Macroeconomics II, Lecture IX: Diamond-Mortensen-Pissarides: Statics

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Recap and motivation

- Last lecture: framework for studying wage dispersion in a fricitional labor market
- Today: framework for studying unemployment in a frictional labor market
- Unemployment dynamics in McCall followed from exogenous arrival rates of offers/separation shocks.
- To understand the underlying forces that govern unemployment dynamics, we need to endogenize these arrival rates.

The Diamond-Mortensen-Pissarides (DMP) model

- DMP emphasizes how jobs are endogeneously created and destructed
- Key references: Diamond (JPE 1981; JPE 1982; ReStud 1982), Mortensen (AER 1982; NBER 1982) and Pissarides (ReStud 1984; AER 1985)
- Sometimes referred to as the "Pissarides model" or the "Mortensen-Pissarides model"
- Large literature testing its assumptions/implications and proposing extensions

DMP: agenda

1. The matching function
2. Static DMP with exogenous separations
3. Static DMP with endogenous separations

The matching function

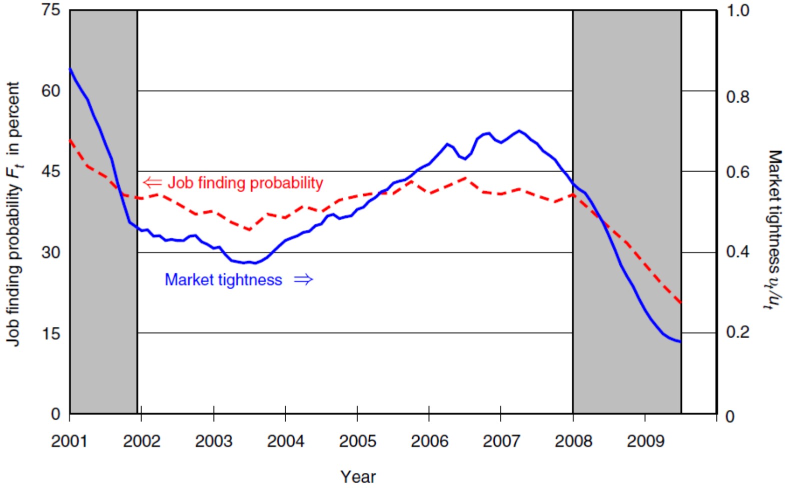
The job-finding rate and market tightness

- Hypothesis: job search is a competitive process - it takes less time for a worker to find a job when demand is abundant and supply is low
- Supply = the number of unemployed U
- Demand = the number of vacancies V
- Define *vacancy rate* as $v = \frac{V}{P}$, where P is number of labor force participants
- Define market tightness:

$$\theta \equiv \frac{V}{U} = \frac{v}{u}$$

  - The market is tight when demand is high relative to supply
- Testable implication: Job-finding rate should be positively correlated with θ

Corr(Job-finding rate, market tightness)



From Rogerson and Shimer (Handbook LE 2011).

The aggregate matching function

- The aggregate matching function endogenizes the job-finding rate via the competition analogy
- $M = M(U, V)$ gives the *flow rate of matches* as a function of current level of unemployment and vacant positions
- $\#$ matches in $\Delta t = M(U, V)\Delta t$
- $M(\cdot)$ should have two basic properties
  - increasing in both arguments
  - concave in both arguments
- Common assumption: $M(\cdot)$ is homogeneous of degree 1
  - Big gain in tractability
  - Support in aggregate time series data (Petrongolo-Pissarides, JEL 2001)
  - Without explicit microfoundations tied to micro evidence, it is difficult to assess this assumption
    - {Carillo-Tudela}-Gartner-Kaas (2020) use German micro data, Skandalis (2019) use French micro data, exciting research area!

