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Author(s): Kenneth Burdett and Dale T. Mortensen
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WAGE DIFFERENTIALS, EMPLOYER SIZE, AND UNEMPLOYMENT*

BY KENNETH BURDETT AND DALE T. MORTENSEN†

University of Essex, U.K.
Northwestern University, U.S.A.

The unique equilibrium solution to a game in which a continuum of individual employers choose permanent wage offers and a continuum of workers search by sequentially sampling from the set of offers is characterized. Wage dispersion is a robust outcome provided that workers search while employed as well as when unemployed. The unique nondegenerate equilibrium distribution of wage offers is constructed for three cases: (i) identical workers and employers, (ii) identical employers and an atomless distribution of worker supply prices, and (iii) identical workers and an atomless distribution of job productivities.

1. INTRODUCTION

Empirical research has documented that inter-industry and cross-employer wage differentials exist, are stable, and cannot be explained by observable differences in worker or job characteristics that might require compensation. Why should workers of apparently equal ability be paid differently on similar jobs? Many have attempted to provide an explanation.

Some writers have argued that workers sort on nonobservable ability in ways that explain the data without contradicting first principles of competitive market analysis, for example, Murphy and Topel (1987). Others appeal to alternative theory with 'efficiency' and 'fair' wage-type arguments, for example, Kreuger and Summers (1987a, 1987b). Utilizing equilibrium sequential search theory, several different authors have provided insights into how a dispersed wage equilibrium can exist, or more precisely, how difficult it is to generate dispersed wages as an equilibrium phenomena. (See, for example, Diamond 1971, Albrecht and Axell 1984, and Burdett and Judd 1983.) Here we show that persistent wage differentials are

* Manuscript received January 1995.
† E-mail: *burdk@essex.ac.uk; d-mortensen@nwu.edu*

consistent with strategic wage formation in an environment characterized by market friction with and without observable heterogeneity across workers and jobs.[1]

In the environment studied, workers randomly search employers for a job that pays a higher wage while employed and an acceptable wage when unemployed, whereas each employer posts a wage conditional on the search behavior of workers and the wages offered by other firms. Given the wages offered by all others and the distribution of worker reservation wage rates, the labor force available to a specific employer evolves in response to the employer's wage. The higher the wage the larger the steady-state labor force, because higher wage firms attract more workers from and lose fewer workers to other employers. The resulting labor supply relation determines the profit of each employer conditional on the wages offered by other employers and the reservation wages demanded by workers. This profit function is the payoff in a 'wage posting' game played by employers. We show that the unique noncooperative steady-state equilibrium to the game can be characterized by a nondegenerate distribution of wage offers even when all workers and jobs are respectively identical if a relatively natural condition holds—the arrival rate of job offers experienced by all workers is strictly positive but finite. As the arrival rate of job offers tends to infinite, the competitive equilibrium results in the limit. However, if employed workers do not receive job offers but unemployed workers face a strictly positive but finite arrival rate of offers, all employers offer the monopsony wage, that is, the monopsony equilibrium obtains.

Wage dispersion exists in equilibrium even when workers are equally productive in all jobs. Three further strong predictions also follow from this simplest version of the model. First, more experienced workers and those with more tenure are more likely to be found in higher paying jobs. Second, there is a positive association between the labor force size and the wage paid. Finally, there is also a negative relationship between wage offers and quit rates across employers.

Several extensions of the basic framework are explored. The presence of search frictions in the form of lags in the arrival of information about the availability and terms of job offers unifies the labor market models analyzed. Each model presented is related to a perfectly competitive counterpart in a natural way in the sense that its solution converges to the competitive equilibrium as frictions vanish. However, the characteristics of equilibrium when frictions are present provide novel theoretical insights and new empirical predictions.

The first model, the pure wage dispersion case analyzed in Section 2, is too simple to address all of the framework's empirical and policy implications of interest. In the first extension, introduced and studied in Section 3, all jobs are equally productive but workers differ with respect to opportunity costs of employment. In this case, equilibrium unemployment exceeds that associated with the operation of an efficient job–worker matching process as some matches that yield gains from trade fail to form. The unemployment inefficiency arising in this case is attributable to monopsony power, which accrues to wage-setting employers when frictions are present.

[1] The origin of this paper is Burdett and Mortensen (1989). We also extend arguments made in Burdett (1990) and Mortensen (1990).

Hence, a mandated minimum wage reduces inefficient unemployment and increases the wage earned by all workers.

In Section 4 employer heterogeneity is introduced. First, we consider the case where there are two types of employer; high productivity and low productivity ones. Any worker employed by a high productivity employer generates more marginal revenue flow than a worker employed by a low productivity employer, by assumption. In this case, more productive employers offer higher wage rates and hire more workers in equilibrium, a result that reinforces the implication that wages offered increase with labor force size in equilibrium. In the second extension, we consider a continuum of employer productivity types. The relationship between the given distribution of productivity types and the resulting equilibrium distribution of wage offers is explored.

