Lecture 2: Incomplete Markets Models: Intro

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Motivation

Many questions that we study in economics have to do with

1. Inequality/heterogeneity/distributions:
   - obvious: inequality in income, wealth, consumption..
   - but also in: ability, productivity, fertility, health, life expectancy, firms' profitability, employment, city sizes, etc.
2. Uncertainty/risk:
   - income risk:
     - e.g., unemployment, mismatch with job/occupation, health/disability shock, etc.
     - changes in skill prices, plant closures/mass layoffs, industry/occupation/region-level shocks, etc.
   - wealth risk: rate of return risk (esp. for retirement saving), housing price shocks,
   - many others: divorce, fertility, gov't policy uncertainty, etc.
   - firms: exchange rates, commodity prices, demand, gov't policy, etc.

Heterogeneity vs. Risk

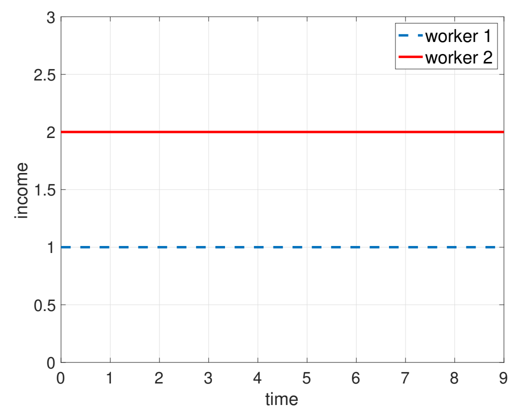
Three key questions in economics:

1. *What is (are) the sources of inequality or risk?*
   - Age old question: "What is the origin of inequality among men and is it authorized by natural law?" —Academy of Dijon, 1754 (Theme for essay competition)

2. *What determines the effects of inequality and risk on individual's behavior and welfare?*
   - A: Interaction of exogenous factors/shocks and the economic/social environment

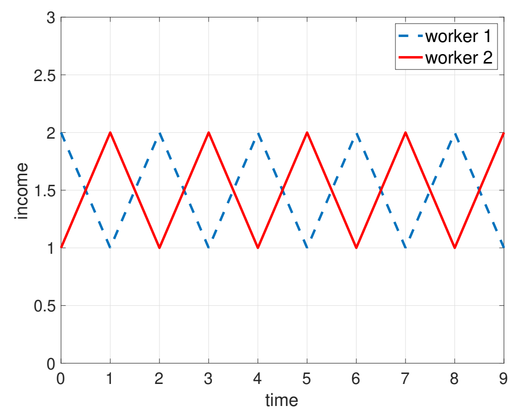
3. *How to separate fixed heterogeneity from risk?*
   - Examples:
     - Inequality without risk.
     - Risk without inequality.

Inequality without Risk



- Can you make an argument that there is still risk here?
  - Yes. Behind the veil of ignorance (at time -1).

Inequality and Risk: Short vs. Long Run



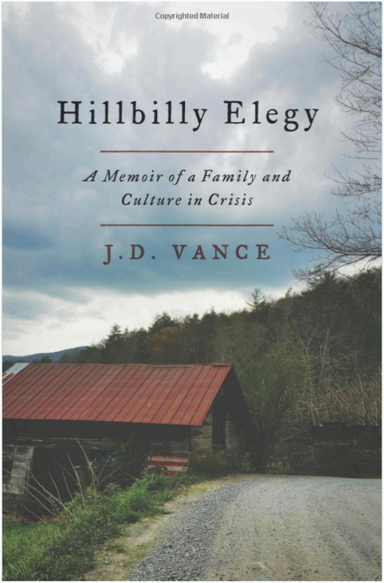
- Inequality in the cross section, but not in lifetime incomes.
- If fluctuations are not deterministic there is risk. But if individuals can trade with each other, they can smooth it completely.

Incomplete Markets Models

Why do we care?