Job-finding and job-filing rates

- With random matching, the aggregate matching function delivers a job-finding and job-filing rate

- Homogeneity of degree 1 implies
  - rate at which unemployed worker meets vacant firms: $\lambda_u = \frac{M(U,V)}{U} = M(1, \frac{V}{U}) = M(1, \theta)$
  - rate at which vacant firm meets unemployed workers: $\lambda_v = \frac{M(U,V)}{V} = M(\frac{U}{V}, 1) = M(\theta^{-1}, 1)$

- $\lambda_u = \lambda_u(\theta)$ increasing and concave, $\lambda_v = \lambda_v(\theta)$ decreasing (and often convex)

- Note that $\lambda_u(\theta) = \theta\lambda_v(\theta)$

Example: Cobb-Douglas

- Common functional form: Cobb-Douglas $M(U, V) = AU^{\alpha} V^{1-\alpha}$

- Satisfies all assumptions: increasing, concave, homogeneous of degree 1

- Job-finding and job-filing rates:

$$\begin{aligned} \lambda_u(\theta) &= A\theta^{1-\alpha} \\ \lambda_v(\theta) &= A\theta^{-\alpha} \end{aligned}$$

- Parameteric interpretation:
  - A: aggregate matching efficiency
  - α: elasticity of matches w.r.t. unemployment

- Note that $\log \lambda_u = \log A + (1 - \alpha) \log \theta$
  - Estimate A and α by regressing the (log) job-finding rate on (log) market tightness

Matching function implies a steady-state relationship between v and u

- Unemployment dynamics:

$$\dot{u} = \sigma(1-u) - \lambda_u(\theta)u$$

- Steady state:

$$u = \frac{\sigma}{\sigma + \lambda_u(\theta)}$$

- With Cobb-Douglas:

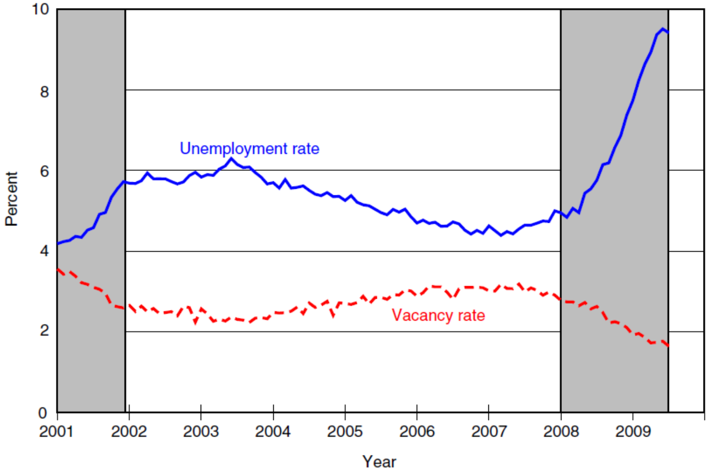
$$u = \frac{\sigma}{\sigma + A\left(\frac{v}{u}\right)^{1-\alpha}}$$

- Rewrite $v = f(u)$:

$$v = \left(\frac{\sigma}{A}\right)^{\frac{1}{1-\alpha}} \left(u^{-\alpha}(1-u)\right)^{\frac{1}{1-\alpha}}$$

- Properties of f: for $0 < u < 1$, we have $f > 0$, $f' < 0$, $f'' > 0$ (Draw graph on Whiteboard)
  - $f' < 0 \Rightarrow Corr(u, v) < 0$

$Corr(u, v)$ in the data



From Rogerson and Shimer (Handbook LE 2011).

- Negative correlation referred to as the **Beveridge curve**

Static Equilibrium

DMP: basic elements

- Continuous time; infinitely lived agents
- Workers: employed or unemployed
- Firms: non-operating, vacant or filled
- A filled firm has one employee, producing instantaneous output flow y
- Exogenous job-separation rate
  - For endogenous separations, see you problem set
- Endogenous job-finding rate
- We will abstract from any heterogeneity, closing down wage dispersion
  - For the case with stochastic firm productivity, see Pissarides (AER 1985)
  - You can also integrate DMP with Burdett-Mortensen

