Exam Ph.D. Macroeconomics II

Department of Economics, Uppsala University

June 2, 2025

Instructions

- Writing time: 5 hours.
- The exam is closed book.
- The exam has 70 points in total
- A passing grade requires a) at least 30 points on the exam, and b) 50 points in total for the course (incl the points you have from your problem sets).
- Start each question on a new paper. Write your anonymous code on all answer pages.
- You may write your solutions by pen or pencil; use your best handwriting.
- Answers shall be given in English.
- Motivate your answers carefully; if you think you need to make additional assumptions to answer the questions, state them.
- If you have any questions during the exam, you may call Erik Öberg (+46 730 606 796) or Teodora Borota (+46 739 262 330) at any time between 3 PM and 5 PM.

1 The Romer (1990) model of expanding varieties (20 points)

Suppose that final output $Y(t)$ is produced according to $Y(t) = \frac{1}{1-\beta} L_E(t)^\beta \int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu$, where $L_E(t)$ is the number of workers engaged in production, $x(\nu,t)$ is the number of machines of variety ν being used, and $N(t)$ is the number of existing varieties, all at time t. Suppose the final good sector is perfectly competitive, but each machine variety is supplied by a monopolist who faces a unit cost of making machines equal to ψ. For convenience, we assume $\psi = 1-\beta$.

1. Show that profit maximization by final good firms yields the demand curve $x(\nu,t) = p(\nu,t)^{-1/\beta} L_E(t)$. Given this, show that $p(\nu,t) = 1$, and show that therefore $Y(t) = \frac{1}{1-\beta} N(t) L_E(t)$. Finally, show that the profits of machine producers are $\pi(t) = \beta L_E(t)$.

Answer: The final goods producers maximization problem at time t is given by

$$\max_{[x(\nu,t)]_{\nu\in[0,N(t)]}} \frac{1}{1-\beta}\left(\int_0^{N(t)} x(\nu,t)^{1-\beta} d\nu\right) L_E(t)^\beta - \int_0^{N(t)} p(\nu,t)x(\nu,t)d\nu - w(t)L_E(t) \tag{1}$$

The first-order condition of this maximization problem with respect to $x(v,t)$ for any $v \in [0, N(t)]$ yields the demand for machines from the final good sector, i.e.

$$x(\nu,t) = p(\nu,t)^{-1/\beta} L_E(t) \tag{2}$$

Machines are produced by monopolists at cost ψ, so given demand $x(\nu,t)$ in (2), optimal price is $p(\nu,t) = \frac{\psi}{1-\beta}$. Using the normalization $1-\beta = \psi$, the profits become

$$\pi(t) = \beta L_E(t) \tag{3}$$

Substituting (2), with $p(\nu,t) = \frac{\psi}{1-\beta}$, in the production of final goods yields $Y(t) = \frac{1}{1-\beta} N(t) L_E(t)$. Finally, the profits of machine producers, $\pi(\nu,t) \equiv p(\nu,t)x(\nu,t) - \psi x(\nu,t)$ are then, given (2) and $p(\nu,t) = 1$, derived as $\pi(\nu,t) = \beta L_E(t)$.

Ideas for new varieties are produced according to $\dot{N}(t) = \eta N(t) L_R(t)$, where $L_R(t)$ is the number of workers doing research at time t. A firm producing the new variety receives a patent and acts as a monopolist. Workers are perfectly mobile across the production and research sectors. Let $w(t)$ be the wage earned in both sectors, and let $V(t)$ be the value of a patent.

2. State and explain the free-entry condition ensuring that workers are indifferent between the two activities.

Answer: Free entry into research implies

$$\eta N(t)V(\nu,t) = w(t). \tag{4}$$

The left-hand side of (4) is the return from hiring one more worker for R&D. The term $N(t)$ is on the left-hand side, because higher $N(t)$ translates into higher productivity of R&D workers. The right-hand side is the flow cost of hiring one more worker for R&D, $w(t)$.

