The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited

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The Mortensen-Pissarides (MP) search and matching model (Christopher Pissarides 1985; Dale Mortensen and Pissarides 1994; Pissarides 2000) has become the standard theory of equilibrium unemployment. It provides an appealing description of the labor market and has been found relevant in quantitative work. For example, Monika Merz (1995) and David Andolfatto (1996) have shown that the performance of the real business cycle model can be improved significantly when the MP model is embedded into it. However, Andolfatto (1996), James S. Costain and Michael Reiter (2005), and Robert Shimer (2005) have argued that the standard calibration of the model fails to account for the cyclical properties of its two central variables—unemployment and vacancies. These variables are much more volatile in US data than in the MP model.

The literature has responded by suggesting that the wage setting mechanism in the MP model has to be altered.[1] We take a different route in this paper. We suggest that the problem lies not in the model itself, but in the way the model is typically calibrated. We consider the MP model to be a linear approximation of a richer model with heterogeneity and curvature in utility and technology. Consistent with this interpretation, we propose a new calibration strategy for the two central parameters of the MP model—the worker's value of nonmarket activity and the worker's bargaining power. Our calibration implies that the model is consistent with the cyclical volatility of unemployment and vacancies.

In the MP model, firms incur costs of posting a vacancy and recover these costs by paying workers less than their marginal product. This gives rise to the period-by-period accounting profits. Free entry ensures that expected economic profits from posting are zero. We measure the costs of posting vacancies in the data and find that they are small, implying small accounting profits in the calibrated model. This estimate uniquely pins down the worker's value of nonmarket activity conditional on a choice of the worker's bargaining power. The choice of the worker's bargaining power determines the elasticity of wages with respect to productivity in the model. Given the attention that has been devoted to the behavior of wages in this literature, we find it natural to explore a specification of the model that matches the elasticity of wages in

[1] Shimer (2004) and Roger E. A. Farmer and Andrew Hollenhorst (2006) suggest that some wage rigidity may be necessary. In Robert E. Hall (2005a) and Mark Gertler and Antonella Trigari (2006), a form of social wage norm renders wages not responsive to productivity changes. Hall and Paul R. Milgrom (2008) modify the bargaining game to limit the influence of labor market conditions on wages. Guido Menzio (2005) and John Kennan (2006) endogenize wage rigidity by modeling asymmetric information about productivity. Andreas Hornstein, Per Krusell, and Giovanni L. Violante (2005) and Eran Yashiv (2006) survey the recent literature.

the data. The fact that wages are only moderately procyclical uniquely pins down the worker's bargaining weight at a relatively low value, implying a value of nonmarket activity in the model that is considerably higher than the typical replacement ratio of unemployment insurance. Thus, low vacancy costs and moderately procyclical wages in the data imply that accounting profits are small and change significantly in percentage terms in response to small changes in productivity. Consequently, firms' incentives to post vacancies also respond strongly to changes in productivity.

Instead, the usual strategy is to choose the bargaining weight in a way that guarantees the efficiency of the model (i.e., to satisfy the Arthur Hosios (1990) condition) and to identify the return to nonmarket activity with receiving unemployment benefits. Our calibration implies that the return to nonmarket activity is substantially higher than the typical unemployment insurance replacement rate. This is the result one would expect in a frictionless competitive environment. For example, in a standard real business cycle model, market and nonmarket productivities are equalized: workers are indifferent between working one more hour at home or in the market (Jess Benhabib, Richard Rogerson, and Randall Wright 1991; Jeremy Greenwood and Zvi Hercowitz 1991) and valuing equally market and nonmarket activities (Gary Hansen 1985: Rogerson 1988). Since the MP model can be considered a linear approximation to a nonlinear real business cycle (RBC) model, it seems reasonable to expect that it exhibits a similar relationship.

The paper is organized as follows. The model is laid out in Section I. In Section II we describe the importance of the values assigned to the return to nonmarket activity and bargaining power in determining the labor market volatility generated by the model. In Section III we describe our proposed calibration strategy, perform a quantitative analysis, and discuss our results. Section IV concludes.

I. The Model

We consider a stochastic discrete time version of the Pissarides (1985, 2000) search and matching model with aggregate uncertainty.

Workers and Firms.—There is a measure one of infinitely lived workers and a continuum of infinitely lived firms. Workers maximize their expected lifetime utility, $E\sum_{t=0}^{\infty} \delta^t y_t$, where y_t represents income in period t and $\delta \in (0,1)$ is workers' and firms' common discount factor. Output per each unit of labor is denoted by p_t. Labor productivity p_t follows a first-order Markov process according to some distribution $G(p',p) = \Pr(p_{t+1} \leq p' | p_t = p)$.

There is free entry of firms. Firms attract unemployed workers by posting a vacancy at the flow cost c_p.[2] Once matched, workers and firms separate exogenously with probability s per period (see Hall (2005b) for the evidence that s is constant over the business cycle). Employed workers are paid a wage w_p, and firms make accounting profits of $p - w_p$ per worker each period in which they operate. Unemployed workers get flow utility z from leisure/nonmarket activity. Workers and firms split the surplus from a match according to the generalized Nash bargaining solution. The bargaining power of workers is $\beta \in (0,1)$.