DMP: defining elements

- **Matching function**
  - Increasing, concave and homogenous of degree 1
  - Determines the job-finding and filing rates as function of θ
- **A wage setting rule**
  - Our starting point: Nash Bargaining
  - Because of matching frictions, a match creates a surplus
  - Wages are set to split the match surplus between workers and firms
  - Contrasts with wage posting, as in Burdett-Mortensen
- **Free entry**
  - The vacancy market is competitive: the value of opening a vacancy must be 0 in equilibrium

Worker values

- Now: static analysis (dynamics later)
- Workers take no decisions and there is no wage distribution.
- Worker values are simply

$$
\begin{aligned}
rW &= w + \sigma (U - W) \\
rU &= b + \lambda_u(\theta) (W - U)
\end{aligned}
$$

- continuous-time structure always the same: flow value = flow benefit + (flow probability of event) $\times$ (change of value from event)

Firm values

- Similarly, we have firm value functions

$$\begin{aligned} rJ &= (y - w) + \sigma (V - J) \\ rV &= -c + \lambda_v(\theta) (J - V) \end{aligned}$$

- $c =$ vacancy posting cost; summarizes all costs related to hiring
- Firms take one decision: should I open a vacancy or not?
- Here is where the free entry assumption come into play
  - Firms open vacancies as long as $V \geq 0$
  - Competitive vacancy market, i.e., *free entry*: $V = 0$ in equilibrium

Wage setting

- The model has one price: w

- Without further assumptions, the equilibrium wage level w is indeterminate

- Combining the worker Bellman equations, we get worker surplus:

$$\begin{aligned} r(W-U) &= w-b-(\sigma+\lambda_u(\theta))\,(W-U) \\ W-U &= \frac{w-b}{r+\sigma+\lambda_u(\theta)} \end{aligned}$$

- From firm job Bellman equations and $V=0$, we get firm suprlus:

$$\begin{aligned} rJ &= (y-w)-\sigma J \\ J-V &= \frac{y-w}{r+\sigma} \end{aligned}$$

- For any wage $w \geq b$, the worker is better off employed than unemployed

- For any wage $w \leq y$, the firm is better off operating than vacant

- Ergo, any wage $w \in [b, y]$ is consistent with individual rationality

Nash bargaining

- When workers and firms meet, they sit down and bargain:

$$\max_{w} \left(W(w) - U\right)^{\gamma} \left(J(w) - V\right)^{1-\gamma}$$

- i.e., they maximize the geometric mean of their surpluses, with bargaining weights $\gamma, 1-\gamma$
- Nash (Ecmtra 1950): the solution to this problem is also the unqiue solution to a general bargaining problem that satisfies 4 reasonable axioms
  - Notably, one axiom is Pareto efficiency
- It is by no means clear that this is a reasonable approximation of the how wages are determined in the data, but it gives us a way to start thinking about it
- First order condition:

$$\gamma(J - V)W' + (1-\gamma)(W - U)J' = 0$$

- From the definition of the value functions, we have $W'(w) = -J'(w)$
  - intuition?
- Hence

$$\gamma(J - V) = (1-\gamma)(W - U)$$

Nash bargaining: an alternative expression

- Define total match surplus S:

$$S = (J - V) + (W - U)$$

- Combine with

$$\gamma(J - V) = (1 - \gamma)(W - U)$$

- to find

$$\begin{aligned} W - U &= \gamma S \\ J - V &= (1 - \gamma) S \end{aligned}$$

- Nash bargaining: contesters split the total surplus according to their bargaining power

Equilibrium definition

- An equilibrium is a collection $\{W, U, J, V, w, \theta\}$ s.t. the following equations hold
  - **Bellman equations:**

$$\begin{aligned} rW &= w + \sigma (U - W) \\ rU &= b + \lambda_u(\theta) (W - U) \\ rJ &= (y - w) + \sigma (V - J) \\ rV &= -c + \lambda_v(\theta) (J - V) \end{aligned}$$

  - **Free entry:**

$$V = 0$$

  - **Wage setting rule:**

$$\gamma(J - V) = (1 - \gamma)(W - U)$$

- Six equations, six unknowns

- Given θ, we can solve for $\{v, u\}$ from

$$\begin{aligned} \dot{u} &= \sigma(1 - u) + \lambda_u(\theta)u \\ u(0) &= \underline{\mathrm{u}} \\ \theta &= \frac{v}{u} \end{aligned}$$

