Macroeconomics II, Lecture XI: Incomplete markets models: basics

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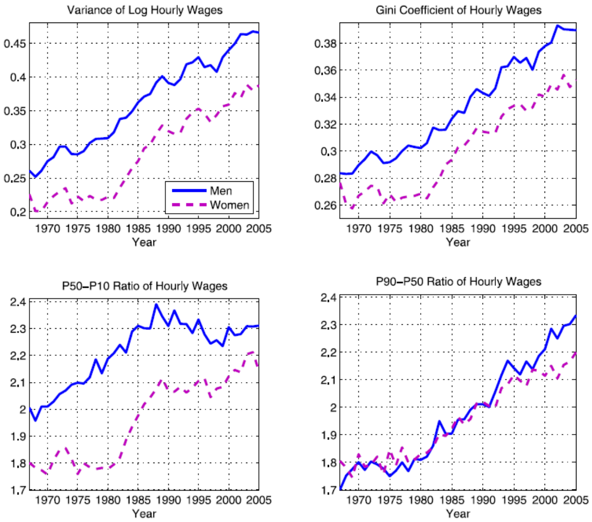
Introduction

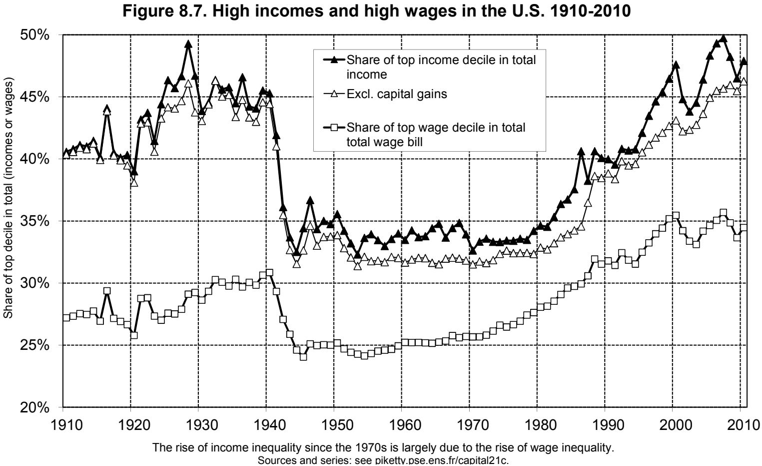
- Traditional macroeconomic models assumes a representative agent: **no room for studying the interaction between macroeconomic dynamics and inequality**
- Inequality/economic heterogeneity is, however, a very salient feature of the macroeconomic environments we seek to understand
- In the last part of this course, we will study macroeconomic models with heterogeneous households
  - Alternatively, macroeconomic models with incomplete asset markets
- We use these models to investigate:
  1. distributional implications of macroeconomic events
     - Does technical change increase income inequality?
     - Does wealth inequality widen when credit become cheaper?
  2. how household heterogeneity matter for macroeconomic transmission
     - Does wider wealth inequality imply more business cycle volatility?
     - Does the existence of more high-debt household imply that monetary policy is more potent?

US cross-sectional wage inequality



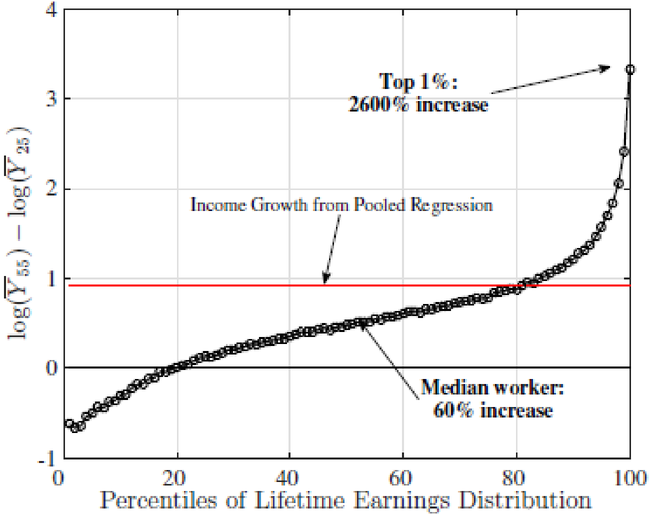
From Heathcote-Perri-Violante (RED 2010) using CPS data

