Macroeconomics II, Lecture VII: Frictional labor markets: basics

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Introduction

- Lectures I-VI: frameworks for business cycle analysis

- We learned a lot, but the models we considered had several problems, and are obviuously to stylized to understand to speak to many macroeconomic phenomena

- In particular, two assumptions seemed very unrealistic while also important for understanding macro dynamics
  1. Labor markets are frictionless
  2. Household have access to complete asset markets (and can therefore be summarized by a representative agent)

- Remainder of the course introduces a set of models that relax these assumptions

- Besides developing a deeper theory of business cycle dynamics, these models will allows us to explore an additional set of questions in macro research:
  - How do macroeconomic conditions affect inequality? And how does inequality affect macroeconomic conditions?

Remainder of course: outline

- Lectures VII-X: Frictional labor markets
  - 4 lectures
  - Digging deepering into the determinants of household income
- Lectures XI-XIII: Incomplete asset markets
  - 3 lectures
  - Digging deeper into the determinants of consumption-savings dynamics, taking the income process as given
- To get us started with frictional labor market, we'll first look at a few key facts

Today's agenda

1. Labor market facts
2. Search models: overview
3. Mathematical Preliminaries
4. The McCall model

Labor market facts

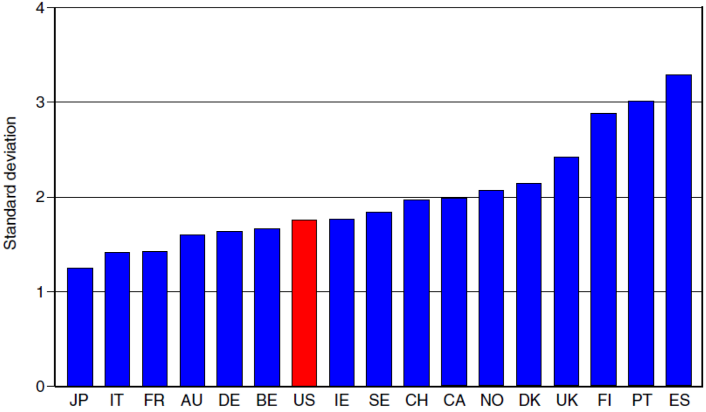
Introduction

- The labor market can be described in terms of
  - Stocks: hours worked, unemployment rate etc.
  - Flows: job-finding rate, separation rate etc.
  - Prices: wages
- We will look at some basic facts relating to each of these categories
- Main focus is US, but also quick glance at other OECD countries
- Main data source: labor force surveys (US: CPS; Sweden: AKU)

Stocks

- Main variables: Population N, Participation P, Hours worked H, Employed E, Unemployed U
- In some time interval Δt, a person is
  - **employed** if hours worked > 0
  - **unemployed** if not working, available for work and actively looking for work
  - **participating** if employed or unemployed
- Hours worked per capita $h^c = \frac{H}{N}$
- Question: how large are fluctuations in hours worked?

SD(hours per capita) cross countries



Standard deviation of detrended log points, using data 1965-2008. From Rogerson-Shimer (Handbook LE 2011).

- Fact 1: SD of cyclical fluctuations $\sim$ 1.5-2 percent for the US
- Fact 2: considerable heterogeneity in fluctations across countries

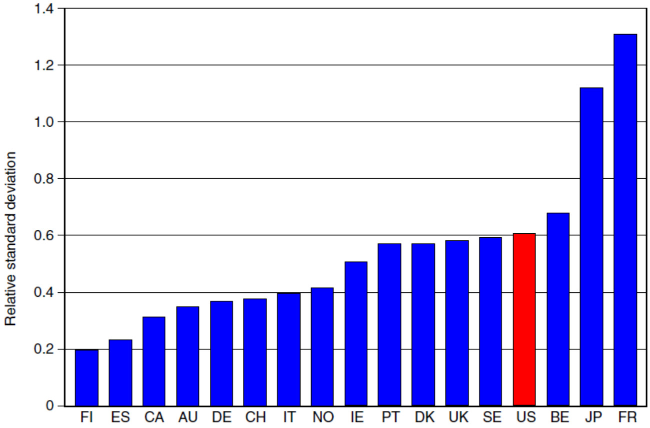
Decomposition

- More definitions
  - Hours worked per worker $h^w = \frac{H}{E}$
  - Employment rate $e = \frac{E}{N}$
- Decomposing hours worked

$$
\begin{aligned}
h_t^c &= \frac{H_t}{N_t} \\
&= \frac{H_t}{E_t} \times \frac{E_t}{N_t} \\
&= \underbrace{h_t^w}_{\text{int. margin}} \times \underbrace{e_t}_{\text{ext. margin}}
\end{aligned}
$$

- Question: How much of fluctuations in hours worked per person is due to intensive vs extensive margin?

$SD(h_t^w)/SD(e_t)$ cross countries



Standard deviation of detrended log points, using data 1965-2008. From Rogerson-Shimer (Handbook LE 2011).