Solving DMP I: job creation

- Start from firm side:

$$\begin{aligned} rJ &= (y-w) + \sigma (V - J) \\ rV &= -c + \lambda_v(\theta)(J - V) \end{aligned}$$

- Free entry in vacancy value equation implies:

$$J = \frac{c}{\lambda_v(\theta)}$$

- Free entry in job value equation implies:

$$J = \frac{y-w}{r+\sigma}$$

- Together:

$$\frac{y-w}{r+\sigma} = \frac{c}{\lambda_v(\theta)} \quad \text{or} \quad w = y - \frac{c(r+\sigma)}{\lambda_v(\theta)}$$

- This is the job-creation curve in $\{w, \theta\}$-space
  - Tells you how many vacancies per unemployed, θ, firms create given the wage w
  - Given $\lambda_v(\theta)$ decreasing, JC curve $w(\theta)$ is decreasing (and typically convex)

Solving DMP II: wage curve

- Plug in firm and worker surplus into wage setting rule

$$\begin{aligned}\gamma(J-V) &= (1-\gamma)(W-U)\\ \gamma\left(\frac{y-w}{r+\sigma}\right) &= (1-\gamma)\left(\frac{w-b}{r+\sigma+\lambda_u(\theta)}\right)\end{aligned}$$

- CRS matching: $\frac{\lambda_u(\theta)}{\lambda_v(\theta)}=\theta$ implies

$$\gamma\left(\frac{y-w}{r+\sigma}\right) = (1-\gamma)\left(\frac{w-b}{r+\sigma+\theta\lambda_v(\theta)}\right)$$

- Use the job creation curve $y-w=\frac{c(r+\sigma)}{\lambda_v(\theta)}$ to substitute for $\lambda_v(\theta)$, eliminate terms and rearrange:

$$w=(1-\gamma)b+\gamma(y+c\theta)$$

- This is the wage curve

- WC and JC: two equations in two unkowns (θ and w)

Solving DMP III: steady state

- Up till now, we have not imposed steady state anywhere in the solution
- In general, the level of unemployment and its dynamics is given by

$$\begin{aligned} \theta &= \frac{v}{u} \\ \dot{u} &= \sigma(1-u) - \lambda_u(\theta)u \\ u(0) &= \underline{\mathrm{u}} \end{aligned}$$

- Since θ is determined without imposing steady state, the transition rates in the unemployment law of motion are constant, also outside the steady state
- In the steady state, $\dot{u} = 0$ and we have the Beveridge curve

$$u = \frac{\sigma}{\sigma + \lambda_u(\theta)}$$

Solving DMP IV: equilibrium charachterization

- Summary: the equilibrium steady state $\{w, \theta, u, v\}$ is characterized by

$$\begin{aligned}
\text{JC:} \quad & w = y - \frac{c(r+\sigma)}{\lambda_v(\theta)} \\
\text{WC:} \quad & w = (1-\gamma)b + \gamma(y + c\theta) \\
\text{BC:} \quad & u = \frac{\sigma}{\sigma + \lambda_u(\theta)} \\
\text{TD:} \quad & v = \theta u
\end{aligned}$$

- Note: the system is *block recursive*, we can solve for θ through combining the first 2 equations

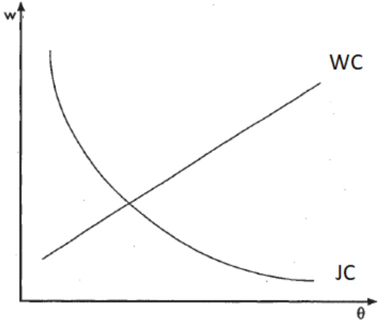
$$\frac{c}{\lambda_v(\theta)} = \frac{(1-\gamma)(y-b)}{r + \sigma + \lambda_u(\theta)\gamma}$$

