Exam Ph.D. Macroeconomics II

Department of Economics, Uppsala University

May 17, 2023

Instructions

- Writing time: 5 hours.

- The exam is closed book.

- The exam has 76 points in total

- A passing grade requires a) at least 30 points on the exam, and b) 50 points in total for the course (incl the points you have from your problem sets).

- Start each question on a new paper. Write your anonymous code on all answer pages.

- You may write your solutions by pen or pencil; use your best handwriting.

- Answers shall be given in English.

- Motivate your answers carefully; if you think you need to make additional assumptions to answer the questions, state them.

- If you have any questions during the exam, you may call me (+46 730 606 796) at any time between 10 AM and noon.

1 Short Questions (4 points each)

Answer the question and provide a short explanation, *emphasizing economic intuition.*

1. The McCall model predicts that match quality decreases when the the separation rate increases - True or False?

   Answer: True. When the separation rate increases, the discounted value of a match decreases. Therefore the value of waiting for a better match decreases, and the reservation wage therefore declines. With lower reservation wages, the observed wages will be lower, i.e., observed match quality will be lower.

2. The basic Burdett-Mortensen model predicts that firm size is negatively correlated with the quit rate - True or False?

   Answer: True. In the BM model, larger firms are firms that pay higher wages. When paying a higher wage, the firm reduces the probability that its workers will get better offers, that is, they reduce the quit rate.

3. In the vanilla NK model, the equilibrium is generally inefficient. Why?

   Answer: There are two frictions in the vanilla NK model: firms have monopoly power, and cannot set prices freely. The first friction implies a static labor wedge, the second friction implies that the labor wedge is time-varying, and also a time-varying efficiency wedge due to missallocation.

4. Consider the steady state of the vanilla RBC model. In $\{K, R\}$-space (where K is the level of capital, and R is the rental rate of capital), draw the demand and supply curve for capital. What is the elasticity of capital supply w.r.t. to the rental rate? (The original exam question asked about capital demand, but this was a mistake from my side)

   Answer: The demand curve for capital is given by the firm F.O.C., which in steady state looks like:

   $$R = F_K(L, K)$$

   which, assuming CRS production function, is a downward-sloping convex curve in $\{K, R\}$-space.

   The supply curve is implied by the household intertemporal optimality condition:

   $$U'(C_t) = \beta E_t \left[(R_t + (1 - \delta) U'(C_{t+1})) \right]$$

   We save that if $\beta R_t + (1 - \delta) > 1$, consumption (and therefore savings in capital) grows without bound. Around the steady state interest rate implied by $\beta(R_t + (1 - \delta) = 1$, we therefore have that the long-run (steady state) capital supply is infinitely elastic with respect to the rental rate.

   See hand-drawn Figure 1 at the end.

2 Cost-push shocks in the New-Keynesian Model (15 points)

Consider the vanilla New-Keynesian model in class, in which we allow for shocks to firm optimal markups, usually referred to as "cost-push" shocks (such shocks can be micro-founded by, e.g., shocks to the elasticity of substitution in the demand function). The log-linearized equilibrium is characterized by the following set of equations:

$$\text{Intratemporal hh optimality:} \quad \hat{\omega}_t = \hat{c}_t + \varphi \hat{n}_t \tag{1}$$

$$\text{Intertemporal hh optimality:} \quad \hat{c}_t = -(\hat{i}_t - E_t \pi_{t+1}) + E_t \hat{c}_{t+1} \tag{2}$$

$$\text{Firm optimality:} \quad \pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t + \nu_t \tag{3}$$

$$\text{Marginal cost:} \quad \widehat{mc}_t = \hat{\omega}_t \tag{4}$$

$$\text{Goods clearing:} \quad \hat{c}_t = \hat{y}_t \tag{5}$$

$$\text{Labor clearing:} \quad \hat{y}_t = \hat{n}_t \tag{6}$$

$$\text{Policy:} \quad \hat{i}_t = \phi \pi_t \tag{7}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\hat{\omega}_t = \hat{w}_t - p_t$ denotes log deivations in the real wage, and the shock process is AR(1): $\nu_t = \rho \nu_{t-1} + \epsilon$.

Figure 1 contains the IRFs to a positive cost push shock with $\rho = 0.5$. The other parameters take the same value as in class: $\varphi = 1, \beta = 0.99, \theta = 2/3, \phi = 1.5$.