3. Infinitely lived consumers own Arrow securities paying a return $r(t)$. State and explain the no-arbitrage condition ensuring that traders in financial markets are indifferent between holding Arrow securities and patents.

Answer: Assuming that the value function $V(\nu,t) = \int_t^\infty \exp\left(-\int_t^s r(s')\,ds'\right)\pi(\nu,s)ds$ is differentiable in time, it could be written in the form of a HJB equation as in Theorem 7.10 in DA Chapter 7:

$$r(t)V(\nu,t) - \dot{V}(v,t) = \pi(\nu,t)$$

or

$$r(t) = \frac{\pi(\nu,t)}{V(\nu,t)} + \frac{\dot{V}(v,t)}{V(\nu,t)}. \tag{5}$$

In equilibrium, the above condition insures that the traders are indifferent between holding sequrities yielding $r(t)$ and holding a patent providing its owner a devidend yield $\frac{\pi(\nu,t)}{V(\nu,t)}$ and loss or gain in the value of the patent.

It can be shown that the wage equals $w(t) = \frac{\beta}{1-\beta}N(t)$ (not needed to derive this result). Finally, assume population is constant and consumers have log utility, so that the growth rate of consumption equals $g_C(t) = r(t) - \rho$.

4. Define the BGP equilibrium of the economy.

Answer: An equilibrium can be defined (somewhat less formally) as

1. time paths of consumption, expenditure, R&D decisions and total number of machine varieties, $[C(t), X(t), L_R(t), N(t)]_{t=}^\infty$
2. time paths of prices and quantities of each machine,
   $[p^x(v,t), x(v,t)]_{v\in N(t),t=0}^\infty$
3. and time paths of interest rate and wages,
   $[r(t), w(t)]_{t=0}^\infty$,

such that all equilibrium conditions (resource constraint, patent value, free-entry, expressions for machine demand and price, total expenditure on machines, the wage rate, the Euler equation and the TVC) hold and markets clear.

A balanced growth path (BGP) is an equilibrium path where consumption $C(t)$ and output $Y(t)$ grow at a constant rate. Then, $N(t)$ must also grow at a constant rate (given $Y(t) = \frac{1}{1-\beta}N(t)L_E(t)$ and constant labor allocation).

5. Take as given that the defined BGP is the unique equilibrium outcome of the model and solve for the BGP r, g (growth rate of output pc), and L_E. What makes this economy grow sustainably?

Answer: First note from the free entry condition that using the equilibrium expression for the wage rate $w(t) = \frac{\beta}{1-\beta}N(t)$ the condition implies that the value of a patent is constant in equilibrium, i.e. $\frac{\dot{V}(v,t)}{V(\nu,t)} = 0$. With the BGP interest constant at some level r^* (to satisfy the constant consumption growth given by the Euler equation), the free entry condition can then be written as

$$\eta N(t)\frac{\beta L_E(t)}{r^*} = \frac{\beta}{1-\beta}N(t) \tag{6}$$

It follows that the BGP equilibrium interest rate must be

$$r^* = (1-\beta)\eta L_E^*, \tag{7}$$

where L_E^* must be constant given the previous condition above (alternatively, observe that with constant growth rate of $N(t)$, the labor allocation between sectors needs to be constant).
Now using the Euler equation of the representative household we obtain

$$\frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta}\left((1-\beta)\eta L_E^* - \rho\right) \equiv g^* \quad \text{for all } t \tag{8}$$

With $\dot{N}(t)/N(t) = \eta L_R^* = \eta\left(L - L_E^*\right)$ and the growth rate of consumption equal to the rate of technological progress $g^* = \dot{N}(t)/N(t)$, one can derive the research labor allocation as

$$L_E^* = \frac{\theta\eta L + \rho}{(1-\beta)\eta + \theta\eta} \tag{9}$$

Using (9), one solves for r^* in (7) and g^* in (8).