US top income share



From Piketty (Book 2014) using US register data

US income inequality over the life-cycle



From Guvenen-Karahan-Ozkan-Song (Ecmtra 2021) using US register data

Income inequality widens in recessions

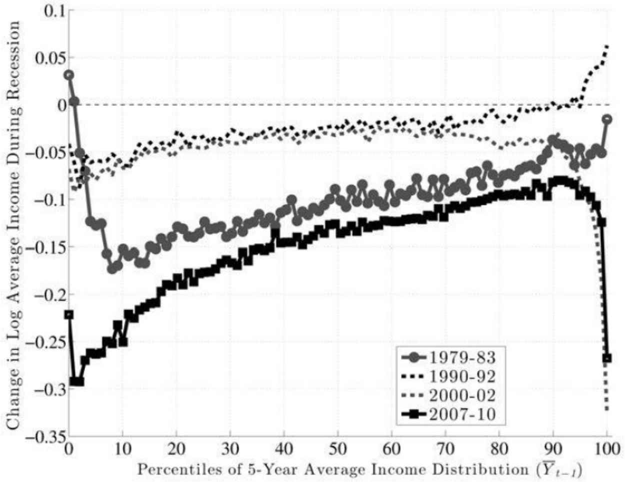
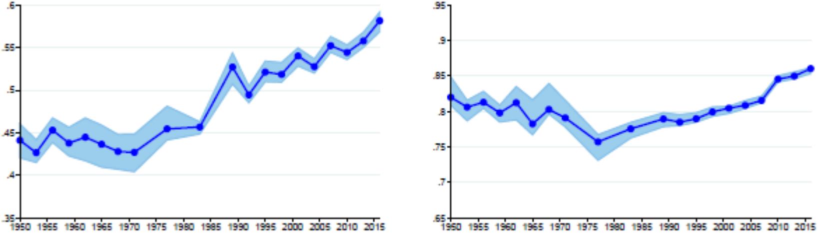


FIG. 13.—Change in log average earnings during recessions, prime-age males

From Guvenen-Ozkan-Song (JPE 2014) using US register data

US income and wealth inequality

Figure 4: Gini coefficients for income and wealth with confidence bands



(a) Income (b) Wealth

From Kuhn-Schularick-Steins (JPE 2020) using SCF+ data

Course part III outline

- Lecture 11: Incomplete markets models: basics
- Lecture 12: Buffer-stock savings: model meet data
- Lecture 13: Incomplete markets in general equilibrium

Today's agenda

1. Aggregation under different market structures
   - Autarky
   - Complete markets
   - Incomplete markets

2. Consumption-savings dynamics with incomplete markets
   - The household problem: setup and characterization
   - Precautionary savings

Aggregation and market structure

A classification of models

- The interaction between aggregate and distributional dynamics depends on the underlying market structure

- Market structure refers to which assets are available to the household — determines how the household can allocate its resources between different states.

- The household budget constraint in three benchmark structures:
  1. **Autarky:**
  $$C_{it}(s^t) \leq Y_{it}(s^t) \quad \forall t, s^t$$
  2. **Complete markets:**
  $$C_{it}(s^t) + \sum_{s_{t+1}|s^t} Q_t(s^t, s_{t+1}) A_{it+1}(s^t, s_{t+1}) \leq Y_{it}(s^t) + A_{it}(s^t, s_t) \quad \forall t, s^t$$
  3. **(Standard) Incomplete markets:**
  $$C_{it}(s^t) + Q_t(s^t) A_{it+1}(s^t) \leq y_{it}(s^t) + A_{it}(s^{t-1}) \quad \forall t, s^t$$