- Fact: for most countries, extensive margin is more important (but still considerable variation due to intensive margin)

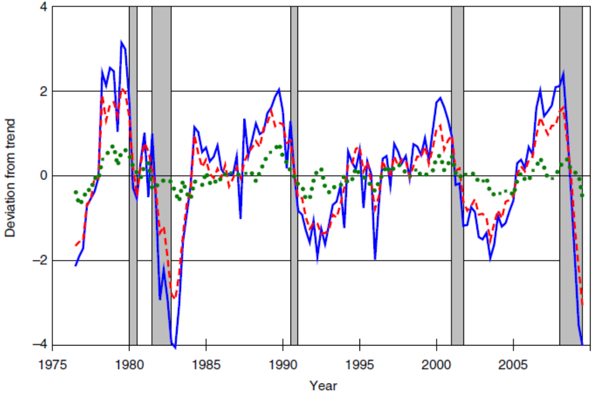
Further decomposition

- Even more defintions
  - Participation rate $p = \frac{P}{N}$
  - Unemployment rate $u = \frac{U}{P}$
- Decomposing hours worked

$$
\begin{aligned}
h_t^c &= \frac{H_t}{E_t} \times \frac{E_t}{N_t} \\
&= h_t^w \times \frac{P_t - U_t}{N_t} \\
&= h_t^w \times \frac{P_t}{N_t} \times (1 - \frac{U_t}{P_t}) \\
&= h_t^w \times p_t \times (1 - u_t)
\end{aligned}
$$

- Question: How much of fluctuations in hours worked is due to participation vs. unemployment?

US time series



Detrended log points. Blue solid - hours worked per person; red dashed - employment rate; green dotted - participation rate. From Rogerson-Shimer (Handbook LE 2011).

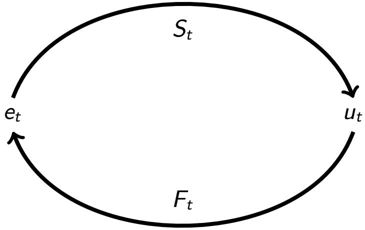
- Fact: In the US, participation accounts only marginally for cyclical variation in hours worked
- ⇒ most cyclical variation in hours worked is due to unemployment

Flows

- To gain further insight about the forces behind these fluctuations, we study the underlying flows
  - Participation fairly constant, so we restrict attention to flows in and out of unemployment

- Denote $F_t(\Delta t), S_t(\Delta t)$ as the job-finding and job-separation probabilities in period $t \to t + \Delta t$
  - $F_t(\Delta t) =$ the fraction of unemployed flowing out of unemployment in period $t \to t + \Delta t$
  - $S_t(\Delta t) =$ the fraction of employed flowing into unemployment in period $t \to t + \Delta t$

- Graphical representation:



Measurement

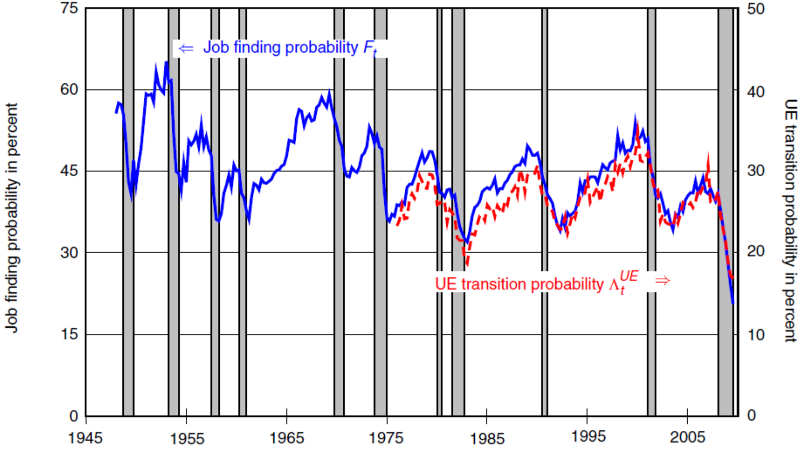
- Assume $\Delta t = 1$ month
- With data on unemployment levels by duration, we estimate the probability F_t for an unemployed worker for finding a job within a month:

$$1 - F_t = \frac{\#\text{Unemployed with duration}>1 \text{ in month t+1}}{\#\text{Unemployed in month t}}$$

- Similary, the monthly job-separation probability can be estimated from

$$S_t = \frac{\#\text{Unemployed with duration}<1 \text{ in month t+1}}{\#\text{Employed in month t}}$$