2. PURE WAGE DISPERSION

Suppose a large fixed number of both employers and workers participate in a labor market, formally a continuum of each. The measure of workers is indicated by m, whereas the measure of employers is normalized to equal the number 1. In the initial model considered, all workers and all firms are respectively identical.

The decision problem faced by a worker is considered first. At a moment in time, each worker is either unemployed (state 0) or employed (state 1). At random time intervals, however, a worker receives information about a new or alternative job opening. Allowing the arrival rate to depend on a worker's current state, let λ_i, $i \in \{0, 1\}$, representing the parameter of a Poisson arrival process, denote the arrival rate of offers while a worker is currently occupying state i. As workers are assumed to randomly search among employers, an offer is assumed to be the realization of a random draw from F, the distribution of wage offers across employers. Workers must respond to offers as soon as they arrive; there is no recall.

As jobs are identical apart from the wage associated with them, employed workers move from lower to higher paying jobs as the opportunity arises. Workers also move from employment to unemployment as well as from job to job. In particular, job–worker matches are destroyed at an exogenous positive rate δ. Any unemployed worker receives utility flow b per instant. All agents discount future income at rate r.

Given the framework briefly outlined above, the expected discounted lifetime income when a worker is unemployed, V_0, can be expressed as the solution to the asset pricing equation

$$(1) \qquad rV_0 = b + \lambda_0\left[\int \max\{V_0, V_1(\tilde{x})\}\, dF(\tilde{x}) - V_0\right].$$

In other words, the opportunity cost of searching while unemployed, the interest on its asset value, is equal to income while unemployed plus the expected capital gain attributable to searching for an acceptable job where acceptance occurs only if the value of employment, $V_1(w)$, exceeds that of continued search. Similarly, the expected lifetime income of a worker currently employed at wage rate w solves

$$(2)\quad rV_1(w) = w + \lambda_1\left[\int \max\{V_1(w), V_1(\tilde{x})\} - V_1(w)\right] dF(\tilde{x}) + \delta[V_0 - V_1(w)].$$

As $V_1(\cdot)$ is increasing in w whereas V_0 is independent of it, a reservation wage, R, exists such that

$$(3)\qquad V_1(w) \gtrless V_0 \quad \text{as } w \gtrless R$$

where $V_1(R) = V_0$. The above, plus equations (1) and (2), and an integration by parts imply.[2]

$$(4)\qquad R - b = [\lambda_0 - \lambda_1]\int_R^\infty [V_1(x) - V_0]\, dF(x)$$

$$= [\lambda_0 - \lambda_1]\int_R^\infty V_1'(x)[1 - F(x)]\, dx$$

$$= [\lambda_0 - \lambda_1]\int_R^\infty \left[\frac{1 - F(x)}{r + \delta + \lambda_1(1 - F(x))}\right] dx.$$

Keeping the analysis as simple as possible, we consider the special case where r is small relative to the offer arrival rate when unemployed. In particular, letting $r/\lambda_0 \to 0$ implies that (4) can be written as

$$(5)\qquad R - b = [k_0 - k_1]\int_R^\infty \left[\frac{1 - F(x)}{1 + k_1[1 - F(x)]}\right] dx$$

where

$$(6)\qquad k_0 = \lambda_0/\delta \quad \text{and} \quad k_1 = \lambda_1/\delta$$

represents the ratios of state-dependent arrival rates to the job separation rate.[3]

Given the reservation wage, the flows of workers into and out of unemployment can be easily specified. Let u denote the steady-state number of workers unemployed. In the steady state, the flow of workers into employment, $\lambda_0[1 - F(R)]u$, equals the flow from employment to unemployment, $\delta(m - u)$, and therefore

$$(7)\qquad u = \frac{m}{1 + k_0[1 - F(R)]}.$$

[2] The details of the derivation can be found in Mortensen and Neuman (1987).

[3] For every strictly positive r/λ_0, no matter how small, (4) holds and therefore (5) holds as the limit. Of course, at $r = 0$, there are many other optimizing strategies, none of which are of particular interest.

Given an initial allocation of workers to firms, the number of employed workers receiving a wage no greater than w at time t, $G(w,t)(m-u(t))$, can be calculated, where $G(w,t)$ is the proportion of employed workers at t receiving a wage no greater than w, and $u(t)$ is the measure of unemployed at t. Its time derivative can be written as

$$(8)\quad dG(w,t)(m-u(t))/dt = \lambda_0 \max\{F(w)-F(R),0\}u(t) - [\delta+\lambda_1(1-F(w))]G(w,t)(m-u(t)).$$

The first term on the right-hand-side of (8) describes the flow at time t of unemployed workers into firms offering a wage no greater than w, whereas the second term represents the flow out into unemployment and into higher paying jobs, respectively. The unique steady-state distribution of wages earned by employed workers can be written as

$$(9)\quad G(w) = \frac{[F(w)-F(R)]/[1-F(R)]}{1+k_1(1-F(w))}$$

by virtue of (7) and (8) for all $w \geq R$.

