

Macroeconomics II Part II, Lecture II: RBC: accounting, measurement, extensions

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Last time

- We set up and learned how to solve the vanilla RBC model
- The calibrated model can seemingly explain several business cycle moments...
- .. but fails in some key aspects
- Today, we will dig deeper into understanding the RBC mechanisms and how the model can be improved

Agenda

- ① Business cycle accounting
- ② Measuring technology shocks
- ③ Two extensions to address the labor wedge
 - ▶ Employment lotteries
 - ▶ GHH preferences

Business cycle accounting

- The plain vanilla RBC model provides an efficient benchmark
- Other BC models can be compared to this benchmark
 - ▶ A succesful model is a model that does something better than the vanilla RBC model in terms of explaining the data
- For this purpose, it is useful to develop methods that make it more precise what the vanilla RBC model does well and what it does not so well
- One such method is called [Business Cycle Accouting](#)
- Developed by Chari-Kehoe-McGrattan (Ecmtra, 2007), further elaborated in Brinca-Chari-Kehoe-McGrattan (HB Macro, 2016)

The idea

- Taking the parameters as given, the RBC model describes relationships between variables that we can measure in the data
- Thus, with given parameters and time series of these variables, we can see exactly which of these relationships fails and by how much
- Let's see how it works

RBC log-linear equilibrium

- Setting TFP shocks to zero $\hat{a}_t = 0$, our log-linear system is

$$\begin{aligned}\hat{w}_t &= \hat{c}_t + \varphi n_t \\ \hat{c}_t &= -\beta R^r E_t \hat{r}_{t+1}^r + E_t \hat{c}_{t+1} \\ \hat{y}_t &= \frac{C}{Y} \hat{c}_t + \frac{I}{Y} \hat{i}_t \\ \hat{y}_t &= \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t \\ \hat{k}_{t+1} &= (1 - \delta) \hat{k}_t + \delta \hat{i}_t \\ \hat{r}_t^r &= -(1 - \alpha)(\hat{k}_t - \hat{n}_t) \\ \hat{w}_t &= \alpha(\hat{k}_t - \hat{n}_t)\end{aligned}$$

where steady state R^r, C, I, Y are determined by the model parameters

- Using that

$$\begin{aligned}\hat{r}_t^r &= \hat{y}_t - \hat{k}_t, \\ \hat{w}_t &= \hat{y}_t - \hat{n}_t,\end{aligned}$$

we can eliminate \hat{r}_t^r, \hat{w}_t from the system

- The reduced system is

$$\hat{y}_t = \hat{c}_t + \varphi(1 + n_t)$$

$$\hat{c}_t = -\beta R^r E_t(\hat{y}_{t+1} - \hat{k}_{t+1}) + E_t \hat{c}_{t+1}$$

$$\hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{I}{Y} \hat{i}_t$$

$$\hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t$$

- This system describes a relationship between NIPA variables Y, C, K, I and hours worked N - stuff we know how to measure in the data
- Q: By how much does these relationship break in the data?
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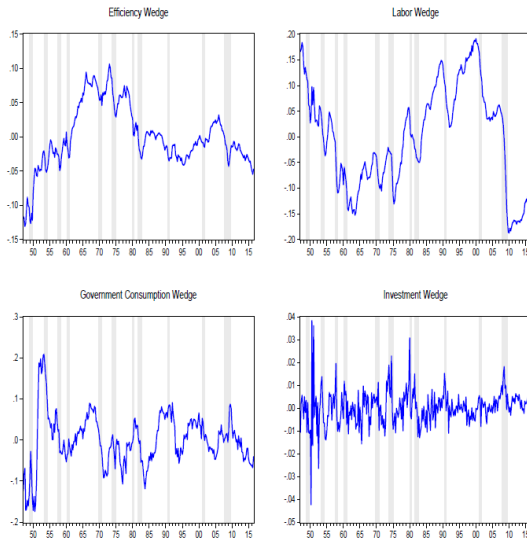
RBC log-linear equilibrium

- The reduced system is

$$\begin{aligned}\hat{y}_t &= \hat{c}_t + \varphi(1 + n_t) + \hat{\psi}_t^N \\ \hat{c}_t &= -\beta RE_t(\hat{y}_{t+1} - \hat{k}_{t+1}) + E_t \hat{c}_{t+1} + \hat{\psi}_t^I \\ \hat{y}_t &= \frac{C}{Y} \hat{c}_t + \frac{I}{Y} \hat{i}_t + \hat{\psi}_t^G \\ \hat{y}_t &= \alpha \hat{k}_t + (1 - \alpha) \hat{n}_t + \hat{\psi}_t^A\end{aligned}$$

- This system describes a relationship between NIPA variables Y, C, K, I and hours worked N - stuff we know how to measure in the data
- Q: By how much does these relationship break in the data?
- Reformulated Q: How large are the **wedges**?