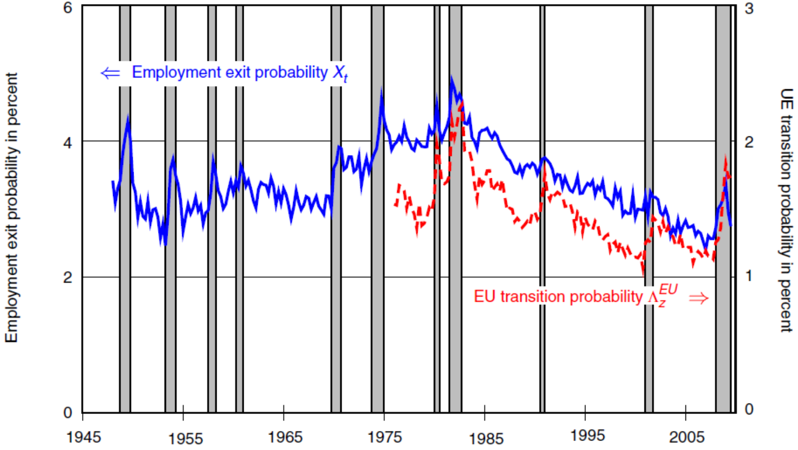
US job-finding probability



Blue solid line: inference from 2-state model; Red dashed line: inference from 3-state model. Rogerson-Shimer (Handbook LE 2011).

- Fact: Job-finding probability decreases (somewhat gradually) during recessions

US job-separation probability



Blue solid line: inference from 2-state model; Red dashed line: inference from 3-state model. From Rogerson-Shimer (Handbook LE 2011).

- Fact: Job-separation probability spikes during recessions

Flows across countries

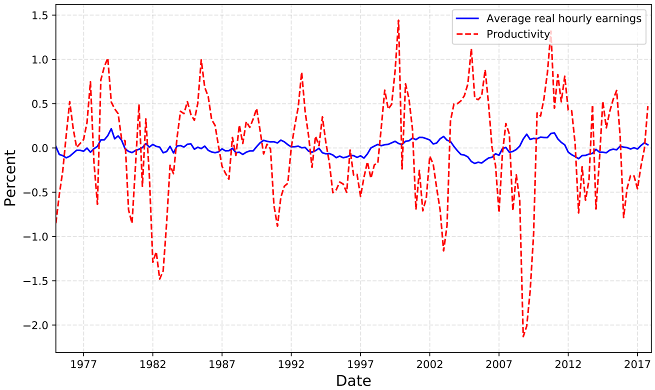
From Elsby-Hobjin-Sahin (ReStat 2013).

- Fact: US labor market is an outlier
- Fact: Strong positive association between job-separation and job-finding probability across countries

Wages

- The allocation of labor is coordinated via the labor price system, that is, the wage distribution
- How large are fluctations in wages?
- Are different workers paid different wages?
- Are similar workers paid different wages?

US wage fluctuations



Detrended (HP-filter) quarterly data. OECD estimate of total labor productivity. Average hourly earnings for total private sector excluding supervisory employees, deflated with PCE. Source: FRED and own calculations.

- Fact: wage fluctuations << productivity fluctuations
- Seemingly at odds with benchmark neoclassical model

US wage dispersion

From Acemoglu (JEL 2002).

- Fact: rising wage inequality since mid-seventies

Residual wage dispersion

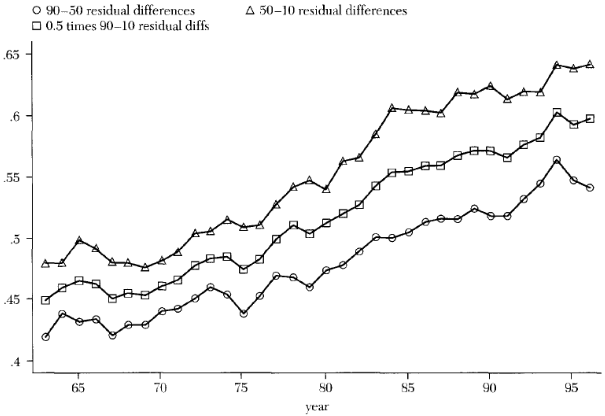
- How much of the observed wage dispersion is due to the differences in observables?
- Cross-sectional household survey data (e.g. PSID, CPS)
- Mincer regression

$$\log y_{it} = \mu_y + \beta \mathbf{X}_{it} + \epsilon_{it}$$

where y_{it} is earnings and $\mathbf{X}_{it}$ includes the age, occupation, industry, tenure, education etc.

- Typically, $R^2 < 0.3$ (Mortensen, Book 2003)
- How to interpret the residual ϵ_{it}?
  - Unobservable productivity differences? Luck?

US residual wage dispersion



Residual Inequality Measures for White Males 1963–97

Y-axis: log points. From Acemoglu (JEL 2002).