Given $Y(t) = \frac{1}{1-\beta}N(t)L_E(t)$, it follows that even though the aggregate production function exhibits constant returns to scale from the viewpoint of final good firms (which take $N(t)$ as given), there are increasing returns to scale for the entire economy. Increase in the variety of machines, $N(t)$, raises the productivity of labor and when $N(t)$ increases at a constant rate so does output per capita. Ideas production is the engine of sustainable growth.

6. Assume now that the population grows at a constant rate n and that you want to remove the scale effect property of the model. How would you modify the R&D technology? What is the BGP growth rate in this economy?

Answer: See AD Chapter 13.3.

7. Suppose that the policy maker wants to promote the BGP growth and introduces a subsidy to R&D cost by paying each research firm a fraction s^R of the wage bill, i.e. the effective wage paid to researchers is now $w^R(t) = (1 - s^R)w(t)$. Will the policy have the intended growth effect? How about the welfare effect?

Answer: The governemnt now pays each research firm a part of the labor cost so the free entry condition in the economy without scale effects now reads

$$\eta N(t)^{\phi} V(\nu, t) = w^R(t) = (1 - s^R)w(t) = (1 - s^R)\frac{\beta}{1 - \beta}N(t). \tag{10}$$

It follows that at the BGP, the patent value grows at the rate $(1 - \phi)g^*$. The value function is then given by $V(\nu, t) = \frac{\beta L_E(t)}{r^* - (1-\phi)g^*}$. The free entry condition can then be written as

$$\eta N(t)^{\phi} \frac{\beta L_E(t)}{r^* - (1 - \phi)g^*} = (1 - s^R)\frac{\beta}{1 - \beta}N(t). \tag{11}$$

Still, at BGP, the free entry implies that $\frac{\dot{N}(t)}{N(t)} = g^* = \frac{n}{1-\phi}$ and the policy has no effect on the BGP growth rate.

Note that the R&D production technology implies the BGP growth rate of $N(t)$

$$\frac{\dot{N}(t)}{N(t)} = g^* = \frac{n}{1 - \phi} = \eta \frac{L(t)\alpha^R}{N(t)^{1-\phi}}, \tag{12}$$

where α^R denotes the share of labor in R&D. The free entry can then be written as

$$\eta \frac{\beta}{r^* - n} \frac{L(t)(1 - \alpha^R)}{N(t)^{1-\phi}} = (1 - s^R)\frac{\beta}{1 - \beta}. \tag{13}$$

Combining (12) and (13) to eliminate $\frac{L(t)}{N(t)^{1-\phi}}$, one obtains

$$\frac{1 - \alpha^R}{\alpha^R} = \frac{(1 - s^R)(1 - \phi)}{n} \frac{r^* - n}{1 - \beta}. \tag{14}$$

A higher subsidy s^R implies a decrease in the lhs, i.e. a higher share of labor employed in research. Given the R&D technology, the subsidy will result in a lower BGP ratio $\frac{L(t)}{N(t)^{1-\phi}}$ in (12), i.e. a higher BGP level of technology $N(t)$ and thus has a positive welfare effect.

Applying a subsidy in the benchmark model with scale effect results in a positive BGP growth effect.

2 Demand shocks in the New-Keynesian model (20 points)

Consider the vanilla New-Keynesian model in class, in which we allow for shocks to the household discount factor ("demand shocks"). The log-linear equilibrium is described by the following set of equations

$$\begin{aligned}
\text{Intratemporal hh optimality:} \quad & \hat{\omega}_t = \hat{c}_t + \varphi \hat{n}_t \\
\text{Intertemporal hh optimality:} \quad & \hat{c}_t = -(\hat{i}_t - E_t \pi_{t+1}) + E_t \hat{c}_{t+1} + \xi_t \\
\text{Firm optimality:} \quad & \pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \\
\text{Marginal cost:} \quad & \widehat{mc}_t = \hat{\omega}_t \\
\text{Goods market clearing:} \quad & \hat{c}_t = \hat{y}_t \\
\text{Labor market clearing:} \quad & \hat{y}_t = \hat{n}_t \\
\text{Policy rule:} \quad & \hat{i}_t = \phi \pi_t + \nu_t \\
\text{Shock process:} \quad & \xi_t = \rho_\xi \xi_{t-1} + \epsilon_{\xi,t}
\end{aligned}$$

where $\xi_t = -\log \beta_t$ is the negative of the log of the household discount factor, $\hat{\omega}_t = \hat{w}_t - p_t$ is log deivations in the real wage, and $\lambda = \frac{(1-\theta)(1-\theta\beta)}{\theta}$.