Matching.—Let u_t denote the unemployment rate, $n_t = 1 - u_t$ the employment rate, and v_t the number of vacancies posted in period t. We refer to $\theta_t = v_t/u_t$ as the market tightness at time t. The number of new matches (starting to produce output at $t + 1$) is given by a constant returns to scale matching function $m(u_t, v_t) \leq \min(u_t, v_t)$. The probability of an unemployed worker being

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[2] Throughout the paper, the notation X_p indicates that a variable X is a function of the aggregate productivity level p, and $E_p X_{p'}$ is next period's expected value of X, conditional on the current state p.

matched with a vacancy next period equals $f(\theta_t) = m(u_t, v_t)/u_t = m(1, \theta_t)$. The probability of a vacancy being filled next period equals $q(\theta_t) = m(u_t, v_t)/v_t = m(1/\theta_t, 1) = f(\theta_t)/\theta_t$. The law of motion for employment is $n_{t+1} = (1 - s)n_t + m(u_t, v_t)$.

Equilibrium.—Denote the firm's value of a job (a filled vacancy) by J, the firm's value of an unfilled vacancy by V, the worker's value of having a job by W, and the worker's value of being unemployed by U. The following Bellman equations describe the model:[3]

$$J_p = p - w_p + \delta(1 - s)E_p J_{p'},$$

$$V_p = -c_p + \delta q(\theta_p)E_p J_{p'},$$

$$U_p = z + \delta\{f(\theta_p)E_p W_{p'} + (1 - f(\theta_p))E_p U_{p'}\},$$

$$W_p = w_p + \delta\{(1 - s)E_p W_{p'} + sE_p U_{p'}\}.$$

Free entry implies $V_p = 0$ for all p and, therefore, $c_p = \delta q(\theta_p)E_p J_{p'}$. Nash bargaining implies that a worker and a firm split the surplus $S_p = J_p + W_p - U_p$ such that $J_p = (1 - \beta)S_p$, $W_p - U_p = \beta S_p$, and wages are given by $w_p = \beta p + (1 - \beta)z + c_p \beta \theta_p$.

II. Business Cycle Properties

In this section, we calibrate all the parameters except for the value of nonmarket activity z and worker's bargaining weight β, and explore how these two parameters affect the business cycle properties of the model.

A. Preliminary Calibration

We choose the model period to be one-twelfth of a quarter ($\approx$ one week), which is lower than the frequency of the employment data we use, but necessary to deal with time aggregation. We aggregate the model appropriately when matching the targets obtained from the data with monthly, quarterly, or annual frequency. We set $\delta = 0.99^{1/12}$. Shimer (2005) estimates the average monthly job finding rate from 1951 to 2003 to be 0.45 and, following Shimer (2005), we estimate the separation rate (not adjusted for time aggregation) to be 0.026. At weekly frequency, these estimates imply a job finding rate $f = 0.139$, a job separation rate $s = 0.0081$, and a steady-state unemployment rate $u = s/(s + f) = 0.055$.[4]

As in Shimer (2005), labor productivity, p, is measured in the data as seasonally adjusted quarterly real average output per person in the nonfarm business sector constructed by the Bureau of Labor Statistics (BLS). We approximate, through a 35-state Markov chain, the continuous-valued AR(1) process $\log p_{t+1} = \rho \log p_t + \varepsilon_{t+1}$, where $\rho \in (0, 1)$ and $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. In the data we find (as do Hornstein, Krusell, and Violante 2005) an autocorrelation of 0.765 and an unconditional

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[3] As in Shimer (2005), we implicitly assume that the value functions depend only on p and not on u. Existence of such an equilibrium is straightforward. Its uniqueness in the Pissarides (1985, 2000) model with aggregate uncertainty was proven in Mortensen and Éva Nagypál (2007).

[4] The probability of not finding a job within a month is 0.55. The probability of not finding a job within a week then equals $0.55^{1/4} = 0.861$, and the probability of finding a job equals $1 - 0.861 = 0.139$. The probability of observing someone not having a job who had a job one month ago equals (counting paths in a probability tree): $s\{(1 - f)(fs + (1 - f)^2) + f(s(1 - f) + (1 - s)s)\} + (1 - s)\{s(fs + (1 - f)^2) + (1 - s)(s(1 - f) + (1 - s)s)\} = 0.026$. Solving for s, we obtain $s = 0.0081$.

standard deviation of 0.013 for the HP-filtered (Edward Prescott 1986) productivity process with a smoothing parameter of 1,600. At weekly frequency, this requires setting $\rho = 0.9895$ and $\sigma_\varepsilon = 0.0034$ in the model. The mean of p is normalized to one.[5]

We need a matching function that ensures that the probability of finding a job and of filling a vacancy lies between 0 and 1. (Since the precise value of θ will be meaningful in our approach to calibrating z below, we cannot conveniently normalize it as was done in Shimer (2005).) We follow Wouter den Haan, Garey Ramey, and Joel Watson (2000) (HRW) and choose $m(u, v) = uv/(u^l + v^l)^{1/l}$. We calibrate the value of the matching function parameter, l, to match the data on the average value for the job finding rate $f = 0.139$.