1. Explain the sign of all responses for all variables displayed in Figure 1. (6p)

   Answer: Given that the shock occurs in the Firm optimality condition (3), lets take as given that inflation growth ($\approx \beta\pi_{t+1} - \pi_t$) declines. The fact that there is a unique bounded equilibrium, which returns to steady state, then implies that inflation π_t increases today. The policy rule (7) then implies that both the nominal and the real interest rate increases, assuming that $\phi > 1$. From (2), we then have that consumption growth is positive, which again implies that consumption decreases today. By (5) and (6), we have that output and hours worked decreases today. By (1), we then have that $\hat{\omega}_t = (1+\varphi)\hat{y}_t$, i.e., that wages also decrease today. By (4), the real marginal cost of production decreases today.

2. Suppose that instead of having log utility in the household problem, we assume a CRRA utlity function $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$. How does the equilibrium characterization change in this case? (4p)

   Answer: The household utility function affects equilibrium conditions (1) and (2). With CRRA utility, the log of marginal utility is $-\sigma\hat{c}_t$, and the two conditions change into

$$\text{Intratemporal hh optimality:} \quad \hat{\omega}_t = \sigma\hat{c}_t + \varphi\hat{n}_t$$

$$\text{Intertemporal hh optimality:} \quad \hat{c}_t = -\frac{1}{\sigma}(\hat{i}_t - E_t\pi_{t+1}) + E_t\hat{c}_{t+1}$$
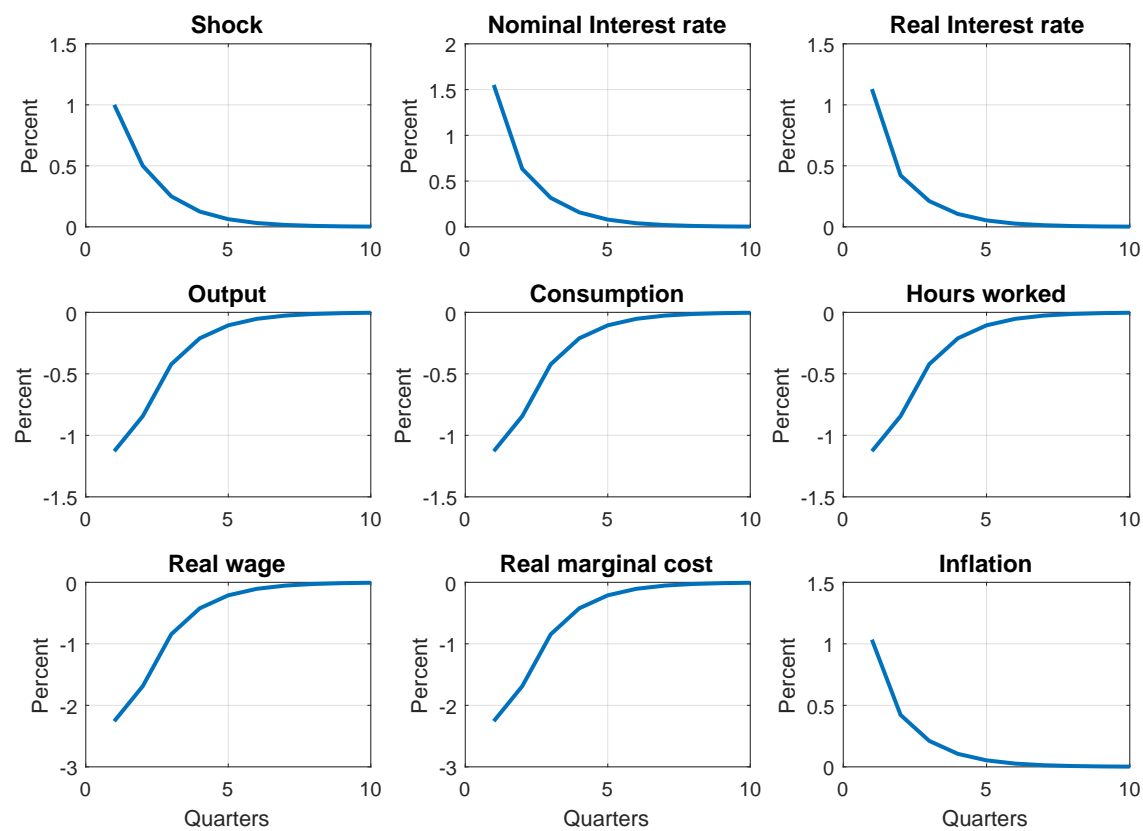
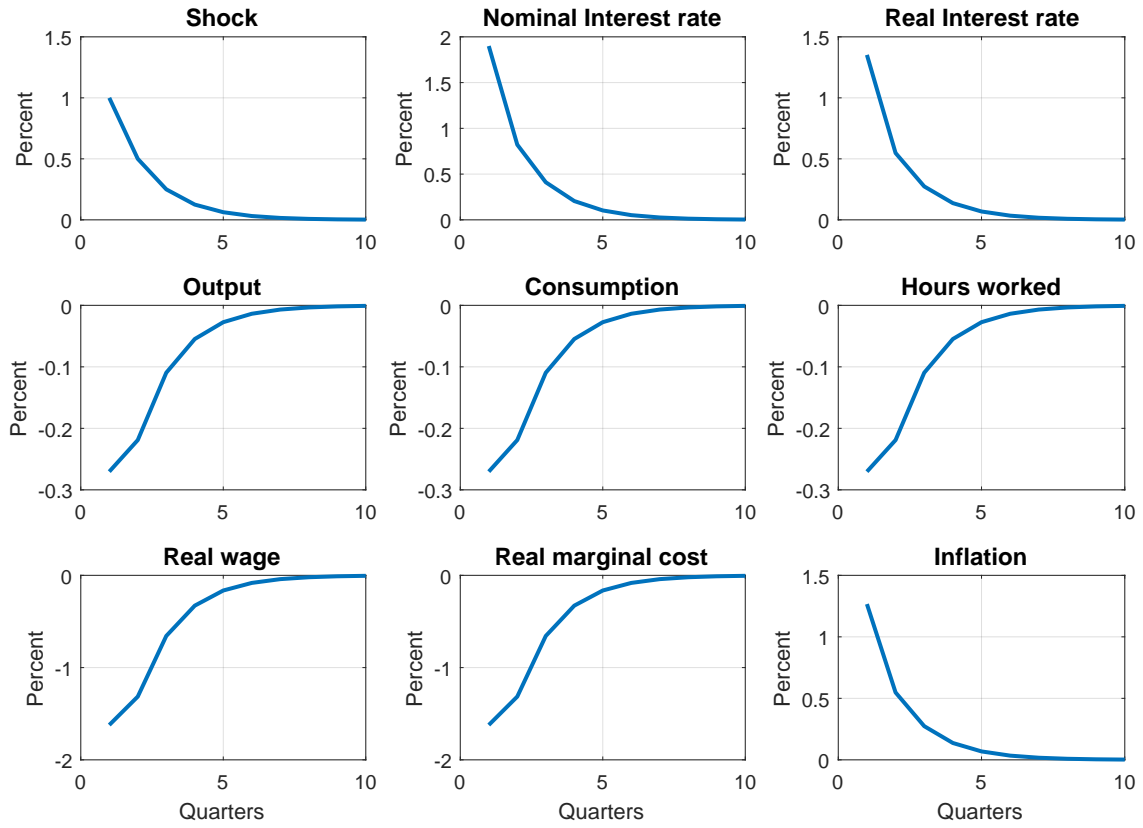Figure 1: IRFs to a cost push shock in the vanilla NK model