Figure 1 contains the IRFs to a positive demand shock with $\rho = 0.5$. The other parameters take the same value as in class: The Frisch elasticity $\varphi = 1$, the price-resetting probability $1 - \theta = 1/3$, the policy rule coefficent $\phi = 1.5$, and the steady state value of the household discount factor $\beta = 0.99$.

1. Explain the sign of the responses for all variables displayed in Figure 1.

   Answer: The demand shock enters the intertemporal hh optmaility condition, so we begin our analysis here. Holding the real rate constant, this condition implies that consumption growth $E_t \hat{c}_{t+1} - \hat{c}_t$ is negative. Given that the equilibrium is unique and bounded, we know that equilibrium path must return to steady state, with $\lim_{T\to\infty} \hat{c}_T = 0$. As such, $\hat{c}_t > 0$ for all t. Given $\hat{c}_t > 0$, Labor market clearing, goods market clearing, intratemporal hh optimality and marginal cost directly imply $\hat{y}_t > 0, \hat{n}_t > 0, \omega_t > 0, \widehat{mc}_t > 0$. From firm optmality, we then have that inflation growth $E_t \pi_{t+1} - \pi_t \approx \beta E_t \pi_{t+1} - \pi_t < 0$. By a unique bounded equilibrium, we again have that $\pi_t > 0$. By the policy rule, we then have that the nominal and the real rate is positive. By intertemporal hh optimality, the positive response of the real rate dampens the response of consumption.

2. Figure 2 contains the IRFs to the same shock, but with $1-\theta = 0.999$. Explain why, in this case, we see close to no response in output.

   Answer: With $1-\theta = 0.999$, almost all firms can reset their prices in every period, and we approximately restore the IRFs under flexible prices. With flexible prices, there is no response of output to a demand shock. With flexible prices, firm optimality simply equates the wage with the MPL, which is constant, and thus we have $\widehat{mc}_t = \omega_t = 0$. From this, market clearing conditions and intratemporal hh optimality directly imply $\hat{c}_t = \hat{n}_t = \hat{y}_t = 0$.

   Another way to think about this is by noting that a positive discount factor shock is equivalent to a negative monetary policy, as shown in Lecture 5. With flexible prices, the vanilla NK model is simply the RBC model without capital, which We know features monetary neutrality.

3. Consider again the benchmark case where $1-\theta = 1/3$. How would the response of output look like if monetary policy were set to maximize social welfare?

   Answer: Optimal monetary policy would restore the response achieved under flexible prices (by implementing a path of the real rate that coincides with the natural real rate). We have just seen that under flexible prices, the output response is zero.

Hiring subsidies to reduce unemployment (15 points)

Consider the standard DMP model of frictional unemployment with Nash bargaining and exogenous separations. A legislator is considering giving a constant subsidy s to any firm that employs a worker in order to promote job creation and reduce the economy's unemployment rate. With the subsidy, the profit flow of a producing firm is $y + s - w$, where y is output and w is the wage level. The legislator is considering two options to finance this subsidy under a balanced budget: 1) by levying a labor income tax τ_e on currently employed households, or 2) by levying an income tax τ_u on currently unemployed households (the second financing option may also be interpreted as a reduction in unemployment benefits).

