Directed Search

Lecture 1: Introduction and Basic Formulations

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Main sources of this lecture:

- Peters, M., 1991, "Ex Ante Price Offers in Matching: Games: Non-Steady State," ECMA 59, 1425-1454.

- Montgomery, J.D., 1991, "Equilibrium Wage Dispersion and Interindustry Wage Differentials," QJE 106, 163-179.

- Moen, E., 1997, "Competitive Search Equilibrium," JPE 105, 694–723.

- Acemoglu, D. and R. Shimer, 1999, "Efficient Unemployment Insurance," JPE 107, 893–927.

Main sources of this lecture (cont'd):

- Burdett, K., S. Shi and R. Wright, 2001,
  "Pricing and Matching with Frictions," JPE 109, 1060-1085.

- Julien, B., J. Kennes and I. King, 2000,
  "Bidding for Labor," RED 3, 619-649.

- Shi, S., 2008, "Search Theory (New Perspectives),"
  in: S.N. Durlauf and L.E. Blume eds.,
  The New Palgrave Dictionary of Economics, 2nd edition,
  Palgrave, Macmillan.

1. Search Frictions and Search Theory

- Search frictions are prevalent:

  - unemployment, unsold goods, unattached singles
  - pervasive failure of the law of one price

- "Undirected search":
  individuals know the terms of trade only AFTER the match

  - bargaining: Diamond (82), Mortensen (82), Pissarides (90)
  - price posting: Burdett and Mortensen (98)
    (price posting $\neq$ directed search!)

"Directed search":

- individuals CHOOSE what terms of trade to search for
- tradeoff between terms of trade and trading probability

Why should we care?

- prices should be important ex ante in resource allocation
- efficiency properties and policy recommendations
- robust inequality and unemployment
- tractability for analysis of dynamics and business cycles

Is directed search empirically relevant?

Casual evidence: People do not search randomly.

- Buyers know where particular goods are sold:

  – If a buyer wants to buy shoes, the buyer does not
    go to a grocery store

- Buyers know the price range:

  – If a buyer wants to buy tailor-made suits,
    the buyer does not go to Walmart.

Is directed search empirically relevant?

- Hall and Krueger (08):
  84% of white, non-college educated male workers
  either "knew exactly" or "had a pretty good idea"
  about how much their current job would pay
  at the time of the first interview.

- Holzer, Katz, and Krueger (91, QJE):
  (1982 Employment Opportunity Pilot Project Survey)
  firms in high-wage industries attract more applicants
  per vacancy than firms in low-wage industries after
  controlling for various effects.

Sketch of the lectures (if time permits):

- basic formulations of directed search

- matching patterns and inequality

- wage ladder and contracts

- business cycles

- monetary economics

2. Undirected Search and Inefficiency

One-period environment:

- workers: an exogenous, large number u

  - risk neutral, homogeneous
  - producing y when employed, 0 when unemployed

- firms/vacancies: endogenous number v

  - cost of a vacancy: $k \in (0, y)$
  - production cost $= 0$

Components of DMP model: (1) - (3)

(1) Matching technology:

- matching function: $M(u, v)$ (constant returns to scale)

- tightness: $\theta = v/u$;   matching probabilities:

$$\text{for a worker:} \quad p(\theta) = \frac{M(u,v)}{u} = M(1, \theta)$$
$$\text{for a vacancy:} \quad q(\theta) = \frac{M(u,v)}{v} = M(\tfrac{1}{\theta}, 1) = \frac{p(\theta)}{\theta}$$

- assumptions:
  $p(\theta)$ is strictly increasing and concave;
  $q(\theta)$ is strictly decreasing; $q(0) = 1, q(\infty) = 0$;
  worker's share of contribution to match:

$$s(\theta) \equiv \frac{\partial \ln M(u, v)}{\partial \ln u} = 1 - \frac{\theta p'(\theta)}{p(\theta)} \quad \in [0, 1]$$

(2) Wage determination (Nash bargaining):

$$\max_{w \in [0,y]} w^{\sigma}(y - w)^{1-\sigma}, \quad \sigma: \text{worker's bargaining power}$$

$$\text{solution: } w = \sigma \, y$$

(3) Equilibrium tightness:

- expected value of a vacancy:

$$J = q(\theta)(y - w) = (1 - \sigma)q(\theta)y$$

- free entry of vacancies: $J = k$

$$\implies w = y - \frac{k}{q(\theta)} \implies q(\theta) = \frac{k}{(1 - \sigma)y}$$

a unique solution for θ exists iff $0 < k < (1 - \sigma)y$.

