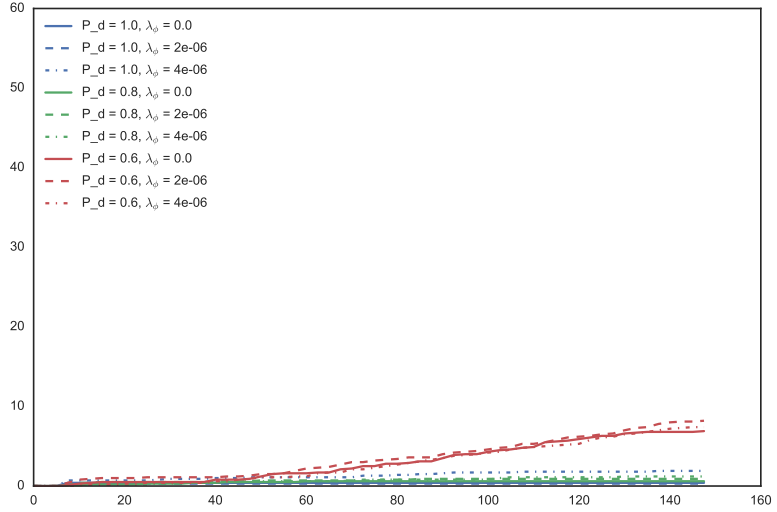
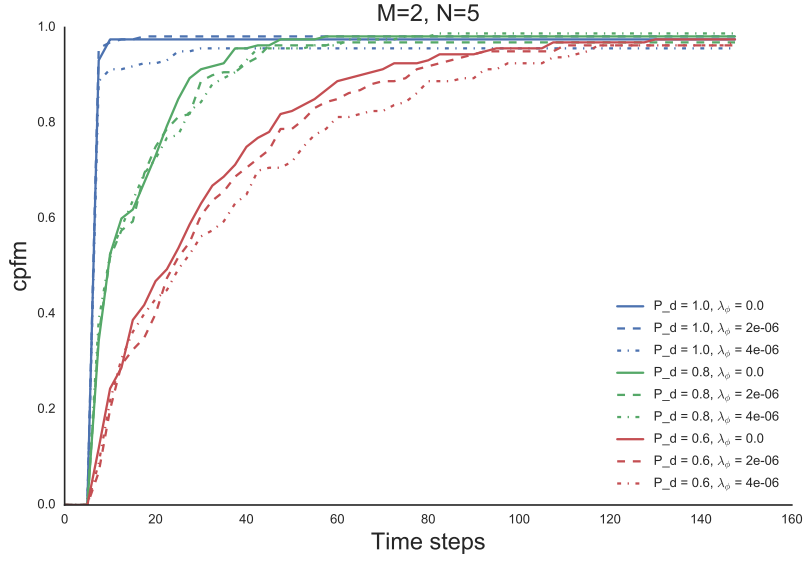


Figure A.8: Initialization time (2/5)

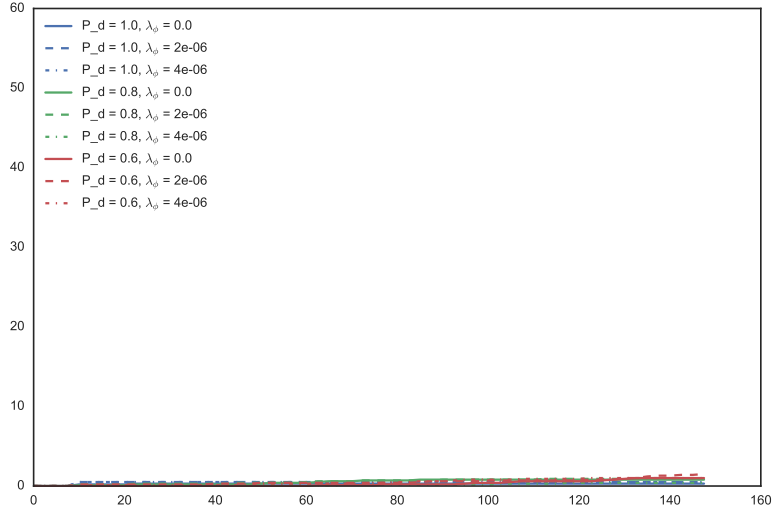
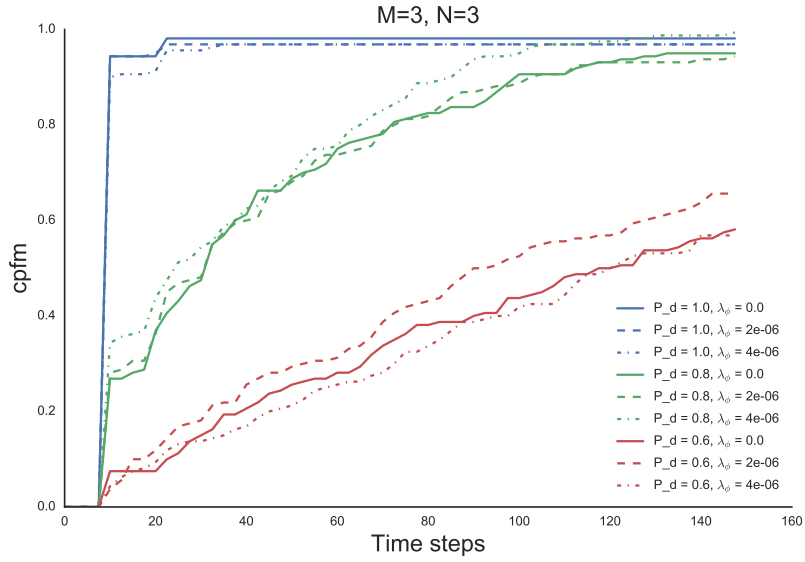


