(2) $$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = ?$$

Seja $f(x,y) = \dfrac{x^3y}{2x^6+y^2}$

(•) Ao longo do eixo x ($y=0$)

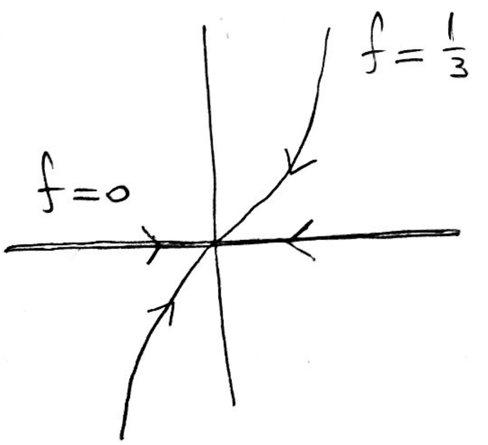
$$f(x,0) = \frac{x^3 \cdot 0}{2x^6+0^2} = 0 \quad ; \text{ para } x \neq 0$$

$\Rightarrow f(x,y) \longrightarrow 0$ qdo $(x,y) \to (0,0)$ ao longo do eixo x

(•) Ao longo da curva $y = x^3$

$$f(x,x^3) = \frac{x^3 \cdot x^3}{2x^6+(x^3)^2} = \frac{x^6}{2x^6+x^6} = \frac{x^6}{3x^6} = \frac{1}{3} \quad ; \text{ para } x \neq 0$$

$\Rightarrow f(x,y) \longrightarrow \frac{1}{3}$ qdo $(x,y) \to 0$ ao longo da curva $y = x^3$



Pelo Teste dos dois caminhos temos que

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} \text{ NÃO existe}$$

(3) $f(x,y) = e^{x} \operatorname{sen} y$

Queremos encontrar a linearização de f em $(0,0)$, ou seja,

$$L(x,y) = f(0,0) + \frac{\partial f}{\partial x}(0,0)\underbrace{(x-0)}_{0} + \frac{\partial f}{\partial y}(0,0)\underbrace{(y-0)}_{0}$$

Como

$$\frac{\partial f}{\partial x}(x,y) = e^{x} \operatorname{sen} y \Longrightarrow \frac{\partial f}{\partial x}(0,0) = e^{0} \operatorname{sen} 0 = 1.0 = 0$$

$$\frac{\partial f}{\partial y}(x,y) = e^{x} \cos y \Longrightarrow \frac{\partial f}{\partial y}(0,0) = e^{0} \cos 0 = 1.1 = 1$$

$$f(0,0) = e^{0} \operatorname{sen} 0 = 1.0 = 0$$

$$\Longrightarrow L(x,y) = 0 + 0.x + 1.y = y \Longrightarrow \boxed{L(x,y) = y}$$

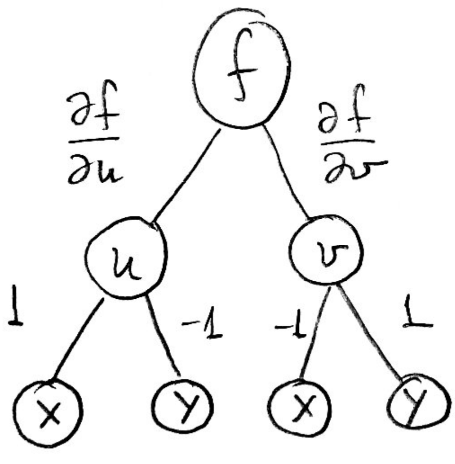
$$\therefore L(-0.01, 0.03) = 0.03 \approx f(-0.01, 0.03)$$

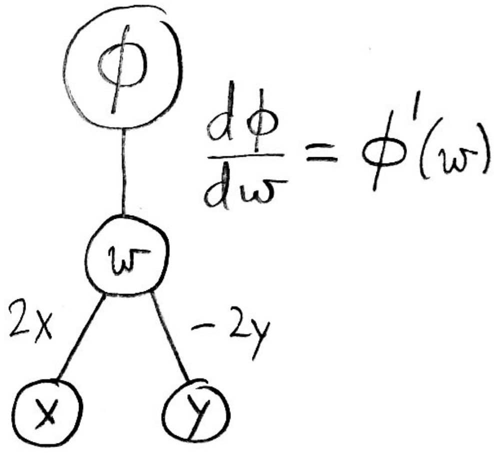
(4) $f: \mathbb{R}^2 \to \mathbb{R}$
$\phi: \mathbb{R} \to \mathbb{R}$ funções diferenciáveis

$$z = f(\underbrace{x-y}_{u}, \underbrace{y-x}_{v}) + \phi(\underbrace{x^2-y^2}_{w})$$

$u = x - y$ $\qquad\qquad$ $w = x^2 - y^2$

$v = y - x$





$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial u} \cdot (1) + \frac{\partial f}{\partial v} \cdot (-1) + \phi'(w) \cdot 2x$$

$$\frac{\partial z}{\partial v} = \frac{\partial f}{\partial u} \cdot (-1) + \frac{\partial f}{\partial v} \cdot (1) + \phi'(w) \cdot (-2y)$$