B. *The Importance of β and z*

Since the business cycle behavior of unemployment, vacancies, and the job finding probability are deterministic functions of labor market tightness θ, we can focus on the latter variable. In Hagedorn and Manovskii (2008) we derive, in the model without aggregate uncertainty, the elasticity of labor market tightness with respect to aggregate productivity:

$$(1) \qquad \varepsilon_{\theta, p} = \frac{p}{p - z} \underbrace{\frac{\beta f(\theta) + (1 - \delta(1 - s))/\delta}{\beta f(\theta) + (1 - \eta)(1 - \delta(1 - s))/\delta}}_{\kappa \, := \,} ,$$

where η is the elasticity of $f(\theta)$ with respect to θ. This expression shows that only changes in z, and not changes in β, have substantial effects on the volatility of market tightness and thus on the volatility of unemployment. Given the calibrated values for δ, s, η, and $f(\theta)$, κ varies only between 1.03 and 2.20 for values of β between 0 and 1. The value of $p/(p - z)$ varies between 2.5 and 20 for values of z between 0.4 and 0.95. Thus $\varepsilon_{\theta, p}$ is large only if $p - z$ is sufficiently small. Equation (1) also confirms that the standard calibration strategy $- z = 0.4$ and β satisfies the Hosios condition—leads to only small fluctuations in θ. It also illustrates that setting $z = 0.955$ and $\beta = 0.052$—the outcomes of the calibration strategy that we propose below—leads to large fluctuations in θ.

These results, however, do not shed light on the economic mechanism behind equation (1). A prominent explanation of the findings in Shimer (2005) is that the elasticity of wages is too high in his model ($\varepsilon_{w, p} = 0.964$). The argument is then that an increase in productivity is largely absorbed by an increase in wages, leaving profits (and, thus, the incentives to post vacancies) little changed over the business cycle. This argument is not quite correct. Consider the experiment of replicating Shimer (2005) with $z = 0.4$ but choosing β to match the moderate productivity elasticity of wages in the data $\varepsilon_{w,p} = 0.449$ (which will also be a target in our calibration). We find std$(\theta) = 0.02$, which is essentially the same as in Shimer (2005) and is low relative to the data (std$(\theta) = 0.259$). This demonstrates that although the elasticity $\varepsilon_{w,p}$ is now much lower, the volatility of market tightness does not rise precipitously.

In the second experiment, we set $z = 0.95$, an outcome in our proposed calibration strategy, but pick β to generate the same high elasticity $\varepsilon_{w,p} = 0.964$ as in Shimer (2005). We find that the volatility of market tightness is now close to what we find in our calibration (std$(\theta) = 0.30$).

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[5] We have defined p as the marginal product of labor. In the data, we observe the average product of labor. We show in Hagedorn and Manovskii (2008) that this difference is inconsequential.

This experiment shows that the model can generate a volatile labor market despite a high volatility of wages.

What explains these results? The correct argument is a subtle but crucial modification of the argument given above. The elasticity of wages does not matter per se. What matters for the incentives to post vacancies is the size of the percentage changes of profits in response to changes in productivity. These percentage changes are large if the size of profits is small and the increase in productivity is not fully absorbed by an increase in wages. In the standard MP model, conditional on the choice of z, the bargaining parameter β determines both the level and the volatility of wages. Thus, if we fix z and raise β, wages rise and become more cyclical, meaning that profits become smaller but less cyclical. These two opposing effects almost exactly cancel each other out. Thus, the volatility of labor market tightness is almost independent of β and is determined only by the level of z. In other words, the elasticity of wages is an important number, but only relative to the size of profits, which depends on z. However, while the value of β plays a minor role in determining labor market volatility, it is important for our calibration strategy because it helps to pin down z.

III. Calibrating β and z

A. The Problem of Linearity and Homogeneity

A strong assumption in the MP model is the absence of curvature: utility is linear, z is constant, and the marginal product of labor moves one for one with average labor productivity. We view the MP model with these assumptions as an approximation of a richer model that incorporates curvature in aggregate productivity and in the utility derived from consumption and leisure, heterogeneity of preferences and workers' productivity, home production, spousal labor supply, etc. This approximation seems appropriate to study business cycles since changes in aggregate productivity are relatively small and not permanent.

In such a nonlinear model without search, indivisibility of labor implies $p = z$ in equilibrium (Hansen 1985; Rogerson 1988).[6] Taking the view that the MP model approximates such a model (with search) constrains the choice of z. Indeed, for the MP model to be consistent with the nonlinear model, the value of nonmarket activity has to be very close to the value of market productivity. Even if the replacement rate of unemployment insurance is as low as 20 percent, z would be close to productivity in the equilibrium of the nonlinear model and thus has to be close to productivity in the equilibrium of the MP model as well. The reason is that households adjust leisure, home production, self-employment, disutility of work, etc.—activities that are all included in z—such that in equilibrium z turns out to be close to p.[7] Thus, if one views the MP model as such an approximation, it would be unwise to identify z as the value of unemployment benefits only. This view also limits the possibility to study the effects of unemployment insurance on the labor market across countries. Since leisure, home production, etc., adjust to changes in unemployment insurance, z is largely invariant with respect to changes in the replacement rate. As a result, even large differences in the generosity of unemployment insurance across

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[6] Consider a family of measure one. The family decides what fraction of its members, L, should work in the market, given that each worker can produce z at home, to $\max_L \{Lp + (1 - L)z\}$, where $p = F_L(L,K)$ denotes the marginal product of labor. Assuming an interior solution, the optimal choice of L implies $p = z$.