Wedges in the data



From Eric Sims' lecture notes, using detrended data from John Fernald and same parameter values as in Lecture 1

Wedges: interpretation

- Efficiency wedge is the same thing as the Solow residual
 - ▶ We treated this as an **input** to the model
- We need large fluctuations in the labor wedge
 - ▶ \Rightarrow Indicates that something is not right about the labor market equilibrium in the model
- We do not need large fluctuations in the investment wedge
 - ▶ This is surprising, the RBC theory of consumption/investment seems very rudimentary...
- Large fluctuations in the government consumption wedge comes as no surprise
 - ▶ We know that both exports/imports and the government expenditure fluctuate a lot over the business cycle
 - ▶ Should perhaps not be interpreted as a failure, we simply didn't attempt to get this right
 - ▶ For a benchmark "international RBC model", see Backus-Kehoe-Kydland (JPE 1992)

Wedges: interpretation II

- Notice that prices do not appear in this accounting method
- The labor market wedge could stem either from
 - ① $W_t \neq$ Marginal rate of substitution (MRS) between consumption and leisure, or
 - ② $W_t \neq$ Marginal product of labor (MPL)
- Similarly, the investment wedge could stem either from
 - ① $R_t^r \neq$ MRS between consumption today and tomorrow, or
 - ② $R_t^r \neq$ MPK
- Nothing stops us from extending the accounting method to include prices
 - ▶ However, there is more uncertainty regarding measurement of prices compared to NIPA variables
 - ▶ Wage fluctuations could, for example, be heavily influenced by cyclical selection
 - ▶ Karabarbounis (RED 2014) shows you can sidestep this issue if assuming a Cobb-Douglas production function, finds that LM wedge is mostly due to $W_t \neq MRS$
- However, without including prices, we gain little information about which economic relationship that accounts for the wedges

Measuring technology shocks

Are Solow residuals a reasonable measure of technology shocks?

- We apparently need large fluctuations in the efficiency wedge to explain the data
- Kydland-Prescott's original approach: interpret the efficiency wedge (or Solow residuals) as **exogenous technology shocks**
- Reasonable that technological progress fluctuates around trend, and that Solow residuals capture some of this
- However, not reasonable that these *detrended* fluctuations can be negative with -5% or $-10\% \Rightarrow$ implies huge technological regress!
 - ▶ Solow residuals imply technological regress 40% of the time in post-war US data (King-Rebelo, HBmacro 1999)
- Technology concerns how factors of production are used for producing specific individual products ("blue-prints"); TFP is an abstract measurement concept
 - ▶ TFP can capture many things
 - ▶ Two examples: **capacity utilization** and **misallocation**

Cleaning “Solow residuals” (I): Utilization

- Firms adjust the usage of their predetermined factors of production over the business cycle
- True for both labor and capital
- Capacity utilization often treated as a key statistic for measuring the state of the business cycle
- Low utilization \Rightarrow lower TFP
- How to clean Solow residuals from utilization? Go and measure utilization!
 - ▶ Burnside-Eichenbaum-Rebelo (EER 1996) uses detrended log electricity usage z_t as a measure of capital utilization, measures technology shocks as
$$a_t = y_t - (1 - \alpha)n_t - \alpha(z_t + k_t)$$
 - ▶ Basu-Fernald-Kimball (AER 2006) and Fernald (2014) uses input data to adjust sectoral Solow residuals from capacity utilization, then aggregate sector-level productivity to TFP
 - ▶ Both find a technology series with 20-25 % less volatility than original Solow residuals

Cleaning “Solow residuals” (II): Misallocation

- Suppose some of the factors of production N_t, K_t are allocated to “under-performing” firms
- This will also look like low TFP in the data
- If missallocation moves with the business cycle, this will “contaminate” Solow residuals
- Cyclical missallocation is actually a key prediction of the New-Keynesian model, introduced in Lecture III
- I’ve not seen it applied in the RBC literature, but general measurement framework provided in Hsieh-Klenow (QJE 2009)

Simulation results with 25 % lower TFP volatility (HP-filtered)