Social welfare and inefficiency:

- welfare function: $\mathcal{W} = u \times V + v \times (J - k) = u\, V$

- value for a worker:

$$V = p(\theta)w = p(\theta)\left[y - \frac{k}{q(\theta)}\right] = p(\theta)y - k\theta$$

- social welfare equals net output:

$$\mathcal{W} = u\, V = u\, p(\theta)y - (u\theta)k$$

- "constrained" efficient allocation:

$$\max_{\theta}\ \mathcal{W} = u\left[p(\theta)y - k\theta\right] \quad \implies\quad p'(\theta) = \frac{k}{y}$$

- rewrite the first-order condition for efficiency:

$$\frac{k}{y} = p'(\theta) = [1 - s(\theta)]\frac{p(\theta)}{\theta} = [1 - s(\theta)]q(\theta)$$

- equilibrium condition for tightness:

$$\frac{k}{y} = (1 - \sigma)q(\theta)$$

- equilibrium is socially efficient if and only if

$s(\theta)$	$=$	σ
worker's share		bargaining
in creating match		power
Hosios (90) condition		

Why is this condition needed for efficiency?

• two externalities of adding one vacancy:

  negative: decreasing other vacancies' matching
  positive: increasing workers' matching

• internalizing the externalities:

$$
\begin{array}{ccc}
\text{private marginal} & & \text{social marginal} \\
\text{value of vacancy} & = & \text{value of vacancy} \\
(y - w)q = (1 - \sigma)qy & & \frac{\partial M(u,v)}{\partial v}y = (1 - s)qy
\end{array}
$$

− if $1 - \sigma > 1 - s$, entry of vacancies is excessive
− if $1 - \sigma < 1 - s$, entry of vacancies is deficient

Efficiency condition, $s(\theta) = \sigma$, is violated generically

- Cobb-Douglas: $M(u, v) = M_0 u^{\alpha} v^{1-\alpha}$

$$p(\theta) = M_0 \theta^{1-\alpha}, \quad s(\theta) = 1 - \frac{\theta p'(\theta)}{p(\theta} = \alpha \quad \text{(a constant)}$$

- telephone matching: $M(u, v) = \frac{uv}{u+v}$

$$p(\theta) = \frac{\theta}{1+\theta}, \quad s(\theta) = \frac{\theta}{1+\theta}$$

$$s(\theta) = \sigma \implies \sigma = 1 - \left(\frac{k}{y}\right)^{1/2} \quad \text{(recall } p'(\theta) = \frac{k}{y})$$

- urn-ball matching: $M(u, v) = v(1 - e^{-u/v})$

Cause of inefficiency:

> search is undirected: wage does not perform the role
> of allocating resources ex ante (before match)

- Nash bargaining splits the ex post match surplus

- it does not take matching prob into account

What about undirected search with wage posting?
(e.g., Burdett-Mortensen 98, with free entry)

- similar inefficiency:
  workers cannot search for particular wages;
  workers receive all offers with the same probability

Criticisms on undirected search models:

- inefficiency arises from exogenously specified elements: Nash bargaining, matching function

- policy recommendations are arbitrary, depending on which way the efficiency condition is violated. E.g.

  - Should workers' search be subsidized?

- can we just impose the Hosios condition and go on?

