Planner's problem:

$$\max_{\{c_{it}(s^t)\}} \sum_i \sum_t \sum_{s^t} \alpha_i \beta^t \pi(s^t) u(c_{it}(s^t))$$

$$\text{s.t.} \quad \sum_i c_{it}(s^t) \leq \sum_i y_{it}(s^t) \quad \forall t, s^t$$

F. O. C.

$$\frac{\alpha_i \beta^t \pi(s^t) u_c(c_{it}(s^t))}{\alpha_j \beta^t \pi(s^t) u_c(c_{jt}(s^t))} = \frac{\lambda_t(s^t)}{\lambda_t(s^t)}$$

$$\Rightarrow \quad \frac{u_c(c_{it}(s^t))}{u_c(c_{jt}(s^t))} = \frac{\alpha_j}{\alpha_i}$$

CRRA:

$$\frac{u_c(c_{it}(s^t))}{u_c(c_{jt}(s^t))} = \frac{c_{it}(s^t)^{-\sigma}}{c_{jt}(s^t)^{-\sigma}}$$

Full insurance implies

$$c_{it}(s^t) = \left(\frac{\alpha_j}{\alpha_i}\right)^{-\frac{1}{\sigma}} c_{jt}(s^t)$$

$$\Rightarrow \quad \sum_i c_{it}(s^t) = \sum_i \left(\frac{\alpha_j}{\alpha_i}\right)^{-\frac{1}{\sigma}} c_{jt}(s^t)$$

$$\Leftrightarrow \quad C_t(s^t) = \alpha_j^{-\frac{1}{\sigma}} c_{jt}(s^t) \cdot \sum_i \left(\frac{1}{\alpha_i}\right)^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow \quad c_{jt}(s^t) = \underbrace{\frac{\alpha_j^{\frac{1}{\sigma}}}{\sum_i \alpha_i^{\frac{1}{\sigma}}}}_{\theta_j} C(s^t)$$

- Define

$$RHS(a_1; \varepsilon) = \frac{1}{2} f(a_1; \varepsilon) + \frac{1}{2} f(a_1; -\varepsilon)$$