In what follows, attention is focused on steady-state behavior. The steady-state number of workers earning a wage in the interval $[w-\varepsilon, w]$ is represented by $[G(w)-G(w-\varepsilon)](1-u)$, while $F(w)-F(w-\varepsilon)$ is the measure of firms offering a wage in the same interval. Thus, the measure of workers per firm earning a wage w can be expressed as

$$l(w|R,F) = \lim_{\varepsilon\to 0} \frac{G(w)-G(w-\varepsilon)}{F(w)-F(w-\varepsilon)}(m-u).$$

Therefore,

$$(10)\quad l(w|R,F) = \frac{mk_0[1+k_1(1-F(R))]/[1+k_0(1-F(R))]}{[1+k_1(1-F(w))][1+k_1(1-F(w^-))]}$$

if $w \geq R$ and $l(w|R,F)=0$, if $w<R$, where

$$F(w) = F(w^-) + \upsilon(w)$$

given $\upsilon(w)$ is the fraction, or mass, of firms offering wage w. Thus, (10) specifies the steady-state number of workers available to a firm offering any particular wage, conditional on the wages offered by other firms, represented by the distribution function F, and the workers reservation wage R. From (10) it follows immediately that $l(\cdot|R,F)$ (for $w \geq R$) is (i) increasing in w; (ii) continuous except where F has a mass point; and (iii) strictly increasing on the support of F and a constant on any connected interval off the support of F.

Firm behavior is now considered. Let p denote the flow of revenue generated per employed worker. Hence, an employer's steady-state profit given the wage offer w

can be written as $(p-w)l(w|R,F)$.[4] Conditional on R and F, each employer is assumed to post a wage that maximizes its steady-state profit flow, that is, an optimal wage offer solves the following problem:

$$\pi = \max_{w}(p-w)l(w|R,F). \tag{11}$$

An equilibrium solution to the search and wage-posting game outlined above can be described by a triple (R, F, π), such that R, the common reservation wage of unemployed workers, satisfies (5), π satisfies (11), and F is such that

$$\begin{aligned}(p-w)l(w|R,F) &= \pi \quad \text{for all } w \text{ on support of } F \\ (p-w)l(w|R,F) &\le \pi \quad \text{otherwise.}\end{aligned} \tag{12}$$

Our first task is to establish the existence of a unique equilibrium solution. To rule out the trivial, assume $\infty > p > b \geq 0$, that is, the productivity of workers is greater than the common opportunity cost of employment. Further, and this is a more critical restriction, assume $\infty > k_i > 0$, $i = 0, 1$. The role this restriction plays is discussed later.

Let $\underline{w}$ and $\overline{w}$ denote the infimum and supremum of the support of an equilibrium F (given one exists). The first thing to note is that no employer will offer a wage less than R in an equilibrium as any employer offering such a wage would have no employees. Hence, without loss of generality, we consider only those distribution functions which have $\underline{w} \geq R$.

Before establishing existence of a unique equilibrium, noncontinuous wage offer distributions are first ruled out as possibilities.[5] As stated previously, $l(w|R,F)$ is discontinuous at $w = \hat{w}$ if and only if $\hat{w}$ is a mass point of F and $\hat{w} \geq R$. This implies any employer offering a wage slightly greater than $\hat{w}$, a mass point where $R \leq \hat{w} < p$, has a significantly larger steady-state labor force and only a slightly smaller profit per worker than an employer offering $\hat{w}$ as $(p-w)$ is continuous in w. Hence, any wage just above $\hat{w}$ yields a greater profit. If there were a mass of F at $\hat{w} \geq p$, all firms offering such a wage make a nonpositive profit. However, any firm offering a wage slightly lower than p will make a strictly positive profit as it still attracts a positive steady-state labor force. In short, offering a wage equal to a mass point $\hat{w}$ cannot be profit maximizing in the sense of (12). Note, *this conclusion rules out a single market wage as an equilibrium possibility.*

As noncontinuous offer distributions have been ruled out, (10) implies that

$$l(\underline{w}|R,F) = mk_0/(1+k_0)(1+k_1) \tag{13}$$

independent of the wage offered as long as $\underline{w} \geq R$. This, of course, implies the employer offering the lowest wage in the market will maximize its profit flow if and only if

$$\underline{w} = R. \tag{14}$$

[4] Steady-state profit is the appropriate criterion in the limiting case as $r/\lambda_0 \to 0$, which is assumed for simplicity. See Cole (1997) for an analysis of the discounted case.

[5] For a more detailed version of the argument, see Burdett and Mortensen (1989).