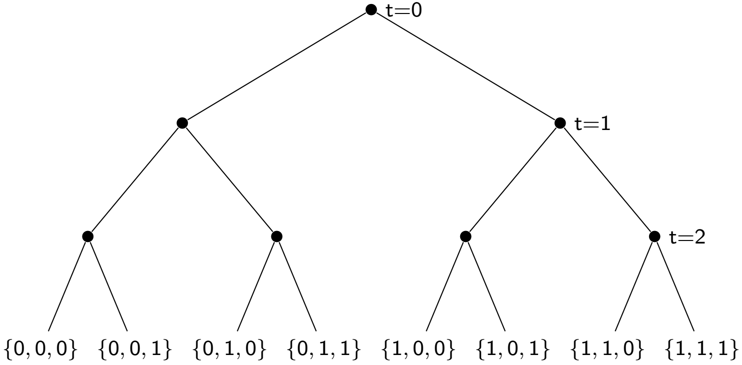
- Now: quick bird's eye's review of these benchmarks

Environment

- Time is discrete and indexed by t
- The economy has N households indexed by i
- All households lives for T periods; $T = \infty$ is possible
- One non-storable consumption good
- In each period, an aggregate event $s_t \in S$ is realized
- At time $t = 0$, each particular event history at time t, $s^t = \{s_0, s_1, ..., s_t\} \in S^t$, is realized with probability $\pi(s^t)$
- Given event history s^t at time t, each event at time $t+1$, $s_{t+1} \in S$, is realized with probability $\pi(s_{t+1}|s^t)$
- For each event history s^t, each household i recieves endowment $Y_t^i(s^t)$

Example event history

- Assume $s_t \in S = \{0, 1\}$
- In period 3, there exists 8 possible "event histories" or "states of the world":



- Note 1: Here, individual endowment depends on entire event history, $Y_t^i = Y_t^i(s^t)$
- Note 2: Even if income only depend on the realized shock today, $Y_t^i = Y_t^i(s_t)$, consumption and savings choices in period t depend on the entire event history s^t

Preferences and beliefs

- All households have identical preferences: time-separable, common discount factor β
- Rational expectations: subjective beliefs about event probabilities coincide with actual probabilities
- Household objective

$$
\begin{aligned}
U_i &= E_0 \sum_{t=0}^{T} \beta^t U(C_{it}(s^t)) \\
&= \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t \pi(s^t) U(C_{it}(s^t))
\end{aligned}
$$

- U has the usual regularity conditions: twice differentiable, strictly increasing, strictly concave and satisfies the Inada conditions

Allocations

- A consumption allocation is a collection $\{C_{it}(s^t)\}_{i=0}^N$
- A consumption allocation is feasible if

$$\begin{aligned} C_{it}(s^t) &\geq 0 \quad \forall i, t, s^t \\ C_t(s^t) &\leq Y_t(s^t) \quad \forall t, s^t \end{aligned}$$

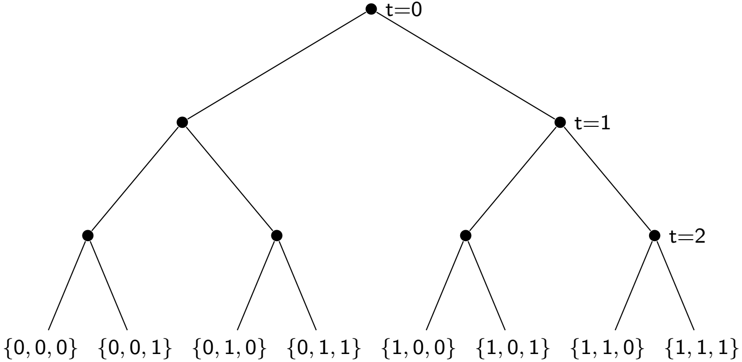
where

$$C_t(s^t) \equiv \sum_{i=1}^N C_{it}(s^t), \quad Y_t(s^t) \equiv \sum_{i=1}^N Y_{it}(s^t)$$

- A feasible consumption allocation $\{C_{it}(s^t)\}_{i=0}^N$ is Pareto efficient if there is no other feasible consumption allocation $\{\hat{C}_{it}(s^t)\}_{i=0}^N$ such that

$$\begin{aligned} U(\hat{C}_{it}(s^t)) &\geq U(C_{it}(s^t)) \quad \text{for all } i \\ U(\hat{C}_{it}(s^t)) &> U(C_{it}(s^t)) \quad \text{for some } i \end{aligned}$$