- In 1995, median is $\sim e^{0.6} \approx 1.8$ times 10th percentile
- In 1995, 90th percentile is $\sim e^{2*0.6} \approx 3.3$ times 10th percentile
- $\Rightarrow$ Fact: high and rising residual wage inequality

Search models: overview

Search models

- Search models provide a theory of the labor market that links stock, flows and the distribution of wages together
- Search markets: markets in which buyers (employers) and sellers (workers) spend time/resources searching for each other and where there is some element of randomness in when and/or for whom a match happens
- Because searching takes time, some workers will experience periods without work: fricitional unemployment
- Because matching has an element of randomness, some workers will be more lucky than others in finding good jobs: fricitional wage dispersion
- Contrast to Walrasian markets (e.g. standard RBC and NK models)
  - Markets clear instantaneously
  - Unemployment can only arise if wages are not clearing the market
  - Law of one price holds: no wage dispersion among identical workers

Search models

- The search approach to labor markets were developed in the 60's, first largely focused on partial equilibrium search behavior, later built into general equilibrium frameworks

- We will cover
  - Lecture VII: Partial-equilibrium search (McCall)
  - Lecture VIII: Job-ladders and wage dispersion (Burdett-Mortensen)
  - Lecture IX-X: Unemployment (Diamond-Mortensen-Pissarides)

Mathematical Preliminaries

Math preliminaries I: Expected value

- For a non-negative random variable x with cdf $F(x)$ on support $[x_1, x_2]$, we can define expected value as

$$Ex = \int_{x_1}^{x_2} x dF(x)$$

- Contrast with "standard" defintion

$$Ex = \int_{x_1}^{x_2} x f(x) dx$$

- For distributions where F is differentiable, these definitions coincide.
  - $\frac{dF(x)}{dx} = f(x) \Leftrightarrow dF(x) = f(x)dx$
  - This includes the usual suspects: Uniform, Normal, Frechet, Pareto, etc...

- However, when F has mass points, the expected value does not exist according to latter definition (Draw example on whiteboard)

- Hence, we will employ the former definition

Math preliminaries II: Riemann-Stieltjes integrals

- Consider our definition or expected value

$$Ex = \int_{x_1}^{x_2} x dF(x)$$

- When the integrator is a function rather than a variable, we call it a Riemann-Stieltjes integral

- Properties or RS integrals:
    1. If F is differentiable: $\int G(x)dF(x) = \int G(x)f(x)dx$
    2. Integration by parts: $\int_a^b G(x)dF(x) = G(x)F(x)|_a^b - \int_a^b F(x)dG(x)$
    3. In particular: $\int_a^b dF(x) = F(x)|_a^b$

- (Compute Ex of example distribution on whiteboard)

Math preliminaries III: Two useful formulas for real analysis

- l'Hôpital's rule: if $f(x)$ and $g(x)$ are two differentiable functions (in an open interval around c) and

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0 \text{ or } \pm\infty$$

then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

- Leibniz' rule: If the functions $f(x,t), \alpha(t), \beta(t)$ are differentiable in t, the function

$$\phi(t) = \int_{\alpha(t)}^{\beta(t)} f(x,t) dg(x)$$

is differentiable and

$$\phi'(t) = f(\beta(t),t)\frac{dg(\beta(t))}{dt} - f(\alpha(t),t)\frac{dg(\alpha(t))}{dt} + \int_{\alpha(t)}^{\beta(t)} f_t(x,t) dg(x)$$

The McCall model

The McCall (QJE 1970) model

- Framework for thinking about optimal search behavior
- Identical households, who search for employment opportunities
- Partial equilibrium: we take prices (and firm behavior) as given

McCall agenda

1. Model setup
2. Reformulation of model in continuous time
3. Solution: reservation wage strategy
4. Implications:
   - Match quality
   - Unemployment duration
   - Unemployment dynamics
   - Wage dispersion
5. The Rothschild critique and the Diamond paradox

McCall assumptions

- An economy populated by a measure P of ex ante identical, infinitely lived agents with linear utility function and discount factor β
- Households can either be employed or unemployed
- An unemployed household recieves benefits b, an employed household recieves wage w
- No savings technology, agents consume hand-to-mouth
- Unemployed household recieve job offer with probability Λ_u in each period, with wages w drawn from i.i.d. $F(w)$ with finite support $\mathbb{W} = [w_{min}, w_{max}]$
- Employed households separate from jobs at exogenous probability S
- Household problem: if unemployed and recieve job offer, accept or wait

Value functions

- Search models: generally easier to work with recursive formulation

- Value (i.e. Present-Discounted Value) of employed agent:

$$W(w) = w + \beta\left[(1-S)W(w) + SU\right]$$

- Value of unemployed agent:

$$U = b + \beta\left[(1-\Lambda_u)U + \Lambda_u \int_{\mathbb{W}} \max\{W(w), U\} dF(w)\right]$$

Value functions in continuous time

- Given these Bellman equations, we could proceed to solve for the worker decision and derive all results
- However, many macro-search models are formulated in continuous time
- We will therefore restate the previous problems in continuous time and work with this formulation from now on
- Same setup, but we denote the time period by Δt (and do not normalize it to 1)
- For given Δt, we distinguish between rates and levels
  - Job-finding rate λ_u; Job-finding probability $\Lambda_u(\Delta t) = \lambda_u \Delta t$
  - Job-separation rate σ; Job-separation probability $S(\Delta t) = \sigma \Delta t$
  - Discount rate r; Discount factor $\beta(\Delta t) = e^{-r\Delta t} \approx \frac{1}{1+r\Delta t}$