1. Will the hiring subsidy be succesful in reducing the unemployment rate? Does it depend on how it is financed?

   Answer: With the hiring subsidy and the taxes, the DMP equilibrium is described by

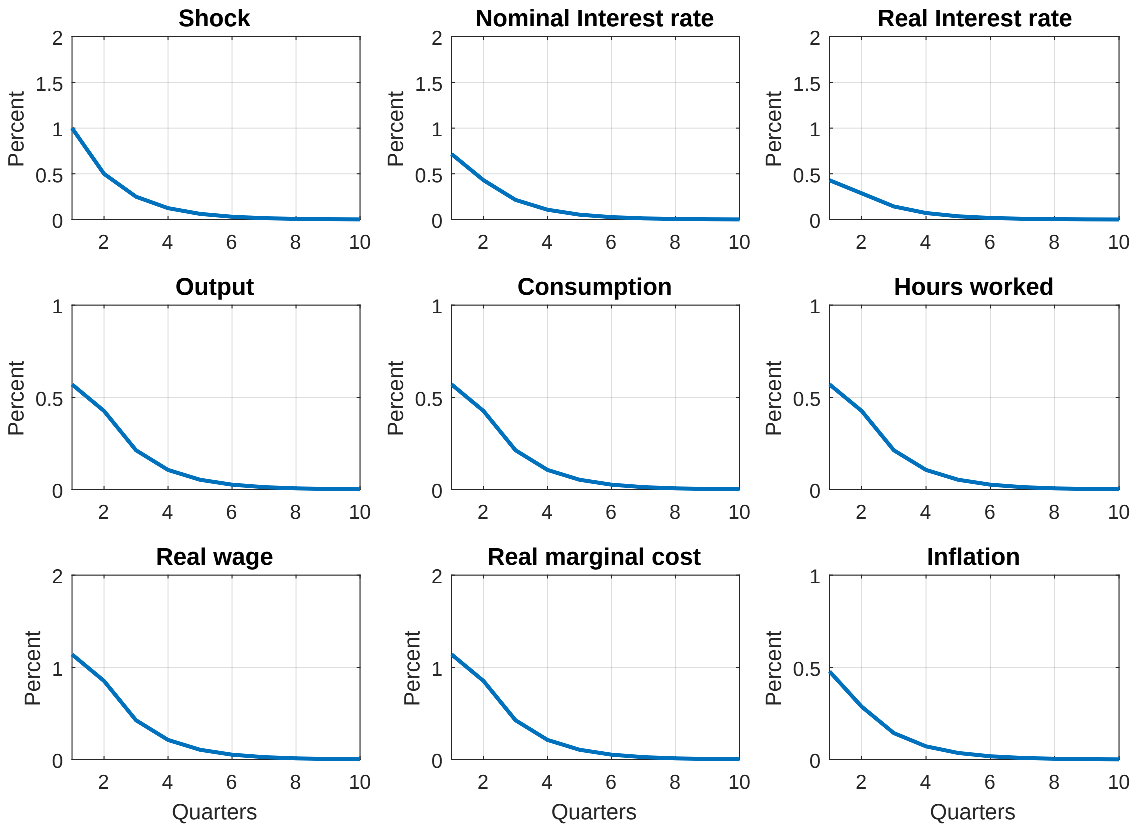

Figure 1: IRFs to a demand shock in the vanilla NK model

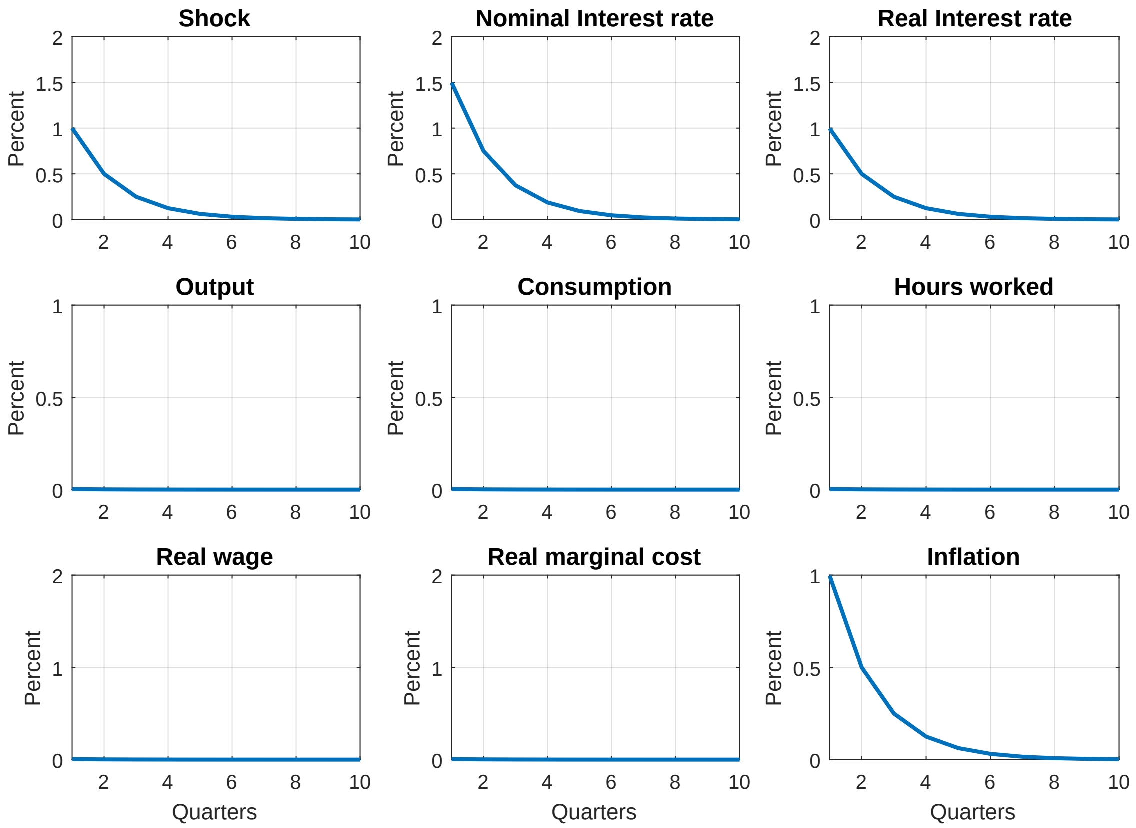

Figure 2: IRFs to a demand shock in the NK model with $1-\theta = 0.999$

- Bellman equations

$$
\begin{aligned}
rW &= w - \tau_e + \sigma\,(U - W) \\
rU &= b - \tau_u + \lambda_u(\theta)\,(W - U) \\
rJ &= y + s - w + \sigma\,(V - J) \\
rV &= -c + \lambda_v(\theta)\,(J - V)
\end{aligned}
$$

- Free entry

$$
V = 0
$$

- Nash Bargaining:

$$
\gamma(J - V) = (1 - \gamma)(W - U)
$$

Consider case 1) with $\tau_u = 0$ and τ_e set to balance the budget. Total government flow expenditure is Ms, where M is the number of matches. Total taxes is $M\tau_e$. As such $\tau_e = s$ to balance the budget. Given this, the subsidy-tax schedule does not affect total match surplus $S \equiv W - U + J - V$. With Nash Bargaining, firm surplus is a constant of total surplus $J - V = (1 - \gamma)S$. Thus, the subsidy-tax schedule does not affect firm surplus, and therefore not vacancy creation, and therefore not the equilibrium unemployment rate.

Consider case 2) with $\tau_e = 0$ and τ_u set to balance the budget. In this case, total match surplus increases, since the flow surplus increases by $(s + \tau_u)$. Nash bargaining implies that firm surplus increases, resulting in more vacancy creation, and lower equilibrium unemployment.

Another way to see this is to use the Equations above to construct the wage curve and the job-creation curve. In case 1) both curves will shift up by exactly s, leaving equilibrium tightness unchanged. In case 2), the Job-creation curve shifts up, and the wage curve shifts down.