[7] Hall (2006) uses empirical results from the labor supply and consumption literature at the household level to obtain a value of leisure relative to productivity of about 43 percent. Adding a conservative estimate of unemployment insurance replacement rate of 0.3 already results in a value of $z = 0.73$. Note, however, that the replacement rate is linked to a worker's productivity in his previous job, which can be, due to the loss of specific human capital, substantially higher than his expected productivity in his next job.

countries do not translate into large differences in z, and thus in unemployment rates. A value of $z \approx 0.4$, typically used in the literature, would also be inconsistent along another labor market dimension. The large and strongly procyclical flows from out-of-the-labor-force into employment can be rationalized only if the value of not working is close to the value of working for these individuals.

B. Proposed Calibration Strategy

Two parameters remain to be determined: the value of nonmarket activity, z, and worker's bargaining weight, β. Thus, we need two targets to identify them. To obtain the first target, we provide a measure of the vacancy posting costs in the data. This estimate uniquely pins down z conditional on a choice of β. The choice of β determines the elasticity of wages with respect to productivity in the model. We explore a specification of the model that matches this target. It turns out that such a specification generates the cyclical properties of the labor market variables that are consistent with the data. Moreover, it implies a value of β that is consistent with the cross-sectional evidence.

Cyclicality of Wages.—We estimate the cyclicality of wages (measured as labor share times labor productivity) from BLS data (1951:I–2004:IV). We find that a 1 percentage point increase in labor productivity is associated with a 0.449 percentage point increase in real wages. Both time series are in logs and HP-detrended with a smoothing parameter of 1,600. The corresponding estimate in the model is one of our calibration targets.[8]

Labor Market Tightness.—To measure the costs of posting vacancies, we need to know the average value of vacancies or, equivalently, the value of θ. Shimer (2005) estimated the average monthly job finding rate, f, to be 0.45. HRW found a monthly job filling rate, q, of 0.71. Since $\theta = f/q$, these numbers imply a value for θ of $0.45/0.71 = 0.634$, which we choose as our calibration target. This number accords well with the direct estimate of 0.539 obtained by Hall (2005a) from the Job Openings and Labor Turnover Survey (JOLTS). As expected, this estimate is slightly lower than 0.634. JOLTS started in December 2000 and covers only a recession and a fraction of the expansion that had slower employment growth than usual. Moreover, some vacancies are not captured by JOLTS: we see firms hiring workers within a month without ever reporting having a vacancy to JOLTS.

Capital Cost of Vacancies and the Interpretation of the Productivity Process.—To account for the capital costs of vacancy creation, we follow Pissarides (2000) and recognize the presence of capital in the model. Making the presence of capital explicit does not change any of the equations in the model and amounts to only a reinterpretation of the productivity process. In the deterministic version of the model, vacancies arise only because firms need to replace

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[8] In Hagedorn and Manovskii (2008), we recalibrated the model targeting a wage cyclicality at the boundary of the 95 percent confidence interval around $\varepsilon_{w,p} = 0.449$ and found that the results are not sensitive to the choice of $\varepsilon_{w,p}$ in the empirically plausible range. We also used the Panel Study of Income Dynamics (PSID) to estimate wage cyclicality from individual data to minimize the selection bias due to the entry of low-wage workers into employment in booms and exit in recessions, and found very similar estimates. This bias is not important in the regression of wages on productivity because both sides are similarly affected: if workers entering in a boom are, say, 10 percent less productive, their wages are also 10 percent lower. Finally, we show that the elasticity of wages in the calibrated model with respect to (un)employment rate and GNP, while not targeted, is consistent with the data.

A standard assumption of the MP model is that wages are renegotiated whenever the aggregate state of the economy changes. An alternative wage determination assumption might be that firms insure workers against aggregate income risk. In Hagedorn and Manovskii (2008), we discuss the evidence and find little empirical support for the latter view.

exogenously separated workers. Thus, we assume that posting firms and operating firms rent the same amount of capital.[9]

Let K denote the aggregate capital stock. The number of active firms equals $v + 1 - u$; $1 - u$ of them are operating and v are looking for a worker. Thus, the amount of operating capital equals $K(1 - u)/(v + 1 - u)$ and the amount of idle capital equals $Kv/(v + 1 - u)$. The aggregate constant returns to scale production function is $F[K(1 - u)/(v + 1 - u), A(1 - u)]$, where A is labor-augmenting productivity. We define $k := K/(A(v + 1 - u))$ and $f(k) := F(k, 1)$. Denote by k^* the value of k that satisfies the equilibrium condition $f'(k) = 1/\delta - 1 + d$, where d is the depreciation rate.

We can now define labor productivity $p := A(f(k^*) - (1/\delta - 1 + d)k^*)$. Assuming that firms can buy and sell capital in a competitive market, the wage bargain is not affected by the presence of capital. The only difference is that A, the exogenous productivity process, is multiplied with the constant $f(k^*) - (1/\delta - 1 + d)k^*$. Thus, p is still an exogenous (productivity) process. The firm's flow capital cost of posting a vacancy is $A(1/\delta - 1 + d)k^*$.