1. *For studying distributional phenomena.*
   - Is this obvious?
     - Can you think of complete mkts models with interesting heterogeneity?
     - In fact, some of these models seem to work better than incomp. mkts models.
2. *For studying aggregate phenomena*
   - Is this obvious?
     - Krusell-Smith (1998): Not for studying aggregates: An incomplete mkts model with lots of heterogeneity can have almost identical implications to rep agent models.
     - Some newer models do have different aggregate implications.

A Real Life Eye Opener



Aggregation Theory

The next several slides provide a summary of what is Chapter 1 of the manuscript. Added here for completeness.

- How to deal with consumer heterogeneity in a tractable way?
  - Basics in Mas-Collel, et al (1995, Ch 4.B).
  - Consider individuals allowed to differ in their preferences and wealth in a **static environment**.
  - For given prices $p \in R^l$ and wealth levels $(w_1, w_2, ..., w_I)$ for the I consumers, aggregate demand can be written as

$$x(p, w_1, w_2, ..., w_I) = \sum_{i=1}^{I} x_i(p, w_i)$$

Aggregation Theory

- When can we write $x(p, w_1, w_2, \cdots, w_n) = x(p, \overline{w})$ where $\overline{w} \equiv \sum w_i$?
- For wealth distribution not to matter, we need $x(p, \sum w_i)$, so for a certain distribution $(w_1, w_2, \cdots, w_n)$ redistribute wealth keeping aggregate constant $\sum dw_i = 0$.

$$\left.\frac{\partial x(p, \overline{w})}{\partial w_i}\right|_{\overline{w}\text{ fixed}} = 0 \Rightarrow \sum_{i=1}^{n} \frac{\partial x_i(p, w_i)}{\partial w_i} dw_i = 0$$

  for all redistributions. This implies

$$\frac{\partial x_i(p, w_i)}{\partial w_i} = \frac{\partial x_j(p, w_j)}{\partial w_j}$$

  wealth effects must be equal.
- Can we relax our demand? Let aggregate demand depend on the statistical distribution of wealth when all individuals have identical (but possibly non-HARA) utilities.

Static (or without background risk):

Gorman (1951, 63) and Rubinstein (74, JFE):

Theorem 1
Demand Aggregation: *(Rubinstein 1974, JFE). Assume all agents have linear risk tolerance (HARA preferences with $T(W_0) = A + BW_0$). Utility is $U(c_0) + \beta V(W_{it})$.* ***Agents consume out of wealth.***
(There are several homogeneity conditions that lead to demand aggregation. The most relevant one for us is:)
All individuals have the same beliefs $\{\pi_s\}, \beta$ and taste parameters $B \neq 0$. A complete market exists and all individuals have the same resources W_0 and taste β, $A = 0$, and $B = 1$.
Then, all equilibrium rates of return are determined as if there exists only composite individuals each with the average resource, belief, and tastes (see Rubinstein for how they are aggregated).

Resource Redistribution Irrelevance

- Corollary: Whenever a composite consumer can be constructed, in equilibrium, rates of return are insensitive to the distribution of resource among individuals.

Discussion about Rubinstein 74:

- Suppose there are J physical assets (and $S > J$ states of the world). If consumers have homogenous tastes, endowments, and beliefs, then markets are (effectively) complete by simply adding enough financial assets.

- There is no loss of optimality and nothing will change by this action, in equilibrium, because identical agents will not trade with each other. (This is what we stated above).

Discussion of Rubinstein

- Do we really need identical preferences?
- Almost, but we can allow for HARA class with identical curvature parameter γ in: $T(y) = \alpha + \gamma y$.
- Or if we write optimal portfolio weights, $\alpha^* = a(b + c\omega_0/\gamma)$ where γ is the curvature parameter. We can allow for differences in b but not in c. In this case Rubinstein (1974) shows that there is demand aggregation.
- This should be obvious since all agents have linear portfolio solutions in wealth, it does not matter who holds the wealth. (Show this by adding up α^*'s for two agents whose total wealth is ω.
- However, if consumers are heterogenous in other dimensions this strategy will not work.

How Good A Benchmark is Complete Mkts?