Figure 2: IRFs to a cost push shock in the NK model with $\sigma = 5$

3. Figure 2 contains the IRFs to a positive cost push shock with the CRRA utility function and where we set $\sigma = 5$. Explain why the response of consumption, the interest rate, real wages and inflation change in the way they do relative to Figure 1. (5p)

Answer: With a higher σ, that is, a lower elasticity of intratemporal substitution, we have that consumption growth responds less to changes in the real interest rate. That is why we see a smaller response in the consumption IRF. By the market clearing conditions, we have a similar smaller response in output and hours worked. The intramteporal optimality condition then gives a smaller response in real wages, which is the real marginal cost of production. With a smaller decline in the marginal cost of production, firm optimality then implies that inflation growth is even more negative, and therefore that initial inflation increases by more.

3 Unemployment on the Coconut Island (25 points)

Consider the following environment: Time is continuous. There is a continuum of agents of mass one on an island. The island is composed of a forest and a beach. The forest is where coconut trees grow, while the beach is where people meet. Coconuts are the only good in the economy. All coconuts are the same, and each agent gets utility y from consuming a coconut. Agents discount utility at rate r.

Production is done by climbing at coconut trees, which are of various height. Agents without coconuts walk in the forest and look for trees. They are said to be unemployed. When they are unemployed, they randomly bump into coconut trees at exogenous rate α. When climbing a tree, they incur a utility cost c, drawn from an i.i.d. distribution with CDF G(c), defined over $[\underline{c}, \bar{c}]$, with $\underline{c} > 0$, reflecting that higher trees are more costly to climb. One cannot collect or carry more than one coconut.

