Mid-Semester Group Project for Math 740/840

I Introduction

The group project consists of designing and running an experiment or experiments to find the optimum configuration for a paper helicopter. The basic helicopter design is described in section III of this document and students must follow the basic design specified in section II. The experimental factors to be used are also described in section II. Be sure to carefully read the entire document. Section II contains important information on the design of the helicopter and the conduct of the experiment.

Two responses are to be recorded for the experiment. The first and primary response is **flight time,** which is how long it takes the helicopter to land after being dropped from a specified height. The second response is **accuracy,** which is the distance the helicopter lands from a specified target point in the landing zone. An inexpensive plumb bob can be used to align the helicopter drop point (above) with the target location (below). The same distance, drop point, and target should be used for the entire project. Your goal in the experiment is to find an optimum helicopter configuration that gives the longest flight time and the most accuracy (closeness to target). In addition to flight time and accuracy consider the qualitative factor stability, it is highly desirable to have a helicopter configuration with a stable flight trajectory rather than one that tumbles and falls to the ground. Finally, in considering flight time, I wish to compare results from different projects, so convert the observed flight time to a **rate of descent in centimeters per second** based upon your drop height – I assume that students will most likely use different heights. **Lower rates (longer flight times) are desirable.**

You may complete the project as an individual; however I do encourage students to work in groups if possible. If you decide to work as a group, then I fully expect everyone in the group to contribute to the conduct, analysis and write-up of the experiment. The same grade is given to each member of a group.

Finally, try having some fun with the project. It is a nice exercise to apply the concepts learned in the class and teach design of experiments, but it also can be a fun exercise. We will publish the drop rates achieved by the individuals or groups submitting a report.

II The Paper Helicopter Experiment

Introduction

The "paper helicopter experiment" is a training exercise, which allows participants to apply the concepts of Experimental Design to an actual physical experiment. The exercise involves the design and construction of paper helicopters with the goal of finding a helicopter design that maximizes total flight time (the time required for a paper helicopter to land after being dropped from a predetermined height) and accuracy (the closeness of the landing point to a specified target in the landing zone). Figure 1 depicts the general shape and experimental design factors that could be considered for the construction of a paper helicopter. The three factors you are to experiment with in this project are bolded in blue in Figure 1. The flight time response should be converted to a rate so that it is possible to compare results of helicopters dropped from different heights by the various groups. As a rule the **minimum drop height should be > 10 feet** (or > 3 meters) in order to give the helicopter time to stabilize and rotate once it is dropped.

Experimental Factors

Figure 1 in section III shows a picture of a typical paper helicopter. Figure 2 in section III shows a schematic of the paper helicopter. Everyone must use a 2^3 full factorial design. The 3 factors are **Rotor Length**, **Rotor Width** and **Nose Length**. All other factors are to be held constant (see the notes below for the complete list of possible factors). It is up to each group to determine the high and low settings for each of these three factors. You may wish to build a few trial helicopters to help decide on the high and low levels. However these trial helicopters should not be used further and are not to be included in the data from the actual experiment. Keep in mind when choosing the high and low settings for each of the three factors that every helicopter design **must fit within an 8.5" by 11" sheet** of paper. The following are a list of all of the experimental factors that exist for the paper helicopter; see Figure 1. Remember you are to **vary only the three factors specified above; all others are to be held constant.**

1. **Paper Weight** – Two types of paper are usually considered. As an example standard copier paper (20 lb.) and heavier paper (e.g., 40-50 lb.). **Hold constant** at a single weight of the groups choice.

2. **Paper Clips** **(ballast)** – The number of paper clips attached to the helicopter nose as stabilizing ballast. **Hold constant** at 1 paper clip.

3. **Nose Length** – The length of the helicopter nose. The group must **choose a high and a low setting** for this factor.

4. **Nose Width** – The nose width of the helicopter. Hold constant at some size determined by the group.