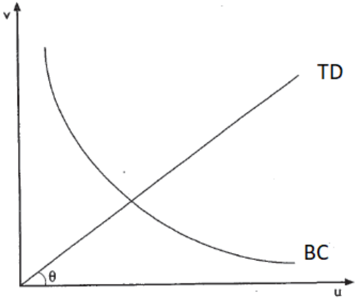
  - LHS: firm cost of posting a vacancy
  - RHS: firm value of a match

- Given θ, we can solve for u and v from the last two equations

- To understand how the equilibrium "work", let's draw some grapgs (Do on whiteboard)

Solving DMP V: graphical view of equilibrium





Comparative statics

- Using our graphs, comparative statics are straightforward:

$$
\begin{aligned}
\text{JC:} \qquad & w = y - \frac{c(r+\sigma)}{\lambda_v(\theta)} \\
\text{WC:} \qquad & w = (1-\gamma)b + \gamma(y + c\theta) \\
\text{BC:} \qquad & u = \frac{\sigma}{\sigma + \lambda_u(\theta)} \\
\text{TD:} \qquad & v = \theta u
\end{aligned}
$$

- $b \uparrow \Rightarrow w \uparrow$ and $\theta \downarrow \Rightarrow u \uparrow$ and $v \downarrow$
  - Intuition? How does this compare to McCall model?
- $\gamma \uparrow \Rightarrow w \uparrow$ and $\theta \downarrow \Rightarrow u \uparrow$ and $v \downarrow$
  - Intuition?
- $y \uparrow \Rightarrow w \uparrow$ and $\theta \uparrow \Rightarrow u \downarrow$ and $v \uparrow$
  - Intuition?
- And so on...
- **A bunch of hypothesis that can be taken to the data!**

Digression: equilibrium effects of extending unemployment benefits

- Did the unemployment benefit extensions in the US during the great recession contribute to the sharp increase in the unemployment rate?
- In lecture 1, we saw that a rich micro literature has estimated the partial equilibrium effect of extending unemployment benefits on unemployment duration
  - Estimates, however, not large enough to explain recession spike (Chetty's rule of thumb: 10 weeks of extra UI gives 1 week of extra unemployment)
- DMP emphasises a different general equilibrium channel through which unemployment benefits affect unemployment: vacany creation
- Recent research employing microeconometric methods to estimate macro effects:
  - US great recession: federal benefit extension program depended on state-level characteristics, such as the elevation in the state-level unemployment rate
  - Hagedorn-Karahan-Manovskii-Mitman (2019) exploit policy discontinuity at US state borders: find big effect
  - Chodorow-Reich-Coglienese-Karabarbounis (QJE 2018) exploit state-level differences in duration extension due to real-time mismeasurement of unemployment rate: find no effect
  - See also Marinescu (JPubE 2017) and Fredriksson-Söderström (JPubE 2020). Ongoing debate — more research needed!

Endogenous separations

Endogenous separations

- How to endogenize the separation decision?

- Separations presumably happen when matches are not beneficial to one of the two parties

- Consider the following twist to the our model:
  - Production is xy, where y is aggregate productivity, and x is match-specific productivity
  - When forming a match, $x = 1$
  - A matched firm-worker pair draw match-specific productivity shocks $x \sim \Gamma(x)$, $x \in [0, 1]$ at arrival rate λ_x
  - If $J(x) < V$, the match is terminated
  - In each period, the firm-worker pair renegoiate the wage according to Nash bargaining

- We focus on steady state

- More generally, we would perhaps also think the match is terminated if $W(x) < U$, but let's start assuming the worker takes no decisions

Firm Values and the reservation productivity

- Firm Bellman equations

$$
\begin{aligned}
rJ(x) &= xy - w(x) + \lambda_x \left[\int_0^1 \max\{V, J(\epsilon)\} d\Gamma(\epsilon) - J(x) \right] \\
rV &= -c + \lambda_v(\theta) \left(J(1) - V \right)
\end{aligned}
$$