Figure A.9: Initialization time (3/3)

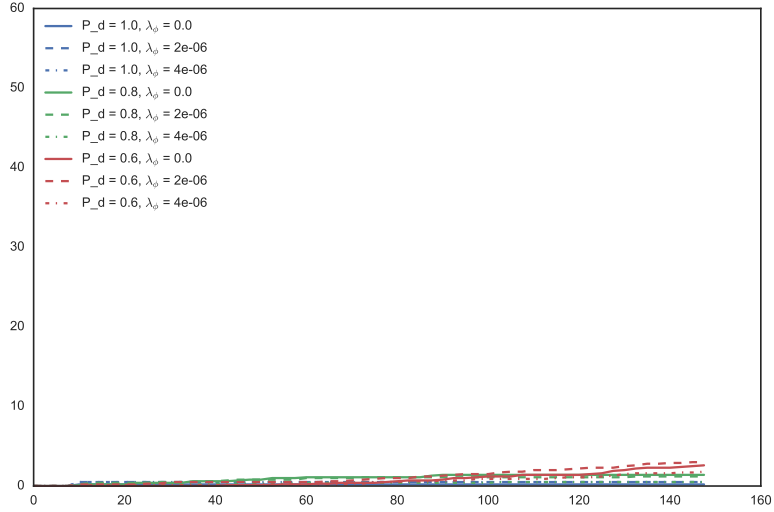
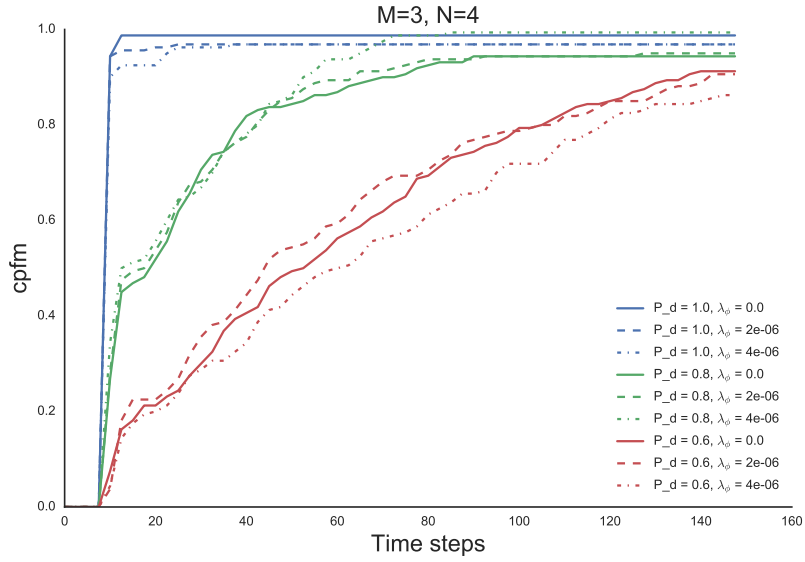


Figure A.10: Initialization time (3/4)