Autarky



- Autarky: nothing can be traded at any node

Autarky

- Household problem:

$$\begin{aligned} \max_{c(s^t)} \quad & \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t \pi(s^t) U(C_{it}(s^t)) \\ \text{s.t.} \quad & c_{it}(s^t) \leq y_{it}(s^t) \quad \forall t, s^t \end{aligned}$$

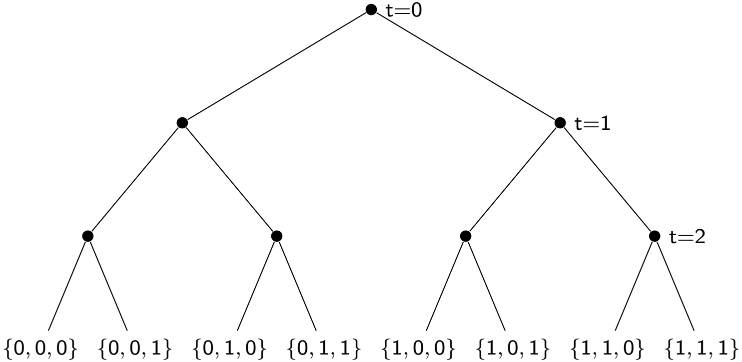
- Solution to household problem is trivial: $c_{it}(s^t) = y_{it}(s^t)$

- Distributional dynamics follows mechanically: individual consumption tracks individual income

- Aggregate dynamics follows mechanically: no prices, and

$$C_t(s^t) = Y_t(s^t) \quad \forall t, s^t$$

Complete markets (with Arrow securities)



- At each node s^t, two assets can be traded:
  1. pays of one consumption good at node $\{s^t, 0\}$ and costs $Q_t(s^t, 0)$
  2. pays of one consumption good at node $\{s^t, 1\}$ and costs $Q_t(s^t, 1)$

Complete markets

- Household problem:

$$\max_{C_{it}(s^t),\{A_{it+1}(s^t,s_{t+1})\}} \quad \sum_{t=0}^{T}\sum_{s^t\in S^t}\beta^t\pi(s^t)U(C_{it}(s^t))$$

$$\text{s.t.} \quad C_{it}(s^t)+\sum_{s_{t+1}|s^t}Q_t(s^t,s_{t+1})A_{it+1}(s^t,s_{t+1})\leq Y_{it}(s^t)+A_{it}(s^t,s_t)$$

$$A_{it+1}(s^t,s_{t+1})\geq -\bar{A}_i \quad \forall t,s^t$$

- First welfare theorem applies: allocation is Pareto efficient and therefore coincides with that of a Social Planner for some Pareto weights $\{\alpha_i\}_1^N$

Complete markets implies full insurance

- Equilibrium features full insurance: (Do on whiteboard)

$$\frac{U_c(C_{it}(s^t))}{U_C(C_{jt}(s^t))} = \frac{\alpha_j}{\alpha_i} \quad \forall s^t$$

- Full insurance: if my gain from consuming more in some state is high, yours must be high in that state too

- With CRRA preferences $U(C_{it}(s^t)) = \frac{(C_{it}(s^t))^{1-\sigma}-1}{1-\sigma}$, full insurance implies that the allocation and prices only depend on aggregate consumption:

$$\begin{aligned} C_{it}(s^t) &= \theta_i C_t(s^t) \\ Q_t(s^t, s_{t+1}) &= \beta\pi(s_{t+1}|s^t)\frac{U_c(C_{t+1}(s^t))}{U_c(C_t(s^t))} \end{aligned}$$

Complete markets: implications

- Hence, the distribution of consumption is constant over time...
- ... and aggregate dynamics ***as if*** representative agent who solves