At any equilibrium every offer must yield the same steady-state profit, which equals

(15) $$\pi = (p-R)mk_0/(1+k_0)(1+k_1) = (p-w)l(w|R,F)$$

for all w in the support of F by equation (13). As $\underline{w} = R$, equations (10) and (15) imply that the unique candidate for F is

(16) $$F(w) = \left[\frac{1+k_1}{k_1}\right]\left[1-\left(\frac{p-w}{p-R}\right)^{1/2}\right].$$

Substituting (16) into (5) yields

$$R-b = \left[\frac{k_0-k_1}{k_1}\right]\int_R^{\overline{w}}\left[\frac{k_1(1-F(x))}{1+k_1(1-F(x))}\right]dx$$

$$= \left[\frac{k_0-k_1}{k_1}\right]\int_R^{\overline{w}}\left[1-\left(\frac{1}{1+k_1}\right)\left[\frac{p-x}{p-R}\right]^{-1/2}\right]dx$$

$$= \left[\frac{k_0-k_1}{k_1}\right]\left[\overline{w}-R+\frac{2(p-R)}{1+k_1}\left(\left(\frac{p-\overline{w}}{p-R}\right)^{1/2}-1\right)\right].$$

However, recognizing that $F(\overline{w}) = 1$, manipulation of equation (16) yields

(17) $$p-\overline{w} = (p-R)/(1+k_1)^2$$

so that

$$R = b + \left[\frac{(k_0-k_1)k_1}{(1+k_1)^2}\right](p-R).$$

Equivalently,

(18) $$R = \frac{(1+k_1)^2 b + (k_0-k_1)k_1 p}{(1+k_1)^2 + (k_0-k_1)k_1}.$$

Equations (17) and (18) imply the support of the only equilibrium candidate, $[R, \overline{w}]$, is nondegenerate and lies strictly below p. Therefore, profit on the support, π, is strictly positive.[6]

To complete the proof that equations (14), (16), (17), and (18) characterize the unique equilibrium, we need only show that no wage off the support of the candidate F yields higher profits. Profits from offers less those on the support

[6] At this stage, one can endogenize the relative measure of firms as reflected in m, the measure of workers per firm, by assuming the existence of a positive fixed cost $c > 0$ and invoking free entry, i.e., $\pi = (p-R)mk_0/(1+k_0)(1+k_1) = c$. All relationships derived above hold in this case as well. Specifically, R is independent of m and therefore is independent of c given free entry.

attract no workers and therefore yield zero profits, whereas a wage offer greater than $\overline{w}$, the supremum of the support, attracts no more workers than $l(\overline{w}|R, F)$, and hence yields a lower profit. Hence, the claim is established.

The equilibrium wage offer distribution derived here contrasts sharply with both the competitive Bertrand solution and Diamond's (1971) monopsony solution to the wage-posting game. Still, both are limiting cases. Specifically, as k_1 tends to zero, (17) implies that the highest wage in the market goes to R. Further, (18) implies an unemployed worker's reservation wage, R, converges to the opportunity cost of employment, b. Hence, Diamond's solution is obtained in this limit. Allowing the possibility that employed workers receive alternative offers and move from lower to higher paying jobs resolves the paradoxical nature of Diamond's (1971) solution. Finally, the equilibrium wage distribution G limits to a mass point at p, the value of revenue product, as frictions vanish in the sense that k_1 tends to infinity. As k_0 tends to infinity as well, the steady-state unemployment rate tends to zero. Hence, the competitive equilibrium is the limiting solution when frictions vanish in the sense that offer arrival rates tend to infinity when employed and not.

A critical feature of the model is the positive relationship between the wage offer and employer labor force size it implies. As the voluntary quit rate, $\lambda F(w)$, decreases with the wage offer, larger firms experience lower quit rates. Because workers only switch employers in response to a higher wage offer, workers with either more experience or tenure are more likely to be earning a higher wage.

3. HETEROGENEOUS WORKERS

Here we extend the basic model by allowing workers to value leisure differently. Assume all job–worker matches are equally productive as before. Workers, however, differ in how they value nonemployment. In particular, let $H(b)$ denote the proportion of workers whose opportunity cost of employment is no greater than b. Assume H is continuous and let $\underline{b}$ and $\overline{b}$ indicate the infimum and supremum of its support. To simplify the analysis, assume that the arrival rate is independent of employment status, that is, $\lambda_0 = \lambda_1 = \lambda$, so that the reservation wage of a type b worker is $R = b$. In this case, the steady-state measure of unemployed workers willing to accept a wage offer less than or equal to x conditional on the wage offer distribution F is

$$u(x|F) = \int_{\underline{b}}^{x} \left(\frac{\delta m}{\delta + \lambda[1 - F(b)]} \right) dH(b) \tag{19}$$

since the unemployment rate of worker of type b is $\delta/(\delta + \lambda[1 - F(b)])$ and the density of type b is $mdH(b)$.