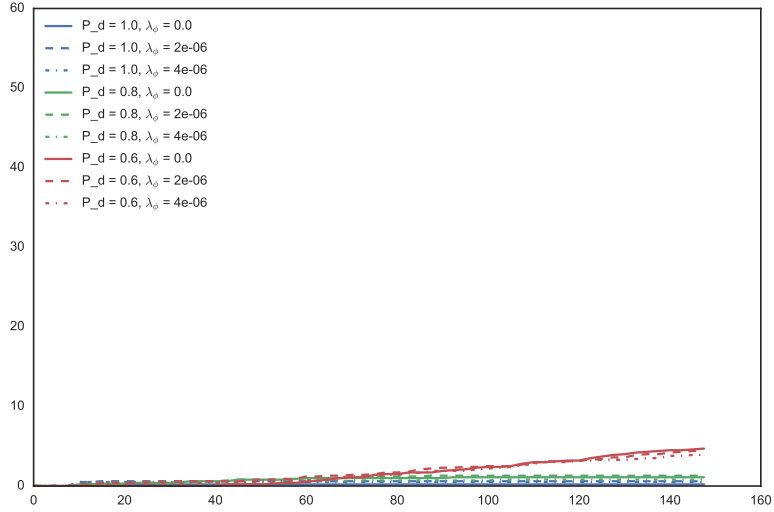
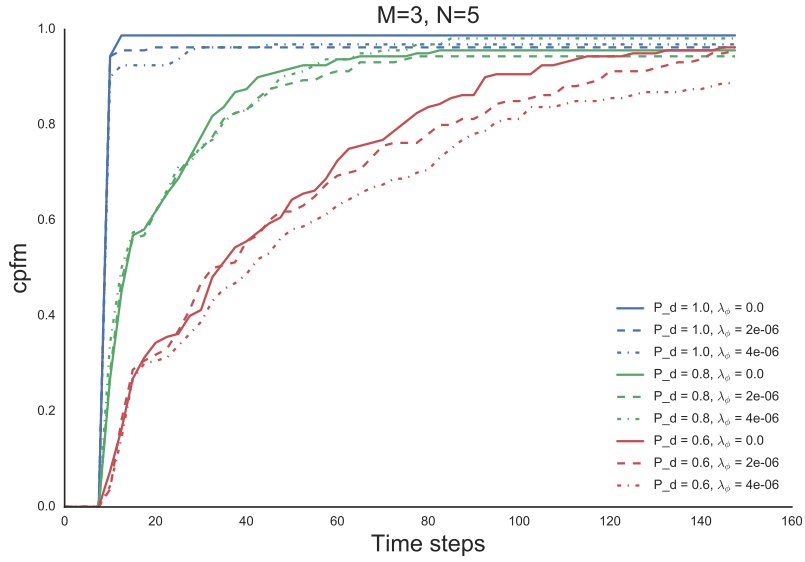


Figure A.11: Initialization time (3/5)

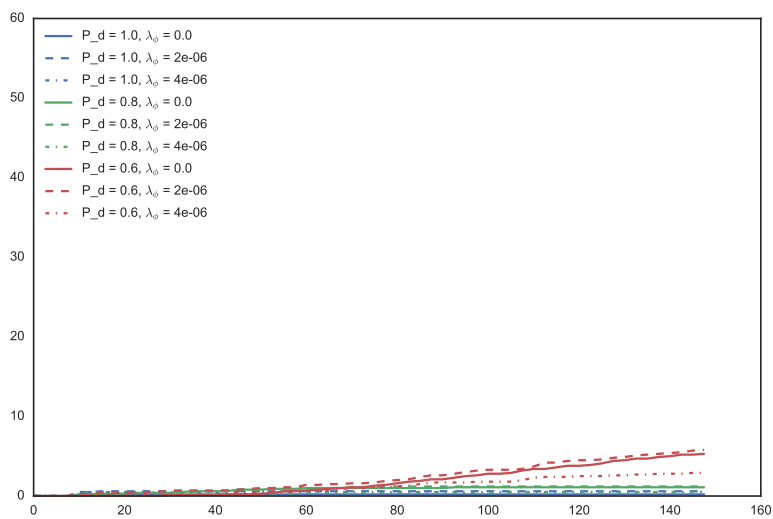
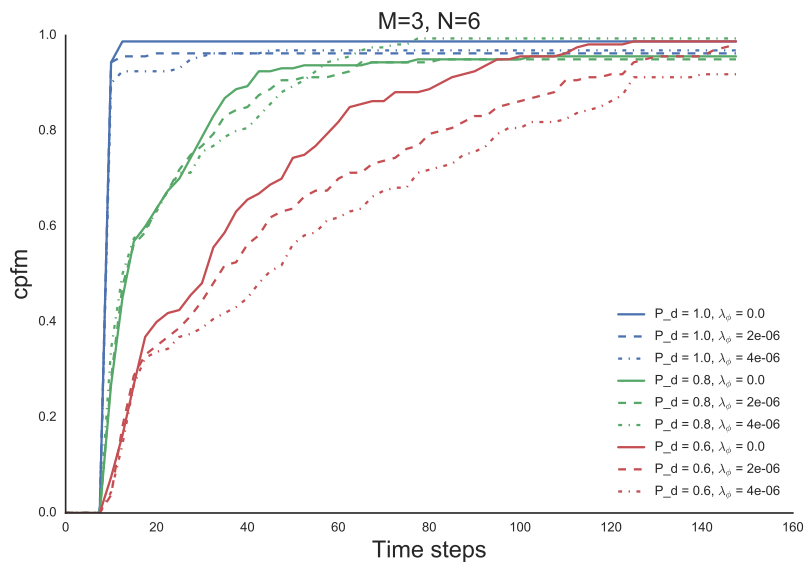