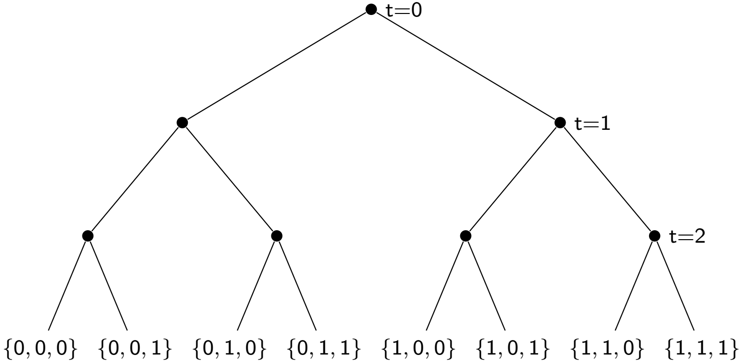
$$\max_{C_t(s^t), A_{t+1}(s^t)} \quad \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t \pi(s^t) U(C_t(s^t))$$

$$\text{s.t.} \quad C_t(s^t) + \sum_{s_{t+1}|s^t} Q_t(s^t, s_{t+1}) A_{t+1}(s^t, s_{t+1}) \leq Y_t(s^t) + A_t(s^t, s_{t-1}) \quad \forall t, s^t$$

$$A_{t+1}(s^t) \geq -\bar{A} \quad \forall t, s^t$$

- $\Rightarrow$ If markets are sufficiently complete, rep-agent macro seems legitimate
  - Large literature testing the extent of consumption insurance, see Violante lecture notes and D Kruegers textbook
  - Violante also discusses more general aggregation results

(Standard) Incomplete markets



- At each node s^t, one asset that pays of one consumption good in period $t+1$ and costs $Q(s^t)$ can be traded

Incomplete markets

- Household problem:

$$
\begin{aligned}
\max_{C_{it}(s^t), A_{it+1}(s^t)} \quad & \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t \pi(s^t) U(C_{it}(s^t)) \\
\text{s.t.} \quad & C_{it}(s^t) + Q_t(s^t) A_{it+1}(s^t) \leq Y_{it}(s^t) + A_{it}(s^{t-1}) \quad \forall t, s^t \\
& A_{it+1}(s^t, s_{t+1}) \geq -\bar{A}_i \quad \forall t, s^t
\end{aligned}
$$

- Due to lack of insurance, ex-ante homogeneous households will be **ex-post heterogeneous** in terms of $\{C, A, Y\}$

- Due to lack of insurance, the allocation is generally inefficient

- The remainder of this course is devoted to explain and derive implications of these two claims

Consumption-savings dynamics with incomplete markets

The Income-Fluctuations Problem

- We will organize our study around the income-fluctuations problem
- Same household problem as above:

$$
\begin{aligned}
\max_{C_{it}(s^t), A_{it+1}(s^t)} \quad & \sum_{t=0}^{T} \sum_{s^t \in S^t} \beta^t \pi(s^t) U(C_{it}(s^t)) \\
\text{s.t.} \quad & C_{it}(s^t) + Q_t(s^t) A_{it+1}(s^t) \leq Y_{it}(s^t) + A_{it}(s^{t-1}) \quad \forall t, s^t \\
& A_{it+1}(s^t, s_{t+1}) \geq -\bar{A}_i \quad \forall t, s^t
\end{aligned}
$$

For simplicitiy, we
- do not keep track of the full history s^t
- assume assets cost 1 and pay R consumption goods (instead of costing Q_t and paying 1 consumption good)
- drop household index i, as we will only study one household
- assume a tight credit constraint $\bar{A} = 0$
- assume income is i.i.d: $Y_t = \bar{Y} + \epsilon_t$ with $\epsilon_t \sim F$ where F has finite support $[\epsilon_{min}, \epsilon_{max}]$

Simplified Income-Fluctuations Problem

- Household problem

$$
\begin{aligned}
\max_{C_t, A_{t+1}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t U(C_t) \\
\text{s.t.} \quad & C_t + A_{t+1} \leq Y_t + RA_t \\
& A_{t+1} \geq 0
\end{aligned}
$$

Recursive forumlation

- For some applications, we will work with a recursive formulation of this problem
- Recursive household problem: find value function V and policy functions C, A' that solves

$$\begin{aligned} V(A, Y) &= \max_{C, A'} U(C) + \beta E V(A', Y') \\ \text{s.t.} \quad & C + A' \leq Y + RA \\ & A' \geq 0 \end{aligned}$$

- We have written this problem as if there are two state variables
- Actually, for iid shocks, there is only one

