Aggregate Labor Market Dynamics
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Problem Set 2
Due on February 25, 2016
Suggested solution

The goal of this problem set is to talk about the McCall search model and Diamond paradox related to it.

1 McCall Search Model

Consider the McCall search model with a mass 1 of risk neutral individuals with discount factor β and an exogenously given stationary distribution of wages $F(w)$. A worker can be either employed or unemployed. An unemployed worker receives unemployment benefits b while an employed worker receives her wage. An unemployed worker receives job offers every period. The job offer is described by its wage, w, which is randomly drawn from $F(w)$. An unemployed worker has to decide whether to accept the job. If she does, she will receive the wage w forever. If she does not, she stays unemployed this period and receives a new offer next period.

Question 1.1 Let $v(w)$ be the value of being unemployed with a job offer with w. Find a recursive formulation for $v(w)$.

Answer It is

$$v(w) = \max\left\{\frac{w}{1-\beta}, b + \beta \int v(\omega)\, dF(\omega)\right\}.$$

■

Question 1.2 Argue that the solution is given by a reservation wage R, so that a worker accepts an offer with $w \geq R$ and rejects otherwise.

Answer The value of staying in unemployment is contact with respect to w, $\bar{v} \equiv b + \beta \int v(\omega)\, dF(\omega)$, the value $w/(1-\beta)$ is increasing in w. Hence, there will be a wage R such that $w/(1-\beta) > \bar{v}$ for any $w \geq R$. ■

Question 1.3 Argue that since all $w < R$ are turned down, $v(w) = R/(1-\beta)$ for all $w < R$. Furthermore, it holds that $v(w) = w/(1-\beta)$ for all $w \geq R$.

Answer Use the value function:

$$v(w) = b + \beta \int v(\omega)\, dF(\omega)$$

■

Question 1.4 Find an equation which implicitely determines R. Use that at R, the worker is indifferent between taking the offer or not. You should get

$$R - b = \frac{\beta}{1-\beta} \int_R^\infty (\omega - R)\, dF(\omega).$$

Answer We have

$$\begin{aligned} v(R) &= \frac{R}{1-\beta} = \bar{v} = b + \beta \left[\frac{R}{1-\beta} F(R) + \int_R^\infty v(\omega)\, dF(\omega) \right] \\ \frac{R}{1-\beta}(1 - \beta F(R)) &= b + \beta \int_R^\infty \frac{\omega}{1-\beta} dF(\omega) \\ R - b &= \beta(R-b) - \beta R (1 - F(R)) + \beta \int_R^\infty \omega dF(\omega) \end{aligned}$$

which after some manipulation gives the desired result. ■

Question 1.5 Intuitively interpret the formula you obtained.

Answer The term $R - b$ is the flow benefit of accepting an offer with R as opposed to staying unemployed. This has to be equal to an "expected cost" of taking such an offer, where these costs is the difference between all potential offers and the one with the reservation wage R. ■

Question 1.6 Let's use this model to derive a simple theory of unemployment. To do so, assume that the setup is the same except for the fact that jobs are destroyed at an exogenous probability s. This effectively means that the discount factor is $\beta(1-s)$ instead of β. The rest is the same. Let U_t be the number of unemployed at time t. Write down the law of motion for unemployment.

Answer We have

$$U_{t+1} = (1 - F(R))\, U_t + s(1 - U_t)$$

■

Question 1.7 Find the steady state unemployment and compare this formula to the one from DMP model.

> **Answer** It is
> $$U = \frac{s}{s + (1 - s)(1 - F(R))},$$
> and it is the same as the DMP formula, since here $(1 - s)(1 - F(R))$ is the job finding rate. ■

2 The Diamond Paradox

The basis of the Diamond paradox is that it is difficult to rationalize the distribution function $F(w)$ as resulting from profit maximizing choices of firms.

Consider an economy with a mass 1 of identical workers and a measure $N >> 1$ of heterogenous firms with a linear production technology. Firms differ in their productivity x (which is output per worker), and we assume that the distribution of x is given by $G(x)$ with support $X \subset R^+$. Suppose that each firm can hire at most 1 worker and can post only 1 vacancy. The vacancy posting cost is γ. For simplicity also assume that $b = 0$.

If a worker accepts its job, it is permanent and is employed forever. A firm commits to the wage it offered at the beginning of the game. To keep the environment stationary, assume that every time a worker accepts a job, a new worker is born.

Let $F(w)$ be the resulting distribution of wages. The question we are going to examine is whether $F(\cdot)$ is going to be non-degenerate.

Let's introduce some more notation. The decision of a firm whether to post a vacancy or not is given by
$$p : X \to \{0, 1\}$$
denoting whether a firm is posting a vacancy or not ($p = 1$ means a vacancy is posted), and
$$h : X \to R^+$$
specifies the wage offers.

