(2) $$\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{(x^2+y^2)\sqrt{x^2+y^2}} = ?$$

Sol 1: Como

$$\frac{x^2y^2}{(x^2+y^2)\sqrt{x^2+y^2}} = \underbrace{\frac{x^2}{x^2+y^2}\cdot\frac{y}{\sqrt{x^2+y^2}}}_{\text{limitado}}\cdot \underset{\to 0}{y} \Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{(x^2+y^2)\sqrt{x^2+y^2}} = 0$$

Pois $$\left|\frac{x^2}{x^2+y^2}\cdot\frac{y}{\sqrt{x^2+y^2}}\right| = \frac{x^2}{x^2+y^2}\cdot\frac{|y|}{\sqrt{x^2+y^2}} \leq 1\cdot\frac{|y|}{\sqrt{x^2+y^2}} \leq 1$$

Sol 2: Escrevendo

$$\frac{x^2y^2}{(x^2+y^2)\sqrt{x^2+y^2}} = \underset{\to 0}{\left(\frac{x^2y}{x^2+y^2}\right)}\cdot\underbrace{\frac{y}{\sqrt{x^2+y^2}}}_{\text{limitado}} \Rightarrow \lim_{(x,y)\to(0,0)} \frac{x^2y^2}{(x^2+y^2)\sqrt{x^2+y^2}} = 0$$

Pois $$\left|\frac{y}{\sqrt{x^2+y^2}}\right| = \frac{|y|}{\sqrt{x^2+y^2}} \leq 1$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2} = \lim \underbrace{\frac{x^2}{x^2+y^2}}_{\text{limitada}}\cdot \underset{\to 0}{y} = 0$$

pois $\frac{x^2}{x^2+y^2} \leq 1$

(3) $f(x,y) = e^{x^2+y}$

Queremos encontrar a linearização de f em $(0,0)$, ou seja,

$$L(x,y) = f(0,0) + \frac{\partial f}{\partial x}(0,0)\underbrace{(x-0)}_{x} + \frac{\partial f}{\partial y}(0,0)\underbrace{(y-0)}_{y}$$

Como

$$\frac{\partial f}{\partial x}(x,y) = e^{x^2+y}\cdot 2x \implies \frac{\partial f}{\partial x}(0,0) = e^{0^2+0}\cdot 2\cdot 0 = 0$$

$$\frac{\partial f}{\partial y}(x,y) = e^{x^2+y}\cdot 1 \implies \frac{\partial f}{\partial y}(0,0) = e^{0^2+0} = e^0 = 1$$

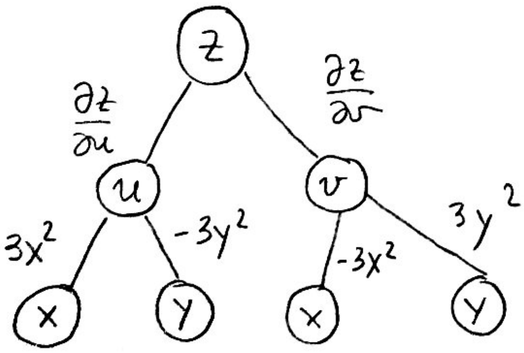
$$f(0,0) = e^{0^2+0} = e^0 = 1$$

$$\implies \boxed{L(x,y) = 1 + 0.x + 1.y = 1+y}$$

$$\therefore L(0.01, -0.02) = 1 + (-0,02) = 0.98 \approx f(0.01, -0.02)$$

(4) $f: \mathbb{R}^2 \to \mathbb{R}$ diferenciável

$$z = f(\underbrace{x^3-y^3}_{u}, \underbrace{y^3-x^3}_{v}) \quad \text{onde} \quad \begin{array}{l} u = x^3 - y^3 \\ v = y^3 - x^3 \end{array}$$



$$\frac{\partial u}{\partial x} = 3x^2 \qquad \frac{\partial v}{\partial x} = -3x^2$$

$$\frac{\partial u}{\partial y} = -3y^2 \qquad \frac{\partial v}{\partial y} = 3y^2$$

$$y^2\frac{\partial z}{\partial x} + x^2\frac{\partial z}{\partial y} = y^2\left(3x^2\frac{\partial z}{\partial u} - 3x^2\frac{\partial z}{\partial v}\right) + x^2\left(-3y^2\frac{\partial z}{\partial u} + 3y^2\frac{\partial z}{\partial v}\right)$$

$$= 3x^2y^2\left(\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}\right) - 3x^2y^2\left(\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}\right) = 0$$

5b) <u>Outra solução</u>

$$f \text{ é dif em } (0,0) \text{ se } \begin{cases} \dfrac{\partial f}{\partial x}(0,0) \text{ e } \dfrac{\partial f}{\partial y}(0,0) \text{ existem} \\ \text{e} \\ \lim\limits_{(h,k)\to(0,0)} \dfrac{E(h,k)}{\sqrt{h^2+k^2}} = 0 \end{cases}$$

Como $E(h,k) = f(0+h,0+k) - \underbrace{f(0,0)}_{0} - \underbrace{\frac{\partial f}{\partial x}(0,0)}_{0} h - \underbrace{\frac{\partial f}{\partial y}(0,0)}_{0} k = \dfrac{5h^2k}{h^6+k^2}$

$$\Downarrow$$

$$\frac{E(h,k)}{\sqrt{h^2+k^2}} = \frac{5h^2k}{(h^6+k^2)\sqrt{h^2+k^2}} = \begin{cases} 0 \; ; & \text{se } h=0 \\ \dfrac{5h^3}{(h^6+h^2)\sqrt{2h^2}} \; ; & \text{se } h=k \end{cases}$$