Capital Costs of Posting Vacancies.—We derived above that the flow capital cost of posting vacancies equals $(1/\delta - 1 + d)\ kA = F_K K/(v + 1 - u)$, where F_K denotes the derivative of F with respect to its first argument. Decompose

$$\frac{F_K K}{v + 1 - u} = \frac{F_K K}{F} \frac{1 - u}{1 - u + v} \frac{F}{1 - u}.$$

We now compute the steady-state values for all three factors. Typical estimates from the national accounts imply a capital income share $F_K K/F = 1/3$. Since $\theta = 0.634$ and $u = 0.055$, the number of vacancies $v = \theta u = 0.03487$. Thus, $(1 - u)/(1 - u + v) = 0.9644$.

In a search model, income and production shares of labor and capital do not coincide. This is because labor is paid below productivity to compensate firms for the costs of vacancy creation. However, since labor productivity is normalized to one $(F_L A = 1)$, it follows that

$$\frac{1 - u}{F} = \frac{F_L A(1 - u)}{F} = 1 - \frac{F_K K \dfrac{1 - u}{v + 1 - u}}{F} = 1 - \frac{1}{3} \frac{1 - u}{1 - u + v} = 1 - 0.321 = 0.679.$$

Thus, the steady-state capital flow cost of posting a vacancy c^K equals 0.474, or 47.4 percent of the average weekly labor productivity.

Labor Costs of Posting Vacancies.—The second part of the cost of filling a vacancy is the opportunity cost of labor effort devoted to hiring activities. John M. Barron, Mark C. Berger, and Dan A. Black (1997) present the evidence. Using the 1982 Employment Opportunity Pilot Project survey of 5,700 employers, they find that, on average, employers spend 10.41 hours per offer and make 1.08 offers per hired worker. This implies a total of 11.24 hours spent on each hire. The corresponding numbers from the 1992 Small Business Administration survey of 3,600 employers are 14.03, 1.14, and 15.99. Thus, the average costs of time spent hiring one worker are

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[9] This assumption seems natural since the one-job-one-worker abstraction of the MP model precludes any reallocation of vacant capital across workers within a firm. In addition, it may not even be in a firm's interest to engage in such, presumably costly, reallocation given the high job-filling rate. To the extent that firms can rent (a fraction of) capital after a worker is found, our assumption provides an upper bound on the capital cost of vacancy creation and, thus, a lower bound on the volatilities of unemployment and vacancies in the model. See Hagedorn and Manovskii (2008) for the sensitivity analysis.

between 2.2 percent to 3.2 percent of quarterly hours. Adjusting, as in José I. Silva and Manuel Toledo (2007), for the possibility that hiring is done by supervisors who receive higher wages than a new hire, the average labor cost of hiring one worker is 3 percent to 4.5 percent of quarterly wages of a new hire. We choose the highest value of 4.5 percent as the benchmark because this generates the lowest volatility.[10]

Let W be aggregate weekly wages. Wages are 2/3 of national income, that is, $W = 2/3F$. Quarterly wages then equal $8F$. Expected labor cost of hiring equals $0.045 \cdot 8F$ in the data and c^W/q in the model. The probability of filling a vacancy q equals $f/\theta = 0.219$, and we have just found that F equals $(1 - u)/0.679 = 1.39$. Thus, the flow labor cost of posting a vacancy c^W equals 0.110, or 11 percent of the average weekly labor productivity.

Cyclicality of Vacancy Posting Costs.—We have computed the average capital and labor costs of hiring. These costs are not constant over the business cycle.

First, capital per worker is procyclical. As derived above, firms use $A k^*$ units of capital in state A, where k^* solves $f'(k) = 1/\delta - 1 + d$. Let $\bar{A}$ and $\bar{p}$ denote the mean levels of A and p, respectively. Then, the steady-state capital cost $c^K = (1/\delta - 1 + d)k\bar{A}$ and the capital cost in state A, $\tilde{c}^K = c^K A/\bar{A}$. Thus, $\tilde{c}^K = c^K A/\bar{A} = c^K p/\bar{p} = c^K p$ in state $p = A(f(k^*) - (1/\delta - 1 + d)k^*)$ since we have normalized $\bar{p} = 1$.

Second, labor costs of hiring change over the business cycle according to $c^W p^{\xi}$. To determine ξ we assume that wages of those engaged in hiring are fluctuating as much over the business cycle as do wages of other workers. As discussed above, the regression coefficient of HP-filtered log wages on HP-filtered log productivity in the data is 0.449. Since the HP-filter is a linear operator, $\xi = \varepsilon_{w,p} = 0.449$.[11] Thus, the costs of posting a vacancy in state A, or equivalently p, equal $c_p = c^K p + c^W p^{0.449} = 0.474p + 0.110p^{0.449}$.

Bargaining Weights and Value of Nonmarket Activity.—Finally, we choose the values for z and β to match the data on the average value for labor market tightness $\theta = 0.634$ and the elasticity of wages with respect to productivity $\varepsilon_{w,p} = 0.449$. As described in Table 1, we are able to match the calibration targets exactly. Calibrated parameter values can be found in Table 2.

We find that $z = 0.955$, which is consistent with our view of the model as a linear approximation of a model with curvature and heterogeneity. We also find a workers' bargaining weight of 0.052. This number is remarkably close to the one identified in the cross-sectional data.[12] Moreover, we will show below that this estimate implies that the model is very close to the efficient benchmark once we account for the level of taxes in the data.