where

$$f(a_1; \varepsilon) = u'(Ra_1 + \bar{y}_1 + \varepsilon)$$

- suppose $a_1(\varepsilon)$ solves

$$u'(y_0 - a_1) = RHS(a_1, \varepsilon) \qquad (EE)$$

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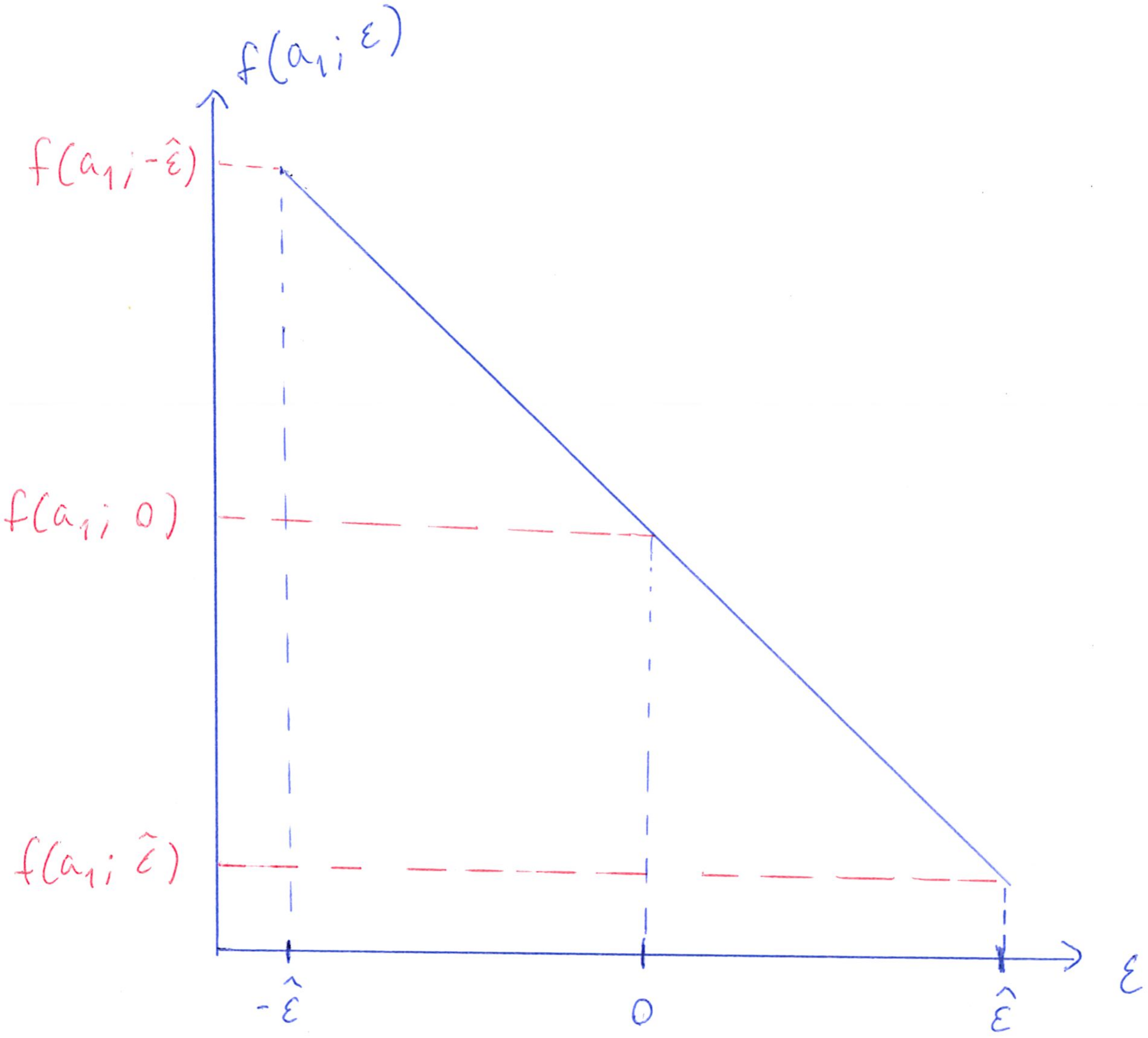
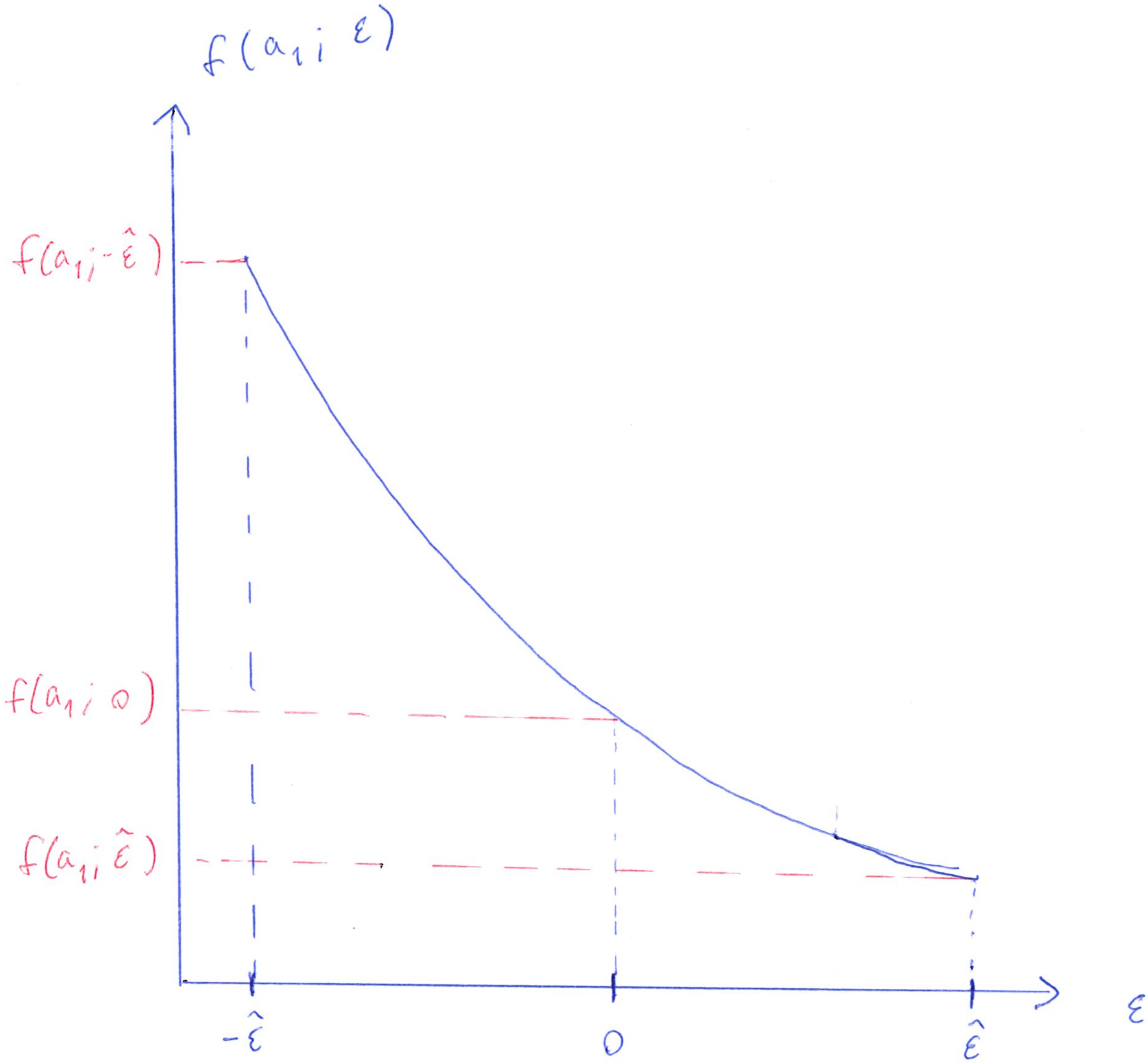
Case 1: $u''' = 0$