Observe que

$$\lim_{h\to 0} \frac{5h^3}{(h^6+h^2)\sqrt{2h^2}} = \lim_{h\to 0} \frac{5h}{\sqrt{2}\,|h|\,(h^4+1)} \quad \text{Não existe}$$

pois os limites laterais são distintos.

$$\therefore \lim_{(h,k)\to(0,0)} \frac{E(h,k)}{\sqrt{h^2+k^2}} \quad \text{NÃO existe}$$

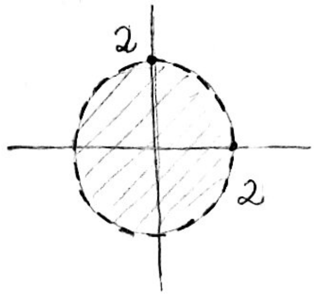
Daí

f NÃO é dif em $(0,0)$

(1) $V(x,y) = \dfrac{1}{\sqrt{4-x^2-y^2}}$

(a) $\text{Dom } V = \{(x,y) \in \mathbb{R}^2 \mid 4-x^2-y^2 > 0\} = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 < 4\}$

Como $x^2+y^2=4$ é uma circunf. c/ centro na origem e raio 2

$\Rightarrow$ 

$\text{Im } V = \{V(x,y) \mid (x,y) \in \text{Dom } V\}$

Seja $(x,y) \in \text{Dom } V \Rightarrow 0 \leq x^2+y^2 < 4$

$\Rightarrow 0 \geq -x^2-y^2 > -4$

$\Rightarrow 4 \geq 4-x^2-y^2 > 0$

$\Rightarrow 2 \geq \sqrt{4-x^2-y^2}$

$\Rightarrow \dfrac{1}{2} \leq \dfrac{1}{\sqrt{4-x^2-y^2}} = V(x,y)$

$\therefore \text{Im } V = [\tfrac{1}{2}, +\infty)$

(b) $C_k = \{(x,y) \in \mathbb{R}^2 \mid V(x,y) = k\} = \{(x,y) \in \mathbb{R}^2 \mid \dfrac{1}{\sqrt{4-x^2-y^2}} = k\}$

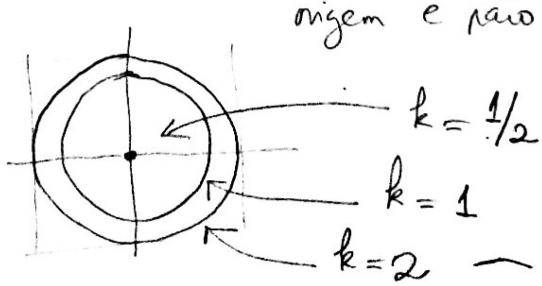
Como $\dfrac{1}{k} = \sqrt{4-x^2-y^2} \Rightarrow \dfrac{1}{k^2} = 4-x^2-y^2 \Rightarrow \underbrace{x^2+y^2 = \left(4-\tfrac{1}{k^2}\right)}_{\text{circunf's c/ centro na origem e raio } \sqrt{4-\frac{1}{k^2}}}$

Por exemplo

$k = 1/2 \Rightarrow x^2+y^2 = 0$

$k = 1 \Rightarrow x^2+y^2 = 3$

$k = 2 \Rightarrow x^2+y^2 = \dfrac{15}{4}$



(5) $f(x,y) = \begin{cases} \dfrac{5x^2y}{x^6+y^2} & \text{se } (x,y) \neq (0,0) \\ 0 & \text{se } (x,y) = (0,0) \end{cases}$

a) $$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h} = \lim_{h \to 0} \frac{\frac{5h^2 \cdot 0}{h^6+0}}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$$

$$\frac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \frac{f(0,h+0) - f(0,0)}{h} = \lim_{h \to 0} \frac{\frac{5 0^2 h}{0^6+h^2}}{h} = 0$$

b) Estudemos a continuidade de f em $(0,0)$

Ao longo do eixo x : $f(x,0) = \dfrac{5x^2 \cdot 0}{x^6+0^2} = \dfrac{0}{x^6} = 0$

$\Rightarrow f(x,y) \to 0$ qdo $(x,y) \to (0,0)$ ao longo do eixo X

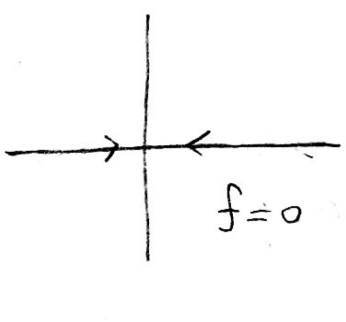
Ao longo da curva $y = x^3$ $(x>0)$: $f(x,x^3) = \dfrac{5x^2 \cdot x^3}{x^6+x^6} = \dfrac{5x^5}{2x^6} = \dfrac{5}{2x}$

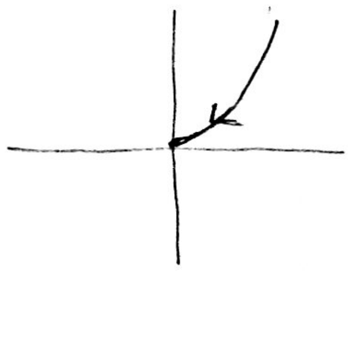
$\Rightarrow f(x,y) \to +\infty$ qdo $(x,y) \to (0,0)$ ao longo da curva $y = x^3$ (para $x > 0$)

Pelo Teste dos dois caminhos temos que

$$\lim_{(x,y) \to (0,0)} f(x,y) \text{ NÃO existe}$$

Logo f não é contínua na origem e assim, NÃO é diferenciável em $(0,0)$





$y = x^3$

$x > 0$