	SD		Rel. SD		Corr Y_t		Autocorr	
	Data	Model	Data	Model	Data	Model	Data	Model
Y_t	0.017	0.011	1.00	1.00	1.00	1.00	0.85	0.72
C_t	0.009	0.005	0.53	0.40	0.76	0.95	0.79	0.78
I_t	0.047	0.031	2.76	2.73	0.79	0.99	0.87	0.72
N_t	0.019	0.004	1.12	0.33	0.88	0.98	0.90	0.72
W_t	0.009	0.008	0.53	0.66	0.10	0.996	0.73	0.74
R_t	0.004	0.011	0.24	1.0	0.00	0.97	0.42	0.71
A_t	0.012	0.09	0.71	0.80	0.76	0.999	0.75	0.72

- Reducing TFP volatility by 25 % does not affect any correlations, but reduces volatility of all variables by 25 % - why?
- Implication: the model needs to add more **endogenous amplification** to fit the data
 - Put differently: the model cannot explain the efficiency wedge any more

Taking stock

- A standard calibration of the vanilla RBC model with a reasonable technology shock process has quite a few problems with matching the data
- The lack of empirical fit is manifested in large fluctuations in the labor wedge and the efficiency wedge (and also government consumption wedge)
 - ▶ Relatedly, too little amplification and lack of persistence
 - ▶ Does not produce reasonable fluctuations in hours worked
 - ▶ \Rightarrow we need more internal propagation, and part of it should come from larger response in hours worked
- Also, recall the problems of getting the price moments right
- Let's start by addressing the labor wedge and the efficiency wedge
 - ▶ Problem set 5: endogenous capacity utilization to address the efficiency wedge
 - ▶ Remainder of this lecture: addressing the labor wedge

Two extensions to address the labor wedge

The role of the Frish elasticity

- The vanilla RBC model needs more amplification, and it should come through hours worked
- How to increase response of hours worked? Lets zoom in on the labor market equilibrium:
- Household intratemporal first order condition describes the labor supply curve (Do diagram on whiteboard):

Labor supply:

Labor demand:



The role of the Frish elasticity

- The vanilla RBC model needs more amplification, and it should come through hours worked
- How to increase response of hours worked? Lets zoom in on the labor market equilibrium:
- Household intratemporal first order condition describes the labor supply curve (Do diagram on whiteboard):

$$\text{Labor supply:} \quad \hat{w}_t = \hat{c}_t + \varphi n_t$$

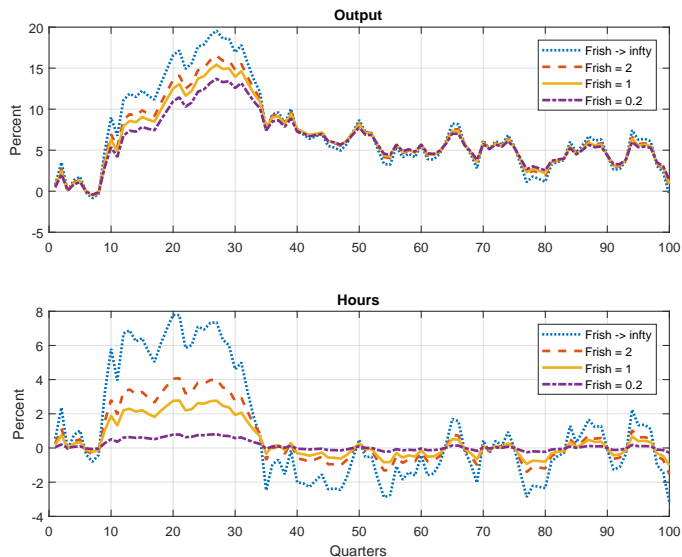
$$\text{Labor demand:} \quad \hat{w}_t = \hat{a}_t + \alpha(\hat{k}_t - \hat{n}_t)$$

- When productivity increases, we get larger response in hours either with larger substitution effect or smaller income effect

The substitution effect

- Holding the marginal utility of consumption constant, i.e. $c_t = 0$, $\eta = 1/\varphi$ measures the elasticity of hours w.r.t. wages
 - ▶ Put differently, it measures the substitution effect of wages on labor supply
- We call η the Frish elasticity
- Our baseline calibration had $\eta = 1$

Varying the Frish elasticity



Is a large Frisch elasticity reasonable?