Employed value function in continuous time

- PDV of employed agent

$$W(w) = w\Delta t + e^{-r\Delta t}\left[(1-\sigma\Delta t)W(w) + \sigma\Delta t U\right]$$

- Rearrange

$$\frac{1-e^{-r\Delta t}}{\Delta t}W(w) = w + \sigma e^{-r\Delta t}(U - W(w))$$

- l'Hôpital's rule says

$$\lim_{\Delta t\to 0}\frac{1-e^{-r\Delta t}}{\Delta t} = \lim_{\Delta t\to 0}\frac{re^{-r\Delta t}}{1} = r$$

- so in the limit $\Delta t \to 0$:

$$rW(w) = w + \sigma(U - W(w))$$

- flow value rW = flow benefit + flow probability of shock $\times$ change in value

Unemployed value function in continuous time

- PDV of unemployed agent:

$$\begin{aligned} U &= b\Delta t + e^{-r\Delta t}\left[(1-\lambda_u\Delta t)U + \lambda_u \Delta t \int_{\mathbb{W}} \max\{W(w), U\} dF(w)\right] \\ &= b\Delta t + e^{-r\Delta t}\left[U + \lambda_u \Delta t \int_{\mathbb{W}} \max\{W(w) - U, 0\} dF(w)\right] \end{aligned}$$

- Rearrange

$$\frac{1-e^{-r\Delta t}}{\Delta t} U = b + e^{-r\Delta t}\lambda_u \int_{\mathbb{W}} \max\{W(w) - U, 0\} dF(w)$$

- Take the limit and use l'Hôpital's rule to find

$$rU = b + \lambda_u \int_{\mathbb{W}} \max\{W(w) - U, 0\} dF(w)$$

Model summary

- Our problem is to find U, W and a wage acceptance rule $a : \mathbb{W} \to \{0,1\}$ that solves our system of two equations

$$\begin{aligned} rW(w) &= w + \sigma(U - W(w)) \\ rU &= b + \lambda_u \int_{\mathbb{W}} \max\{W(w) - U, 0\} dF(w) \end{aligned}$$

- That's it

- Now we 1) solve this problem and 2) study the implications of the solutions

Solution I: the reservation wage strategy

- We solve for W in terms of U:

$$rW(w) = w + \sigma(U - W(w)) \Rightarrow W(w) = \frac{w + \sigma U}{r + \sigma}$$

- Interpretation?

- Using this, we rewrite unemployed value

$$\begin{aligned} rU &= b + \lambda_u \int_{\mathbb{W}} \max\{\frac{w + \sigma U}{r + \sigma} - U, 0\} dF(w) \\ &= b + \lambda_u \int_{\mathbb{W}} \max\{\frac{w - rU}{r + \sigma}, 0\} dF(w) \end{aligned}$$

- $\frac{w-rU}{r+\sigma}$ increasing, 0 is a constant (Draw graph on whiteboard)

- Ergo, the wage acceptance rule is a reservation wage strategy: accept all wages $w \geq w_R$

- First testable hypothesis: households use reservation wage strategies

Solution II: characterizing the reservation wage strategy

- Because of reservation strategy, unemployed value can be rewritten

$$
\begin{aligned}
rU &= b + \lambda_u \int_{w<w_R} 0 dF(w) + \lambda_u \int_{w \geq w_R} \left(\frac{w - rU}{r + \sigma} \right) dF(w) \\
&= b + \frac{\lambda_u}{r + \sigma} \int_{w \geq w_R} (w - rU) dF(w)
\end{aligned}
$$

- The reservation wage satisfies $w_R = rU$:

$$
w_R - b = \frac{\lambda_u}{r + \sigma} \int_{w \geq w_R} (w - w_R) dF(w)
$$

- We name this the reservation wage equation
  - LHS = marginal cost of waiting
  - RHS = marginal gain of waiting
  - The equation solves for w_R

- Why is $w_R > b$?

Solution retrieved, what now?

- Reminder: The problem was to find the acceptance rule, which we have shown to be cutoff strategy with reservation wage w_R
- This is all there is to it
- Solving explicitly for w_R requires knowledge about F
- We will remain agnostic about F here, but we can still derive a lot of properties and implications of the solution

Comparative statics I

- How does w_R vary with b? Our solution satisfies

$$w_R - b = T(w_R) \equiv \frac{\lambda_u}{r+\sigma} \int_{w \geq w_R} (w - w_R) dF(w)$$

- Differentiate w.r.t. b:

$$\frac{dw_R}{db} - 1 = T'(w_R) \frac{dw_R}{db}$$

- Rearrange:

$$\frac{dw_R}{db} = \frac{1}{1 - T'(w_R)}$$

- Apply Leibniz's rule:

$$\begin{aligned} T'(w_R) &= \frac{\lambda_u}{r+\sigma} \left[(w_{max} - w_R) \frac{\partial F(w_{max})}{\partial w_R} - (w_R - w_R) \frac{\partial F(w_R)}{\partial w_R} + \int_{w \geq w_R} -1 dF(w) \right] \\ &= -\frac{\lambda_u}{r+\sigma} \int_{w \geq w_R} dF(w) \\ &< 0 \end{aligned}$$

- Ergo, reservation wages increases with unemployment benefits

Comparative statics II

- Similarly, one can show

$$\frac{dw_R}{d\lambda_u} > 0, \qquad \frac{dw_R}{dr} < 0, \qquad \frac{dw_R}{d\sigma} < 0$$

- Intuition?