Let the steady-state number of workers employed by employers offering a wage no greater than w be given by $G(w)(m - u)$, where $u = u(\overline{b}|F)$ is total unemployment. In a steady-state the flow of workers leaving employers offering a wage no greater than w equals the flow of workers entering such employers,

$$(\delta + \lambda[1 - F(w)])(m - u(\overline{b}|F))G(w) = \lambda \int_{\underline{b}}^{w} [F(w) - F(x)]\, du(x|F), \tag{20}$$

where $du(b|F)$ is the measure of unemployed workers with reservation wage b and $F(w)-F(b)$ is the probability that an offer received by a type b worker is acceptable and less than or equal to w. Solving (20) yields

$$\text{(21)} \qquad G(w)\big(m-u(\bar{b}|F)\big)=\frac{k\int_{\underline{b}}^{w}[F(w)-F(x)]\,du(x|F)}{1+k[1-F(w)]}$$

where $k=\lambda/\delta$. As $[1+k(1-F(x))]\,du(x|F)=mdH(x)$ from (19), the steady-state number of workers available to an employer offering wage w given the offer distribution F can be written as

$$\text{(22)} \qquad l(w,F)=\frac{\big(m-u(\bar{b}|F)\big)\,dG(w)}{dF(w)}=\frac{kmH(w)}{[1+k[1-F(w)]]^2}$$

at least when F is continuous.[7]

In this case, *an equilibrium solution to the search and wage posting game* is a triple (R,F,π) composed of a reservation wage function $R(b)=b$ from (5), a maximal profit π as defined by (11), and an offer distribution F that satisfies

$$\text{(23)} \qquad \begin{aligned} &(p-w)l(w,F)=\pi \quad \text{for all } w \text{ on the support of } F\\ &(p-w)l(w,F)\le\pi \quad \text{otherwise.}\end{aligned}$$

Equations (23) and (22) imply

$$\text{(24)} \qquad \begin{aligned}\pi&=(p-\underline{w})kmH(\underline{w})/[1+k]^2\\ &=(p-w)kmH(w)/[1+k[1-F(w)]]^2\end{aligned}$$

where $\underline{w}$ is the infimum of the support of F. Thus, all candidates for F must satisfy

$$\text{(25)} \qquad F(w)=\left[\frac{1+k}{k}\right]\left[1-\left[\frac{(p-w)H(w)}{(p-\underline{w})H(\underline{w})}\right]^{1/2}\right]$$

for all w on its support. Of course, in the case where workers have the same opportunity cost of employment (25) reduces to (16).

Let F represent a candidate for an equilibrium offer distribution and let w_l and w_h indicate the infimum and supremum of its support. We have established that (25) describes F on its support. We now characterize the support of an equilibrium distribution of offers. For this purpose, define

$$\text{(26)} \qquad \phi(w,H)=(p-w)H(w).$$

[7] Noncontinuous wage offer distributions can be ruled out as an equilibrium phenomena utilizing arguments essentially the same as those used in the previous section.

Note, $\phi(\cdot, F)$, which is equal to monopsony profit were there only one employer, is continuous in w as $H(\cdot)$ is continuous. Assume that $H(b) > 0$ for some $b < p$, that is, some workers are willing to accept a wage equal to match productivity. Then, $\underline{b} < w < p \Rightarrow \phi(w, H) > 0$. Further, $\phi(w, H) = 0$ if $w \leq \underline{b}$ and $\phi(w, H) \leq 0$ if $w \geq p$. As $\pi(w, H) = k\phi(w, H)/[1 + k[1 - F(w)]]^2$, profit maximization requires $\underline{b} < w_l \leq w_h < p$.

We show that the support of the unique equilibrium wage offer distribution is characterized by the following three conditions:

(i) $w_l = \underline{w}$ where $\underline{w}$ is the largest wage that satisfies

$$\underline{w} = \arg\max_{w} \{\phi(w, H)\}. \tag{27}$$

(ii) $w_h = \overline{w}$ where $\overline{w}$ is the largest wage that satisfies

$$\frac{\phi(\overline{w}, H)}{\phi(\underline{w}, H)} = \frac{1}{[1 + k]^2}. \tag{28}$$

(iii) $w \in$ support $\{F\}$ if and only if $\underline{w} < w \leq \overline{w}$ and

$$\phi(w, H) > \phi(w', H) \text{ for all } w' > w. \tag{29}$$

Suppose $w_l < \underline{w}$. As

$$\pi(w_l, H) = \frac{k\phi(w_l, H)}{[1 + k]^2} < \frac{k\phi(\underline{w}, H)}{[1 + k]^2} \leq \frac{k\phi(\underline{w}, H)}{[1 + k(1 - F(\underline{w}))]^2} = \pi(\underline{w}, H)$$

an employer's steady-state profit at wage $\underline{w}$ is greater than at w, which violates the profit maximization condition (23). Now assume $w_l > \underline{w}$. As

$$\pi(w_l, H) = \frac{k\phi(w_l, H)}{[1 + k]^2} < \frac{k\phi(\underline{w}, H)}{[1 + k]^2} = \pi(\underline{w}, H),$$

the lowest priced firm can make more steady-state profit offering $\underline{w}$ than w_l, the profit maximization condition (23) is again violated. As $k\phi(w_h, H) = \pi(w_h, H)$, condition (ii) is implied by (24).

