General Equilibrium with Incomplete Markets

1 General Equilibrium

- An economy populated by households without insurance against income risk
- They use borrowing-saving to self-insure
- Factor prices:
  - general level of wages that shifts labour income proportionally
  - rental rate of capital $\Leftrightarrow$ interest rates

  come from production side

1.1 Households

- Represent labour-income risk as a process for the effective supply of labour:

$$y^i = wl^i$$

  where l^i is household-specific

- Shocks can represent
  - Unemployment
  - Shocks to individual ability
  - Shocks to the quality of the employer-employee match
- Households solve income-fluctuations problem taking w and R as given and constant
- State variables for individual household problem can be

  - A and s

or

  - $x \equiv RA + y(s)$ and s as we did before

- As long as $\beta R < 1$ and $\lim_{c\to\infty} -\frac{u''(c)}{u'(c)} = 0$, there will be an invariant joint distribution of A and s for given factor prices. Call this

$$F(A, s; R, w)$$

and denote the marginal distribution of A by

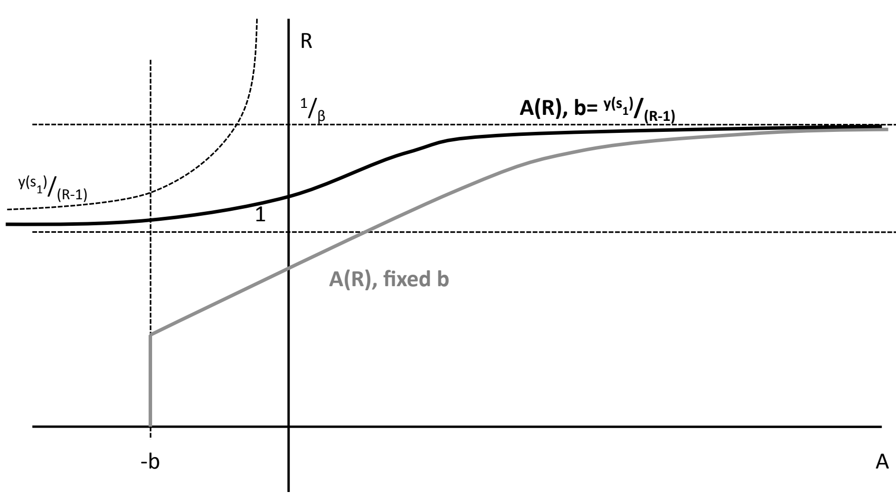
$$F(A; R, w)$$

- Total assets held by all the individuals of the economy in steady state are:

$$A(R, w) = \int A dF(A; R, w)$$

- Properties of $A(R, w)$:

  - $\lim_{R\to\frac{1}{\beta}^-} A(R, w) = \infty$
  - $\lim_{R\to 0^+} A(R, w) = -b$
  - Not necessarily monotonic in R



1.2 Firms

- Competitve firms, so factor prices are

$$
\begin{aligned}
w &= F_L(K, L) \\
R &= F_K(K, L) + (1-\delta)
\end{aligned}
$$

1.3 Market clearing

- Analyze steady state (transitional dynamics and aggregate shocks not easy to compute)
- Aggregate labour supply is fixed (no aggregate risk)

$$
L = \sum_i l^i = 1
$$

- Capital market clearing:

$$
A(R, w) = K \tag{1}
$$

- Equation (1) is the whole point of Aiyagari [1994]
- Express (1) in terms of capital supply and demand, i.e. call the LHS capital supply and the RHS capital demand
  - Demand:

$$
K^D(R) = F_K^{-1}(R - 1 + \delta)
$$

  - Supply:

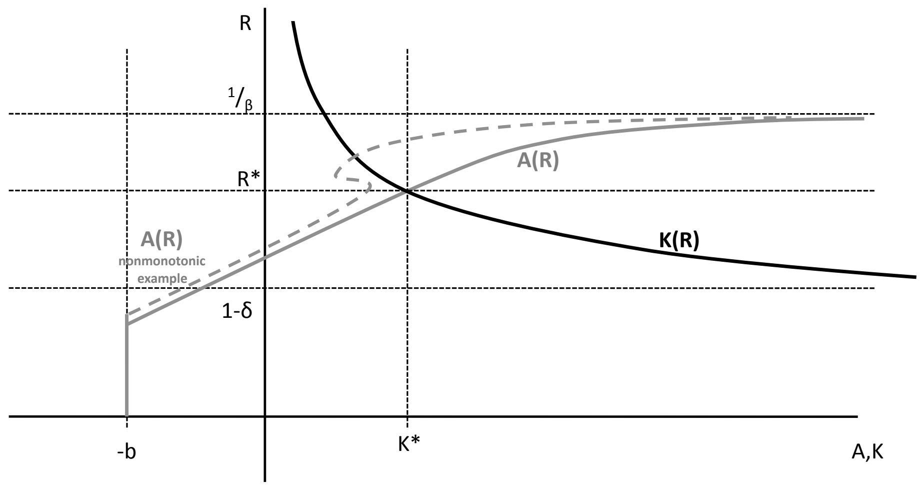
$$
K^S(R) = A(R, w(R))
$$

    where

$$
w(R) = F_L\left(K^D(R), 1\right)
$$

    i.e. we also implicitly clear the labour market

- $w(R)$ is decreasing $\Rightarrow$ additional source of possible nonmonotonicity in $K^S(R)$!
- (but monotone in most examples)
- Note that we immediately know that in equilibrium $R < \frac{1}{\beta}$. Why?



1.4 Computation

An algorithm:

1. Guess R
2. Compute $K^D(R)$ and $w(R)$
3. Solve consumer's dynamic programing problem $\Rightarrow$ obtain policy function $c\,(x, s)$ or $c\,(A, s)$
4. Simulate the life of one consumer for many periods $\Rightarrow$ obtain invariant distribution of assets
   - or, alternatively, compute the invariant distribution directly from the transition matrix of the consumer's state
5. Use invariant distribution of assets to compute $K^S\,(R) = A(R, w\,(R))$
6. Adjust the guess of R and repeat 1-5 until $K^S(R) = K^D(R)$
   - One way to update the guess of R is $R^{NEW} = \mu R + (1-\mu)\left[F_K\left(A(R) + 1 - \delta, 1\right)\right]$

1.5 Aiyagari's calibration

- $\beta = 0.96$
- $F = K^\alpha$ with $\alpha = 0.36$
- $\delta = 0.08$
- $u = \frac{c^{1-\sigma}}{1-\sigma} \quad \sigma = 1, 3, 5$
- Labour income process:

$$\log l_t = \rho \log l_{t-1} + \sqrt{(1-\rho^2)}\varepsilon_t$$

  with $\rho = \{0, 0.3, 0.6, 0.9\}$ and $Var(\varepsilon_t) = \{0.2, 0.4\}$, approximated with a discrete Markov chain

- $b = 0$
- Results:

TABLE II

A. Net return to capital in %/aggregate saving rate in % (Var(ε) = 0.2)

ρ	σ = 1	σ = 3	σ = 5
0	4.1666/23.67	4.1456/23.71	4.0858/23.83
0.3	4.1365/23.73	4.0432/23.91	3.9054/24.19
0.6	4.0912/23.82	3.8767/24.25	3.5857/24.86
0.9	3.9305/24.14	3.2903/25.51	2.5260/27.36

B. Net return to capital in %/aggregate saving rate in % (Var(ε) = 0.4)

ρ	σ= 1	σ = 3	σ = 5
0	4.0649/23.87	3.7816/24.44	3.4177/25.22
0.3	3.9554/24.09	3.4188/25.22	2.8032/26.66
0.6	3.7567/24.50	2.7835/26.71	1.8070/29.37
0.9	3.3054/25.47	1.2894/31.00	−0.3456/37.63

-
- Compare to full insurance:

$$R - 1 = \frac{1}{\beta} - 1 = 4.17\%$$
$$\frac{s}{Y} = 23.67$$

- The savings rate comes from:

$$
\begin{aligned}
\alpha K^{\alpha-1}+(1-\delta) &= \frac{1}{\beta} \\
\Rightarrow K &= \left(\frac{\frac{1}{\beta}-1+\delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \\
sK^{\alpha} &= \delta K \\
\Rightarrow s &= \delta K^{1-\alpha} \\
&= \delta \frac{\alpha}{\frac{1}{\beta}-1+\delta} \\
&= 0.08 \frac{0.36}{1.0417-1+0.08}
\end{aligned}
$$

- Conclusion: for middle-of-the-range parameters, not a big deal!
- Self-insurance quite effective: welfare gain of 14 percent of consumption compared to autarky (for a single individual, no GE effects)
- In the cross section, $Var(c) < Var(y) < Var(A)$, which coincides with data. Quantitatively, model underpredicts wealth inequality
- Is this the wrong model for the top of the wealth distribution?
- Entrepreneurship and uninsured capital-income risk [Angeletos, 2007, Gentry and Hubbard, 2000, Moskowitz and Vissing-Jorgensen, 2002, Quadrini, 2000, Hurst and Lusardi, 2004]
- Carroll [1997]: The right calibration of this type of model has:
  - Temporary and permanent income shocks
  - $\beta \ll \frac{1}{R}$

