Macroeconomics II: Problem set 7

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Send your solutions to Andrii by **April 10, 12.00** at the latest.

Exercise 1: Government spending multipliers

Since the days of Keynes, there's been the hypothesis that you can use government spending to stimulate the economy. Let's study this in the vanilla NK model.

Assume the government finances wasteful government spending G_t (by "wasteful", we mean that G_t does not enter anyone's utility nor any production function) by non-distortionary lump-sum taxes TAX_t levied on households. We assume the government budget is balanced, so $G_t = TAX_t$ in all periods. The household problem is still

$$\begin{aligned} \max_{\{C_t, N_t, B_{t+1}\}} \quad & E_O \sum_{t=0}^{\infty} \beta^t \left[\log C - \theta \frac{N^{1+\varphi}}{1+\varphi} \right] \\ \text{s.t} \quad & P_t C_t + Q_t B_{t+1} \leq W_t N_t + B_t + T_t \\ & C_t, N_t, B_{t+1} \geq 0 \end{aligned}$$

but now, $T_t = D_t - TAX_t$ is the sum of firm dividends and taxes.

1. In our baseline model with $G_t = 0$, the log-linear equilibrium was described by

$$\begin{aligned} \text{Intratemporal hh optimality:} \quad & \hat{\omega}_t = \hat{c}_t + \varphi \hat{n}_t \\ \text{Intertemporal hh optimality:} \quad & \hat{c}_t = -(\hat{i}_t - E_t \pi_{t+1}) + E_t \hat{c}_{t+1} \\ \text{Firm optimality:} \quad & \pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t \\ \text{Marginal cost:} \quad & \widehat{mc}_t = \hat{\omega}_t \\ \text{Goods clearing:} \quad & \hat{c}_t = \hat{y}_t \\ \text{Bonds clearing:} \quad & \hat{b}_t = 0 \\ \text{Labor clearing:} \quad & \hat{y}_t = \hat{n}_t \\ \text{Policy rule:} \quad & \hat{i}_t = \phi \pi_t + \nu_t. \end{aligned}$$

Argue that introducing positive and time-varying government spending only changes one of these equations. Which, and how?

2. Suppose government spending follows the process

$$g_t = \rho_g g_{t-1} + \epsilon_t^g$$

with $\rho_g = 0.5$ and the government spending share of output is 30 percent in steady state. Construct graphs of the response of output, inflation and interest rates to 1 percent positive government spending shock.

3. To start interpreting these results, let's first start with constructing the same graphs, but with flexible prices. With flexible prices, the firm optimality condition changes to $\widehat{mc}_t = 0$, as discussed in class. You should find that output increases in response to the shock. What is the mechanism?

4. Let's go back to sticky prices. With sticky prices, you should have found a larger response of output to the same shock. What is the intuition for this? To uncover the mechanism, you might find it helpful to investigate how the response of output changes when you vary the monetary policy response parameter ϕ.

Note: If you are interested in digging deeper, check out Woodford (AEJmacro, 2011).

Exercise 2: Forward Guidance

Optimal policy under commitment typically feature what is called *Forward Guidance*. This tool was forcefully used by the Fed and several other central banks in the aftermath of the Financial Crisis 2007-2008, during which the zero lower bound became binding. The idea is that although you're not able to affect current interest (due to the zero lower bound), you can still affect aggregate demand by promising to keep future interest rates low.

1. Consider the linearized Euler equation in the vanilla NK model

$$\hat{c}_t \quad = \quad -(\hat{i}_t - E_t\pi_{t+1}) + E_t\hat{c}_{t+1}.$$

Iterate on this condition, and use the boundary condition $\lim_{T\to\infty} \hat{c}_T = 0$ to show that

$$\hat{c}_t = -E_t \sum_{s=0}^{\infty} (\hat{i}_{t+s} - \pi_{t+s+1})$$

2. Suppose the central bank can commit to a policy that lowers the future real interest rate $E_t[\hat{i}_{t+s} - \pi_{t+s+1}]$ by 1 percent for some $s > 0$. How will that affect consumption demand in period t in comparison to changing the current real interest rate $\hat{i}_t - E_t\pi_{t+1}$ by 1 percent? Does it matter if $s = 1, 10, 1000$?

3. Explain the intuition to this result.

4. Suppose the central bank makes a commitment today to keep real rates low for the next ten years. Is it credible? Explain your reasoning.