2. Suppose the policy is successful in reducing the economy's unemployment rate. Is it clear that the policy have also raised social welfare?

   Answer: No, it is not clear. Whether creating more vacancies and lowering unemployment increase welfare or not depends on whether the congestion externality dominates the thick-market externality or not. This, in turn, depend on the size of worker bargaining power γ relative to the elasticity of the matching function w.r.t. tightness.

A Huggett model with government debt (15 points)

Consider an infinite-horizon economy with a continuum (measure 1) of ex-ante identical households each having efficiency units of labor ϵ_{it}, drawn from distribution F with finite support $[\epsilon_{min}, \epsilon_{max}]$ and mean 1, i.i.d. across households and time. Consumers can trade a risk-free bond subject to a credit constraint. Each household i solves

$$\begin{aligned} \max_{a_{it+1}, c_{it}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_{it}) + v(G_t) \right] \\ \text{s.t.} \quad & c_{it} + a_{it+1} \leq \epsilon_{it} w_t - T_t + (1 + r_t) a_{it} \\ & a_{it+1} \geq -\underline{a} \end{aligned}$$

where u and v satisfy standard conditions, $\underline{a} > 0$ is some exogenous debt limit, G_t is government consumption and T_t is lump-sum tax. A representative firm hires labor in a competitive labor market and employs the production function $Y_t = L_t$, where L_t is the aggregate labor endowment. We denote total bond demand with $A_{t+1} = \int_{i=0}^{1} a_{it+1} di$.

The bonds are issued by the government. The government sets the consumption level G_t and the tax rate T_t exogenously. Bond supply B_{t+1} are resiudally determined from the government budget constraint:

$$T_t + B_{t+1} = R_t B_t + G_t$$

1. Define a competitive equilibrium.

   Answer: The firm problem is to in every period solve

   $$\begin{aligned} \max_{L_t} \quad & Y_t - w_t L_t \\ \text{s.t.} \quad & Y_t = L_t \end{aligned}$$

   A competitive equilibrium is an allocation $\{c_{it}, a_{it+1}, L_t, B_{t+1}\}$, a price system $\{w_t, r_t\}$ and a policy T_t, G_t s.t.

   - Given prices and the policy, the household problem is solved
   - Given prices, the firm problem is solved
   - Given prices and the policy, the government budget constraint is satisfied
   - The markets for goods, labor and assets clear:

   $$\begin{aligned} \int_{i=0}^{1} c_{it} di &= Y_t \\ \int_{i=0}^{1} \epsilon_{it} di &= L_t \\ \int_{i=0}^{1} a_{it+1} di &= B_{t+1} \end{aligned}$$

2. Consider a steady state, in which government surplus is some positive constant: $T - G > 0$. Derive an equation for bond supply, and draw it together with bond demand in an "Ayiagari diagram".

   Answer: Denote steady state bond supply and net interest rate with B and r. The government budget constraint imply

   $$B = \frac{T - G}{r}$$

   which defines the asset supply curve $B(r)$. This is a downward-sloping curve curve with $\lim_{r \to \infty} B(r) = 0$ and $\lim_{r \to 0} B(r) = \infty$.

   The Asset demand curve $A(r)$ comes form aggregating the household policy functions. It is clear that $\lim_{r \to -1} A(r) = -\bar{a}$, as all households would go up against the borrowing constraint if borrowing was free. In class, we have also proved that $\lim_{r \to \frac{1}{beta} - 1} A(r) = \infty$ due to the precautionary-savings motive in the presence of idiosyncratic income risk (no proof needed here).

   Putting this together, we get the diagram in Figure 3. Asset supply and demand intersects at the equilibrium interest rate r^*.

3. What happens to the steady state interest rate if the steady state government suprlus increases?

   Answer: If $T - G$ increases, the asset supply curve shifts out, and we get a higher equilibrium real interest rate.

   Not required for full score but for completeness, it is worth noting that the increase in $T - G$ also affects asset demand if it is caused by an increase in T.