There is a taboo on this island, according to which one cannot eat self-collected coconuts. Therefore, once a coconut is collected, an agent has to walk on the beach carrying her coconut, and she will randomly meet another agent with a coconut. When walking in search for a trade partner, agents are said to be employed. When a meeting happens, the pair of agents that meets exchange their coconuts (one against one) and eat them. We denote by e (employment) the measure of agents that are holding a coconut and looking for a partner. $e \in [0, 1]$ is endogenous. Meeting someone else on the beach happens at rate $\beta(e)$. It is assumed that $\beta(e)$ is increasing and concave, and that $\beta(0) = 0$.

The life of an agent can therefore be described as follows: once agents have a coconut, they simply walk on the beach until they meet another agent with a coconut, and trade. Without a coconut, they walk in the forest until they run into the coconut tree, and then they decide whether to collect the coconut given the cost.

We will only be concerned with the steady state of this island economy.

1. The assumption that β is increasing reflects a so called "thick market externality". Explain why, especially the "externality" part. (2p)

   Answer: When deciding to climb a tree and collect a coconut, an agent marginally increases the employment rate e, and therefore the contact rate at which all other employed agents meet other agents, through $\beta(e)$. This is an externality, as the deciding agent does not consider how her choice affects the utility of all other agents.

2. Let V_E and V_U denote the value of being employed and unemployed, respectively. The Bellman equation for an employed agent is given by

   $$rV_E(e) = \beta(e)\left(y + (V_U(e) - V_E(e))\right).\tag{8}$$

   Explain in words what this equation says. (2p)

   Answer: It says that the flow value of being employed equals that flow rate of meeting another employed agent $(\beta(e))$ times the change in value when such a meeting occurs. The change in value comes from the direct flow benefit y, and from the changing state from employed to unemployed.

3. Argue that optimizing unemployed agents follow a *reservation cost strategy*: they will only climb the tree they bump into if the cost of doing so is smaller than some reservation cost $c_R(e)$. Why does the reservation cost depend on e? (2p)

   Answer: Given some employment rate e, the expected utility from climbing a tree is decreasing in c. The expected utility from not climbing is zero. These two lines intersect at most once, implying a cutoff $c_R(e)$ under which is better to climb the tree. This depends on e, as the expected utility from owning a coconut depends on the contact rate at which you can trade, $\beta(e)$.

4. Write the Bellman equation for an unemployed agent. (3p)

   Answer: The Bellman equation is

   $$rV_U(e) = \alpha \int_{\underline{c}}^{c_R} \left(V_e(e) - V_U(e) - c\right) dG(c)\tag{9}$$

5. Show that $c_R(e)$ satisfies (3p)

   $$\beta(e)y - (\beta(e) + r)c_R(e) = rV_U(e)\tag{10}$$

Answer: From the Bellman equation for the employed, we have

$$V_E(e) = \frac{\beta(e)(y + V_U(e))}{r + \beta(e)}. \tag{11}$$

Using this in the Bellman equation for the unemployed, we get

$$rV_U(e) = \alpha \int_{\underline{c}}^{c_R} \left(\frac{\beta(e)(y + V_U(e)) - (r + \beta(e))(V_U(e) + c)}{r + \beta(e)} \right) dG(c) \tag{12}$$

or

$$rV_U(e) = \frac{\alpha}{r + \beta(e)} \int_{\underline{c}}^{c_R} (\beta(e)(y - c) - r(V_U(e) + c)) \, dG(c) \tag{13}$$

At the reservation cost $c = c_R$, the agent must be indifferent between climbing or not, from which we get

$$\beta(e)y - (\beta(e) + r)c_R(e) = rV_U(e). \tag{14}$$

6. Using the previous results, solve for an equation that implicitly solves for $c^R(e)$. Let's name this equation the *reservation cost equation*. (3p)

Answer: Using the result in question 5 in the Bellman equation for the unemployed, we get

$$\frac{\beta(e)}{r + \beta(e)} y - c_R(e) = \frac{\alpha}{r + \beta(e)} \int_{\underline{c}}^{c_R(e)} (c^R(e) - c) dG(c) \tag{15}$$

which is the reservation cost equation

7. Using the reservation cost equation, show that $c^R(e)$ is increasing and concave in e whenever $y > c_R(e)$, and that $c_R(0) = 0$. What is the intuition for the "increasing" part? (3p)

For answering this question you might find Leibniz' rule useful. Recall that this rule says that if the functions $f(x, t), \alpha(t), \beta(t)$ are differentiable in t, the function

$$\phi(t) = \int_{\alpha(t)}^{\beta(t)} f(x, t) dg(x)$$

is differentiable, and

$$\phi'(t) = f(\beta(t), t) \frac{dg(\beta(t))}{dt} - f(\alpha(t), t) \frac{dg(\alpha(t))}{dt} + \int_{\alpha(t)}^{\beta(t)} f_t(x, t) dg(x).$$