C. Implied Labor Market Volatilities

The statistics of interest, computed from US data, are presented in Table 3. Hornstein, Krussel, and Violante (2005) report similar numbers. Table 4 describes the results generated by the standard model calibrated using the proposed strategy: the model matches the key business cycle

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[10] In Hagedorn and Manovskii (2008) we show that results are not very sensitive to this choice.

[11] Linearity means $HP(\log p^{\xi}) = \xi HP(\log p)$. HP-filtering an isoelastic time series does not affect the regression coefficient: regressions of $HP(\log p^{\xi})$ on $HP(\log p)$ and $\log p^{\xi}$ on $\log p$ give the same coefficient ξ.

[12] Several papers (e.g., Louis N. Christodes and Andrew J. Oswald 1992; David G. Blanchflower, Oswald, and Peter Sanfey 1996; Andrew K. G. Hildreth and Oswald 1997) found using cross-sectional US data that, controlling for outside labor market conditions, a 1 percentage point increase in rm's probability leads to an increase in wages of ≈ 0.05 percent. This value is remarkably close to our finding of $\beta = 0.052$. Since they control for our outside labor market conditions, their estimate corresponds to β in our model and not to the wage elasticity. Note that the identification in those papers does not rely on the cyclical volatility of wages. (A higher estimate of ≈ 0.2 percent was obtained by John A. Abowd and Thomas Lemieux (1993) in a sample of Canadian collective bargaining agreements.)

TABLE 1—MATCHING THE CALIBRATION TARGETS

	Value	
Target	Data	Model
Elasticity of wages w.r.t. productivity, $\varepsilon_{w,p.}$	0.449	0.449
Average job finding rate, f,	0.139	0.139
Average market tightness, θ,	0.634	0.634

Note: The table describes the performance of the model in matching the calibration targets.

TABLE 2—CALIBRATED PARAMETER VALUES

Parameter	Definition	Value
z	Value of nonmarket activity	0.955
β	Workers' bargaining power	0.052
l	Matching parameter	0.407
c	Cost of vacancy when $p = 1$	0.584
δ	Discount rate	$0.99^{1/12}$
s	Separation rate	0.0081
ρ	Persistence of productivity process	0.9895
σ_ε^2	Variance of innovations in productivity process	0.0034

Note: The table contains the calibrated parameter values in the benchmark calibration.

facts quite well. The volatility of labor market tightness, unemployment, and vacancies is higher but close to the data.[13]

D. Analysis

The Values of β and z.—We first establish that, since our estimate of the vacancy posting costs implies small accounting profits in the calibrated model (2.255 percent of labor productivity on average), and wages are moderately procyclical in the data, the value of nonmarket activity, z, has to be close to the productivity level, p, and workers' bargaining weight, β, has to be relatively small.

Without aggregate uncertainty, it holds that

$$w = p - \frac{(1 - \beta)(1 - \delta(1 - s))}{1 - \delta(1 - s) + \delta f(\theta)\beta} (p - z),$$

and

$$\Pi = p - w = \frac{(1 - \beta)(1 - \delta(1 - s))}{1 - \delta(1 - s) + \delta f(\theta)\beta} (p - z).$$

[13] Table 4 reveals two well-known shortcomings of the MP model. The correlation of labor market tightness and productivity is too high compared to the data, and vacancies are more persistent in the data. The findings in Shigeru Fujita and Ramey (2007) and Hagedorn and Manovskii (2007) suggest that these problems can be fixed without dampening of the volatility of market tightness in the model.

TABLE 3—SUMMARY STATISTICS, QUARTERLY US DATA, 1951:I TO 2004:IV

		u	v	v/u	p
Standard deviation		0.125	0.139	0.259	0.013
Quarterly autocorrelation		0.870	0.904	0.896	0.765
	u	1	−0.919	−0.977	−0.302
	v	—	1	0.982	0.460
Correlation matrix	v/u	—	—	1	0.393
	p	—	—	—	1

Notes: Seasonally adjusted unemployment, u, is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index, v, is constructed by the Conference Board. Both u and v are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the nonfarm business sector, constructed by the BLS from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 1,600.

TABLE 4—RESULTS FROM THE CALIBRATED MODEL

		u	v	v/u	p
Standard deviation		0.145	0.169	0.292	0.013
Quarterly autocorrelation		0.830	0.575	0.751	0.765
	u	1	−0.724	−0.916	−0.892
	v	—	1	0.940	0.904
Correlation matrix	v/u	—	—	1	0.967
	p	—	—	—	1

Notes: All variables are reported in logs as deviations from an HP trend with smoothing parameter 1,600. Calibrated parameter vaues are described in Table 2.

Finally, consider the derivative of wages with respect to productivity:

$$\frac{\partial w}{\partial p} = 1 - \frac{(1 - \beta)(1 - \delta(1 - s))}{1 - \delta(1 - s) + \delta f(\theta)\beta} + (p - z)\frac{\delta\beta(1 - \beta)(1 - \delta(1 - s))}{(1 - \delta(1 - s) + \delta f(\theta)\beta)^2}\frac{\partial f(\theta)}{\partial p}.$$

Since $\partial f(\theta)/\partial p$ is positive, $\partial w/\partial p$ is small if $(1 - \beta)[1 - \delta(1 - s)]/[1 - \delta(1 - s) + \delta f(\theta) \beta]$ is large, i.e., when β is small. Accounting profits, on the other hand, are small only if $(p - z) \times (1 - \beta)[1 - \delta(1 - s)]/[1 - \delta(1 - s) + \delta f(\theta)\beta]$ is small. Thus, $p - z$ also has to be small. The explanation is easy. Small profits mean that $p - w$ is small, and moderately procyclical wages mean that $w - z$ is small.