- Suppose $J(x)$ is increasing in x (will be prooved later)
- Then, there is reservation productivity x_R below which all matches are dissovled, satisfying $J(x_R) = V$
- Rewritten Bellman equation for job values

$$
\begin{aligned}
rJ(x) &= xy - w(x) + \lambda_x \left[\int_0^1 \max\{V, J(x')\} d\Gamma(x) - J(x) \right] \\
\Rightarrow rJ(x) &= xy - w(x) + \lambda_x \left[\int_{x_R}^1 (J(x') - J(x)) d\Gamma(x') + \int_0^{x_R} (V - J(x)) d\Gamma(\epsilon) \right] \\
\Leftrightarrow rJ(x) &= xy - w(x) + \lambda_x \left[\int_{x_R}^1 (J(x') - J(x)) d\Gamma(x') + (V - J(x)) \Gamma(x_R) \right]
\end{aligned}
$$

Worker values

- Taking x_R as given, worker Bellman equations are

$$
\begin{aligned}
rW(x) &= w(x) + \lambda_x \left[\int_{x_R}^{1} (W(x') - W(x)) d\Gamma(x') + (U - W(x))\Gamma(x_R) \right] \\
rU &= b + \lambda_u(\theta) \left(W(1) - U \right)
\end{aligned}
$$

Nash bargaining and the wage curve

- With Nash rebargaining every time a match-specific shock arrives implies that

$$\gamma(J(x) - V) = (1 - \gamma)(W(x) - U)$$

  or

$$\begin{aligned} W(x) - U &= \gamma S(x) \\ J(x) - V &= (1 - \gamma) S(x) \end{aligned}$$

- Implication: $J(x) - V < 0 \Rightarrow W(x) - U < 0$, i.e., terminations are mutually benificial
  - This is only true because of the rebargaining assumption ("flexible wage setting")

Equilibrium definition

- An equilibrium is a collection $\{W, U, J, V, w, x_R, \theta\}$ s.t. the following equations hold
  - **Bellman equations:**

$$\begin{aligned} rW(x) &= w(x) + \lambda_x \left[\int_{x_R}^{1} (W(x') - W(x)) d\Gamma(x') + (U - W(x))\Gamma(x_R) \right] \\ rU &= b + \lambda_u(\theta) (W(1) - U) \\ rJ(x) &= xy - w(x) + \lambda_x \left[\int_{x_R}^{1} (J(x') - J(x)) d\Gamma(x') + (V - J(x))\Gamma(x_R) \right] \\ rV &= -c + \lambda_v(\theta) (J(1) - V) \end{aligned}$$

  - **Separation decision:**

$$J(x_R) = V$$

  - **Free entry:**

$$V = 0$$

  - **Wage setting rule:**

$$\gamma(J(x) - V) = (1 - \gamma)(W(x) - U)$$

- 7 equations, 7 unknowns

Equilibrium unemployment

- Given θ, x_R, we can solve for $\{v, u\}$ from

$$\begin{aligned} \dot{u} &= \sigma(x_R)(1-u) + \lambda_u(\theta)u \\ u(0) &= \underline{u} \\ \theta &= \frac{v}{u} \end{aligned}$$

where $\sigma(x_R) = \lambda_x \Gamma(x_R)$

- In steady state:

$$u = \frac{\lambda_x \Gamma(x_R)}{\lambda_x \Gamma(x_R) + \lambda_u(\theta)}$$

Computing the equilibrium I: the wage curve

- Combine Bellman equations, the surplus splitting rule and the free entry condition $V = 0$ to find the wage curve (Do at home!):

$$w(x) = (1 - \gamma)b + \gamma(xy + c\theta)$$

Computing the equilibrium II: the surplus function

- Use the surplus splitting rule, the wage curve and the free entry condition to find the Bellman equation for total match surplus $S(x)$: (Do at home!)

$$rS(x) = xy - b - \frac{\gamma}{1-\gamma}c\theta + \lambda_x \left[\int_{x_R}^{1} S(x')d\Gamma(x') - S(x) \right]$$

- Evaluate at $x = x_R$:

$$0 = x_R y - b - \frac{\gamma}{1-\gamma}c\theta + \lambda_x \left[\int_{x_R}^{1} S(x')d\Gamma(x') \right]$$

- Take difference to get

$$\begin{aligned} rS(x) &= rS(x) - rS(x_R) \\ &= (x - x_R)y - \lambda_x S(x) \end{aligned}$$

or

$$S(x) = \frac{y(x - x_R)}{r + \lambda_x}$$

- Interpretation?