Answer: By total differentation (and using Leibniz' rule), we get

$$\beta'(e)y - \beta'(e)c_R(e) - (r + \beta(e))c_R'(e) = \alpha G(c_R(e))c_R'(e) \tag{16}$$

or

$$c'_R(e) = \frac{\beta'(e)(y - c_R(e))}{(r + \beta(e)) + \alpha G(c_R(e))} \tag{17}$$

which shows that $c'_R(e) > 0$ whenever $y > c_R(e)$, i.e., that $c_R(e)$ is increasing whenever $y > c_R(e)$. The intuition is that if e is higher, the probability of meeting a trading partner is higher, which increases the marginal value of climbing a tree. Total differentiation one more time yields

$$c''_R(e)(r + \beta(e)) + \alpha G(c_R(e)) + c'_R(e)(\beta'(e)) + \alpha g(c_R(e)c'_R(e)) = \beta''(e)(y - c_R(e)) - \beta'(e)c'_R(e) \tag{18}$$

or

$$c''_R(e) = \frac{\beta''(e)(y - c_R(e)) - c'_R(e)(2\beta'(e) + \alpha g(c_R(e)c'_R(e)))}{r + \beta(e)) + \alpha G(c_R(e)} \tag{19}$$

which shows that $c''_R(e) < 0$ whenever $y > c_R(e)$, i.e., that $c_R(e)$ is concave whenever $y > c_R(e)$.

8. Write the law of motion for employment in this economy, and solve for the steady state value of employment in terms of α and $\beta(e)$. Let's name this steady state relation to the "Beveridge curve". (3p)

   Answer: The LOM is

   $$\frac{d}{dt}e = \alpha G(c^R(e))(1 - e) - \beta(e)e.$$

   Steady state is given by

   $$e = \frac{\alpha G(c^R(e))}{\alpha G(c^R(e)) + \beta(e)}.$$

9. The "Beveridge curve" implies yet another relation between c^R and e, which is increasing and convex and with $c_R(0) = 0$. By drawing an appropriate graph, show that the model may have two steady states: one associated with zero emplyment and one with positive employment. Explain how this can be the case. (4p)

   Answer: Hand-drawn Figure 2 shows a graph of the Beveridge curve and the Reservation cost equation. Since one is increasing and convex, and the other is increasing and concave, and both has an intercept at 0, the will intersect eachother at 0, and, at most, one more time where $e > 0$.

   We can understand the non-employment steady state by recognizing that if no other agent carries a coconut, there is no point for me in climbing at a tree to collect a cocnut - it is costly, and does not grant me any opportunity to trade.

   On the other hand, if there are sufficiently many other agents that have climbed at trees and carry cocnuts, it is also rational for me to climb at some trees, as this gives me an opportunity to trade coconuts.

A two-period Aiyagari model (20 points)

Consider an economy with a continuum (measure 1) of ex-ante identical households, each living for two periods. Each household i has utility given by

$$\log(c_{i1}) + \beta E \log(c_{i2}) \tag{20}$$

where c_{i1}, c_{i2} are period 1 and 2 consumption, β is the discount factor and E is the expected value operator. In period 1, each household is endowed with y_1 units of output that can either be consumed, c_{i1}, or invested, k_i. In period 2, households recieve income from the capital they saved in period 1 and from wages earned from supplying l_i efficiency units of labor. l_i is a random variable, i.i.d. across households and equals $1 + \epsilon$ with probability $1/2$ and $1 - \epsilon$ with probability $1/2$, with $0 < \epsilon < 1$. The Law of large numbers imply that the aggregate efficiency units of labor supply $L = 1$. In period 2, output is produced by a competitive representative firm which operate a Cobb-Douglas production function $K^\alpha L^{1-\alpha}$, renting capital and labor services from the households at rate r and w, respectively.

1. Write the household and firm problems and define a competitive equilibrium for this economy. (3p)

   Answer: The households' problem are

   $$\max_{c_{i1},c_{i2},k_i} \quad \log(c_{i1}) + \beta \log(c_{i2})$$
   $$\text{s.t.} \quad c_{i1} + k_i = y_1$$
   $$c_{i2} = (1+r)k_i + wl_i.$$

   The firm's problem is

   $$\max_{K,L} K^\alpha L^{1-\alpha} - (1+r)K - wL.$$

   A competitive equilibrium is an allocation $\{c_{i1}, c_{i2}, k_i, K, L\}_{i=0}^1$ and prices $\{r, w\}$ such that