- Large literature since late 1980s rejected perfect risk sharing (PRS).
- Test if marginal utility growth is equalized across households

$$\beta^i \frac{U^i\left(c^i_{t+1}, x^i_{t+1}\right)}{U^i\left(c^i_t, x^i_t\right)} = \beta^j \frac{U^j\left(c^j_{t+1}, x^j_{t+1}\right)}{U^j\left(c^j_t, x^j_t\right)} = \frac{\lambda_{t+1}}{\lambda_t} \tag{1}$$

- Take a functional form for U and what goes into x^i_t and get micro data on c^i_t and x^i_t.
  - Altug and Miller (1990) was the first test and could not reject.
  - Hayashi, Altonji, Kotlikoff (1996), Attanasio and Davis (1996), Mace (1991 as corrected by Nielsen), etc.
- **Aside:** Harder to reject PRS in poor small villages than in the US data. Harder to reject for stockholders than non-stockholders.
- HAK and AD look at multi-year changes to get more power.

Attanasio and Davis (1996, JPE)

Figure 1: Long-Run Consumption vs Wage Growth, by Group

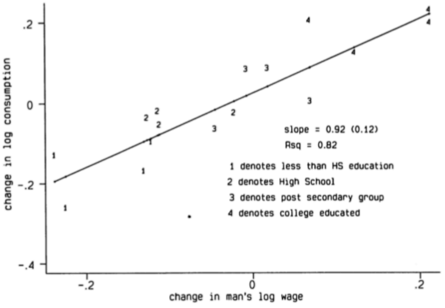


FIG. 2.—Household consumption vs. man's wage, 1980–90 log change residuals. Groups are defined by four-way education crossed with 5-year birth cohorts. Plotted values are residuals from regressions on a cubic in age.

- **New take on this problem:** Perfect insurance is an extreme/ideal benchmark. Maybe we should try to measure the extent of **partial insurance.**
- Very active topic in economics and is a key topic in this class.

Attanasio and Davis (1996, JPE)

- **Question:** Can you think of a complete mkts model explaining this observation?
  - One would be heterogeneity in time discounting
  - Another: would be non-separable leisure and rising wage inequality.
- The most straightforward explanation is that markets are incomplete.
  - So income risk and other idiosyncratic shocks do matter.
- Q: How to model decisions in such an environment?

The Income Fluctuation Problem

Consider the following problem:

$$\mathcal{V}(a_t; y_t, R_t) = \max_{c_t} U(c_t) + \delta \mathbb{E}\left[\mathcal{V}(a_{t+1}; y_{t+1}, R_{t+1}) | y_t\right]$$

$$\text{s.t.} \quad c_t + a_{t+1} = y_t + R_t a_t,$$

$$\log y_t = \rho \log y_{t-1} + \eta_t, \tag{2}$$

$$a_t \geq -B_{\min}, \tag{3}$$

- When the stochastic element is
  1. y_t^i, this is the income fluctuations problem for an individual.
  2. R_t^i or R_t, this is the risky return problem for an individual.
- When we embed
  - (1) into GE, it becomes Bewley-Huggett-Aiyagari models: y_t^i affects everybody, can study income inequality, but no Pareto tail.
  - (2) into GE, it becomes, Angeletos models or power law models: R_t^i affects the rich more, Pareto wealth tail, other interesting implications.

Hall and Mishkin (1982, ECMA)

- Income process:

$$\begin{aligned} y_t &= y_t^P + \eta_t \\ y_t^P &= y_{t-1}^P + \epsilon_t \end{aligned} \tag{4}$$

- Quadratic Preferences:

$$\max E_t \left[-\frac{1}{2} \sum_{\tau=0}^{T-t} (1+\delta)^{-\tau} \left(c^* - \widetilde{c}_{t+\tau} \right)^2 \right]$$

$$s.t$$

$$\sum_{\tau=0}^{T-t} (1+r)^{-\tau} \left(y_{t+\tau} - \widetilde{c}_{t+\tau} \right) + \widetilde{A}_t = 0$$