- Larger Frisch elasticity \Rightarrow more amplification through hours worked
- Problem: most micro-level evidence suggest $\eta \in (0.0, 0.5)$
 - ▶ Classics: MacCurdy (JPE, 1981); Ashenfelter (JME, 1984); Angrist (JE, 1991)
 - ▶ Evidence reviewed in Chetty-Guren-Manoli-Weber (NBER annual, 2013)
 - ▶ Martinez-Saez-Siegenthaler (AER, 2020) exploit tax holiday in Austria, find $\eta = 0.02$
 - ▶ Stefanson (2019) and Sigurdsson (2022) exploit tax holiday in Iceland, find $\eta = 0.1 - 0.4$
- Moreover, most variation in hours worked over the business cycle is along the extensive margin, our model does not speak to this
- How to resolve?

Employment lotteries

- Hansen (JME 1985)/Rogerson (JME 1988) insight: **indivisible labor** disconnects the aggregate from the individual Frisch elasticity
 - ▶ In fact, with indivisibility, aggregate Frish elasticity can be *infinitely large*, although micro elasticities might be very small (even 0...)
- Model: Suppose there exist a measure 1 of ex-ante identical households, who have the same MacCurdy preferences:

$$U(C_t, N_t) = \log C_t - \theta \frac{N_t^{1+\varphi}}{1+\varphi}$$

- Assume choice set $N_t = \{0, \bar{N}\}$, but if households could pick freely, they would actually prefer some $N_t^* \in (0, \bar{N})$
- With **complete markets**, these households can be made better off if they enter a lottery with other households on whether to work or not

Employment lotteries II

- Denote the probability of working π_t
- In expectation, households work $N_t = \pi_t \bar{N}$ hours
- N_t therefore also denotes aggregate hours worked
- Complete markets imply $U_c(C_1, 0) = U_c(C_2, \bar{N})$
- With separable preferences, complete markets simply imply $C_1 = C_2 = C$
- With appropriate choice of π_t , households maximize expected utility

Employment lotteries III

- Household per-period utility is

$$\begin{aligned}U(C_t, N_t) &= \log C_t - \pi_t \theta \frac{\bar{N}^{1+\varphi}}{1+\varphi} - (1 - \pi_t) \frac{\theta \times 0}{1+\varphi} \\&= \log C_t - \pi_t \theta \frac{\bar{N}^{1+\varphi}}{1+\varphi} \\&= \log C_t - \frac{N_t}{\bar{N}} \theta \frac{\bar{N}^{1+\varphi}}{1+\varphi} \\&= \log C_t - B N_t\end{aligned}$$

$$\text{where } B = \frac{\theta}{\bar{N}} \frac{\bar{N}^{1+\varphi}}{1+\varphi}$$

- \Rightarrow the lottery model is thus **isomorphic** to our rep-agent model

$$U(C_t, N_t) = \log C_t - \theta \frac{N_t^{1+\varphi}}{1+\varphi}$$

with $\varphi = 0$ and with B calibrated to match the same average hours worked

- Now we have that the aggregate Frish elasticity $= \infty$

Employment lotteries: discussion

- Labor markets are obviously not organized through lotteries, but this is more of a technical trick to work analytically with indivisible labor
 - ▶ Indivisible labor \Rightarrow discrete choice problem \Rightarrow F.O.C.s are neither sufficient nor sufficient for characterizing the optimum
 - ▶ The lottery convexifies the choice set \Rightarrow we can proceed with our F.O.C.s as usual
- Modern approaches instead assume a fixed cost of working and deals with the non-convexity up front (using numerical methods)
 - ▶ Rogerson-Wallenius (JET 2009; AER 2013) show that the same discrepancy between micro-level and macro-level elasticities come out from quantitative life-cycle models
- At a more fundamental level, do we really believe the bulk of fluctuations in hours worked stem from households' labor supply choices? Isn't *involuntary unemployment* the main reason we are concerned with recessions?
- Lectures 5-7 present a theory of unemployment that can be readily integrated into RBC and other business cycle models

The income effect

- The income effect on labor supply is, with additively separable preferences, not separable from the elasticity of intertemporal substitution
- Household F.O.C.s:

$$\begin{aligned}\hat{w}_t &= \hat{c}_t + \varphi n_t \\ \hat{c}_t &= -\beta R^r E_t \hat{r}_{t+1}^r + E_t \hat{c}_{t+1}\end{aligned}$$

- If instead assuming CRRA consumption utility $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$:

$$\begin{aligned}\hat{w}_t &= \sigma \hat{c}_t + \varphi n_t \\ \hat{c}_t &= -\beta R^r \frac{1}{\sigma} E_t \hat{r}_{t+1}^r + E_t \hat{c}_{t+1}\end{aligned}$$

- Higher elasticity of intertemporal substitution $\frac{1}{\sigma} \Leftrightarrow$ smaller income effect on labor supply