Comparative statics III

- What about changes in F?
- A uniform increase in $F(w)$ increases the reservation wage
- How does w_R respond to an increase in dispersion?
  - see your problem set

Match quality

- The observed wage distribution is $G(w; w_R) = F(w|w > w_R)$
- An increase in w_R implies that observed wages are higher
- More precisely: if $w_R^1 > w_R^2$, then $G_1(w; w_R^1)$ first order stochastically dominates $G_2(w; w_R^2)$
  - The only difference between $G_1(w; w_R^1)$ and $G_1(w; w_R^2)$ is that lower wage offers are discarded in $G_1(w; w_R^1)$
- Hypothesis: Everything that increases w_R (e.g. benefits b) will also increase observed wages
- Reformulated: higher unemployment benefits should increase observed match quality

Unemployment dynamics

- The economy is populated with P workers
- In a time period Δt, a fraction $\sigma\Delta t$ of employed workers $E_t = P - U_t$ separate and a fraction $\lambda\Delta t$ of unemployed workers U_t find a job
  - $\lambda = \lambda_u(1 - F(w_R))$
- Law of motion for unemployment

$$U_{t+\Delta t} = U_t + \sigma\Delta t(P - U_t) - \lambda\Delta t U_t$$

- Divide by P:

$$u_{t+\Delta t} = u_t + \sigma\Delta t(1 - u_t) - \lambda\Delta t u_t$$

- Rearrange:

$$\frac{u_{t+\Delta t} - u_t}{\Delta t} = \sigma(1 - u_t) - \lambda u_t$$

- Take the limit $\Delta t \to 0$:

$$\dot{u}_t = \sigma(1 - u_t) - \lambda u_t$$

Unemployment dynamics II

- Law of motion for unemployment:

$$\dot{u}_t = \sigma(1 - u_t) - \lambda u_t$$

- Given σ, λ, this ordinary differential equation has a unqiue solution converging to steady state value u.

- Steady state u defined by $\dot{u}_t = 0$:

$$u = \frac{\sigma}{\sigma + \lambda}$$

- Hypothesis: Everything that increases w_R (e.g. benefits b) will also increase the unemployment rate

- How does the unemployment rate relate to unemployment duration here?

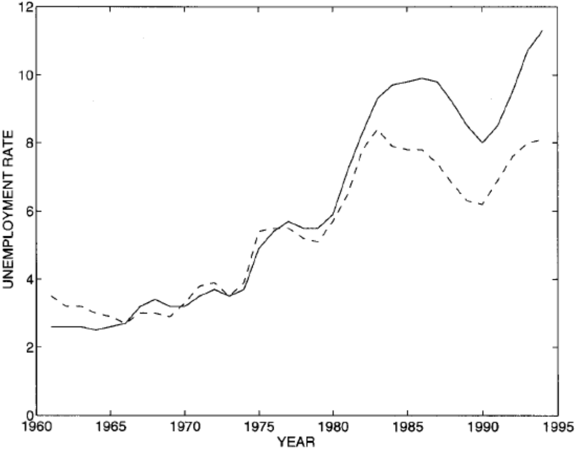
Digression: micro-level evidence of model predictions I

- Following these insights, large empirical literature have used micro data to investigate the effect of unemployment benefits on unemployment incidence, unemployment duration and match quality
- Reservation wages and job-search behavior
  - A Krueger-Mueller (AEJep 2016) use interview data to document self-percieved reservation wages
  - Marinescu-Skandalis (QJE 2020) use online application data to document how job-search behavior change with benefit duration
- Unemployment duration and benefit generosity:
  - Older literature summarized in A Kreuger and Meyer (Handbook PE 2002)
  - Card-Lee-Pei-Weber (Ecmtra, 2015) exploit kinks in Austrian UI
  - Kolsrud-Landais-Nilsson-Spinnewijn (AER 2017) exploit kinks in Swedish UI
  - Many more papers...
  - Raj Chetty's summary: 10 weeks of extra UI $\Rightarrow$ 1 week of extra unemployment (of course, the effect depends on all kinds of things)
  - Key input for evaluating optimal benefit provision: see, e.g., Chetty (JPubE, 2006), Chetty (JPE, 2008)

Digression: micro-level evidence of model predictions II

- Match quality and benefit generosity:
  - van Ours-Vodopivec (JLE 2006); Lalive (AER 2007); Card-Chetty-Weber (QJE 2007); Schmieder-von Wachter-Bender (AER 2016): zero effect
  - Nekoei-Weber (AER 2017) uses austrian micro data and RD design: positive effect
- Lesson that appears in many of these papers: duration dependence matter
  - Reservation wage falls as unemployment benefit period is ending; human capital depreciates while unemployed
- Ljungqvist-Sargent (JPE 1998) argue that duration dependence is key for understanding difference in unemployment dynamics in Europe vs. US

Digression: Ljungqvist-Sargent (JPE 1998)



Solid line: OECD Europe. Dashed line: OECD average.