  - no, if σ and parameters in $s(\theta)$ change with policy
  - no, if there is heterogeneity (more on this later)

3. Directed Search and Efficiency

Directed search:

- Basic idea: individuals explicitly take into account the relationship between wage and the matching probability

- A more detailed description:

  - a continuum of "submarkets", indexed by w
  - market tightness function: $\theta(w)$
  - matching inside each submarket is random
  - matching probability:
    for a worker $p(\theta(w))$;   for a vacancy: $q(\theta(w))$

Market tightness function: $\theta(w)$

- free entry of vacancies into each submarket

- complementary slackness condition for all w:

$$J(w) = q(\theta(w))(y - w) \leq k, \qquad \text{and } \theta(w) \geq 0$$

  - if there is potential surplus $(y - w > k)$, then $J(w) = k$: firms are indifferent between such submarkets
  - if there is no potential surplus $(y - w \leq k)$, then $\theta(w) = 0$

- solution:

$$\theta(w) = q^{-1}\left(\frac{k}{y-w}\right) \text{ whenever } w < y - k;$$
$$\theta(w) \text{ is strictly decreasing in } w$$

Worker's optimal search:
(This decision would not exist if search were undirected.)

- A worker chooses which submarket w to enter:

$$\max_{w} \; p(\theta(w)) \; w \quad \text{where } \theta(w) = q^{-1}\left(\frac{k}{y - w}\right)$$

- tradeoff between wage w and matching prob $p(\theta(w))$:
  higher wage is more difficult to be obtained: $\frac{dp(\theta(w))}{dw} < 0$

- optimal choice:

$$w = -\frac{\tilde{p}(w)}{\tilde{p}'(w)}, \quad \tilde{p}(w) \equiv p(\theta(w))$$

Efficiency of directed search equilibrium:
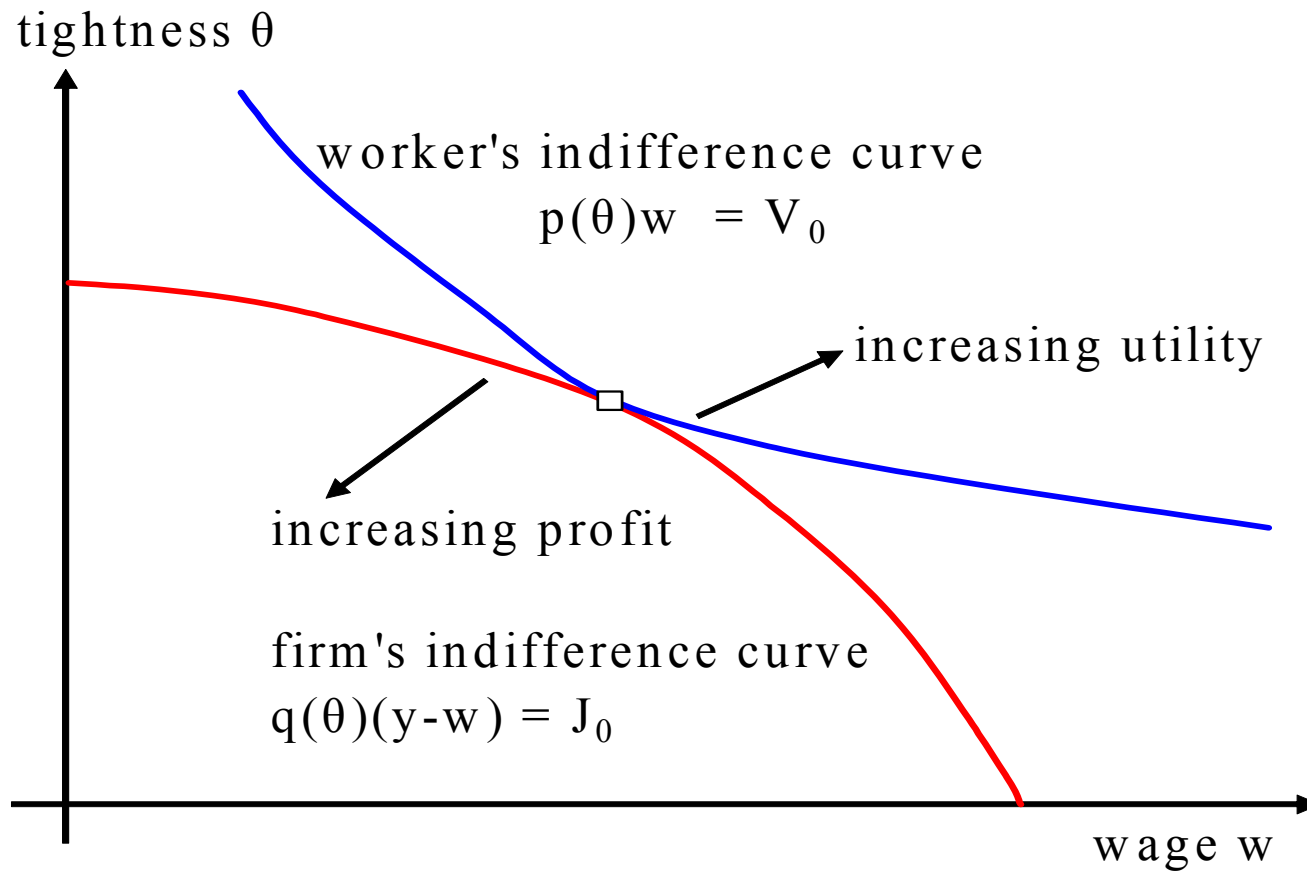
Optimal directed search implies the Hosios condition:

$$\frac{w}{y} = s(\theta), \quad \text{where } s(\theta) = 1 - \frac{\theta p'(\theta)}{p(\theta)}$$

Proof:

$$\theta(w) = q^{-1}\left(\frac{k}{y-w}\right) \implies \theta'(w) = \frac{q(\theta(w))/(y-w)}{q'(\theta(w))}$$

$$\text{sub. } q(\theta) = \frac{p(\theta)}{\theta} \implies \theta'(w) = \frac{\theta p(\theta)/(y-w)}{\theta p'(\theta) - p(\theta)}$$

$$\text{FOC: } w = -\frac{p(\theta)}{p'(\theta)\theta'(w)} = \left(\frac{p}{\theta p'} - 1\right)(y - w)$$

$$= \left(\frac{1}{1-s(\theta)} - 1\right)(y - w)$$

$$\implies \frac{w}{y} = s(\theta). \quad \blacksquare$$

Hedonic pricing

tightness θ

worker's indifference curve
$$p(\theta)w = V_0$$

increasing utility

increasing profit

firm's indifference curve
$$q(\theta)(y-w) = J_0$$

wage w

Remarks:

- Because search is directed by the "price" function $\theta(w)$, this formulation is called **competitive search** (Montgomery 91, Moen 97, Acemoglu and Shimer 99)

- The price function, $\theta(w)$, is not unique: but they all have the same value at equilibrium w

- Efficiency is "constrained" efficiency:

  - the planner cannot eliminate search frictions
  - unemployment still exists

4. Strategic Formulation of Directed Search

Motivation:

- The formulation above endogenizes the wage share;
  but the matching function is still a black box

- Is there a way to endogenize the mf as well?

- In a strategic formulation, total # of matches is
  an aggregate result of workers' application decisions

- some papers:
  Peters (91, ECMA),
  Burdett-Shi-Wright (01, JPE), Julien-Kennes-King (00, RED)

One-period game with directed search: BSW 01
(for fixed numbers u and v, for now)

- firms simultaneously post wages

- workers observe all posted wages

- each worker chooses which firm to apply to:
  no multiple applications (interviews)

- each firm randomly chooses one among
  the received applicants to form a match

No coordination among firms or workers
$\Longrightarrow$ a worker and a vacancy may fail to match

Focus on symmetric equilibrium:

- all workers use the same strategy,
  including responses to a firm's deviation

- this implies that all firms post the same wage w

Why such a focus?

- symmetric equilibrium emphasizes lack of coordination

- tractability: even in the case $u = v = 2$, there are many
  asymmetric equilibria which involve trigger strategies

A worker's strategy:
(when firm A posts x and other firms post w)

- each worker applies to firm A with probability a, and applies to each of the other firms with prob $\pi(a) = \frac{1-a}{v-1}$

- an applicant's indifference condition:

$$\underbrace{p(a)}_{\substack{\text{prob. of being} \\ \text{chosen by firm } A}} x \quad = \quad \underbrace{p(\pi(a))}_{\substack{\text{prob. of being} \\ \text{chosen elsewhere}}} w$$

function $p(.)$: to be determined.