   - Given $\{r, w\}$, $\{c_{i1}, c_{i2}, k_i\}_{i=0}^1$ solves the households' problem.
   - Given $\{r, w\}$, $\{K, L\}$ solves the firm's problem.
   - The markets for capital, labor and consumption clear:

   $$\int_0^1 c_{i1} di = \int_0^1 y_1 di$$
   $$\int_0^1 c_{i2} di = K^\alpha L^{1-\alpha}$$
   $$\int_0^1 k_i di = K$$
   $$\int_0^1 l_i di = L$$

2. For the case $\epsilon = 0$ (no uncertainty), solve for the equilibrium level of capital and interest rate. (3p)

Answer: The Law of large numbers imply $L = 1$, using this in the firm F.O.C., we find

$$
\begin{aligned}
1 + r &= \alpha K^{\alpha - 1} \\
w &= (1 - \alpha) K^{\alpha}
\end{aligned}
$$

The households' problem are charachterized by the Euler equation:

$$
\frac{1}{c_{i1}} = \beta(1 + r)\frac{1}{c_{i2}}
$$

or

$$
c_{i2} = \beta(1 + r)c_{i1}
$$

Inserting the budget constraints, we find

$$
(1 + r)k_i + wl_i = \beta(1 + r)(y_1 - k_i)
$$

Integrating across all households, we find

$$
(1 + r)K + w = \beta(1 + r)(y_1 - K)
$$

Using the firm's F.O.C., we find

$$
K^{\alpha} = \beta\alpha K^{\alpha - 1}(y_1 - K)
$$

or

$$
K = \alpha\beta(y_1 - K)
$$

or

$$
K = \frac{\alpha\beta y_1}{1 + \alpha\beta}
$$

The equilibrium interest rate is then given by

$$
1 + r = \alpha\left[\frac{\alpha\beta y_1}{1 + \alpha\beta}\right]^{\alpha - 1}
$$

3. For the case $\epsilon > 0$, solve for the equilibrium level of capital and interest rate and show that the interest rate is decreasing in ϵ. (3p)

Answer: The Law of large numbers imply $L = 1$, using this in the firm F.O.C., we find, as before, that

$$1 + r = \alpha K^{\alpha - 1}$$

$$w = (1 - \alpha)K^\alpha$$

Denote household i's consumption in the good and bad state with c_{i2}^g and c_{i2}^b respectively. The households' problem are charachterized by the Euler equation:

$$\frac{1}{c_{i1}} = \beta(1 + r)\left[\frac{1}{2}\frac{1}{c_{i2}^g} + \frac{1}{2}\frac{1}{c_{i2}^b}\right]$$

Inserting the budget constraints, we find

$$\frac{1}{y_1 - k_i} = \frac{1}{2}\beta(1 + r)\left[\frac{1}{(1+r)k_i + (1+\epsilon)w} + \frac{1}{(1+r)k_i + (1-\epsilon)w}\right]$$

Since all households are identical in period 1, $k_i = K$ for all i. Hence,

$$\frac{1}{y_1 - K} = \frac{1}{2}\beta(1 + r)\left[\frac{1}{(1+r)K + (1+\epsilon)w} + \frac{1}{(1+r)K + (1-\epsilon)w}\right]$$

Using the firm's F.O.C., we find

$$\frac{1}{y_1 - K} = \frac{1}{2}\beta\alpha K^{\alpha-1}\left[\frac{1}{(1+\epsilon)K^\alpha} + \frac{1}{(1-\epsilon)K^\alpha}\right]$$

or

$$\frac{K}{y_1 - K} = \frac{1}{2}\alpha\beta\left[\frac{1}{(1+\epsilon)} + \frac{1}{(1-\epsilon)}\right]$$

or

$$\frac{K}{y_1 - K} = \alpha\beta\frac{1}{(1+\epsilon)(1-\epsilon)}$$

or

$$\frac{K}{y_1 - K} = \alpha\beta\frac{1}{1 - \epsilon^2}$$

or

$$K = \alpha\beta\frac{1}{1 - \epsilon^2}(y_1 - K)$$

or

$$\left(1 + \frac{\alpha\beta}{1 - \epsilon^2}\right)K = \frac{\alpha\beta}{1 - \epsilon^2}y_1$$

or

$$K = \frac{\alpha\beta}{1 - \epsilon^2 + \alpha\beta}y_1$$

which shows that the equilibrium level of K is increasing in ϵ. From the firm F.O.C., we see that the interest rate is then a decreasing function in ϵ.

4. Explain the intuition for why the interest rate is decreasing in ϵ. (3p)

Answer: Due to prudence in the utility fuunction, the households have a precautionary savings motive. With a precautionary savings motive, increasing idionsyncratic income risk increases the demand for savings, or equivalently, the supply of capital. In equilibrium, this translates into a lower level of interest rate, to clear the market for capital.