$$
\begin{aligned}
RHS(a_1; 0) &= f(a_1; 0) \\
&= \frac{1}{2} f(a_1; -\hat{\varepsilon}) + \frac{1}{2} f(a_1; \hat{\varepsilon}) \\
&= RHS(a_1; \hat{\varepsilon})
\end{aligned}
$$

$$\Rightarrow a_1(\hat{\varepsilon}) = a_1(0)$$

Case 2: $u''' > 0$



$$
\begin{aligned}
RHS(a_1, 0) &= f(a_1; 0) \\
&< \tfrac{1}{2} f(a_1; \hat{\varepsilon}) + \tfrac{1}{2} f(a_1; -\hat{\varepsilon}) \\
&= RHS(a_1, \hat{\varepsilon})
\end{aligned}
$$

$\Rightarrow a_1(\hat{\varepsilon}) > a_1(0)$

Case 3: $u''' < 0$

$\Rightarrow a_1(\hat{\varepsilon}) < a_1(0)$

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At the optimum:

$c = c(x)$

$$\Rightarrow \quad V(x) = u(c(x)) + \beta E V\left(R(x - c(x)) + y'\right)$$

$$\Rightarrow \quad V_x(x) = u_c(c)\, c_x(x) + R\underbrace{\beta E V_x(x')}_{u_c(c) - R\mu} \left[\cancel{R}(1 - c_x(x))\right]$$

$$\Rightarrow \quad V_x(x) = u_c(c)\, c_x(x) + u_c(c)\, (1 - c_x(x)) - R\mu\,(1 - c_x(x))$$

$$= u_c(c) - R\mu\,(1 - c_x(x))$$

If cc binding: $c_x(x) = 1$

If not: $\mu = 0$

$$\Rightarrow \quad V_x(x) = u_c(c)$$