Figure A.12: Initialization time (3/6)

Appendix B

Track loss plot

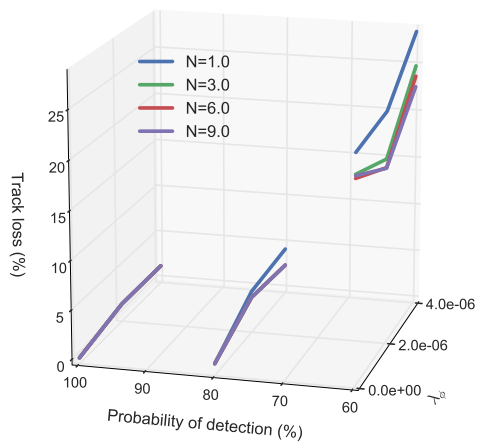


Figure B.1: Scenario 0 — Track loss

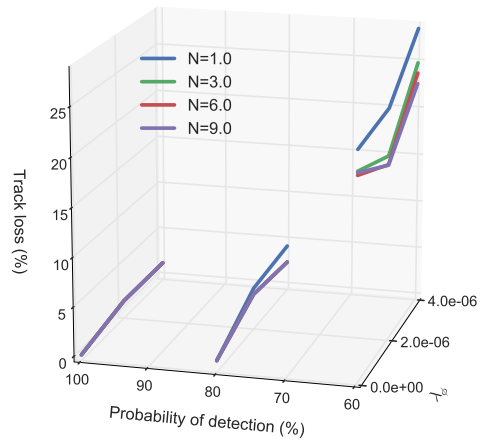


Figure B.2: Scenario 1 — Track loss

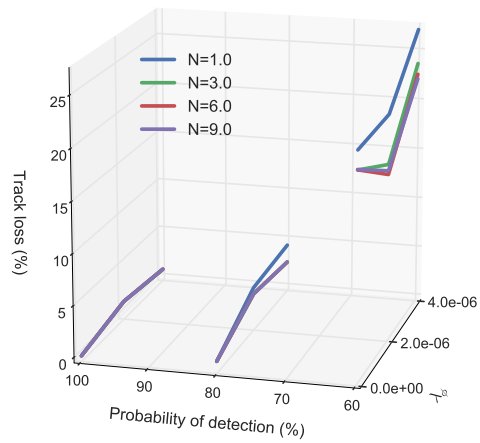


Figure B.3: Scenario 2 — Track loss

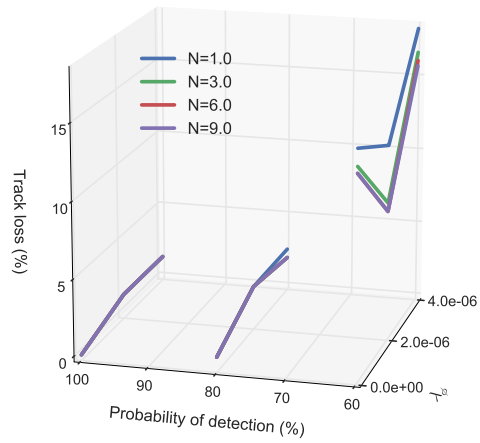


Figure B.4: Scenario 3 – Track loss

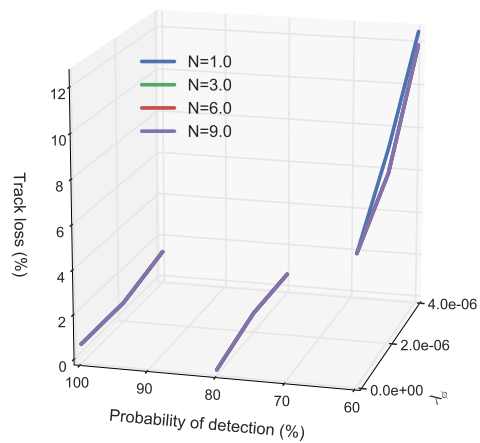


Figure B.5: Scenario 4 – Track loss

Appendix C

Tracking percentage plot