- this solves $a = f(x, w)$:
  workers' best response to firm A's deviation to x

A worker B's matching probability with firm A:

# of other app. to A	prob. of this event	conditional prob. that B is chosen
n	$C^n_{u-1}a^n(1-a)^{u-1-n}$	$\frac{1}{n+1}$

unconditional prob. that B matches with firm A:

$$\sum_{n=0}^{u-1}\frac{1}{n+1}C^n_{u-1}a^n(1-a)^{u-1-n} = \sum_{n=0}^{u-1}\frac{(u-1)!\,a^n(1-a)^{u-1-n}}{(n+1)!(u-1-n)!}$$

$$= \frac{1}{ua}\sum_{n=1}^{u}\frac{u!}{n!(u-n)!}a^n(1-a)^{u-n} = \frac{1-(1-a)^u}{ua}\quad(\equiv p(a))$$

Firm A's optimal choice:

● queue length (expected #) of applicants to firm A:

$$\sum_{n=1}^{u} n\ C_u^n a^n (1-a)^{u-n} = \sum_{n=1}^{u} \frac{u!\ a^n (1-a)^{u-n}}{(n-1)!(u-n)!}$$

$$= ua \sum_{n=0}^{u-1} \frac{(u-1)!}{n!(u-n)!} a^n (1-a)^{u-1-n} = ua.$$

● tightness for firm A, $\frac{1}{uf(x,w)}$, is indeed a function of x

● firm A's matching probability:

$$\sum_{n=1}^{u} C_u^n a^n (1-a)^{u-n} = 1 - (1-a)^u$$

Firm A's optimal choice:

- choosing wage $x = g(w)$ (and a) to solve:

$$\max_{(x,a)} \left[1 - (1-a)^u\right](y-x)$$

$$\text{s.t.} \quad \frac{1-(1-a)^u}{ua}x = \frac{1-[1-\pi(a)]^u}{u\pi(a)}w$$

- tradeoff with a higher x:

  - lower ex post profit $(y-x)$
  - higher matching probability $[1-(1-a)^u]$:
    * $a = f(x,w)$ satisfies the constraint;
    * it is an increasing function of x

Symmetric equilibrium:

$$\text{wage } w \text{ that satisfies } w = g(w).$$

- worker's application prob.: $a = \pi(a) = \frac{1}{v}$

- queue length for each firm: $ua = \frac{u}{v} = \frac{1}{\theta}$

- firm's matching probability:

$$q(u,v) = 1 - (1-a)^u = 1 - (1 - \frac{1}{v})^u$$

- firm's first-order condition yields:

$$w = y \left[\frac{(1-1/v)^{-u} - 1}{u/v} - \frac{1}{v-1} \right]^{-1}$$

Differences from competitive search:

- endogenous matching function:

$$M(u,v) = v\, q(u,v) = v \left[1 - (1 - \frac{1}{v})^u\right]$$

  − decreasing returns to scale:

$$q(2u,2v) < q(u,v) \implies M(2u,2v) < 2M(u,v)$$

  − coordination failure is more severe
    when there are more participants on each side

- deviating firm can affect a worker's payoff elsewhere:

$$\frac{1 - [1 - \pi(a)]^u}{u\pi(a)} w, \quad \text{where } \pi(a) = \frac{1-a}{v-1}$$

All works out well in the limit $u, v \to \infty$:
[denote $\theta = \lim \frac{v}{u} \in (0, \infty)$]

• constant returns to scale in matching:

$$q(u, v) = 1 - (1 - \tfrac{1}{v})^u$$
$$= 1 - (1 - \tfrac{1}{\theta u})^u \to 1 - e^{-1/\theta}$$

$$p(u, v) = \frac{1 - (1 - \tfrac{1}{v})^u}{u/v} \to \theta \left(1 - e^{-1/\theta}\right)$$

• a firm's deviation no longer affects the
queue length of applicants elsewhere:

$$u\pi(a) = u\frac{1 - a}{v - 1} \to \frac{1}{\theta}$$

The limit $u, v \to \infty$: (continued)

