# Section 1.10

- solar radiation incident on a horizontal plane outside of the atmosphere: $\boxed{G_o = G_{sc}\left[1+0.033\cos\left(\frac{360}{365}n\right)\right]\cos\theta_z}$   solar constant $G_{sc} = 1367\ \frac{W}{m^2}$

(1.6.1a) $\delta = 23.45\sin\left[360\left(\frac{284+n}{365}\right)\right]$

(1.6.1b) - more accurate equation but need another equation (1.4.2)

- for horizontal surface, $\boxed{\cos\theta_z = \cos\phi\cos\delta\cos\omega + \sin\phi\sin\delta}$

ex: June 27, 2021 - Phoenix, Arizona $(\overset{lat}{33.44°}, \overset{long}{-112.07°})$ @ 1pm

$\Rightarrow \phi = 33.44°,\ \delta = 23.3352°,\ \omega = 15°$

$\Rightarrow \boxed{\cos\theta_z = 0.9584}$

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| |
|---|
| $\phi$ = latitude (-90° ~ 90°) |
| $\delta$ = declination (-23.45° ~ 23.45°) - north is positive |
| $\beta$ = angle of plate (0° ~ 180°) |
| $\omega$ = hour angle (angle of sun off local meridian) → 15° per hour off solar noon (morning negative, afternoon positive) |

$G_o = G_{sc}\left[1+0.033\cos\left(\frac{360}{365}n\right)\right]\cos\theta_z$

$\Rightarrow \boxed{G_o = 1.2670 \times 10^3\ \frac{W}{m^2}}$

- integrated daily extraterrestrial radiation on a horizontal surface: $\boxed{H_o = \frac{24 \times 3600\, G_{sc}}{\pi}\left[1+0.033\cos\left(\frac{360}{365}n\right)\right]\left[\cos\phi\cos\delta\sin\omega_s + \frac{\pi\omega_s}{180}\sin\phi\sin\delta\right]}$ - not necessary
  - sunset hour angle: $\boxed{\omega_s = \cos^{-1}\left[-\tan\phi\tan\delta\right]}$

$\Rightarrow \boxed{\omega_s = 106.5520°}$

$\Rightarrow \boxed{H_o = 4.1463 \times 10^7\ \frac{J}{m^2}}$ → or can look at Figure (1.10.1) or Table (1.10.1) to get $H_o$

- can integrate $G_o$ to get $I_o$ and get anisotropy index, $A_i$

$A_i = \frac{I_b}{I_o} = \frac{G_b}{G_o}$, from Excel sheet, DNI $(G_b) = 876\ \frac{W}{m^2}$

$\Rightarrow \boxed{A_i = 0.6914}$   DHI $(G_d) = 147\ \frac{W}{m^2}$

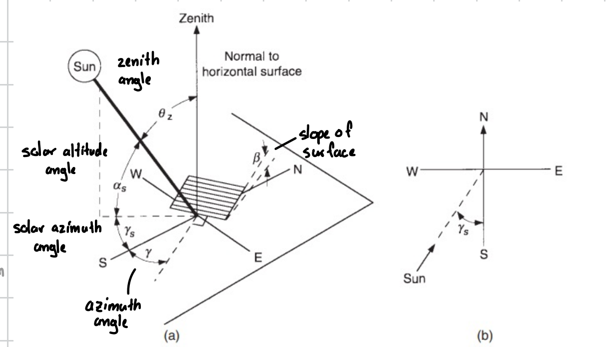


**Figure 1.6.1** (a) Zenith angle, slope, surface azimuth angle, and solar azimuth angle for a tilted surface. (b) Plan view showing solar azimuth angle.

## Example 2.16.1

- from excel sheet, solar zenith angle, $\gamma_s = 11.93°$ - changes every hr
- for max DNI, need $\beta = \theta_z \longrightarrow \cos\beta = \cos\theta_z$ - only for calculating $\delta$ value at 1pm
- for max DNI, need $\gamma = \gamma_s$

$R_b = \frac{\cos\theta}{\cos\theta_z} \overset{\text{Eqn (1.6.3)}}{=} \frac{\cos\theta_z\cos\beta + \sin\theta_z\sin\beta\cos(\gamma_s-\gamma)}{\cos\theta_z} = \frac{\cos^2\theta_z + \sin^2\theta_z}{\cos\theta_z} = \frac{1}{\cos\theta_z}$

$\Rightarrow \boxed{R_b = 1.0434}$ - ratio of beam radiation on tilted surface to that on a horizontal surface