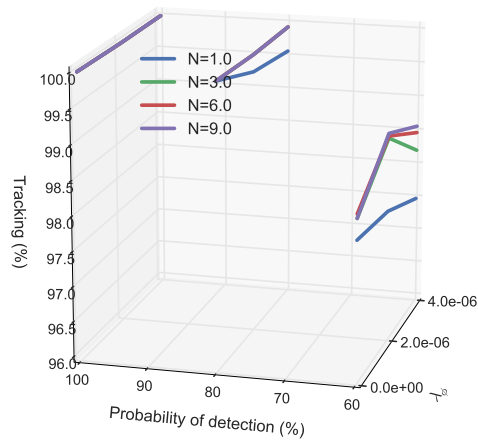


Figure C.1: Scenario 0 — Tracking percentage

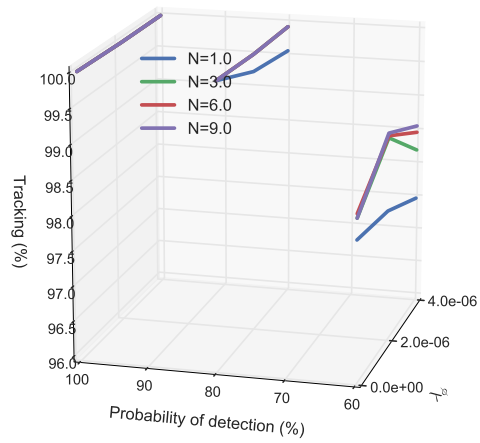


Figure C.2: Scenario 1 — Tracking percentage

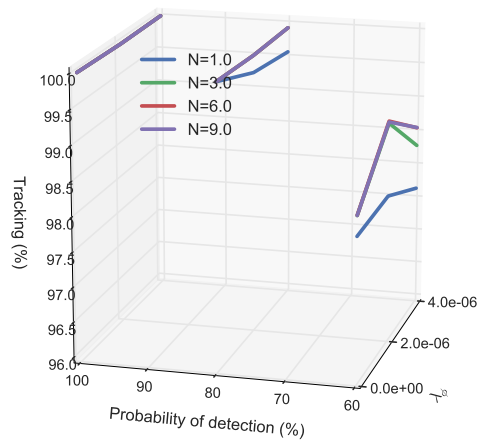


Figure C.3: Scenario 2 — Tracking percentage

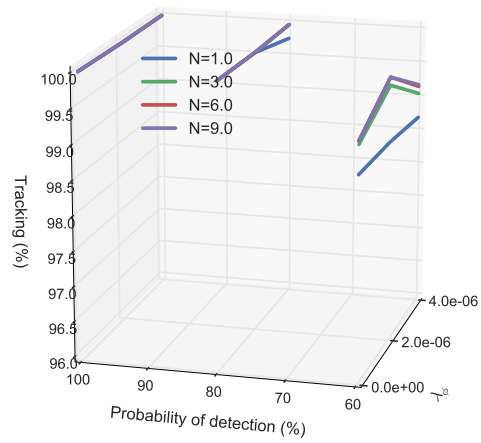


Figure C.4: Scenario 3 — Tracking percentage

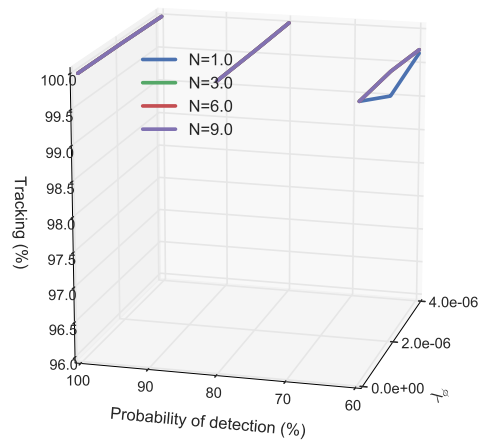


Figure C.5: Scenario 4 — Tracking percentage

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