The offer w in the support $\{F\} \Rightarrow \underline{w} < w \leq \overline{w}$ is a corollary of (i) and (ii) and the profit maximization condition. Now, if there exists a $w' > w$ where $\underline{w} < w \leq \overline{w}$ and $\phi(w, H) \leq \phi(w', H)$, then w is not profit maximizing, that is,

$$(p - w)l(w, F) = \frac{k\phi(w, F)}{[1 + k[1 - F(w)]]^2} < \frac{k\phi(w', F)}{[1 + k[1 - F(w')]]^2} = (p - w')l(w', F)$$

from (23), (26), and (25). Finally, if $w < w'$ and both are profit maximizing then $\phi(w, F) > \phi(w', F)$ because w' attracts more workers and both yield the same profit. Hence, condition (iii) holds.

When workers are heterogenous with respect to the opportunity cost of employment, wage dispersion implies that some efficient matches do not form in the sense that the wage offered is less than the reservation wage, even though the worker's contribution to employer revenue exceeds the worker's opportunity cost of employment. For the sake of the argument, let frictional unemployment be defined as the level that prevails if every employer were to offer marginal revenue product, p, and every worker with opportunity cost of employment less or equal to that of the competitive wage, $b \leq p$, were to form a match when an opportunity arises. Total unemployment minus frictional unemployment is then a measure of inefficient unemployment in the sense that it reflects failures to capture all gains from trade. The steady-state density of workers of type b who are unemployed is $mdH(b)/(1 + k[1 - F(b)])$, whereas the frictional unemployment density is $mdH(b)/(1 + k)$ provided that $p \geq b$. As efficiency requires that only those workers with an opportunity cost of employment b less than or equal to the value of marginal product p participate and the lower support of the wage offer distribution $\underline{w}$ is bounded below by the lower support of the distribution of opportunity costs of employment $\underline{b}$, total equilibrium inefficient unemployment, given offer distribution F, is appropriately defined as

$$(30) \qquad \mu = \int_{\underline{b}}^{p} \left[\left[\frac{1}{1+k(1-F(b))} \right] - \left[\frac{1}{1+k} \right] \right] mdH(b)$$

$$= \frac{m}{1+k} \int_{\underline{w}}^{p} \left[\frac{1 - \left[\dfrac{(p-w)H(b)}{(p-\underline{w})H(\underline{w})} \right]^{1/2}}{\left[\dfrac{(p-w)H(b)}{(p-\underline{w})H(\underline{w})} \right]^{1/2}} \right] dH(b)$$

after substitution from equation (25) for F.

Given a differentiable distribution of worker types $H(b)$, the lower support of the wage offer distribution $F(w)$ satisfied the first-order condition

$$(31) \qquad \phi_w(w, H) = [(p-w)H'(w) - H(w)]m = 0 \quad \text{by (27)}.$$

As the solution is the monopsony wage, $\underline{b} < \underline{w} < p$ and $\underline{w}$ is independent of k. Hence, inefficient unemployment increases with the extent of friction as indexed by the value of $1/k = \delta/\lambda$ and vanishes, along with frictional unemployment, in the competitive limit, that is, $\mu \to 1/(1+k) \to 0$ as $k \to \infty$.

Note that a minimum wage in excess of the monopsony wage, the solution to (31), would be the lowest wage offered were it mandated. Because condition (ii) implies that the highest wage offer $\overline{w}$ increases with the mandated minimum wage when binding in this sense, earnings of all employed workers increase with the minimum wage: that is, the equilibrium wage offer distribution F is stochastically increasing in $\underline{w}$. Furthermore, because $d(p-w)H(w)/dw < 0$ for all w above the monopsony wage in the support of F, an increase in an effective minimum decreases inefficient unemployment by equation (30). Hence, *employment increases with the minimum* *wage* even though atomistic wage competition, not classic monopsony in the formal sense of one buyer, characterizes the market structure.[8] The result is the consequence of monopsonistic competition conditions generated by the existence of search friction.

4. JOB PRODUCTIVITY DIFFERENTIALS

Consider the case of identical workers and two types of employers, one more productive than the other. In particular, let p_i denote a type i employer's revenue flow per worker, $i \in \{1,2\}$, and assume $p_2 > p_1$. The fraction of employers that have a high productivity is indicated by σ. Given all other aspects of the model are as presented in the previous section, an equilibrium in this case can be described by $(F_1, F_2, R, \pi_1, \pi_2)$, where R, is the (common to all workers) reservation wage satisfying (5), F_i, an offer distribution of type i employers, $i = 1, 2$, and

(32)
$$(p_i - w)l(w|R,F) = \pi_i, \quad \text{on support of } F_i$$
$$(p_i - w)l(w|R,F) \le \pi_i, \quad \text{otherwise}$$

where the market distribution, F, is the following mixture:

(33)
$$F(w) = (1-\sigma)F_1(w) + \sigma F_2(w).$$

Existence of such an equilibrium can be established using arguments similar to those used in Section 2.