  ⇒ "Buffer stock" behaviour
    * Hold a small amount of assets to smooth temporary shocks
    * But not large because $\beta R < 1$
    * Do not smooth permanent shocks (you saw this with Rui in class)
    * An individual's consumption closely tracks their income

2 Welfare implications of market incompleteness

- Market incompleteness: 1st welfare theorem does not hold

- Formally, it comes from no single budget constraint (because it must hold state by state)
- Average consumption higher than with complete markets (unless past golden rule)
- *Steady state* utility may be higher than with complete markets (higher average consumption but more risk)
  - (but take into account transition)

2.1 Constrained efficiency

- Allocation not Pareto efficient
- When 1st welfare theorem holds, it's a very strong result:
  - We don't really think a planner could solve the "planner problem"
  - But *even* if they could, they still wouldn't improve on the market
- When 1st welfare theorem doesn't hold, should we conclude that the market does not allocate resources well?
- Clearly a planner who *could* solve the planner problem would do better
- But what if we made the planner less powerful?
- What is a fair comparison between what the markets and a hypothetical planner would do?
- Suppose we impose "the same" constraints on the planner as on the market
- What do we mean by "the same constraints"?
  - The planner can tell people how much to trade in the existing markets
  - The planner cannot create new markets
- Justification:
  - Maybe there is a reason why markets are not complete
  - Comparing equilibrium to first-best allocation may be too much to ask of the markets
  - If we could prove that the planner cannot improve upon the market, this would be a constrained form of the 1st welfare theorem
  - If we were persuaded that we could not fix the market incompleteness, there would be a good case not to intervene in the market

- "Constrained efficiency"
  - Could a planner make everyone be made better off *by using only the existing markets*?
  - In general, yes! Hart [1975], Stiglitz [1982], Geanakoplos and Polemarchakis [1986]
  - i.e. incomplete market allocations are *not* constrained efficient in general
- Constrained efficiency is more of a case-by-case definition of what powers we imagine the planner has than a generally-well-defined concept that can be used in any context.
  - Need to be careful in spelling out what powers the planner is assumed to have and what powers the planner does not.
  - Policy implications may or may not follow. i.e. to go from "constrained inefficiency" to "this policy should be pursued" one needs to make the case that the planner's powers correspond to some plausible policy instruments.
- In insurance example, because marginal rates of substitution are not equalized, prices affect the degree of insurance. A planner could take that into account, while the market does not
- Davila et al. [2005] apply this reasoning to the Aiyagari [1994] model.

2.2 Two period case

- Households maximize

$$\max_{c_1,c_{2s},K} u(c_1) + \beta \left[\pi u\left(c_{2H}\right) + (1-\pi)u\left(c_{2L}\right)\right]$$
$$s.t. \quad K = y - c_1$$
$$c_{2s} = RK + e_s w$$

- Firms:

$$R = F_K(K, L)$$
$$w = F_L(K, L)$$

- Labour market

$$L = \pi e_H + (1-\pi)e_L$$

- Market incompleteness: no insurance for labour-endowment shocks

- Household FOC:

$$u_c(y-K) = \beta R\left[\pi u_c\left(RK + e_H w\right) + (1-\pi)u_c\left(RK + e_L w\right)\right]$$

- Social planning problem: assume planner can tell household how much to save:
  - (but not, for instance, tell firms they should change their factor demand curves so that factor prices are different)
  - (and especially not make state-contingent transfers, which is what we are assuming markets don't do)

$$\max_{c_1, c_{2s}, K} W\left(c_1, c_{2s}\right) = u(c_1) + \beta\left[\pi u\left(c_{2H}\right) + (1-\pi)u\left(c_{2L}\right)\right]$$
$$s.t. \quad K = y - c_1$$
$$c_{2s} = F_K\left(K, L\right)K + e_s F_L\left(K, L\right)$$

- Suppose the planner decides to engineer an increase in K starting from the equilibrium allocation. What is the change in ex-ante welfare?

$$\frac{dW}{dK} = -u_c(y-K) + \beta\left[\pi u_c\left(c_{2H}\right)\frac{dc_{2H}}{dK} + (1-\pi)u_c\left(c_{2L}\right)\frac{dc_{2L}}{dK}\right]$$

where

$$\frac{dc_{2H}}{dK} = F_K\left(K, L\right) + F_{KK}(K,L)K + e_H F_{LK}(K,L)$$
$$\frac{dc_{2L}}{dK} = F_K\left(K, L\right) + F_{KK}(K,L)K + e_L F_{LK}(K,L)$$