$f = \sqrt{\frac{I_b}{I}} = \sqrt{\frac{G_b}{G_b+G_d}} = \sqrt{\frac{920}{126+920}} \Rightarrow \boxed{f = 0.9254}$ - modulating factor (accounting for cloudiness)

albedo (reflectance), $\rho_g = 0.31$ - avg. for earth

**Albedo of Earth Materials**

| Substance | Albedo (% reflectance) |
|---|---|
| Whole Planet | 0.31 |
| Cumulonimbus Clouds | 0.9 |
| Stratocumulus Clouds | 0.6 |
| Cirrus Clouds | 0.5 |
| Water | 0.06 - 0.1 |
| Ice & Snow | 0.7 - 0.9 |
| Sand | 0.35 |
| Grass lands | 0.18 - 0.25 |
| Deciduous forest | 0.15 - 0.18 |
| Coniferous forest | 0.09 - 0.15 |
| Rain forest | 0.07 - 0.15 |

https://www.e-education.psu.edu/earth103/node/1002#:~:text=In%20the%20above%20table%2C%20we,shown%20in%20the%20figure%20below

$I_d \approx G_d \times 3600s \quad I_b \approx G_b \times 3600s$

$\boxed{I_b = 3.1536 \times 10^6\ \frac{J}{m^2} \quad I_d = 5.292 \times 10^5\ \frac{J}{m^2}}$

$\boxed{I = 3.6828 \times 10^6\ \frac{J}{m^2}}$

$I_T = (I_b + I_d A_i)R_b + I_d(1-A_i)\left(\frac{1+\cos\beta}{2}\right)\left[1+f\sin^3\left(\frac{\beta}{2}\right)\right] + I\rho_g\left(\frac{1-\cos\beta}{2}\right)$

$\boxed{I_T = 4.7066 \times 10^6\ \frac{J}{m^2}}$

## Example 5.9.1

$(\tau\alpha) \cong 1.01\tau\alpha$   (5.5.2)

- assuming that incident angle $\theta = 0°$ (exactly perpendicular) $\longrightarrow \frac{\alpha}{\alpha_n} = 1$ ; $T = 45.6°C = 318.75K \longrightarrow$ using methods of HW2 #1, $\alpha = 0.94$
- given that $KL = 0.0125 \xrightarrow{\text{Fig 5.3.1}}$ for 2 sheets, $\tau = 0.83$

$\Rightarrow \boxed{(\tau\alpha)_b = 0.788}$

- previously, found that $\beta = 16.5883°$

$\longrightarrow$ from figure 5.4.1, $\alpha_d = 58°$ $\longrightarrow$ from eqn. 4.11.1, $\frac{\alpha}{\alpha_n} = 0.939$; from figure 5.3.1, $\tau = 0.75$ $>$ $\boxed{(\tau\alpha)_d = 0.6686}$

$\longrightarrow$ from figure 5.4.1, $\alpha_g = 81°$ $\longrightarrow$ from eqn. 4.11.1, $\frac{\alpha}{\alpha_n} = 0.5982$; from figure 5.3.1, $\tau = 0.44$ $>$ $\boxed{(\tau\alpha)_g = 0.2499}$

- putting it all together, $S = (I_b + I_d A_i) R_b (\tau\alpha)_b + I_d (1 - A_i)(\tau\alpha)_d \left(\frac{1+\cos\beta}{2}\right)\left[1 + f \sin^3\left(\frac{\beta}{2}\right)\right] + I\rho_g (\tau\alpha)_g \left(\frac{1-\cos\beta}{2}\right)$

$\Rightarrow \boxed{S = 3.1607 \times 10^6 \frac{J}{m^2}}$

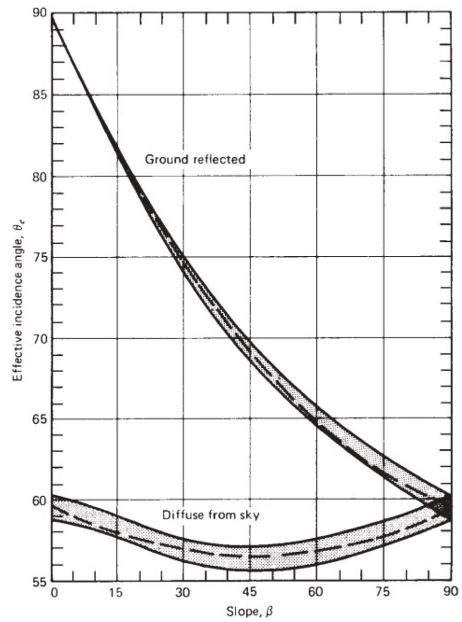


**Figure 5.4.1** Effective incidence angle of isotropic diffuse radiation and isotropic ground-reflected radiation on sloped surfaces. From Brandemuehl and Beckman (1980).