5. For the case $\epsilon > 0$, define individual consumption risk as the ratio between individual period 2 consumption in the "good" state and the "bad" state. Show that a) household i percieves that its individual consumption risk is lower if it chooses a higher k_i and b) that, in equilibrium, household i's individual consumption risk is not affected by its choice of k_i. Explain how this can be case. (4p)

Answer: Using the budget constraint, individual consumption risk is given by

$$
\begin{aligned}
\frac{c_{i2}^g}{c_{i2}^b} &= \frac{(1+r)k_i + (1+\epsilon)w}{(1+r)k_i + (1-\epsilon)w}. \\
&= \frac{(1+r)k_i + (1-\epsilon)w + 2\epsilon w}{(1+r)k_i + (1-\epsilon)w}. \\
&= 1 + \frac{2\epsilon w}{(1+r)k_i + (1-\epsilon)w}.
\end{aligned}
$$

which shows that a) households perceive that their individual consumption risk is decreasing in their choice of k_i. To show b), we compute how the households' choice of k_i affects equilibrium prices. First, we rewrite the above relationship as

$$
\frac{c_{i2}^g}{c_{i2}^b} = \frac{\frac{1+r}{w}k_i + (1+\epsilon)}{\frac{1+r}{w}k_i + (1-\epsilon)}.
$$

All households are identical in period 1, hence $k_i = K$ for all i. Thus

$$
\frac{c_{i2}^g}{c_{i2}^b} = \frac{\frac{1+r}{w}K + (1+\epsilon)}{\frac{1+r}{w}K + (1-\epsilon)}.
$$

Using the firm F.O.C., we find that $\frac{1+r}{w}K = \frac{\alpha}{1-\alpha}$ and thus

$$
\frac{c_{i2}^g}{c_{i2}^b} = \frac{\frac{\alpha}{1-\alpha} + 1 + \epsilon}{\frac{\alpha}{1-\alpha} + 1 - \epsilon}.
$$

which shows that the individual consumption risk in equilibrium is independent of the level K, and since $k_i = K$, also independent of any individual household's choice of k_i. The explanation for this result is the following. Individual consumption risk is reduced if households recieve relatively more capital income than labor income in the bad state compared to the good state. Holding prices constant, this is acheived if households save more in period 1 s.t. $\frac{(1+r)}{w}k_i$ increases. However, since all households choose the same level of savings (they are ex ante idenitical), if any households increases k_i, then all households do so, and this results in a drop in r and an increase in w. Since firms employ a Cobb-Douglas production function, factor shares are constant in this economy, meaning that $\frac{(1+r)}{w}K$ is a

constant. The drop in r and the increase in w thus perfectly offset any partial equilibirum increase in K, which holds $\frac{(1+r)}{w}k_i$ constant in equilibrium.

6. For the case $\epsilon > 0$, define the ex-ante social welfare function as

$$W = \int_{i=0}^{1} \log(c_{i1}) + \beta \log(c_{i2}) di$$

Show that, when evaluating W at the decentralized equilibrium allocation, W would increase if all households where to save a little bit less (while maintaining that markets clear). Explain the intuition for this result and how it relates to the notion of "constrained efficiency". Hint: compute $\frac{\partial W}{\partial K}$ using the fact the households behave individually optimal at the equilibrium allocation. (4p)

Answer: We have that

$$\frac{\partial W}{\partial K} = \frac{1}{c_1}\frac{\partial c_{i1}}{\partial K} + \beta\frac{1}{2}\left[\frac{1}{c_2^g}\frac{\partial c_2^g}{\partial K} + \frac{1}{c_2^b}\frac{\partial c_2^b}{\partial K}\right]$$

Where c_1, c_2^g, c_2^b are individual consumption in period 1, period 2 good state and period 2 bad state respectively. At the decentralized equilibrium allocation, we have that

$$c_1 + K = y_1$$
$$c_2^g = (1+r)K + (1+\epsilon)w$$
$$c_2^b = (1+r)K + (1-\epsilon)w$$
$$\frac{1}{c_1} = \beta(1+r)\left[\frac{1}{2}\frac{1}{c_2^g} + \frac{1}{2}\frac{1}{c_2^b}\right]$$

Thus

$$
\begin{aligned}
\frac{\partial W}{\partial K} &= \frac{\beta}{2}\left[-(1+r)\left(\frac{1}{c_2^g} + \frac{1}{c_2^b}\right) + \left(\frac{1}{c_2^g}\frac{\partial c_2^g}{\partial K} + \frac{1}{c_2^b}\frac{\partial c_2^b}{\partial K}\right)\right] \\
&= \frac{\beta}{2}\left[\left(\frac{\partial c_2^g}{\partial K} - (1+r)\right)\frac{1}{c_2^g} + \left(\frac{\partial c_2^b}{\partial K} - (1+r)\right)\frac{1}{c_2^b}\right]
\end{aligned}
$$