Digression: Ljungqvist-Sargent (JPE 1998)

- Hypothesis: human capital depreciates over unemployment spell
  - Consistent with studies on mass layoffs: earnings losses after unemployment are very persistent (Lalonde-Jacobsen-Sullivan, AER 1993; Davis-von Wachter BPEA 2012)
  - Evidence based on mass layoffs might confound partial and general equilibrium effects (Cederlöf 2019) - see lecture 3 on DMP model
- In 70-80's: European countries had generous replacement rates tied to previous wage with close-to unlimited duration
- Human capital depreciation + unlimited duration may result in $w_R > w_{offered} \Rightarrow$ long-term unemployment
- Makes little difference when macroeconomic volatility is low
- In late 70's/80's: macroeconomic volatility increased (oil price shocks) $\Rightarrow$ difference in UI institutions can explain why Europe but not US got stuck in high (long-term) unemployment equilibrium

Frictional wage dispersion

- Because of the randomness of wage offers, models of this sort generate frictional wage dispersion
- Hypothesis: some of the empirical wage dispersion that cannot be accounted for by observed hetereogeneity is due to this kind of randomness
- Remember: Mincer regression typically produce $R^2 < 0.3$ (Mortensen, Book 2003) and substantial residual dispersion: Acemoglu (JLE 2002) reports 90/10-ratio ≈ 3.3 and 50/10-ratio ≈ 1.8
- Can our model account for these numbers?

Frictional wage dispersion: how to the test the model?

- To answer this question, the obviuous approach seems to be
  1. match the model parameters $\{r, \lambda_u, F(w), b\}$ to the data
  2. solve for the reservation wage
  3. compute the wage distribution implied by the model $G(w) = F(w|w > w_R)$
  4. compare to the data
- Are there any problems with this approach?

Frictional wage dispersion: an alternative approach

- Hornstein-Krusell-Violante (AER 2011): use the information implicitly contained in households' search behavior

- Again, the reservation wage equation is:

$$w_R - b = \frac{\lambda_u}{r+\sigma} \int_{w \geq w_R} (w - w_R) dF(w)$$

- The observed mean wage is $\bar{w} = E[w | w > w_R]$

- Rearrange

$$w_R - b = \frac{\lambda}{r+\sigma} \frac{\int_{w \geq w_R} (w - w_R) dF(w)}{1 - F(w_R)}$$

where $\lambda = \lambda_u (1 - F(w_R))$ is the observed job-finding rate

- What is the last ratio?

Frictional wage dispersion: a useful formula

- Step-by-step:

$$\begin{aligned}\frac{\int_{w\geq w_R}(w-w_R)dF(w)}{1-F(w_R)} &= \frac{\int_{w\geq w_R} wdF(w)}{1-F(w_R)} - w_R\frac{\int_{w\geq w_R} dF(w)}{1-F(w_R)} \\ &= E[w|w>w_R] - w_R\frac{1-F(w_R)}{1-F(w_R)} \\ &= \bar{w} - w_R\end{aligned}$$

- and so the reservation equation can be written

$$w_R - b = \frac{\lambda}{r+\sigma}(\bar{w} - w_R)$$

- Write $b = \rho\bar{w}$, divide by w_R and rearrange

$$Mm = \frac{\bar{w}}{w_R} = \frac{\frac{\lambda}{r+\sigma} + 1}{\frac{\lambda}{r+\sigma} + \rho}$$

- Mm is a measure of wage dispersion and **the formula does not have F in it!**
- Can the model match the data?
  - All parameters in our formula are observed in the data!
  - Plug in parameter values and compare Mm ratio in model to data

Frictional wage dispersion: testing the model

- US monthly data: $r = 0.0041$ (5% yearly interest rate), $\sigma = 0.03$, $\lambda = 0.43$, $\rho = 0.41$ (average replacement rate) $\Rightarrow$

$$Mm = \frac{\frac{\lambda}{r+\sigma} + 1}{\frac{\lambda}{r+\sigma} + \rho} = \frac{\frac{0.43}{0.0041+0.03} + 1}{\frac{0.43}{0.0041+0.03} + 0.41} = 1.046$$