5. Do you think this result is a reasonable prediction of the vanilla NK model?

6. Gabaix (AER 2020) shows that if assuming bounded rationality in the form of "cognitive discounting"- the idea that agents understand current economic conditions better than future ones - the linearized Euler equation changes to

$$\hat{c}_t \quad = \quad -(\hat{i}_t - E_t\pi_{t+1}) + \delta E_t\hat{c}_{t+1}.$$

where $\delta \in [0, 1)$ is the discounting parameter. In this model, does it now matter if the central bank changes the interest rates in period $s = 1, 10$ or $s = 1000$?

Exercise 3: Identification of the Phillips curve

During the period of elevated inflation in the EU and the US 2021-2024, several academics and policy commentators argued that the data does not support a clear tradeoff between real economic activity and inflation (=a Phillips curve). An exmaple is the Claudia Sahm's opinion piece in the Financial Times on Jul 27, 2022, which contained Figure 1 and argued that since the 70's, US data show little empirical support for a positive comovement between unemployment/output and inflation. The author concluded that the concept of a Phillips curve is dead.

Let's examine the argument from the point of view of the New Keynesian model. Consider the dynamics of the following version of the linearized NK model:

$$\begin{aligned} \text{DIS curve:} \quad & \hat{y}_t = -(\hat{i}_t - E_t\pi_{t+1}) + E_t\hat{y}_{t+1} + \nu_t \\ \text{Phillips curve:} \quad & \pi_t = \beta E_t\pi_{t+1} + \kappa\hat{y}_t + \xi_t \\ \text{Policy rule:} \quad & \hat{i}_t = \phi E_t\pi_{t+1} \end{aligned}$$

where ν_t is a "demand shock" and ξ_t is a "supply shock". There are no "policy shocks", and to simplify some algebra, we assume that monetary policy responds to expected inflation rather than current inflation.

1. Suppose there are no supply shocks, $\xi_t = 0$, but only AR(1) demand shocks: $\nu_t = \rho\nu_{t-1} + \epsilon_t$, where ϵ_t is exogenous. Guess that in a solution to a positive demand shock ϵ_t, output and inflation responds linearly to ν_t:

$$\hat{y}_t = \theta_y\nu_t$$
$$\pi_t = \theta_\pi\nu_t$$

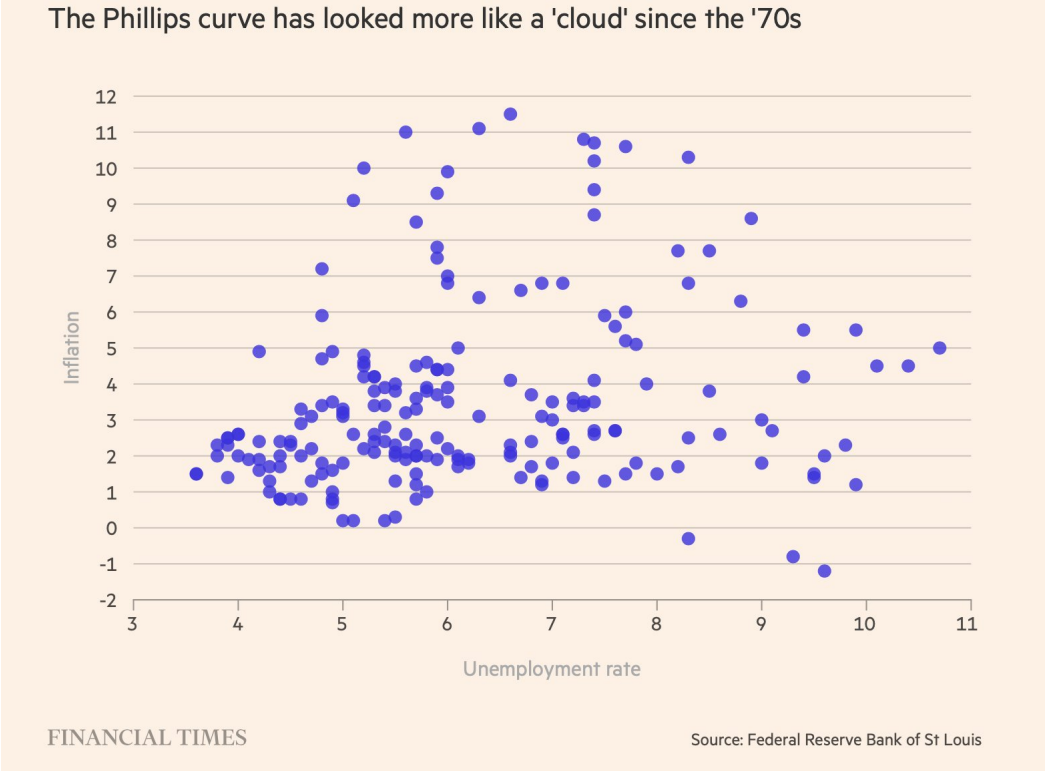

Figure 1: From FT, Jul 27 2022

By using this guess, solve for θ_y, θ_π in terms of the model parameters (and thereby verify that the guess is correct). Is there a positive correlation between $\hat{y}_t$ and π_t?