- FOC: $E_t \left[(1+\delta)^{-\tau} \left(c^* - \widetilde{c}_{t+\tau} \right) \right] = (1+r)^{-\tau} \left(c^* - \widetilde{c}_t \right)$
- Assume $\delta = r$, and we get the Euler equation:

$$E_t \left[\left(c^* - \widetilde{c}_{t+\tau} \right) \right] \quad = \quad E_t \Longrightarrow \widetilde{c}_{t+\tau} \left(c^* - \widetilde{c}_t \right) = \widetilde{c}_t \tag{5}$$

Consumption Function

- Take the conditional expectation of budget constraint and use (5):

$$\sum_{\tau=0}^{T-t} (1+r)^{-\tau} \left(E_t y_{t+\tau} - \widetilde{c}_t \right) + \widetilde{A}_t = 0$$

- Define: $\widetilde{H}_t = E_t \left(\sum_{\tau=0}^{T-t} (1+r)^{-\tau} y_{t+\tau} \right)$ and $\gamma_t = \frac{1}{\left(\sum_{\tau=0}^{T-t} (1+r)^{-\tau} \right)}$ and write:

$$\widetilde{c}_t = c^* + \gamma_t \left(\widetilde{H}_t + \widetilde{A}_t \right)$$

- Define: $\overline{H}_t = E_t \left(\sum_{\tau=0}^{T-t} (1+r)^{-\tau} \overline{y}_{t+\tau} \right)$ and $\overline{c}_t = c^* + \gamma_t \left(\overline{H}_t + \overline{A}_t \right)$ and $c_t = \widetilde{c}_t - \overline{c}_t$ and $A_t = \widetilde{A}_t - \overline{A}_t$, we get the consumption function

$$c_t = \gamma_t \left(H_t + A_t \right) \tag{6}$$

Consumption Function, cont'd

- Although this is a nice equation, in many empirical applications we do not have data on wealth, so A_t creates a problem.
- First difference this equation, and use the specification of income to get:

$$\Delta c_t = \epsilon_t + \gamma_t \eta_t$$

- **Caution:** This is not the Euler equation. It requires the derivation of the consumption function (which means you need to take a stand on budget constraint, the income process, etc.
- Now how to use this for empirical work because we do not observe η_t and ϵ_t in the data?
- Plus: consumption is measured with error: $c_t = c_t^{**} + \nu_t$

Empirical Implementation

- Use covariances:

$$\begin{aligned} cov\left(\Delta y_t, \Delta y_{t-1}\right) &= -\sigma_\eta^2 \\ C_0 &= cov\left(\Delta y_t, \Delta c_t\right) = \sigma_\epsilon^2 + \beta\sigma_\eta^2 \\ C_1 &= cov\left(\Delta y_{t+1}, \Delta c_t\right) = -\beta\sigma_\eta^2 \\ cov\left(\Delta c_t, \Delta c_{t-1}\right) &= -\sigma_\nu^2 \end{aligned}$$

- Blundell and Preston (1998): used this observation to back out whether the rise in income inequality was persistent or temporary.
  - The different behavior of rise in consumption and income inequality gives information about this.

Figure 2: Blundell and Preston (1998, QJE)

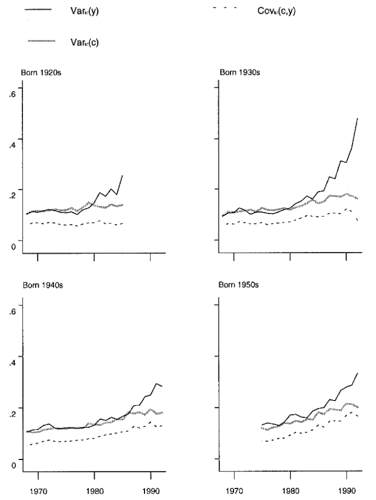


FIGURE I
Variances of Expenditures and Income and Their Covariance by Cohort, 1968–1992

BPP (2008, AER)

- Take the same equation in HM and modify:

$$\Delta c_t = \theta \epsilon_t + \phi \gamma_t \eta_t$$

- Interpret θ and ϕ as the response parameters and interpret $(1-\theta)$ a measure of partial insurance of permanent shocks, and $(1-\phi)$ partial insurance of temporary shocks.
- They find θ to be significantly less than 1 so argue there is a lot of partial insurance.
- **Issues:**
  1. If income shocks are less than persistent, even if $\rho = 0.95$ then you can match the θ they find with PIH.
  2. If you have retirement again the response of consumption to income is not 1 for 1.
  3. Precautionary savings can make the response smaller.
  4. If permanent and transitory shocks are not separately observable and there is estimation risk, again not valid.