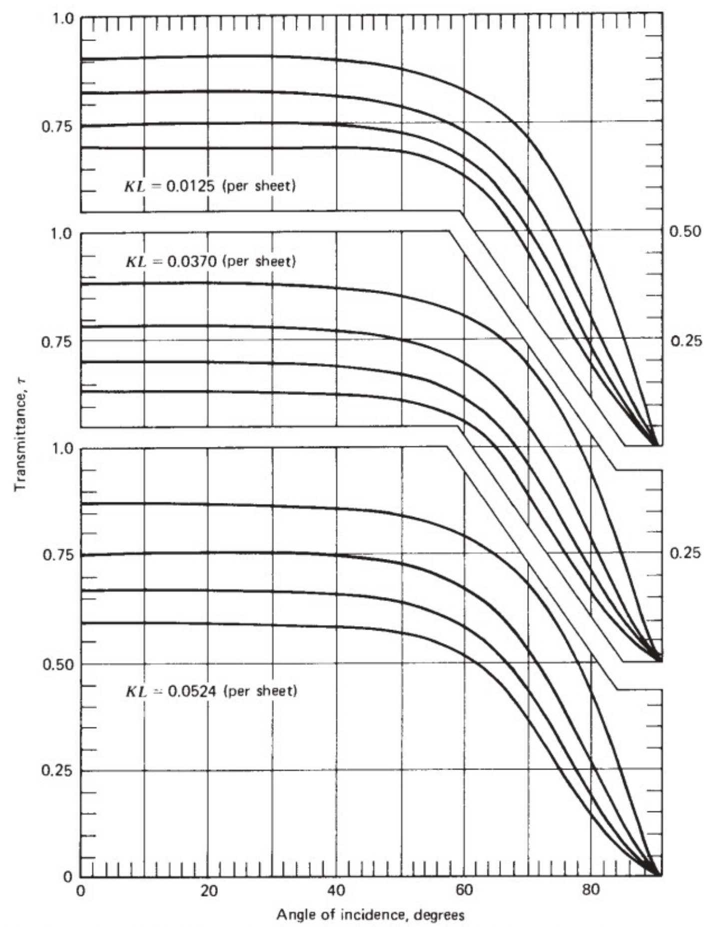


**Figure 5.3.1** Transmittance (considering absorption and reflection) of one, two, three, and four covers for three types of glass.

| | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Source | Location I | City | State | Country | Latitude | Longitude | Time Zone | Elevation | Local Time Zone | Clearsky D | Clearsky DNI Units | Clearsky G | Dew Point | DHI Units | DNI Units | GHI Units | Solar Zenith Angle Units | Tempe |
| 2 | NSRDB | 1307275 | - | b'Arizona' | b'United S | 33.44 | -112.07 | 0 | 332 | -7 | w/m2 | w/m2 | w/m2 | c | w/m2 | w/m2 | w/m2 | Degree | c |
| 3 | Year | Month | Day | Hour | Minute | DHI | Temperature | Clearsky DHI | Clearsky DNI | Clearsky GHI | Cloud Typ | Dew Point | DNI | Fill Flag | GHI | Ozone | Relative Humidity | Solar Zenith Angle | Surfac |
| 4255 | 2021 | 6 | 27 | 3 | 0 | 0 | 34.9 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0.315 | 11.77 | 94.08 | |
| 4256 | 2021 | 6 | 27 | 4 | 0 | 0 | 33.9 | 0 | 0 | 0 | 0 | 0.2 | 0 | 0 | 0 | 0.317 | 11.74 | 104.18 | |
| 4257 | 2021 | 6 | 27 | 5 | 0 | 0 | 33.1 | 0 | 0 | 0 | 0 | -0.5 | 0 | 0 | 0 | 0.318 | 11.62 | 112.78 | |
| 4258 | 2021 | 6 | 27 | 6 | 0 | 0 | 32.5 | 0 | 0 | 0 | 0 | -1.1 | 0 | 0 | 0 | 0.317 | 11.58 | 119.23 | |
| 4259 | 2021 | 6 | 27 | 7 | 0 | 0 | 32 | 0 | 0 | 0 | 0 | -0.7 | 0 | 0 | 0 | 0.315 | 12.19 | 122.77 | |
| 4260 | 2021 | 6 | 27 | 8 | 0 | 0 | 31.2 | 0 | 0 | 0 | 0 | 0.5 | 0 | 0 | 0 | 0.313 | 14 | 122.85 | |
| 4261 | 2021 | 6 | 27 | 9 | 0 | 0 | 30.4 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0.311 | 16.32 | 119.46 | |
| 4262 | 2021 | 6 | 27 | 10 | 0 | 0 | 29.7 | 0 | 0 | 0 | 0 | 3.2 | 0 | 0 | 0 | 0.31 | 18.39 | 113.13 | |
| 4263 | 2021 | 6 | 27 | 11 | 0 | 0 | 28.9 | 0 | 0 | 0 | 0 | 4.1 | 0 | 0 | 0 | 0.31 | 20.64 | 104.61 | |
| 4264 | 2021 | 6 | 27 | 12 | 0 | 0 | 28.1 | 0 | 0 | 0 | 3 | 5 | 0 | 0 | 0 | 0.311 | 22.97 | 94.57 | |
| 4265 | 2021 | 6 | 27 | 13 | 0 | 41 | 29.9 | 41 | 191 | 63 | 0 | 5.7 | 191 | 0 | 63 | 0.312 | 21.66 | 83.38 | |
| 4266 | 2021 | 6 | 27 | 14 | 0 | 97 | 33.2 | 97 | 471 | 245 | 0 | 5.6 | 471 | 0 | 245 | 0.313 | 17.85 | 71.7 | |
| 4267 | 2021 | 6 | 27 | 15 | 0 | 126 | 37.4 | 126 | 642 | 451 | 0 | 4.9 | 642 | 0 | 451 | 0.311 | 13.5 | 59.53 | |
| 4268 | 2021 | 6 | 27 | 16 | 0 | 140 | 40.3 | 140 | 750 | 650 | 0 | 4.6 | 750 | 0 | 650 | 0.309 | 11.33 | 47.09 | |
| 4269 | 2021 | 6 | 27 | 17 | 0 | 143 | 42.6 | 143 | 818 | 816 | 0 | 4.5 | 818 | 0 | 816 | 0.306 | 9.97 | 34.6 | |
| 4270 | 2021 | 6 | 27 | 18 | 0 | 162 | 44 | 162 | 832 | 930 | 0 | 4.5 | 832 | 0 | 930 | 0.306 | 9.24 | 22.46 | |
| 4271 | 2021 | 6 | 27 | 19 | 0 | 154 | 45 | 154 | 863 | 997 | 0 | 4.3 | 863 | 0 | 997 | 0.305 | 8.65 | 12.28 | |
| 4272 | 2021 | 6 | 27 | 20 | 0 | 147 | 45.6 | 147 | 876 | 1004 | 0 | 4 | 876 | 0 | 1004 | 0.304 | 8.23 | 11.93 | |
| 4273 | 2021 | 6 | 27 | 21 | 0 | 153 | 45.9 | 153 | 847 | 939 | 0 | 3.6 | 847 | 0 | 939 | 0.304 | 7.87 | 21.89 | |
| 4274 | 2021 | 6 | 27 | 22 | 0 | 161 | 45.7 | 161 | 789 | 816 | 0 | 3.2 | 789 | 0 | 816 | 0.305 | 7.74 | 34 | |
| 4275 | 2021 | 6 | 27 | 23 | 0 | 171 | 45.2 | 171 | 679 | 639 | 0 | 2.9 | 679 | 0 | 639 | 0.306 | 7.75 | 46.49 | |
| 4276 | 2021 | 6 | 28 | 0 | 0 | 166 | 44.2 | 166 | 520 | 435 | 0 | 2.5 | 520 | 0 | 435 | 0.308 | 7.95 | 58.93 | |
| 4277 | 2021 | 6 | 28 | 1 | 0 | 122 | 41.8 | 122 | 327 | 228 | 0 | 2.7 | 327 | 0 | 228 | 0.31 | 9.16 | 71.13 | |
| 4278 | 2021 | 6 | 28 | 2 | 0 | 38 | 38 | 38 | 119 | 53 | 0 | 2.3 | 119 | 0 | 53 | 0.311 | 10.88 | 82.85 | |
| 4279 | 2021 | 6 | 28 | 3 | 0 | 0 | 36.3 | 0 | 0 | 0 | 3 | 1.6 | 0 | 0 | 0 | 0.312 | 11.34 | 94.07 | |
| 4280 | 2021 | 6 | 28 | 4 | 0 | 0 | 35.6 | 0 | 0 | 0 | 3 | 1.5 | 0 | 0 | 0 | 0.313 | 11.7 | 104.18 | |
| 4281 | 2021 | 6 | 28 | 5 | 0 | 0 | 34.2 | 0 | 0 | 0 | 3 | 3 | 0 | 0 | 0 | 0.313 | 14.1 | 112.79 | |