Recursive forumlation II

- Our problem

$$\begin{aligned} V(A, Y) &= \max_{C, A'} U(C) + \beta E V(A', Y') \\ \text{s.t.} \quad & C + A' \leq Y + RA \\ & A' \geq 0 \end{aligned}$$

- When studying a recursive problem, always ask youself what is the minimal set of state variables!
  - Here, we seemingly have two state variables: A, Y
  - But, to choose how much to consume/save, the household only needs to know its total cash on hand $M = Y + RA$
  - And conditional on knowing M, knowing A or Y does not help to forecast M' and therefore neither the continuation value $EV(\cdot)$
  - Note: this is only true when Y_t is iid
    - ★ Not true with persistent shocks! E.g. if $Y' = \rho Y + \epsilon$

Recursive forumlation III

- Reformulating our constraints with cash-on-hand $M = Y + RA$ and without A

$$
\begin{aligned}
& C + A' \leq Y + RA \\
\Rightarrow \qquad & A' \leq M - C \\
\Rightarrow \qquad & M' \leq R(M - C) + Y'
\end{aligned}
$$

  and

$$
\begin{aligned}
& A' \geq 0 \\
\Rightarrow \qquad & C \leq M
\end{aligned}
$$

- Reformulated recursive problem:

$$
\begin{aligned}
V(M) \quad = \quad & \max_{C,M'} U(C) + \beta E V(M') \\
\text{s.t.} \qquad & M' \leq R(M - C) + Y' \\
& C \leq M
\end{aligned}
$$

- Solution given by value function $V(M)$ and consumption policy function $C(M)$

Recursive forumlation IV (Do on whiteboard)

Solution is characterized by

1. an Euler equation,

$$U_c(C) \quad = \quad \beta RE\left[V_m(M')\right] + \mu$$

2. complementary slackness conditions,

$$\begin{aligned} \mu(C-M) &= 0 \\ \mu &\geq 0 \\ \lambda(M' - R(M-C) - Y') &= 0 \\ \lambda &\geq 0 \end{aligned}$$

Note:

- Non-satiated preferences imply that budget constraint always binds: $\lambda > 0$.
- The credit constraint may either
  1. bind $\Rightarrow$ exterior solution with $\mu > 0$
  2. not bind $\Rightarrow$ interior solution with $\mu = 0$

3. an envelope condition

$$V_m(M') \quad = \quad U_c(C').$$

Recursive forumlation V

- Put together, an interior solution satisfies

$$\begin{aligned} U_c(C(M)) &= \beta R E V_m(R(M - C(M) + Y')) \\ &= \beta R E U_c(C') \end{aligned}$$

- an exterior solution satisfies

$$C(M) = M$$

- If interior solution is feasible for M, then also feasible for any $\hat{M} > M$
- If interior solution not feasible for M, then neither feasible for any $\hat{M} < M$
- Ergo, global solution $C = C(M)$ therefore satisfies

$$C(M) = \begin{cases} M & \text{if } M \leq M^* \\ (U_c)^{-1}\left[\beta R E U_c(C')\right] & \text{if } M > M^* \end{cases}$$

for some cutoff M^*

Consumption-Savings Dynamics

- So far: how to technically characterize consumption-savings problems with uninsurable income risk
- Now: what does uninsurable income risk imply for consumption-savings dynamics?
- One lesson we've already learnt: Households may face a binding credit constraint, in which households have a high Marginal Propensity to Consume (MPC)
- Now: Uninsurable income risk produces an additional savings motive: the precautionary-savings motive

Precautionary savings

- With uninsurable income risk, precautionary savings arise if
  1. Household preferences exhibit prudence, or
  2. Households face a potentially binding credit constraint

Prudence

- Prudent preferences: preferences with $U_{ccc} > 0$
- Consider again our household problem:

$$\begin{aligned} V(M) &= \max_{C,M'} U(C) + \beta E V(M') \\ \text{s.t.} \quad & M' \leq R(M-C) + Y' \\ & C \leq M \end{aligned}$$

- W.L.G. assume $\beta R = 1$
- Assume the credit constraint is never binding, problem is characterized by

$$U_c(C) = EU_c(C')$$

Prudence II

- Suppose $U_{ccc} = 0$ (as with quadratic preferences), then U_c is a linear function, and

$$\begin{aligned} U_c(C) &= E_t U_c(C') \\ &= U_c(EC') \end{aligned}$$

  implying $C = EC'$

- Now suppose $U_{ccc} > 0$ (as with CRRA preferences), then U_c is convex

- Using Jensen's inequality, we have

$$\begin{aligned} U_c(C) &= EU_c(C') \\ &> U_c(EC') \end{aligned}$$

  implying $C < EC'$

- Lesson: For the same income stream, prudent preferences implies consumption is expected to grow over time
  - i.e., the household saves more today!