More Recently

Newer papers find smaller room for partial insurance:

- Kaplan and Violante (AEJ: Macro, 2010): makes points 1 to 3 above.
- Blundell, Pistaferri, Saporta-Eksten (AER, 2016): Model family time use and other details.
- Heathcote, Storesletten, Violante (AER, 2014): Model partial insurance at a deeper level. Add taxes and labor supply.
- Guvenen and Smith (ECMA, 2014): Separate risk from anticipated income changes. Model partial insurance directly.

Bewley-Huggett-Aiyagari Models

Aiyagari (1994)

Individual's Problem:

$$\begin{aligned} & E_0\left\{\sum_{t=0}^{\infty}\beta^t U\left(c_t\right)\right\} \\ & s.t \\ c_t + a_{t+1} &= wl_t + (1+r)\, a_t \\ c_t &\geq 0,\; a_t \geq -b \\ & l_t \text{ is a stochastic w/ bdd support} \end{aligned}$$

- b can be either the natural limit: $wl_{\min}/r$
- Note that if $l_{\min}$ is zero, natural borrowing limit is also zero.
- or some ad hoc one stricter than the natural one that you make up.

Aiyagari (1994)

- Define:

$$\widehat{a}_t \equiv a_t + \phi$$
$$z_t = wl_t + (1+r)\,\widehat{a}_t - r\phi$$

- Asset demand is: $\widehat{a}_{t+1} = A\,(z_t, b, w, r)$:

Figure 3: Aiyagari (1994, QJE)

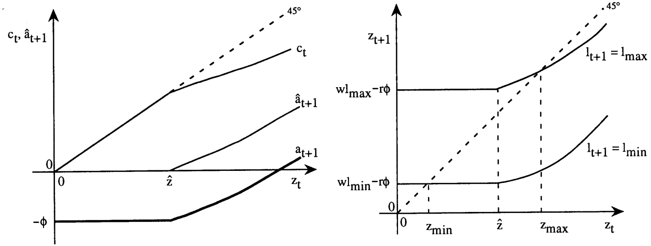


FIGURE Ia
Consumption and Assets as Functions of Total Resources

FIGURE Ib
Evolution of Total Resources

Figure 4: Aiyagari (1994, QJE)

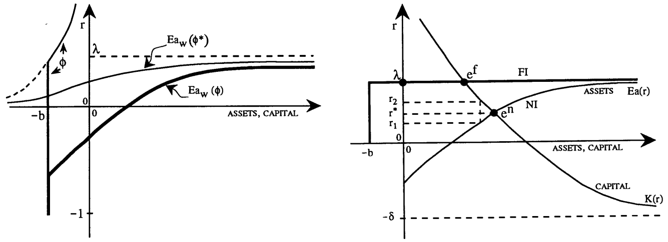


FIGURE IIa
Interest Rate versus Per Capita Assets

FIGURE IIb
Steady-State Determination

- Notice that when $r = \lambda$ long-run asset demand goes to infinity.

Comments on Aiyagari

- Chamberlain and Wilson (2000, RED) show that this result extends to the case where asset return is stochastic. In that case what matters is the geometric average of return and how that compares to λ.

- What do we learn from Aiyagari?
  - Gini for consumption, wealth, and income:
    - Aiyagari: 0.06, 0.12, 0.32 (see Fig. 6 in WP version)
    - US data: 0.35, 0.45, 0.85.
  - With incomplete markets, K^* is higher than under complete mkts (but not by much)
  - And r^* is lower (many papers tried to explain the equity premium puzzle by this).
  - Figure IIb very useful for other incomplete mkts models too (for example, Krusell-Smith (1998) stochastic beta model, or Guvenen (2006) limited participation model, or Laitner (2002) bequests model.