A critical characteristic of an equilibrium in this case is that more productive employers offer higher wages. Formally, we claim that $w_2 \ge w_1$ if w_i is on the support of F_i, $i = 1, 2$. This follows as

(34)
$$\begin{aligned}\pi_2 &= (p_2 - w_2)l(w_2|R,F) \ge (p_2 - w_1)l(w_1|R,F)\\ &> (p_1 - w_1)l(w_1|R,F) = \pi_1 \ge (p_1 - w_2)l(w_2|R,F)\end{aligned}$$

where the first inequality and last inequality are implied by (32). Comparing the difference between the first and last terms of (34) with the difference between the middle two yields the inequality $(p_2 - p_1)l(w_2|R,F) \ge (p_2 - p_1)l(w_1|R,F)$. This inequality and the fact that $l(\cdot|R,F)$ is increasing in w imply $w_2 \ge w_1$ as claimed. It follows that F_1 and F_2 can be written as

(35)
$$F_i(w) = \left[\frac{k_1}{1+k_1}\right]\left[1 - \left(\frac{p_i - w}{p_i - \underline{w}_i}\right)^{1/2}\right]$$

on its support $[\underline{w}_i, \overline{w}_i)$, $i = 1, 2$.

$$\underline{w}_1 = R, \quad \text{where } R \text{ satisfies (5)}$$

$$\overline{w}_1 = \underline{w}_2, \quad \text{where } p_1 - \overline{w}_1 = (p_1 - \underline{w}_1)/(1+k_1)^2$$

$$p_2 - \overline{w}_2 = (p_2 - \underline{w}_2)/(1+k_1)^2.$$

[8] Still, the result is global only in the constant returns case.

There are several relatively obvious implications of the above result. As the more productive employers offer higher wages, they have larger workforces, make more profit, and keep workers longer than less productive firms. Thus, the existence of productivity differences and matching friction together provide an explanation for the persistence in cross-firm and inter-industry wage and profit differentials that has recently been highlighted in the empirical literature.

Note, in the above example the equilibrium distribution of wages offered is a mixture of the two underlying distributions of wages across employers of the same type and the proportion of high productivity employers. As shown in Mortensen (1990), this result holds for any finite number of productivity types. In other words, when there are n productivity types among employers, the equilibrium distribution of wage offers is a mixture of n distributions, one for each type, with weights equal to the relative frequency of types. An employer with productivity p_j will offer a wage at least as great as one with productivity p_{j-1} ($p_j > p_{j-1}$): that is, the support of the distribution of wage offers associated with employers with productivity p_j, $[\underline{w}_j, \overline{w}_j)$ is such that $\overline{w}_{j-1} = \underline{w}_j$. Hence, when there are n productivity types, the market distribution of wage offers has $n-1$ 'kinks.' Between these 'kinks,' the distribution satisfies (35).

Below we briefly consider the case of a continuum of productivity types. In this case there is a unique wage associated with each productivity type. This fact implies the market distribution of wage offers is a transformation of the underlying distribution of productivity types.[9]

Let $J(p)$ denote the proportion of employers with productivity no greater than p. Assume J is continuous and differentiable with support $[\underline{p}, \overline{p}]$. Keeping things as simple as possible, we again assume $\lambda_1 = \lambda_0 = \lambda$, which implies $R = b$ from (5). As all wage offers must be at least as great as the common worker reservation wage, b, only employers with productivity $p \geq b$ can make a profit and hence will participate. Hence, without loss of generality assume that $\underline{p} \geq b$.

As the argument made above in the case of two employer types applies here as well, the employer making the lowest offer will be the least productive. Because all unemployed workers accept if the offer exceeds $R = b$ and all the employer's workers will leave in response to an outside offer, the lowest offer is profit maximizing if and only if it equals b, that is,

$$w(\underline{p}) = b. \tag{36}$$

For any employer of productivity type $p > \underline{p}$, steady-state profit is determined by a wage choice. Hence,

$$\pi(p) = \max_{w \geq b} \{(p - w) l(w | b, F)\}. \tag{37}$$

[9] Bontemps et al., (1997) independently derive and characterize the same equilibrium solution.

The *first-order condition* for an interior solution is

$$(38) \qquad (p-w)\left[\frac{l'(w|b,F)}{l(w|b,F)}\right]=(p-w)\left[\frac{2kF'(w)}{1+k(1-F(w))}\right]=1$$

from (10). Finally, the *sufficient second-order condition* is

$$(39) \qquad F''(w)[1+k(1-F(w))]-2k[F'(w)]^2<0.$$

A wage offer distribution F is an *equilibrium solution to the wage posting game* if and only if it satisfies (38) and (39) for all $p \in (b, \bar{p}]$ with lower support equal to b from (36). Below we construct a unique distribution function with this property. The principal insight used is that all employers with the same productivity must offer the same wage in the case of a continuum of productivity types. The wage offer correspondence that maps productivity to the wage offer is a function $w(p)$ that satisfies the first-order profit maximization condition (38), so that

$$(40) \qquad F(w(p))=J(p) \quad \text{for all } p \in [b, \bar{p}].$$

Substituting from (40) into (38), the first-order ordinary differential equation follows

$$(41) \qquad w'(p)=\frac{[p-w(p)]2kJ'(p)}{1+k[1-J(p)]}.$$

As (36) provides a boundary condition, its solution $w(p)$ is unique and, consequently, the equilibrium offer distribution candidate is unique.