- Use household's FOC and factor prices:

$$\begin{aligned}
\frac{dW}{dK} &= -\beta R\left[\pi u_c\left(c_{2H}\right) + (1-\pi)u_c\left(c_{2L}\right)\right] + \beta\left[\pi u_c\left(c_{2H}\right)\frac{dc_{2H}}{dK} + (1-\pi)u_c\left(c_{2L}\right)\frac{dc_{2L}}{dK}\right] \\
&= -\beta R\left[\pi u_c\left(c_{2H}\right) + (1-\pi)u_c\left(c_{2L}\right)\right] \\
&\quad + \beta\left[\pi u_c\left(c_{2H}\right)\left(R + F_{KK}K + e_H F_{LK}\right) + (1-\pi)u_c\left(c_{2L}\right)\left(R + F_{KK}K + e_L F_{LK}\right)\right] \\
&= \beta\left[\pi u_c\left(c_{2H}\right)\left(F_{KK}K + e_H F_{LK}\right) + (1-\pi)u_c\left(c_{2L}\right)\left(F_{KK}K + e_L F_{LK}\right)\right] \\
&= \beta\left[\pi u_c\left(c_{2H}\right)\left(-LF_{LK} + e_H F_{LK}\right) + (1-\pi)u_c\left(c_{2L}\right)\left(-LF_{LK} + e_L F_{LK}\right)\right] \\
&= \beta F_{LK}\pi\left(1-\pi\right)\left(e_H - e_L\right)\left[u_c\left(c_{2H}\right) - u_c\left(c_{2L}\right)\right] < 0
\end{aligned}$$

- Note: this uses the fact that

$$
\begin{gathered}
F_K K + F_L L = F \\
F_{KK} K + F_K + F_{LK} L = F_K \\
\Rightarrow F_{KK} K + F_{LK} L = 0
\end{gathered}
$$

- At the margin, decreasing the capital stock increases the return to capital and depresses the return to labour
- Sometimes this is called a "pecuniary externality"
- Pecuniary externality:
  - Person A changes decisions at the margin
  - This affects prices
  - This affects the utility of other people
- Warning:
  - pecuniary externalities are present in complete market models as well!
  - (the difference is that with incomplete markets they don't wash out, so Pareto improvements are possible)
- Let's see this in the equations. With complete markets, we know that the consumers will get full insurance, so

$$c_{2H} = c_{2L} = RK + \left[\pi e_H + (1-\pi)\, e_L\right] w$$

and

$$u_c(y-K) = \beta R u_c \left(RK + \left[\pi e_H + (1-\pi)\, e_L\right] w\right)$$

- Set up the social planner problem:

$$
\begin{gathered}
\max_{c_1, c_{2s}, K} W\left(c_1, c_{2s}\right) = u(c_1) + \beta\left[\pi u\left(c_{2H}\right) + (1-\pi) u\left(c_{2L}\right)\right] \\
s.t. \quad K = y - c_1 \\
c_{2H} = c_{2L} = F_K\left(K, L\right) K + \left[\pi e_H + (1-\pi)\, e_L\right] F_L\left(K, L\right)
\end{gathered}
$$

- Suppose the planner decides to engineer an increase in K starting from the equilibrium allocation. What is the change in ex-ante welfare?

$$\frac{dW}{dK} = -u_c(y-K) + \beta u_c\left(c_2\right)\left[F_{KK} K + F_K + L F_{LK}\right] \tag{2}$$

- Equation (2) shows that the pecuniary externalities are still there: changing allocations changes equilibrium prices, which has indirect effects on welfare. The key is that they cancel out:
- Use household FOC:

$$\begin{aligned}\frac{dW}{dK} &= -\beta F_K u_c(c_2) + \beta u_c(c_2)[F_{KK}K + F_K + LF_{LK}] \\ &= \beta u_c(c_2)[-F_K + F_{KK}K + F_K + LF_{LK}] \\ &= 0\end{aligned}$$

- With incomplete markets, the pecuniary externality does not cancel out because it indirectly increases insurance
- (Without making state-contingent transfers)
- (Wrong) intuition about $K^{SB} < K^{EQ}$
  - Precautionary savings lead to increased savings
  - Equilibrium capital stock is higher than first best $K^{FB} < K^{EQ}$
  - Constrained efficient allocation ("second best") features $W^{FB} > W^{SB} > W^{EQ}$, so it makes sense that $K^{FB} < K^{SB} < K^{EQ}$. Second best in "in between"
  - Multiperiod case shows that this is completely misleading
- The key to whether we want to encourage more or less capital is whether the consumption-poor households (whom we would like to insure if there was a market for that) have labour-intensive or capital-intensive income
  - In the two-period case, the consumption-poor had capital-intensitve income compared to the consumption-rich
  - Therefore we wanted to lower wages and increase interest rates
  - In general, this will depend on the stochastic process for labour-endowment shocks, especially its persistence.

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