Computing the equilibrium III: the job-creation curve

- Nash bargaining:

$$\begin{aligned} J(x) - V &= (1-\gamma)S(x) \\ &= \frac{(1-\gamma)y(x - x_R)}{r + \lambda_x} \end{aligned}$$

- Free entry in Bellman for vacancy value:

$$J(1) - V = \frac{c}{\lambda_v(\theta)}$$

- Together:

$$\frac{c}{\lambda_v(\theta)} = (1-\gamma)y\frac{1 - x_R}{r + \lambda_x}$$

- Let's name this the *Job-creation curve* in θ, x_R-space

Computing the equilibrium IV: the job-destruction curve

- Bellman for Surplus:

$$rS(x) = xy - b - \frac{\gamma}{1-\gamma}c\theta + \lambda_x \left[\int_{x_R}^{1} S(x')d\Gamma(x') - S(x) \right]$$

- Evaluated at $x = x_R$:

$$0 = x_R y - b - \frac{\gamma}{1-\gamma}c\theta + \lambda_x \left[\int_{x_R}^{1} S(x')d\Gamma(x') \right]$$

- Using our solution for $S(x)$:

$$0 = x_R y - b - \frac{\gamma}{1-\gamma}c\theta + \lambda_x \left[\int_{x_R}^{1} \frac{y(x - x_R)}{r+\lambda_x} d\Gamma(x') \right]$$

or

$$x_R y = b + \frac{\gamma}{1-\gamma}c\theta - y\frac{\lambda_x}{r+\lambda_x} \int_{x_R}^{1} (x' - x_R)d\Gamma(x')$$

- Let's name this the *Job-destruction curve* in θ, x_R-space

Equilibrium characterization

- Summing up, $\{x_R, \theta\}$ is solved from the job-creation and the job-destruction curve:

$$\frac{c}{\lambda_v(\theta)} = (1-\gamma)y\frac{1-x_R}{r+\lambda_x}$$
$$x_R y = b + \frac{\gamma}{1-\gamma}c\theta - y\frac{\lambda_x}{r+\lambda_x}\int_{x_R}^{1}(x'-x_R)d\Gamma(x')$$

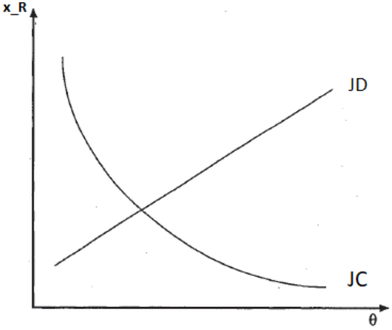
- Given x_R, θ, we can solve for $\{v, u\}$ from

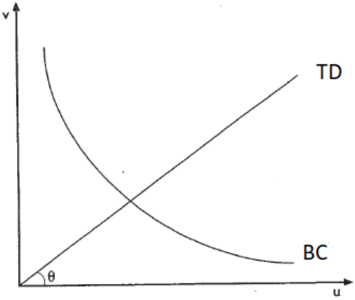
$$v = \theta u$$
$$u = \frac{\lambda_x \Gamma(x_R)}{\lambda_x \Gamma(x_R) + \lambda_u(\frac{v}{u})}$$

- In the background, match-specific wages are given by

$$w(x) \quad = (1-\gamma)b + \gamma(xy + c\theta)$$

Graphical representation





Comparative statics?

- See your problem set

Summary

- DMP: A GE theory of unemployment
  - emphasizes vacancy creation and job destruction as key mechanisms
- Defining elements: Matching function + Wage-setting rule + Free-entry condition
- Can be integrated in growth and business cycle frameworks
- Today we focused on steady state equilibrium and comparative statics
- Next lecture: welfare and dynamics