Efficiency.—When evaluating the efficiency properties of the calibrated model, one cannot ignore taxes. Adding taxes to the model has two consequences. First, the Hosios (1990) condition ceases to imply efficiency. Second, with taxes, market activity provides much higher incremental value over nonmarket activity than our estimate of z appears to imply. However, as Hagedorn and Manovskii (2008) show, given our calibration strategy, all equations (free entry condition, solution for wages, etc.) are identical in the model with and without taxes. Thus, the presence of taxes does not affect the dynamics of the endogenous variables, such as market tightness and unemployment, and there is no need to recalibrate and recompute the model. Only the efficiency

properties are affected, since taxes are taken into account in a decentralized economy but not in a planner's solution.

Let τ_f be the wage tax paid by the firm and τ_w be the wage tax paid by the worker, respectively. Set $\tilde{w}_p = w_p(1 - \tau_w)$ and $\hat{w}_p = w_p(1 + \tau_f)$. Nash bargaining implies that

$$\hat{w}_p = \beta \tilde{p} + (1 - \beta) \frac{1 + \tau_f}{1 - \tau_w} z + c_p \beta \theta_p,$$

$$\tilde{w}_p = \beta \frac{1 - \tau_w}{1 + \tau_f} \tilde{p} + (1 - \beta)z + c_p \beta \frac{1 - \tau_w}{1 + \tau_f} \theta_p,$$

$$\Pi = \tilde{p} - \hat{w}_p = (1 - \beta)\tilde{p} - (1 - \beta) \frac{1 + \tau_f}{1 - \tau_w} z - c_p \beta \theta_p,$$

where $\tilde{p}$ is the after sales tax revenue/productivity. Using 1987 effective average tax rates provided in Enrique Mendoza, Assaf Razin, and Linda Tesar (1994), we set $\tau_f = 0$, $\tau_w = 0.291$ and $\tilde{p} = (1 - 0.051)p$.[14] When we estimate z, we really estimate $(1 + \tau_f) z / (1 - \tau_w)$. Our estimate for z is 0.955, but the true value of z is 0.677. Instead of normalizing p to 1, we really normalize $\tilde{p}$ to be 1. The implicit normalization on p is then $p = 1/0.949 = 1.054$. Thus, $p - z = 0.375$. This calculation implicitly assumes that unemployed workers do not pay a consumption tax on z. This would be true if z represented only the value of leisure. Under the alternative assumption that the consumption of z is fully taxed, consumption taxes do not create a wedge between the values of market and nonmarket activities. Therefore, we can ignore them and have $\tilde{p} = p$. In this case $p - z = 0.323$.

Next, we show that the bargaining power that maximizes social welfare is lower than the unemployment elasticity of the matching function. The efficient levels of θ's are the solution to the following optimization problem:

$$SW_p(u) = \max_{\theta} \; [zu + p(1 - u) - cu\theta + \delta E_p SW_{p'}(s + (1 - s)u - f(\theta)u)].$$

Hagedorn and Manovskii (2008) show that, in a deterministic version, the optimal market tightness, θ^*, solves

$$\frac{c}{\delta f'(\theta^*)} = (p - z) + c \left(\theta^* - \frac{f(\theta^*)}{f'(\theta^*)} + \frac{(1 - s)}{f'(\theta^*)} \right).$$

For $\delta = 0.9992$, $s = 0.0081$, $c = 0.584$, $p = 1.054$, $z = 0.677$, and $l = 0.407$, we find $\theta^* = 0.670$. To solve for the bargaining power such that the efficient amount of vacancies is posted, we derive the equation that determines labor market tightness for a given bargaining power of a worker in a deterministic version of the model:

$$\frac{c}{\delta q(\theta^*)} - \frac{(1 - s)c}{q(\theta^*)} = (1 - \beta) \left(\tilde{p} - \frac{1 + \tau_f}{1 - \tau_w} z \right) - c\beta\theta^*.$$

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[14] We take the level of taxes as given. Hagedorn (2007) studies optimal taxation in models with search frictions.

The result is $\beta = 0.152$. If the consumption of z is taxed as well, we would find $\theta^* = 0.596$, and the efficient $\beta = 0.056$. This result means that the calibration strategy that we are proposing implies that the model is much closer to the efficient benchmark than what is implied by the standard calibration, which, paradoxically, is targeting efficiency.

IV. Conclusion

We have proposed a new way to calibrate the parameters of the Mortensen-Pissarides model and found that a reasonably calibrated model is consistent with the key business cycle facts. In particular, it generates volatilities of unemployment, vacancies, and labor market tightness that are very close to those in the data.

We find a relatively low value for workers' bargaining weight. Despite the low bargaining weight, workers' bargaining position is not weak because outside opportunities have significant effects in a dynamic model. Thus, the low bargaining weight does not imply that wages are either substantially below the marginal product or that wages do not change with changes in productivity. We show that such a low bargaining weight is needed to restore efficiency in the MP model, once we account for the level of taxes observed in the data.