What's the effect of having lower income effects? Introducing GHH preferences

- To admit separation, Greenwood-Hercowitz-Huffman (1988) proposed the following preference specification

$$U(C, N) = U(C - V(N))$$

- With this specification, we still have $U_c(C, N) = U'(\cdot)$ but $U_n(C, N) = -U'(\cdot)V'(N)$
- The household's F.O.C. with respect to N_t becomes

$$-\beta^t U'(C_t - V(N_t))V'(N) = W_t \lambda_t$$

and thus

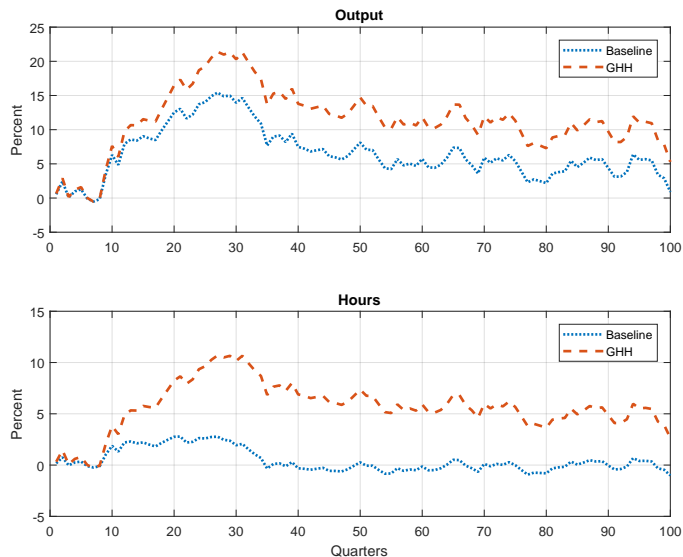
$$V'(N) = W_t$$

- With $V(N) = \frac{N^{1+\varphi}}{1+\varphi}$, we get

$$\hat{w}_t = \varphi n_t$$

- Now, the income effect is zero!

Baseline vs. GHH



Is GHH a reasonable preference specification?

- Cesarini-Lindqvist-Notowidigdo-Östling (AER 2017): labor supply responses to windfall lottery gains very small
- But if this is true, how to resolve with (almost) constant labor supply in the long run?
- Possible explanation: income effects are small in the short run, while still large in the long run
 - ▶ Jaimovich-Rebelo (JPE 2009): with habits in household preferences, you can actually achieve this
 - ▶ Broer-Harmenberg-Krusell-Öberg (AER:insights 2023): this is what you should expect if households are subject to rigid wage contracts

Rigid-wage contracts a la Broer-Harmenberg-Krusell-Öberg

- “Rigid” wages = wage contracts cannot be contingent on the shock, nor can they be renegotiated once a shock happens
- Instead of having a competitive market for hours in exchange for fixed per-hour wages, suppose we have a competitive market for rigid wage-hour contracts: $W(N)$
- If firms have the “right to manage”, then conditional on a TFP shock, we get $A_t F'(N_t) = W'(N_t)$
 - ▶ Firms equate the marginal rate of transformation to the *marginal wage*
- If the asset market is complete, and aggregate shocks are small relative to idiosyncratic shocks, households marginal utility of consumption is constant when writing the contract
 - ▶ Ex ante, households will agree to the contract if $W'(N_t) = -\frac{V'(N_t)}{U'(C)}$ and $\mathbb{E}W(N_t)$ (“base pay”) is large enough
 - ▶ However, if given the opportunity to rewrite the contract after seeing the shock, households recognize that their consumption has changed, and demand $W'(N_t) = -\frac{V'(N_t)}{U'(C_t)}$.
- \Rightarrow as if GHH within the contract, as if KPR when rewriting the contract
 - ▶ Frequency of recontracting determines how short the short run is

Summing up

- Business cycle accounting (BCA): useful diagnosis tool \Rightarrow we learned that vanilla RBC is associated with large labor and efficiency wedges
 - ▶ It is not reasonable to interpret the entire efficiency wedge as exogenous technological shocks
 - ▶ After cleaning Solow residuals from utilization, amplification problem becomes bigger
- Motivated by BCA, we learned that model fit is improved if making labor supply more elastic and muting income effects
 - ▶ Highly elastic labor supply a natural implication of indivisible labor
 - ▶ Small short-run income effects is a natural implication of rigid wage contracts
- The lessons from lecture II will serve as benchmarks when we introduce models of frictional labor markets
- But first: something about investment