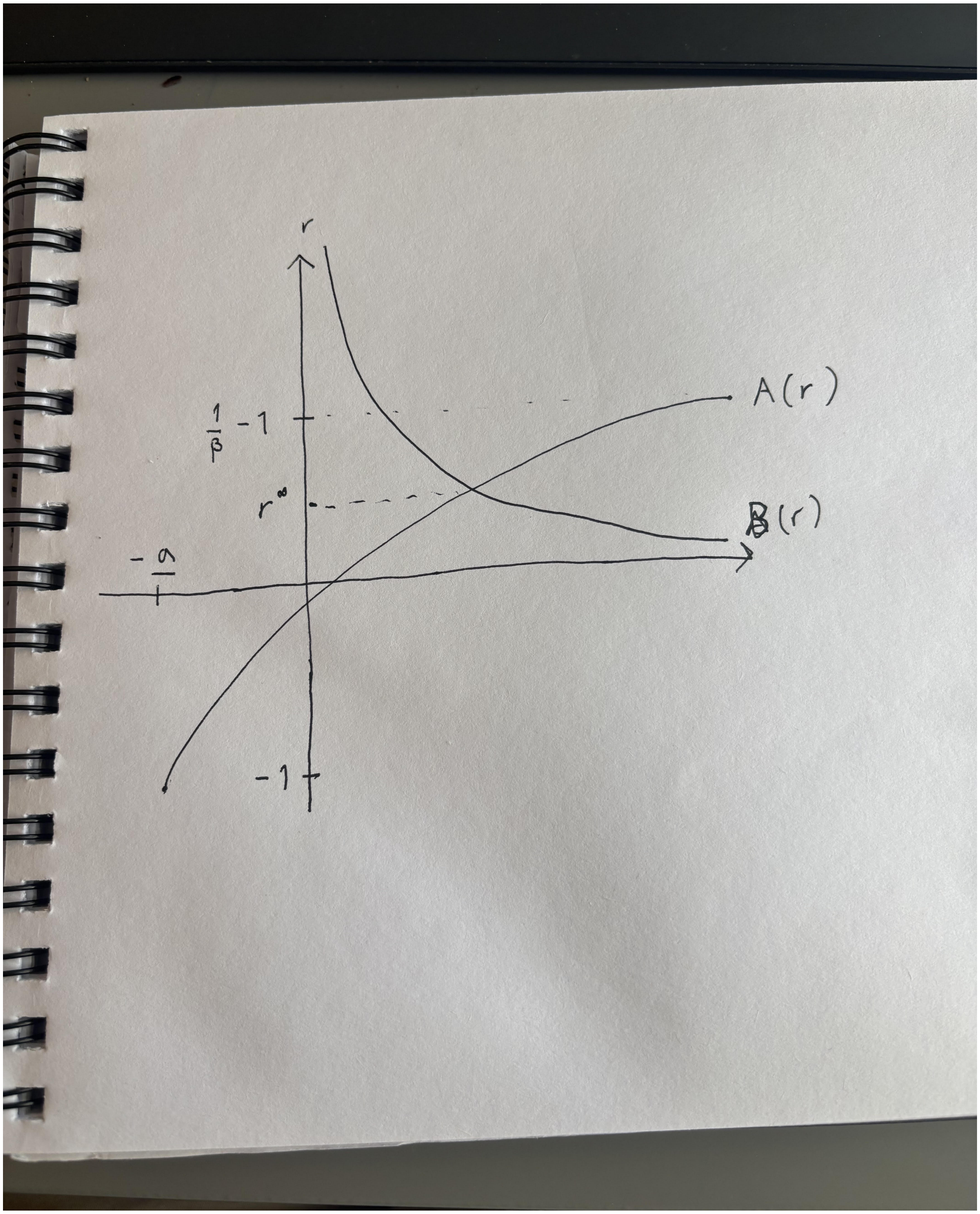

Figure 3: Asset demand diagram