(5) $f(x,y) = \begin{cases} \dfrac{5xy^2}{\sqrt{x^2+y^2}} & ; \text{ se } (x,y) \neq (0,0) \\ 0 & ; \text{ se } (x,y) = (0,0) \end{cases}$

(a) Como $\dfrac{5xy^2}{\sqrt{x^2+y^2}} = \underbrace{5y^2}_{\to 0} \cdot \underbrace{\dfrac{x}{\sqrt{x^2+y^2}}}_{\text{limitada}} \longrightarrow 0$

pois $\left| \dfrac{x}{\sqrt{x^2+y^2}} \right| = \dfrac{|x|}{\sqrt{x^2+y^2}} \leq 1$

$\Rightarrow \lim\limits_{(x,y)\to(0,0)} f(x,y) = 0 = f(0,0)$

$\therefore$ f é contínua em $(0,0)$

(b) $\dfrac{\partial f}{\partial x}(0,0) = \lim\limits_{h\to 0} \dfrac{f(0+h,0) - f(0,0)}{h}$

$= \lim\limits_{h\to 0} \dfrac{f(h,0) - 0}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{5h.0^2}{\sqrt{h^2+0^2}}}{h}$

$= \lim\limits_{h\to 0} \dfrac{0}{h} = 0$

$\dfrac{\partial f}{\partial y}(0,0) = \lim\limits_{h\to 0} \dfrac{f(0,h)}{h} = \lim\limits_{h\to 0} \dfrac{0}{h} = \lim\limits_{h\to 0} 0 = 0$

c) f é dif em (0,0) se

(i) $\frac{\partial f}{\partial x}(0,0)$ e $\frac{\partial f}{\partial y}(0,0)$ existem (ok)

e

(ii) $\lim\limits_{(h,k)\to(0,0)} \frac{E(h,k)}{\sqrt{h^2+k^2}} = 0$

$$E(h,k) = f(0+h, 0+k) - \underbrace{f(0,0)}_{0} - \underbrace{\frac{\partial f}{\partial x}(0,0)}_{0}(x-0) - \underbrace{\frac{\partial f}{\partial y}(0,0)}_{0}(y-0)$$

$$= f(h,k)$$

$$= \frac{5hk^2}{\sqrt{h^2+k^2}}$$

$$\Rightarrow \frac{E(h,k)}{\sqrt{h^2+k^2}} = \frac{\frac{5hk^2}{\sqrt{h^2+k^2}}}{\sqrt{h^2+k^2}} = \frac{5hk^2}{h^2+k^2} = \underbrace{5h}_{\to 0} \cdot \underbrace{\frac{k^2}{h^2+k^2}}_{\text{limitada}}$$

pois $\left|\frac{k^2}{h^2+k^2}\right| = \frac{k^2}{h^2+k^2} \leq 1$

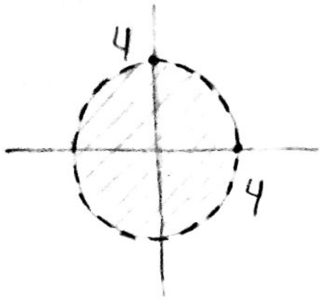
$$\Rightarrow \lim_{(h,k)\to(0,0)} \frac{E(h,k)}{\sqrt{h^2+k^2}} = 0$$

∴ f é dif em (0,0)

① $f(x,y) = \dfrac{4}{\sqrt{16-x^2-y^2}}$

(a) $\text{Dom } V = \{(x,y)\in\mathbb{R}^2 \mid 16-x^2-y^2>0\} = \{(x,y)\in\mathbb{R}^2 \mid x^2+y^2<16\}$

Como $x^2+y^2=16$ é uma circunf. c/ centro na origem e raio 4

$\Rightarrow$ 

$\text{Im } f = \{f(x,y) \mid (x,y)\in \text{Dom } f\}$

Seja $(x,y)\in \text{Dom } f \Rightarrow 0 \le x^2+y^2 < 16$

$\Rightarrow 0 \ge -x^2-y^2 > -16$

$\Rightarrow 16 \ge 16-x^2-y^2 > 0$

$\Rightarrow 4 \ge \sqrt{16-x^2-y^2}$

$\Rightarrow 1 = \frac{4}{4} \le \dfrac{4}{\sqrt{16-x^2-y^2}} = f(x,y)$

$\therefore \text{Im } f = [1, +\infty)$

(b) $C_k = \{(x,y)\in\mathbb{R}^2 \mid f(x,y)=k\} = \{(x,y)\in\mathbb{R}^2 \mid \dfrac{4}{\sqrt{16-x^2-y^2}} = k\}$

Como $\dfrac{4}{k} = \sqrt{16-x^2-y^2} \Rightarrow \dfrac{16}{k^2} = 16-x^2-y^2 \Rightarrow \underbrace{x^2+y^2=\left(16-\frac{16}{k^2}\right)}$

circunf's c/centro na origem e raio $\sqrt{16-\frac{16}{k^2}}$

Por exemplo

$k=1 \Rightarrow x^2+y^2=0$

$k=2 \Rightarrow x^2+y^2=16-\frac{16}{4}=12$

$k=4 \Rightarrow x^2+y^2=16-1=15$