5. **Rotor Length** – The length of the helicopter rotors. The group must **choose a high and a low setting** for this factor.

6. **Rotor Width** – The width of the helicopter rotors. The group must **choose a high and a low setting** for this factor.

7. **Block Length** – The length of the block (in the long direction of the paper). Note that the rotor width defines the block width as well. **Hold constant** at some size determined by the group.

8. **Block Bevel** – The block is either beveled or left square (90°). **Hold constant** at square (no bevel).

9. **Rotor Bevel** – The rotors are either beveled or left square. **Hold constant** at no bevel.

10. **Rotor Struts** **(supports)** – One can add struts to support the rotors by cutting slits in the block/rotor. See the diagram. **Hold constant** at no struts.

Designing the Experiment

Once you have determined the high and low settings for each of the three factors, and then use the **Full Factorial** platform in the JMP DOE menu to design the experiment (**DOE → Full Factorial**). In the **Factor Table** be certain to enter your predetermined high and low settings for each factor, do not use the generic -1, +1 settings provide by JMP. **Do not replicate** the entire experiment; however **add three center points (runs)** to the design before making the design table. Please note that for each center point run you need to construct a new helicopter. In order to capture experimental error three separate helicopters (EUs) need to be constructed. Your final design table should have 11 total runs in random order and each run requires the construction of an individual helicopter – do not reuse helicopters from other runs.

Prior to the actual experimentation phase of the project, spend some time brainstorming various unwanted sources of variation that might mask significant experimental effects. Discuss strategies in the execution of the experimentation that could remove or at least control these extraneous sources of variation. An example of unwanted variation might be differences in the use of a stopwatch for different members of the team or changes in air pressure or temperature in the room might be a couple others. **In your write up explicitly state the identified sources of nuisance variation and how the team controlled for the sources of variation in the conduct of the experiment.**

Finally, before actually performing the experiment the group must decide how the responses will be measure, what task each member of the team will perform, and the exact technique to be used in dropping the helicopters. Plus handling any other concerns related to the conduct of the experiment that the team discussed during the planning of the experiment.

Subsampling vs. Replication

Recall, in the discussion of one factor experiments we introduced the concept of replication vs. subsampling. Replication of an experimental trial involves applying those experimental settings to a new experimental unit (EU). In our case, the EU is an individual paper helicopter. So, replication of a trial requires building a new paper helicopter with the same settings. Subsampling occurs when we take multiple observations on the same EU, so in our case the same helicopter would be dropped multiple times during a single experimental trial and the response measured for each drop; the multiple drops of the same helicopter do not constitute replication.

For this experiment, it is up to the teams to decide whether or not they wish to employ subsampling. Individual helicopter drops can be quite variable, so making multiple drops of each individual helicopter may be advantageous to achieving better experimental results. If the team decides to employ subsampling, simply add multiple columns to the data table for each response and record the subsample observations in those columns for each experimental trial. If you do incorporate subsampling in the experiment, then I recommend that you use the average of the subsample drops for each trial; this keeps the analysis straightforward. Remember, subsampling is not replication, so you still need to build your replicate helicopters as directed in the instructions. In your write-up please specify whether or not subsampling was employed. It is not a requirement, but is a good experimental practice when feasible.

In your write up specifically state whether or not subsampling (essentially dropping the same helicopter multiple times in the same run) was employed.

Analysis of the Experimental Results

The analysis of the experimental data and optimization are to be performed using the **Fit Model** platform in JMP. The following is the sequence of steps to performing the analysis. You will need to do this for each of the two responses keeping in mind that **Flight Time** (or the **calculated rate**) is the by far the primary response. **Analyze Flight Time and Accuracy separately**; do not analyze them simultaneously even though JMP allows one to do so.

- In the Fit Model launch window specify a full factorial model – include the 3-way interaction.