2. Now, suppose there are no demand shocks, $\nu_t = 0$, but only AR(1) supply shocks: $\xi_t = \rho\xi_{t-1} + \epsilon_t$, where ϵ_t is exogenous. Analogously, guess that in the solution to a positive supply shock ϵ_t:

$$\hat{y}_t = \theta_y \xi_t$$
$$\pi_t = \theta_\pi \xi_t$$

Solve for θ_y, θ_π in this case too. Is there a positive correlation between $\hat{y}_t$ and π_t?

3. Much research have showed in the late 70's, US monetary policy shifted from being "passive" to "active". Put differently, since the late 70's, the Fed has tried to raise the real interest rate in response to inflation rising above target (and lowering rates in response to inflation below target). We can interpret this in our model as if ϕ shifted from $\phi < 1$ to $\phi > 1$. Ignoring determinacy issues, how does this parameter shift affect the correlation between inflation and output in response to demand and supply shocks?

4. In general, do you think that we should expect a positive correlation between inflation and real activity in the data? Can we retrieve the slope of the Phillips curve from the reduced-form relationship in Figure 1? Use your findings from subquestions 1-3 to motivate your answer.

Note: If you are interested in digging deeper, check out McLeay-Tenreyro (NBER Macro Annual 2020) and Rognlie (NBER Macro Annual 2020).

Exercise 4: Rigid wages instead of rigid prices in the NK model

Let's consider the basic NK model discussed in class, but instead of rigid prices, let's assume rigid wages (together with fully flexible price setting). Specifically, let's be a bit extreme and assume that while resting in the steady state, the representative household and the firms have agreed to honor a labor contract with the following features:

- If a transitory shocks occurs, the nominal wage will not change.
- Firms are always free to choose how much labor services to hire at the going wage
- Households are committed to supply whatever hours worked is demanded by the firms

In contrast to the NK model discussed in class, we assume that the intermedidate goods market is fully competitive (meaning that the final goods producer operates a linear production function) and also that

intermediate goods production function is DRS:

$$Y_{it} = N_{it}^{1-\alpha}$$

where TFP $A = 1$.

1. Show that in this model, optimal firm behavior prescribes that the real wage equals
$$\frac{W_t}{P_t} = (1-\alpha)Y_t^{-\frac{\alpha}{1-\alpha}}$$
where, Y_t is the production of final goods.

2. Denote log deviations in the real wage with $\hat{\omega}_t = \hat{w}_t - p_t$. Argue that the growth rate in log real wages must equal the negative of the inflation rate, i.e., $\Delta\hat{\omega}_t = -\pi_t$,

3. Show that combining the results from question 1 and 2 yields the following Phillips curve
$$\pi_t = \frac{\alpha}{1-\alpha}\Delta\hat{y}_t$$

4. Interpret the previous equation, why does firm optimality imply a positive relationship between inflation and output growth in this model?

5. Show that household optimality together with goods market clearing imply the following IS curve:
$$\Delta E_t\hat{y}_{t+1} = \hat{i}_t - E_t\pi_{t+1}$$
You don't need to set up the entire household problem, but can depart from log-linearized optimality condition directly.

6. Suppose the central bank policy rule responds to inflation expectations rather than realized inflation (this is argubly pretty close to how actual monetary policy is conducted):
$$\hat{i}_t = \phi E_t\pi_{t+1} + \nu_t$$
Show that if $\phi \leq \frac{1}{\alpha}$, then a positive monetary policy shock will result in positive expected output growth $\Delta E_t\hat{y}_{t+1} > 0$.

7. One can show that a unique bounded solution is garuanteed if also $\phi > 1$. Argue that, given a unique bounded solution and $\phi < \frac{1}{\alpha}$, output today (in the period in which the shock occurs) must be negative: $\hat{y}_t < 0$

8. Summing up, explain the mechanism, step-by-step, whereby a positive monetary contraction results in a negative output response in this model.

9. Finally, compare the welfare properties in this model to the rigid-price model discussed in this class. Specifically, indicate in what manner the allocation in this model is inefficient, and compare that to the inefficiencies in the rigid-price model discussed in class.