Problems

The following problems are representative of the type of problems you will have to solve in the exam (some have actually been used in past incarnations of this course). Most are based on actual research papers, and as such are also meant to introduce extensions of the basic models seen in class.

Problem 1: workers' choices of search intensity. In this problem we take up the (partial equilibrium) wage posting model covered in Lecture 2. Workers sample wage offers from the sampling distribution $F(\cdot)$, *which we will take as given throughout the problem*, without asking where it comes from (this is the sense in which the model is partial equilibrium). The model and notation are exactly the same as in the course handouts, except for the following change. We now assume that *workers can choose how hard to search for jobs*. Search intensity (or effort) is denoted by s, and works as follows: a worker choosing search intensity s incurs a flow cost of $c(s)$, and receives offers at Poisson rate $\lambda \times s$. The cost function $c(s)$ is continuously differentiable, increasing and strictly convex. The parameter λ is strictly positive. Both $c(\cdot)$ and λ are *independent of a worker's employment status* (i.e. unemployed and employed jobseekers have access to the same search technology).

1. Explain *intuitively* (i.e. without equations) why you expect workers earning different wages to choose different levels of search effort. In the rest of the problem, the optimal choice of search effort for a worker earning wage w is denoted by $s^\star(w)$. Moreover, the optimal choice of search effort for an unemployed worker is denoted by $s_0^\star$

2. Explain why the worker holding a job paying wage w and choosing search effort s enjoys utility $V(w;s)$ solving:
$$rV(w;s) = w - c(s) + \delta\left[U(s_0^\star) - V(w;s)\right] + \lambda s \cdot \int_w^{\overline{w}} \left[V(x;s^\star(x)) - V(w;s)\right] dF(x). \quad (1)$$

3. Using (1), maximize $V(w;s)$ with respect to s to show that the optimal search effort is implicitly defined by:
$$c'(s^\star(w)) = \lambda \int_w^{\overline{w}} \left[V(x;s^\star(x)) - V(w;s^\star(w))\right] dF(x). \quad (2)$$
Using (2), show that $s^\star(w)$ is a decreasing function of w. Explain why intuitively (if you haven't already done so at question 1).

4. Define the *unemployed workers' reservation wage*, ϕ. In the absence of any institutional minimum wage, why is ϕ the lowest wage offered in this labor market?

5. Given search intensity s, explain why the worker's valuation of unemployment, $U(s)$, solves:

$$rU(s) = b - c(s) + \lambda s \cdot \int_{\phi}^{\overline{w}} [V(x; s^\star(x)) - U(s)]\, dF(x). \tag{3}$$

6. Using (3), maximize $U(s)$ with respect to s to show that the optimal search effort of unemployed workers is implicitly defined by:

$$c'(s_0^\star) = \lambda \int_{\phi}^{\overline{w}} [V(x; s^\star(x)) - U(s_0^\star)]\, dF(x). \tag{4}$$

7. Using all the above questions, show that $\phi = b$ in this model. How (and why) does this result differ from the wage posting model with exogenous search intensities seen in class?

Problem 2: wage posting conditional on employment status. In this problem we modify the wage posting model covered in Lecture 2 by assuming that employers observe the *employment status* of applicants, i.e. whether an applicant is employed or not. In case the applicant is employed, however, the prospective employer does not observe at what wage. It is then natural to assume that employers will make different offers to employed and unemployed workers. In what follows, we will denote the wage sampling distribution of offers made to *employed* workers by $F_e(w)$, and that of offers to *unemployed workers* as $F_u(w)$.

As in the basic model, firms maximize their steady-state profit flow. Consider a firm with productivity p offering wages w_u to unemployed applicants and w_e to employed applicants. This firm's profit $\pi(p; w_u, w_e)$ can be decomposed as:

$$\pi(p; w_u, w_e) = \pi_u(p; w_u) + \pi_e(p; w_e) = (p - w_u)\,\ell_u(w_u) + (p - w_e)\,\ell_e(w_e), \tag{5}$$

where π_u is the profit from hiring unemployed workers and π_e is the profit from hiring employed workers, and $\ell_u(w_u)$ [resp. $\ell_e(w_e)$] is the stock of previously unemployed [resp. previously employed] workers that the firm is able to hire and retain given a posted wage offer of w_u [resp. w_e].

1. We first focus on previously unemployed workers hired at a type-p firm. Using a flow-balance equation, show that:

$$\ell_u(w_u) = \frac{\lambda_0 u}{\delta + \lambda_1 \overline{F}_e(w_u)}. \tag{6}$$

(For simplicity, the total numbers of firms and workers in the economy have been normalized at 1.)