Post-Aiyagari Models

Many papers extended Aiyagari to generate more wealth inequality:

- Huggett (1996, JME): Introduce life-cycle, so wealth variation across ages also contributes to wealth inequality.
- Hubbard-Skinner-Zeldes (1995, JPE): Introduce government welfare programs and insurance to explain very low wealth holding at the bottom end.
- Krusell-Smith (1998, JPE): stochastic-beta. Powerful mechanism for generating lots of inequality in steady state.
- Castaneda, Diaz-Jimenez, Rios-Rull (2003, JPE): Parametrize the income process and pick its parameters to match wealth inequality.

Comments: Post-Aiyagari Models

- Models building on Aiyagari framework face some common challenges.
- Even when they match the Gini and top 1% share of wealth it misses several things:
  - It takes an extremely long time to get to steady state where such inequality exists. Several hundreds years or more, or dozens of generations.
    - But in the data, many super wealthy are self made: e.g., 54% of Forbes 400 billionaires.
    - Evidence on bequests inconsistent with transmission of such large wealth (e.g. Kopczuk's survey)
  - Income shocks needed to generate top 1% and Gini is unrealistically large.
  - Even when top 1% is matched, nobody in simulated data has more than $20M or so.
  - Most very wealthy do not work for wages. They are entrepreneurs.

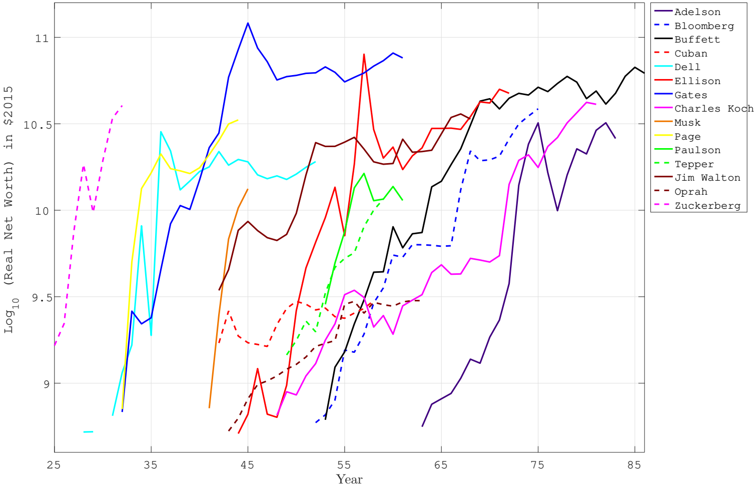
Wealth Concentration by Assets

Table 1: Wealth Concentration by Asset Type

	Stocks w/o pensions	*All stocks*	*Non-equity financial*	*Housing equity*	*Net Worth*
Top 0.5%	41.4	37.0	24.2	10.2	25.6
Top 1%	53.2	47.7	32.0	14.8	34.0
Top 10%	91.1	86.1	72.1	51.7	68.7
Bottom 90%	8.9	13.9	27.9	49.3	31.3
			Gini Coefficients		
		Financial Wealth			*Net Worth*
		0.91			0.82

Source: Poterba (2000) and Wolff (2000)

Evolution of Net Worth Among Forbes 400



How Much Inequality in Aiyagari-Style Models?

	U.S. Data	Gaussian	GKOS benchmark
Parametrization:		$\rho = 0.985,\ \sigma^2 = 0.0234$	Rich process
Gini	0.85	0.58	0.66
Top 0.1%	**14.8%**	**1.1%**	**2.2%**
Frac > $10M	**0.4–0.5%**	≈ 0	**0.02%**
Top 1%	35.5%	7.0%	9.2%
Top 10%	75.0%	37.9%	41.6%
Top 20%	87.0%	48.2%	52.8%

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