- In the Fit Model report window, using the **Actual by Predicted** plot and **Lack of Fit** report determine if any curvature exists in the relationship of the response to the factors. Clearly report on your findings.

- Even if significant Lack of Fit appears present continue on with the analysis. Using the **Parameter Estimates** table determine the best model to predict the response. One can do this by looking at p-values for the hypothesis tests and the relative magnitudes of the estimated effects. In making decisions with regard to retaining terms in the model, at this point we are enforcing the **heredity principle**. If an interaction is significant or active then retain any lower order terms that are incorporated in to the interaction. Once you specified a final model be certain to carefully document your analysis and result.

- Once you have found a best model for each of the responses, then save the JSL script for the analysis (open the main report menu at the top of the report window and to the left of **Response**, then click on the **Script** submenu and then click on **Save Script to Data Table**).

- Finally, once you have determined the best model for each response, save the prediction formula for that response to the data table. Click on the main report menu, click on the **Save Columns** submenu, and then click on **Prediction Formula** at the top of that drop down menu. You will now have two prediction columns, one for each response, added to the original data table.

Optimizing Helicopter Performance

Once you have determined the best model to predict each response, it is time to determine the optimum helicopter design. For this phase of the project we will use the **Desirability Functions** located in the **Prediction Profiler** menu. The goal is to minimize the rate of descent (we want the helicopter to stay in flight as long as possible). The goal for accuracy depends upon how you measured it. If you measured accuracy in deviations from target (say positive and negative deviations), then your response goal is to match a target of 0; however if you measured it in absolute deviation, then the goal is minimize the deviation from target.

Since you have previously saved your prediction formulas for each response to the data table – if you have not done this, then revisit the previous section and review the required analysis tasks – we do not need to use Fit Model. We will access the Profiler directly through the Graph menu (**Graph → Profiler**). Once in the Profiler launch window, place the two prediction formula columns in the Y box (the box only accepts prediction formula columns as inputs); the formula columns are how JMP determines the models associated with each response.

Once in the Profiler report window, then complete the following tasks. Refer to the Two Level Design Parts 1 and 2 notes for help with the Profiler and Desirability.

- Click on the **Profiler** Menu and select **Desirability Functions**. The Desirability Profiler Column and row should now be added to the Prediction Profiler display.

- Next, in the Desirability Function display (last column in the row for the Rate response) double click, then in the **Response Goal** window that appears, change the goal to **Minimize** for Rate (whatever name you use) and set the Importance = 3. Do the same for the Accuracy, but base the goal on how you measured accuracy (see comment above) and leave the Importance = 1. You are now ready for optimization.

- Once both response goals have been properly set, it is time to find the helicopter configuration that will achieve the best results in Rate and Accuracy. Click on the **Prediction Profiler** menu and select **Maximize and Remember** from the drop down menu. JMP now searches for helicopter design parameter settings that achieve the response goals. Be certain to carefully document your Prediction Profiler display and optimum settings in your write up.

Provide the Prediction Profiler Report, the results of the additional helicopter drops, and a brief analysis of whether or not you were able to improve the performance. You have now determined the settings of the three factors that achieve the predicted optimum results for your two responses. However these are predictions and the settings need to be validated in practice.

Validating the Results

In the previous section we have determined the settings of our three experimental factors that predict the optimum helicopter performance. However, it is always a sound experimental practice to validate or confirm that these results can be achieved in practice. In this section we now validate our experimental results.

- Build three helicopters to the optimum settings determined in the previous section. It is important to build three distinct helicopters (EUs) and not drop the same helicopter multiple times.

- Using the same technique you used for the original experiment drop each helicopter and measure the Rate and Accuracy. Be certain to report on the results in your write –up.

- Finally, discuss whether or not the results of the three drops confirm or validate the results of your analysis in the previous section. In other words did the observed Rate and Accuracy values come reasonably close to the predicted target values? If the results do not seem to confirm (this can happen), then discuss what might be the causes. There is no absolutely right or wrong answer here.