2. *We now assume that, in equilibrium, the lowest wage offered to employed workers (i.e. the lower support of F_e) is greater than the highest wage offered to unemployed workers (i.e. the upper support of F_u).* Using (5) and (6), show in this case that F_u is in fact degenerate, and that all firms offer unemployed workers their reservation wage.

3. Let N_u [resp. N_e] denote the number of previously unemployed [reps. previously employed] workers in the employed labor force, so that $N_u + N_e = 1 - u$. Further denote the CDF of the cross-section wage distribution among the N_e previously employed workers by $G_e(w)$. Using a flow-balance equation (and the result of question 2), show that:

$$N_e G_e(w) = \frac{\lambda_1 N_u F_e(w)}{\delta + \lambda_1 \overline{F}_e(w)}, \tag{7}$$

and conclude that:

$$\ell_e(w) = \frac{\lambda_1 N_u (\delta + \lambda_1)}{\left[\delta + \lambda_1 \overline{F}_e(w)\right]^2}. \tag{8}$$

4. Let $w_e(p)$ denote the optimal wage offer to employed applicants chosen by a type-p firm. We look for an equilibrium where $w_e(p)$ is increasing in p. Following the same steps as in the basic model seen in class, show that:

$$w_e(p) = p - \left[\delta + \lambda_1 \overline{F}_p(p)\right]^2 \left\{ \int_{\underline{p}}^{p} \frac{dx}{\left[\delta + \lambda_1 \overline{F}_p(x)\right]^2} + \frac{\underline{p} - \underline{w}_e}{\delta + \lambda_1} \right\}, \tag{9}$$

where F_p is the exogenous sampling distribution of firm types. Compare the above wage equation to the basic wage posting model with heterogeneous firms.

5. Suppose a government enacts a stringent antidiscrimination law that forces firms to make the same offers to employed and unemployed workers. Will that decrease the typical firm's profit?

Problem 3: The sequential auction model with homogeneous firms. We consider the Sequential Auction model seen in class with $\beta = 0$ and *homogeneous firms*: there is only one type of firm in the economy, with constant marginal labor productivity equal to p.

For simplicity we also assume that *workers are homogeneous*, i.e. in the notation of the course handouts, all workers have $\varepsilon = 1$. We can thus simplify the notation for the value function of an employed worker earning some wage w to $V(w)$ (as the other two arguments p and ε present in the course handouts are now irrelevant).

The rest of the model is left unchanged. The economy is at a steady state. Firms maximize steady-state profit flows. The notation is taken up from the course handouts.

Importantly, throughout the problem, we assume that the firm's residual value of a job after separation (i.e. after the job has been destroyed by a δ shock, or after the worker has left voluntarily for another job) is zero.

1. Explain *intuitively* (without using any mathematical notation) why the equilibrium wage distribution of this economy will only have two points of support (i.e. in equilibrium workers can only earn one of two different wage levels).

2. Call the two wages in the support of the equilibrium distribution w_ℓ and w_h (with $w_\ell \leq w_h$). Show *without using any mathematics* that $w_h = p$.

3. Write the asset pricing equation that characterizes the typical worker's value of earning a wage of w

4. Using the answers to questions 2 and 3, show that:

$$w_\ell = b - \frac{\lambda_1}{\rho + \delta} (p - b) \, .$$

   Carefully interpret this result. Explain why $w_\ell \to b$ as $\rho \to +\infty$.

5. Having thus characterized the support of the equilibrium wage distribution, we now turn to probability masses.

   (a) Show that the steady state fraction of employed workers earning w_h is equal to $p_h = \frac{\lambda_1}{\delta + \lambda_1}$.

   (b) Discuss the cases $\lambda_1 \to +\infty$ and $\lambda_1 \to 0$. Explain why $\frac{\lambda_1}{\delta}$ is often referred to as an "index of search frictions".

6. Derive the mean wage in this economy.

7. We now discuss some of the model's comparative static properties.

   (a) Give an expression of the ratio $R = \frac{w_h}{w_\ell}$ as a function of the model's parameters (including p and b, the unemployment income flow). R will be our *measure of wage inequality.*

   (b) What is the impact on the *mean wage* and on *wage inequality* of an increase in λ_1? An increase in δ? An increase in aggregate productivity (increase in p)? Give an intuitive explanation in each case.