Aggregate Labor Market Dynamics
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Problem set 1
Due on February 18, 2016

1 Endogenous separations in the DMP model

In the DMP model we discussed in class, we assumed that all separations are exogenous. In this problem set we will endogenize the job destruction decision by introducing a richer structure of productivity shocks. At some of the idiosyncratic productivities the match is profitable, but at some others it is not. A firm will now choose a reservation productivity at which it will terminate the match.

The setup is the same as the DMP model we studied in class, with one twist. Assume that a worker-firm match produces yx units of output where x is match-specific productivity and y is aggregate productivity. We call x *match-specific*, because workers and firms do not carry this productivity across matches, unmatched agents are all homogenous. We assume that shocks to x arrive at a Poisson rate λ, independent over time and across matches, and are drawn from distribution $G(x)$ with the support $[0, 1]$. For simplicity, we assume that new jobs start with the highest productivity $x = 1$. The rest of the setup is the same as we had in class. Firms and workers are risk-neutral and discount future at the rate ρ. Unemployed workers get flow benefit z while employed receive a wage $w(x)$. The wage is determined by Nash bargaining, where workers' bargaining power is γ. There is a free entry of firms, and the flow cost of posting a vacancy is c. Vacancies and unemployed workers match in a market where the total matches is described by a matching function $m(U, V)$. As before, use $p(\theta), q(\theta)$ to denote a probability that an unemployed worker meets a vacancy, and that a vacancy meets a worker, respectively.

Question 1.1 Let $W(x), U, J(x), V$ be value functions for an employed and unemployed worker, and filled and unfilled jobs. Argue that $J(x), W(x)$ are increasing in x. ■

Question 1.2 Job destruction will happen when the value of a job falls below that of a vacancy, $V = 0$. Since $J(x)$ is increasing in x, the decision to separate is represented by a reservation productivity R below which jobs are destroyed. Taking R as given, formulate the Bellman equations, focusing on a steady-state analysis.

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Question 1.3 The wage is determined by Nash bargaining. We make the additional assumption that renegotiation takes place when a new idiosyncratic productivity x is drawn. Show that it leads to the same surplus splitting rule as in class:

$$\begin{aligned} W(x) - U &= \gamma S(x) \\ J(x) - V &= (1-\gamma) S(x) \end{aligned}$$

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Question 1.4 Find an expression for the wage $w(x)$ as a function of parameters of the model. To do it, combine value functions and surplus splitting rules. You should get an expression which is analogous to the one in class.

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Question 1.5 Write down a Bellman equation for the total surplus of the job, $S(x)$.

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Question 1.6 Write down the law of motion for unemployment. Find the steady-state unemployment rate.

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Question 1.7 Now we need to determine R. Argue that R is such that $S(R) = 0$. Further, argue that all separations are mutually agreeable, meaning that a worker and a firm agree to terminate the match.

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Question 1.8 Use the value function for $S(x)$ to find a value of $S(R)$. Use that $S(R) = 0$ to find an equation for R.

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Question 1.9 Use the expression for R and the value function for $S(x)$ to solve for $S(x)$. You should get

$$S(x) = \frac{y(x-R)}{\rho + \lambda}. \tag{1}$$

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Question 1.10 Combine the value for $S(x)$, the surplus splitting rule and the value function for $J(x)$ to find a new job creation curve,

$$\frac{c}{q(\theta)} = (1-\gamma)\, y \frac{1-R}{\rho+\lambda}.$$

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Question 1.11 Next, we will derive a job destruction curve which determines how firm choose the reservation productivity R. Substitute the expression for $S(x)$ from (1) into the integral term in the value function for $S(x)$, and evaluate it at $x = R$. You should get a condition which contains R,θ and then only parameters of the model.

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Question 1.12 The equilibrium with endogenous separations is determined by three equations in three unknowns (θ, R, u)

$$\begin{aligned}
[JC] &: \quad \frac{c}{q(\theta)} = (1-\gamma)\frac{1-R}{\rho+\lambda} \\
[JD] &: \quad 0 = yR - z + \frac{\lambda}{\rho+\lambda} y \int_R^1 \left(x' - R\right) dG\left(x'\right) - c\theta \frac{\gamma}{1-\gamma} \\
[BC] &: \quad u = \frac{\lambda G(R)}{\lambda G(R) + p(\theta)}
\end{aligned}$$

Sketch two graphs. One with JC and JD curve in the (θ, R) space, with θ on the horizontal axis. Another one with BC and JC in the (u, v) space, with u on the horizontal axis. When drawing BC, assume that R is given.

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Question 1.13 Use the figures above to examine how the steady state values of $R, w(x), \theta, u, v$ change if y increases.

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Question 1.14 Let's now think about out-of-steady-state dynamics in this model. Assume that unprofitable jobs can be shut down at any time without delay. Argue that R, θ and $w(x)$ are jump variables, and that they must be at their steady state values at all times.

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Question 1.15 Consider now dynamic adjustment to the new steady state after a *negative* aggregate productivity shock y. Argue that i) there will be a spike in the job destruction rate, ii) there will be a discrete jump in the unemployment rate after the shock.

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Question 1.16 Does a model with endogenous job destruction rate imply a slower convergence to a new steady state after a negative aggregate productivity shock that the standard DMP? Suppose you calibrated the model with exogenous separations and the model with endogenous separations using the same moments, so that the steady state (before the unexpected shock) is the same.

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Question 1.17 This model implies an asymmetric response of the unemployment rate and job destruction to aggregate productivity shock – why? Think about how unemployment and job destruction respond after a positive aggregate productivity shock y and compare it to the response after a negative productivity shock.

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Question 1.18 What are the benefits of having endogenous separations in the model? Is this version of the model helping us explain some data better than the model with exogenous separations?

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Question 1.19 Is the equilibrium efficient? Does Hosios condition continue to hold here? *HINT*: Write down the objective function for the social planner as

$$\max_{\theta_t, R_t} \int_0^\infty e^{-\rho t} \left(y\bar{X}_t + u_t z - c\theta_t u_t \right) dt$$

where $y\bar{X}_t$ is the total output of employed workers. Write down the law of motion for $\bar{X}_t$, which depends on θ and R. Treat $\bar{X}_t$ and u_t as state variables, formulate the current value Hamiltonian with μ_1 and μ_2 as co-states, and derive the optimality conditions. Then proceed as in class to derive the Hosios condition. Since we are interested in the steady state analysis, you can assume that the co-states are constant over time, $\dot{\mu}_1 = \dot{\mu}_2 = 0$.

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