Our calibration also implies that the value of nonmarket activity is fairly close to market productivity.[15] This is the result one would expect in a frictionless competitive environment. Furthermore, our estimate appears reasonable since z is a sum of the value of leisure, unemployment benefits, home production, self-employment, disutility of work, etc. The finding that a typical unemployed worker does not suffer a large decline in utility has to be interpreted with some caution, however. We make a strong assumption that z does not depend on the length of the unemployment spell. In our calibration we (implicitly) estimate the average z of all unemployed. Since the job finding rate equals 45 percent per month on average, short-term unemployed make up the bulk of observations. Thus, our estimate of z represents the value of unemployment for the representative unemployed worker and is uninformative about the value of long-term unemployment, since it is a low probability event.

Costain and Reiter (2005) suggest that a high z implies that changes in unemployment insurance would have counterfactually strong effects on unemployment.[16] Unfortunately, the effects of changes in unemployment insurance are hard to measure in the data. One possibility is to use microeconomic studies, surveyed in Bruce Meyer (1995). However, these studies are informative about unemployed workers' search incentives, but not about firms' incentives to post vacancies. In a typical microeconomic study, a small fraction of the unemployed are given a bonus if they find a job fast. Consistent with the MP model, their expected duration of unemployment remains little changed. The reason is that firms' vacancy posting decisions are virtually unaffected

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[15] Note that the value of being unemployed is close to the value of working, both in our calibration and in Shimer (2005) where $(W - U) / W \approx 0.003$. In addition, our finding does not rule out that becoming unemployed can cause noticeable distress for some displaced workers, as found in Louis S. Jacobsen, Robert J. LaLonde, and Daniel G. Sullivan (1993). This distress is caused not by the search frictions of the MP model but, more likely, by the loss of the worker's union status or the loss in the value of the worker's occupation-specific human capital (see Gueorgui Kambourov and Manovskii 2008). In other words, in a world with worker heterogeneity, there may be individuals with p much higher than z, whose p declines substantially upon displacement. Given that our model does not consider heterogeneity in p values, it does not speak to this issue.

[16] Any model where shocks to productivity are strongly amplified is likely to exhibit strong effects of policies as well. The argument is simple. Any sequence of productivity shocks can be replicated through a sequence of sales taxes. In a basic RBC model, productivity and tax changes have identical effects both on first-order conditions and on households' budget constraint—the conditions that characterize the equilibrium.

because matching is random and expected profits do not change when a small fraction of the unemployed has a higher z.[17]

Whereas using the linear MP model seems appropriate for the analysis of business cycles, it may not be for other experiments, such as large and permanent changes in policy. For example, $p = F_L$ is a process that moves with changes in technology, capital, and employment. The variation of employment and capital over the business cycle creates some curvature in p, which is absent in our analysis since we take p to be an exogenous process. This is fine for our purposes in this paper: what matters is how much p varies over the business cycle (measured in the data) and not whether technology, capital, or employment cause this movement. With curvature in labor in production, however, one cannot treat p as an exogenous process when studying the effects of changes in policy, especially if large changes in the employment level are considered.[18]

As another example, consider the response to an increase in unemployment benefits in a model where z is decreasing with the length of the unemployment spell. Firms would respond through posting fewer vacancies, which leads to an increase in the average duration of unemployment accompanied by a decline in the average z of the unemployment pool. This works against the direct effect of the policy and moves the economy closer to the equilibrium prior to the change in the policy.[19]

To study the effects of policies, it may be productive to embed the MP model into the RBC framework instead of resorting to a linear approximation. As Merz (1995) and Andalfatto (1996) have shown, this significantly improves the performance of the real business cycle model as well. An incomplete list of successes includes the findings that productivity leads total hours, unemployment, and vacancies are negatively correlated (Beveridge curve), and total hours and output fluctuate substantially more than wages. But the RBC model (with MP embedded and calibrated in the standard way) exhibits the same empirical shortcoming as the MP model itself. Unemployment and vacancies are not volatile enough. Applying our calibration strategy within an RBC framework resolves this problem.

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[17] We are also skeptical that the macro effects of unemployment insurance can be isolated and that endogeneity problems can be overcome in cross-country regressions. Such regressions, for example, do not take into account the extent to which the consumption of z is taxed (by consumption taxes), that spousal labor supply responds to changes in unemployment insurance (Julie Berry Cullen and Jonathan Gruber 2000), or that a higher replacement rate crowds out private (precautionary) savings (Eric M. Engen and Gruber 2001).

[18] In a model with a Cobb-Douglas production and homogeneous labor, capital eventually fully adjusts to changes in employment, leaving productivity unchanged. However, in a richer model with a production function that combines capital structures, capital equipment, and labor with heterogeneous skills (Krusell et al. 2000), this result does not hold. Indeed, policy changes have permanent effects on productivity in such an environment, as we show in Hagedorn, Manovskii, and Sergiy Stetsenko (2007).

[19] The model's ability to replicate business cycle facts is, however, unlikely to be affected. The model with duration dependence in z will exhibit procyclicality in z. Applying our calibration strategy to such a model would result in a lower bargaining power in order to match the cyclicality of wages.

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