• equilibrium wage share satisfies Hosios condition:

$$\frac{w}{y} = \left[\frac{(1-1/v)^{-u}-1}{u/v} - \frac{1}{v-1} \right]^{-1}$$

$$\to \frac{1}{\theta[e^{1/\theta}-1]} = 1 - \frac{\theta p'(\theta)}{p(\theta)} \equiv s(\theta)$$

recall: $p(\theta) = \theta(1 - e^{-1/\theta}), \quad q(\theta) = 1 - e^{-1/\theta}$

• expected payoff equals the expected social value:

a worker: $pw \to y\, e^{-1/\theta}$

a firm: $\quad q(y - w) \to y\left[1 - (1 + \frac{1}{\theta})e^{-1/\theta}\right]$

Explain eqm expected payoff as social marginal values:

- A worker's expected payoff

$$pw = y \times \underbrace{e^{-1/\theta}}_{\text{prob. that a firm fails to match}}$$

  Adding a worker to match with a firm creates social value only when the firm does not have a match.

- A firm's expected payoff

$$q(y - w) = y \underbrace{(1 - e^{-1/\theta})}_{\substack{\text{firm's matching} \\ \text{probability}}} - y \underbrace{\frac{1}{\theta}e^{-1/\theta}}_{\substack{\text{crowding-out} \\ \text{on other firms}}}$$

Endogenize the tightness (in the limit $u, v \to \infty$):

- free entry of vacancies implies: $q(y - w) = k$

$$\text{i.e.} \quad \underbrace{1 - (1 + \frac{1}{\theta})e^{-1/\theta}}_{\text{strictly decreasing in } \theta} = \frac{k}{y}$$

- for any $k \in (0, y)$, there is a unique solution $\theta \in (0, \infty)$

A game with first-price auctions: JKK 00
(for fixed numbers u and v)

- firms post auctions with reserve wages
  above which a firm does not hire a worker

- workers observe all posted reserve wages

- each worker chooses which firm to apply to

- after receiving a number $n \geq 1$ of applicants:

  − if $n \geq 2$, the applicants bid in first-price auction
    (i.e., the worker with the lowest wage offer wins)
  − if $n = 1$, the worker is paid the reserve wage

Consider firm A that posts reserve wage x
(while all other firms post reserve wage r)

- each worker visits firm A with prob. $a = f(x, r)$

- payoff to a worker (B) who visits firm A:

# of other visitors, n	prob. of the event	worker B's payoff
$n = 0$	$(1 - a)^{u-1}$	x
$n \geq 1$	$1 - (1 - a)^{u-1}$	0

- $a = f(x, r)$ solves a worker's indifference condition:

$$(1 - a)^{u-1} x = [1 - \pi(a)]^{u-1} r, \text{ where } \pi(a) = \frac{1 - a}{v - 1}$$

- payoff to firm A:

# of visitors, n	prob. of the event	payoff
$n = 1$	$ua(1-a)^{u-1}$	$y - x$
$n \geq 1$	$1 - (1-a)^u - ua(1-a)^{u-1}$	y

- firm A's optimal choice of x, together with a, maximizes

$$ua(1-a)^{u-1}(y-x) + \left[1 - (1-a)^u - ua(1-a)^{u-1}\right] y$$

$$\text{s.t. } (1-a)^{u-1}x = [1 - \pi(a)]^{u-1} r$$

- solution (firm A's best response to other firms): $x = g(r)$

Symmetric equilibrium: $r = g(r)$

- the limit when $u, v \to \infty$:

  - queue length: $ua = u/v \to 1/\theta$

  - reserve wage: $r \to y$

  - equilibrium wage distribution:

wage	prob
y	$(1-a)^{u-1} \to e^{-1/\theta}$
0	$1 - (1-a)^{u-1} \to 1 - e^{-1/\theta}$

- equivalence to wage posting in expected payoff:

$$\text{a worker: } y \, e^{-1/\theta}; \quad \text{a firm: } y \left[1 - (1 + \frac{1}{\theta}) e^{-1/\theta} \right]$$

General lessons:

- directed search makes sense:
  ex ante tradeoff between terms of trade and probability

- directed search can attain constrained efficiency
  in the canonical search environment

- the mechanism to direct search is not unique:
  price/wage posting, auctions, contracts

  - commitment to the trading mechanism is the key
  - uniform price is not necessary for efficiency
    when agents are risk-neutral