Question 2.1 Argue that it is reasonable to assume that $h(\cdot)$ is non-decreasing. We will assume that it is the case in what follows.

> **Answer** High productivity firms can afford paying high wages. ■

Question 2.2 Find a formula for the wage distribution $F(w)$ in terms of h and p. Denote the inverse of h as h^{-1}.

Answer We have

$$F(w) = \frac{\int_{-\infty}^{h^{-1}(w)} p(x)\,dG(x)}{\int_{-\infty}^{\infty} p(x)\,dG(x)}.$$

■

Question 2.3 Let the strategy of a worker be represented by a function $a : R^{+} \to [0,1]$, denoting the probability that the worker accepts a wage in the support of the wage distribution. We consider a subgame perfect Nash equilibrium, where the strategies of a firm (p, h) is the best response to a and vice-versa in all subgames. Argue that the strategy of a worker will be given by a reservation wage R and that R is the same for all workers. The characterization of R is the same as in McCall.

Answer This is the same as in the McCall model. Since all workers are identical, they will have the same reservation wage. This gives us

$$\begin{aligned} a(w) &= 1 \text{ if } w \geq R \\ a(w) &= 0 \text{ if } w < R \end{aligned}$$

We get

$$R - b = \frac{\beta}{1-\beta}\left[\int_{w \geq R} (w - R)\,dF(w)\right]$$

■

Question 2.4 Now consider a firm's problem. Take a firm with productivity x offering a wage $w' > R$. Write down the net present value of profits for this firm. Remember that the matching is random, the measure of workers is 1 and the measure of active firms is $n \equiv \int_{-\infty}^{\infty} p(x)\,dG(x)$. Denote the profit $\pi(p = 1, w' > R, x)$.

Answer We have

$$\pi\left(p = 1, w' > R, x\right) = -\gamma + \frac{1}{n}\frac{x - w'}{1-\beta}.$$

■

Question 2.5 Consider now a deviation of this firm. Suppose the firm offers a wage $w' - \varepsilon$. What is the firms profit in this case? Is it a profitable deviation?

Answer The profit is

$$\pi\left(p = 1, w' - \varepsilon > R, x\right) = -\gamma + \frac{1}{n}\frac{x - (w' - \varepsilon)}{1-\beta} > \pi\left(p = 1, w' > R, x\right)$$

Noticet that workers will still accept the wage because it is still above the reservation wage R. The search is random hence the probability of matching is the same as before. ■

Question 2.6 Argue that there should be no wage strictly above R and hence $w \leq R$.

Answer It follows immediately from above. ■

Question 2.7 Next, consider a firm offering a wage $\tilde{w} < R$. What is the profit of the firm in this case?

Answer No worker will accept the wage and hence $\pi\left(p = 1, w' < R, x\right) = -\gamma$. ■

Hence we established the following theorem: In an environment with homogenous workers and undirected search, all equilibrium distributions will have a mass point at the reservation wage.

In what follows we will argue that in any subgame perfect equilibrium, $R = 0$.

Suppose that almost all firms offer wage R – "almost all" is going to be important in the argument. We will establish that it is profitable for a worker to decrease his acceptance threshold. In particular, consider another acceptance strategy $\tilde{a}\left(w\right)$ given by

$$\begin{aligned}\tilde{a}\left(w\right) &= 1 \text{ if } w \geq R - \varepsilon \\ \tilde{a}\left(w\right) &= 0 \text{ if } w < R - \varepsilon\end{aligned}$$

Question 2.8 Suppose a worker faces the wage $R - \varepsilon$. Write down worker's utility from following the strategy a and strategy $\tilde{a}$.

Answer If he follows $\tilde{a}$, he accepts the wage and gets utility $\tilde{u}$

$$\tilde{u} = \frac{R - \varepsilon}{1 - \beta}.$$

If he follows a, he gets utility u

$$u = 0 + \frac{\beta R}{1 - \beta}.$$

Tomorrow the worker gets a wage offer R because almost all firms offer R. Clearly, $\tilde{u} > u$ and hence a worker should follow $\tilde{a}$. ■

Question 2.9 Argue that if all workers follow $\tilde{a}$, then starting from a situation where all firms offer R, it is profitable for a firm to deviate and offer a wage $R - \varepsilon$.

Answer Deviating will increase firm's profits:

$$\pi\left(p=1, R-\varepsilon, x|\tilde{a}\right) = -\gamma + \frac{1}{n}\frac{x-(R-\varepsilon)}{1-\beta} > -\gamma + \frac{1}{n}\frac{x-R}{1-\beta}.$$

■

Any firm can do this, and hence no $R > 0$ can be an equilibrium. Hence, regardless of $G\left(x\right)$ and β, not only there is no non-degenerate distribution, but also all firms offer the lowest possible wage, the "monopsony wage", and the whole search model collapses to a simple labor market monopsony model.