Prudence III

- Preferences that exhibit prudence
  - CRRA: $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$
  - CARA: $U(C) = 1 - \frac{1}{\gamma} e^{-\gamma C}$
  - Most other preferences you see in macro models
- Preferences that do not exhibit prudence:
  - Linear: $U(C) = C$
  - Quadratic: $U(C) = -C^2 + \bar{C}$
- Standard reference: Carroll-Kimball (Ecmtra 1996)

Credit constraints I

- Precuationary savings may also arise due to a potentially binding credit constraint
- Our household problem

$$\begin{aligned} V(M) &= \max_{C,M'} U(C) + \beta E V(M') \\ \text{s.t.} \quad & M' \leq R(M-C) + Y' \\ & C \leq M \end{aligned}$$

- As before, W.L.G. assume $\beta R = 1$
- Euler equation and complementary slackness:

$$\begin{aligned} U_c(C) &= \beta R E U_c(C') + \mu \\ \mu(C-M) &= 0 \\ \mu &\geq 0 \end{aligned}$$

Credit constraints II

- For this exercise, I reintroduce time subscripts
- Assume quadratic preferences, such that prudence-induced savings are excluded. Euler equation:

$$-C_t = -EC_{t+1} + \mu_t$$

or

$$C_t = EC_{t+1} - \mu_t$$

- If the credit constraint is not binding in period t: $C_t = EC_{t+1}$
- If binding: $C_t < EC_{t+1}$
- If the credit constraint is binding, the household consumes less today than what it would like given its consumption-smooting motive

Credit constraints III

- Suppose credit constraint is not binding today, but that it might be binding tomorrow:

$$\begin{aligned} C_t &= E_t C_{t+1} \\ C_{t+1} &= E_{t+1} C_{t+2} - \mu_{t+1} \end{aligned}$$

- By the law of iterated expectations

$$C_t = E_t [C_{t+2}] - E_t [\mu_{t+1}]$$

- If $\mu_{t+1} > 0$ in some states of the world, then $E_t [\mu_{t+1}] > 0$

- Compare to the solution when the constraint is never binding:

$$C_t = E_t [C_{t+2}]$$

- Lesson: If the household anticipates that the credit constraint will be binding in some states tomorrow, the household also consumes less today than what it would like given its consumption-smooting motive
  - I.e. anticipating binding credit constraint induces the household to save more
  - By saving more, the household can bring consumption closer to its unconstrained optimum in the states where the constraint binds

The natural credit constraint

- Previous example had constraint $A_{t+1} \geq 0$
- More general credit constraint $A_{t+1} \geq -\bar{A}$
- Which $\bar{A}$ are allowed?
- See you problem set

Precautionary savings: comments

- Precautionary-savings motive imtimitely linked to the marginal propensity to consume (MPC)
  - We'll talk about this more in the next lecture

- Prudence and potentially binding credit constraints can interact in non-trivial fashion, see Carroll-Kimball-Holm (JET 2021)

- Other standard reason why income risk may affect consumption dynamics: irreversible durable purchases
  - See Bernanke (AER 1982); Dixit-Pindyck (Book 1994); Harmenberg-Öberg (JME 2021)

Summary

- Complete markets: ex-ante homeogeneous households will be ex-post homogeneous
- Incomplete markets: ex-ante homeogeneous households will be ex-post heterogeneous
- Much of the incomplete-markets literature is organized around the income-fluctuations problem
- Features of income-fluctuations problem
  - Stochastic income
  - Limited set of assets (typically: one risk-free assset)
  - Potentially binding credit constraint
- With incomplete markets, household have a precuatinary-savings motive
  - Prudence
  - Potentially binding credit constraint