- Reminder: 50/10-ratio in data ≈ 1.8
- **The model comes now way near the data!**
- Intuition: high empirical λ puts tight constraint on the value of expected wage offer
  - High $\lambda \Rightarrow$ households must think it is not worth waiting long to get a better offer
  - $\Rightarrow$ expected wage from a new offer $\bar{w}$ cannot be much higher than outside option $\rho\bar{w}$
- Only ingredient is optimal search behavior of households $\rightarrow$ very robust result
- However, we shall see that one interesting avenue to think about fricitional wage dispersion is models in which households also perform on-the-job search

Taking stock

- Key idea: unemployed households reject some wage offers because the option value of waiting for better offers dominates
- Generates predictions on unemployment rate, unemployment duration, match quality, wage dispersion
- Motivated by these predictions, large empirical literature has emerged
- But the question that begs to be answered: where did the offer distribution F and contact rate λ_u come from?
  - Presumably vacancy/wage posting by firms?
- Also, we would presume that firm vacancy/wage posting behavior depends on worker search behavior, so can we really treat F and λ_u as exogenous?
  - We need a GE model to explore this!

General equilibrium? The Rothshild (JPE 1973) critique

- Suppose that the wage distribution F is generated by wage-posting firms with different productivities $p > b$
- Suppose that the firms know that all workers are identical and all workers follow the same reservation wage strategy (accept all $w > w_R$)
- What is the optimal wage posting rule of the firms?
- If posting $w > w_R$, you always get someone to work for you after a contact
- If posting $w < w_R$, you never get someone to work for you
- Optimal wage posting strategy is simply $w = w_R$, independent of p
- $\Rightarrow F$ is degenerate with unit mass point at $w = w_R$!

General equilibrium? The Diamond (JET 1971) paradox

- The problem goes deeper still. If there is no wage distribution, there is no point for an unemployed household to wait

- Our reservation wage equation:

$$\begin{aligned} w_R - b &= \frac{\lambda_u}{r+\sigma} \int_{w \geq w_R} (w - w_R) dF(w) \\ &= \frac{\lambda_u}{r+\sigma} \int_{w \geq w_R} (w_R - w_R) dF(w) \\ &= 0 \end{aligned}$$

- Ergo, optimal search behavior implies $w_R = b$ and all firms offer the monopoly contract $w = b$

- If workers have to pay a tiny fixed cost $\epsilon > 0$ for participating in search, nobody would search for jobs and there would be no production!

- Clearly, firm behavior and wage formation need to be addressed. Cliffhanger...

Bonus material: mapping data probabilties to model flow rates

- How to retrieve continuous-time transition rates from discrete-time transition probabilities (which are measurable in the data)?

- Our model says that in an interval $\delta t \to 0$
  1. you can get at most one job
  2. the probability of finding and accepting a job is $\lambda \delta t$

- Suppose we allowed workers to accept > 1 jobs within some empirically measurable time period Δt and that the arrival rate of job offers is independent of time
  - This does not change anything in our continuous-time model, as the probability of finding > 1 jobs in any given instant is neglible

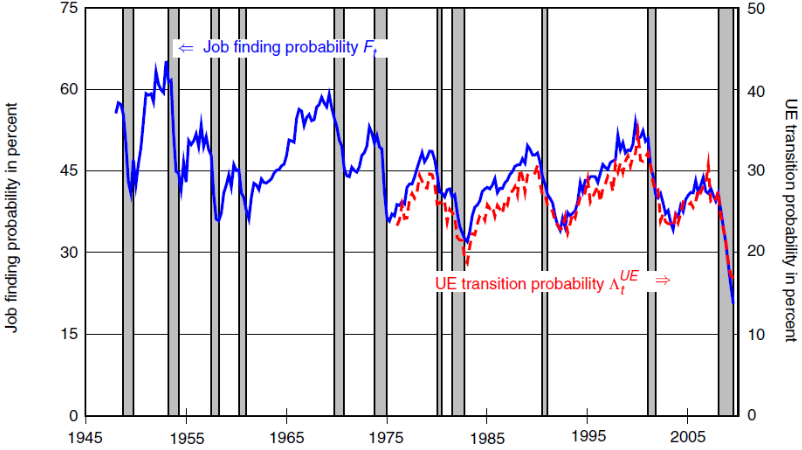
- Then, the probability of finding k jobs in period Δt is $P(k|\Delta t)$, where P is given by the Poisson distribution with arrival rate λ
  - Hence, $P(k|\Delta t) = \frac{(\lambda \Delta t)^k}{k!} e^{-\lambda \Delta t}$

- What we can measure in the data is $F_t(\Delta t)$ — this is related to the continuous-time job-finding rate according to

$$F_t(\Delta t) = 1 - P(0|\Delta t) = 1 - e^{-\lambda \Delta t}$$

- **If F_t is the monthly job-finding probability, then $\lambda = -\log(1 - F_t)$ is the monthly job-finding rate**

Bonus material: mapping data probabilties to model flow rates



Blue solid line: inference from 2-state model; Red dashed line: inference from 3-state model. From Rogerson and Shimer (Handbook LE 2011).

- $F_t \approx 0.45 \Rightarrow \lambda_t \approx -\log(1-0.45) \approx 0.6$