Macroeconomics II: Problem Set 10

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Send your solutions to Andrii by **Friday, May 23, 12.00,** at the latest.

The natural borrowing limit

What is the maxmimum amount of debt that a household can purchase without risking default? Consider an infinetly lived household that earns income stream $\{y_t\}_0^\infty$, retrieves utility from consumption, which is not allowed to be negative, and who can, in each period t, borrow/save in a risk-free bond a_{t+1} that pays of $(1+r)a_{t+1}$ in period $t+1$.

1. Write the budget constraint of the household.
2. Suppose that the income stream $\{y_t\}_0^\infty$ is deterministic. Show that the household can repay its debt a_{t+1}if and only if:

$$a_{t+1} \geq -\sum_{k=0}^{\infty} \frac{y_{t+k+1}}{(1+r)^{k+1}}$$

3. What is economic interpretation of the right-hand side of this equation?
4. Suppose $\{y_t\}_0^\infty$ is stochastic: $y_t \sim F$ where F has support $[y_{min}, y_{max}]$. What is the maximum amount of debt that the household can repay?
5. Suppose $y_{min} = 0$. What is the maximum amount of debt that the household can repay?
6. Suppose a household faces the borrowing constraint derived in question 4. Under what (standard) condition on the household's utility function u does the borrowing constraint never bind in the solution to the household's problem?

A Huggett economy with a tight borrowing constraint

Consider an infinite-horizon economy with a continuum (measure 1) of ex-ante identical households each having efficiency units of labor ϵ_{it}, drawn from distribution F with finite support $[\epsilon_{min}, \epsilon_{max}]$ and mean 1, i.i.d. across households and time. Consumers can trade a non-contingent bond but cannot borrow. Each household i solves

$$\begin{aligned} \max_{a_{it+1}, c_{it}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}) \\ \text{s.t.} \quad & c_{it} + a_{it+1} \leq \epsilon_{it} w_t + (1 + r_t) a_{it} \\ & a_{it+1} \geq 0 \end{aligned}$$

where u satisfies standard conditions. A representative firm employs production function $Y_t = L_t$, where L_t is the aggregate labor endowment. There is no goverment and assets are in zero net supply.

1. Define a competitive equilibrium
2. Argue that any equilibrium allocation features autarky, i.e., that $c_{it} = \epsilon_{it} w_t$ for all i, t.
3. Argue that any real interest rate that satisfies

$$1 + r_t \leq \frac{1}{\beta} \frac{u_c(\epsilon_{max})}{E_t u_c(\epsilon_{it+1})} \tag{1}$$

   is consistent with a competitive equilibrium.
4. Focus on the equilibrium in which Equation (1) holds with equality. Why does the curvature of the utility function affect the equilibrium real interest rate, but not the equilibrium allocation, in this equilibrium?
5. Is this equilibrium allocation efficient?
6. Is this equilibrium allocation constrained efficient?

An Ayiagari model with an exogeneous savings rule

Consider a stationary economy with a continuum (measure 1) of ex-ante identical households each having efficiency units of labor ϵ which is drawn from a discrete distribution with PDF $\pi(\epsilon)$, i.i.d. over time and across households. The distribution has non-negative support and mean 1. Households can trade a risk-free asset a but cannot borrow $a \geq 0$. Assume that the households' decision rule for savings a' has this form

$$a' = (1 + r)a + \phi w \epsilon,$$

where r, w are the interest rate and the wage rate, respectively, and $0 < \phi < 1$. That is, in each period, they add a constant fraction ϕ of their current-period labor income to their savings account. In the economy, there are also competitive firms which hire labor and capital at prices w and r and operate a standard production function $K^\alpha L^{1-\alpha}$, where $0 < \delta < 1$ is the depreciation of capital. The economy is closed, and in equilibrium, the sum of household asset holdings must equal the capital stock.

1. Set up the firm problem and show the first order conditions.

2. Show that a stationary distribution of a cannot exist if $r \geq 0$. (Hint: study the evolution of aggregate asset supply $A' = \int_i a_i' di$ where i denotes an individual household)

3. For $r < 0$, solve for the long-run aggregate asset supply $A(r)$. Draw a graph of $A(r)$ together with capital demand $K(r)$.

4. Discuss what happens to output, the interest rate and the wage level if the savings rate ϕ increases in the stationary state of this economy.