To complete the existence proof, it is sufficient to show that the candidate satisfies the second-order profit maximization condition (39). That it does follows from a more explicit representation of the solution to (41). For this purpose, define

$$(42) \qquad T(p)=-2\log(1+k(1-J(p))).$$

It follows that

$$(43) \qquad T'(p)=\frac{2kJ'(p)}{[1+k(1-J(p))]}.$$

Substituting (43) into (41) it follows that

$$(44) \qquad w'(p)+T'(p)w(p)=T'(p)p.$$

Any solution to this differential equation must satisfy

$$(45) \qquad w(p)e^{T(p)}=\int_b^p xT'(x)e^{T(x)}\,d\varepsilon+A$$

where A is the constant of integration. However, $de^{T(x)}/dx = T'(x)e^{T(x)}$ and therefore

(46) $$\int_b^p xT'(x)e^{T(x)}\,dx = pe^{(p)} - be^{T(b)} - \int_b^p e^{T(x)}\,dx.$$

Hence, (45) can be written as

$$w(p) = p + e^{-T(p)}[A - be^{T(b)}] - e^{-T(p)}\int_b^p e^{T(x)}\,dx.$$

As we have already established that $w(b) = b$, condition (36), it follows that

$$A = be^{T(b)}$$

and

$$w(p) = p - e^{-T(p)}\int_b^p e^{T(x)}\,dx.$$

Finally, as

$$e^{T(p)} = \frac{1}{[1 + k(1 - J(p))]^2}$$

from definition (42), the wage-productivity profile is

(47) $$w(p) = p - \int_b^p \left(\frac{1 + k(1 - J(p))}{1 + k(1 - J(x))}\right)^2 dx.$$

By differentiation

(48) $$w'(p) = 2kJ_e'(p)\int_b^p \left(\frac{1 + k(1 - J(p))}{[1 + k(1 - J(x))]^2}\right) dx > 0 \quad \text{for all } p > b.$$

Consequently, equations (40) and (48) imply

(49) $$F'(w(p)) = \frac{1}{2k\int_b^p \left(\frac{1 + k(1 - J(p))}{[1 + k(1 - J(x))]^2}\right) dx} > 0 \quad \text{for all } p > b.$$

By completely differentiating the first-order condition for profit maximization (38) with respect to p, equations (48) and (49) imply

(50) $$F''(w(p))[1 + k(1 - F(w(p)))] - 2k[F'(w(p))]^2$$
$$= \frac{-(2kF'(w(p)))^2}{w'(p)} < 0 \quad \text{for all } p \in (b, \bar{p}).$$

In other words, the second-order condition for profit maximization holds on the support of $J(p)$ and consequently on the support of $F(w)$.

When jobs have the same productivity, the equilibrium wage distribution has an increasing density, a fact implied by equation (16) in the case of a degenerate distribution of reservation wage rates, and equations (25) and (29) in the more general case. Indeed, for large values of the offer arrival rates, most wage offers are bunched in a neighborhood to the left of the common value of marginal revenue, p. This property, one shared with the perfectly competitive case, implies that the long flat right tail observed in the earnings distributions must be explained by heterogeneity in worker productivity or job productivity. The model in this section implies that job productivity differences can indeed generate a right skew even without skewing in the distribution of productivities. For example, if the distribution of productivity $J(p)$ is uniform on the support $[b, \bar{p}]$, where, without loss of generality for the point at issue $b = 0$ and $\bar{p} = 1$, then

$$w(p) = \frac{kp^2}{1+k}$$

by equation (47). By differentiating (40) and the equation above twice and then substituting appropriately, one finds that the offer density declines throughout its support. Namely,

$$F''(w(p)) = \frac{J''(p) - F'(w(p))w''(p)}{w'(p)} = -\frac{F'(w(p))}{p} < 0.$$

5. CONCLUSION

The unique equilibrium solution to a wage posting game in which a continuum of individual employers choose permanent wage offers and a continuum of workers search by randomly and sequentially sampling from the set of offers is characterized. In contrast to Diamond's (1971) conclusion, the principal result of the paper is that wage dispersion is a robust outcome when information about the individual offers is incomplete, provided that workers search while employed as well as when unemployed. The unique solution for the equilibrium distribution of wage offers is constructed for three cases: (i) identical workers and employers, (ii) identical employers and an atomless distribution of worker supply prices, and (iii) identical workers and an atomless distribution of employer productivities.

A question arises whether the solution derived in the paper is also an equilibrium of a more general repeated game in which employers cannot precommit to a wage offer once and for all. Coles (1997) provides an affirmative answer to the question by showing that our solution is the limit of a particular sequential equilibrium of the more general game as the common rate of time discount tends to zero. Cole also shows that this equilibrium solution is supported by a 'reputation' for not cutting wage rates. However, other equilibria also exist once the precommitment requirement is relaxed.

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