Projected Potential Improvement

Often the analysis of the experimental results suggests that even better performance can be achieved by extending the original ranges of the experimental factors. Most likely the settings for the three experimental factors for optimal performance (the most Desirable performance) occurred on the boundaries of the original experimental region. The Prediction Profiler is a very nice tool to visualize what direction one might move out of the original experimental region in order to achieve even better performance.

As final stage of this project, use the Prediction Profiler to extend the ranges of the three factors. You can even click on the axes at the bottom of the Prediction Profiler in order to get the Profiler to predict performance beyond the original experimental region. Try building a few helicopters with new settings outside of the original experimental region and see if you are able to achieve even better performance. In general, we do not extrapolate our models out of the original experimental region, because we do not know if our estimated models continue to work beyond that region. However, if feasible it is a good practice to try and expand the ranges to achieve better performance, then perform some additional experimental trials and see if better performance is achieved. So in this section try some extended ranges for the three factors and report on the observed results. Were you able to improve performance?

Format for the Project Report

The final report should be concise and to the point. Do not include extraneous data tables and JMP output in the body of the report. The final written report should be **around 2 to 3 pages** and may include a brief appendix if you wish to include pictures, a JMP data table, etc. Any JMP output used to support your findings should be copy and pasted into the body of the report near the location where it is referenced. As an example, if you are referring to a Prediction Profiler display, then copy and paste that display (not the entire JMP report window) into the location in your report where the reference is made. Please do not simply include JMP output at the end of the report; this makes grading next to impossible. Remember the **Selection** cursor tool in JMP can be used to copy and paste selected output into other documents; it is also acceptable to use standard screen capture software. I am not defining a specific line spacing, font size, margin, etc.; with this exception, Wingdings are not acceptable as a font. Readably of the report is the key.

III The Paper Helicopter Design

Figure 1 Picture of a Typical Paper Helicopter

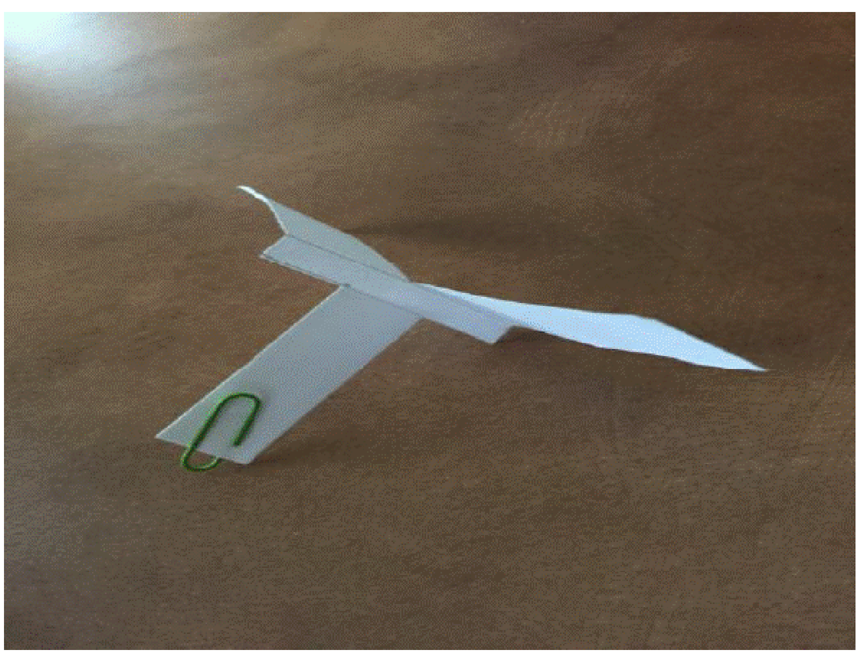

Figure 2 Layout Diagram for a Paper Helicopter