From the budget constraint and the Firm F.O.C., we have that

$$
\begin{aligned}
\frac{\partial c_2^g}{\partial K} - (1+r) &= \frac{\partial r}{\partial K}K + (1+\epsilon)\frac{\partial w}{\partial K} \\
&= \alpha(\alpha-1)K^{\alpha-1} + (1+\epsilon)\alpha(1-\alpha)K^{\alpha-1} \\
&= \alpha(1-\alpha)\epsilon K^{\alpha-1} \\
\frac{\partial c_2^b}{\partial K} - (1+r) &= -\alpha(1-\alpha)\epsilon K^{\alpha-1}
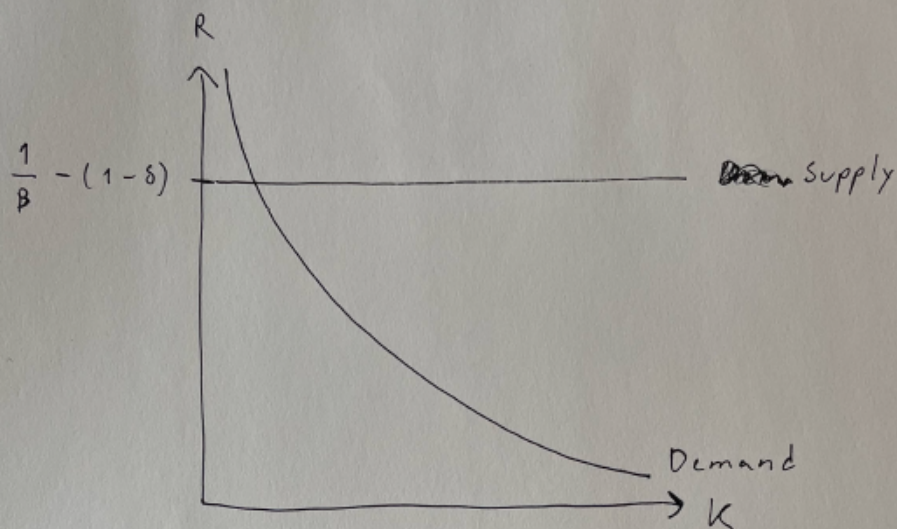\end{aligned}
$$

and that

$$
\begin{aligned}
c_2^g &= (\alpha + (1-\alpha)(1+\epsilon))K^\alpha \\
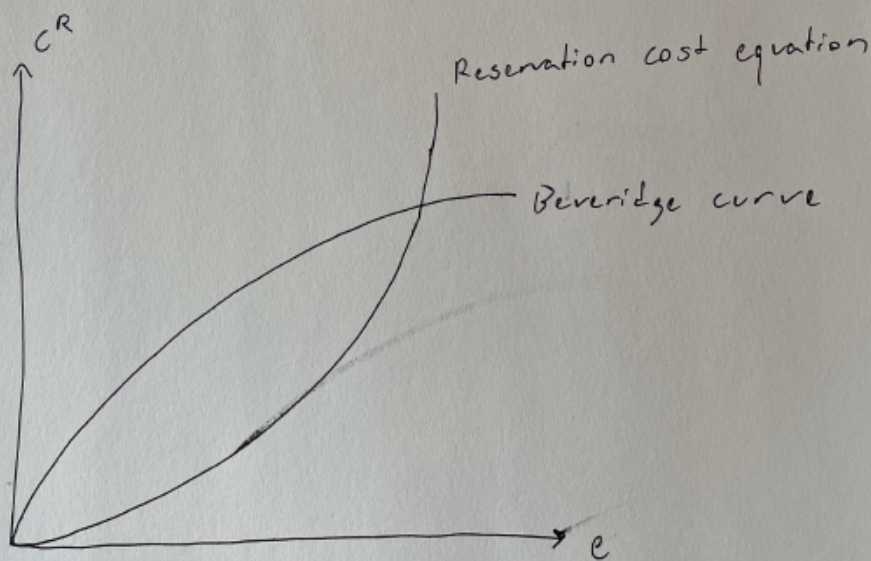c_2^b &= (\alpha + (1-\alpha)(1-\epsilon))K^\alpha
\end{aligned}
$$

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so that

$$
\begin{aligned}
\frac{\partial W}{\partial K} &= \frac{\beta}{2}\left[\frac{\alpha(1-\alpha)\epsilon K^{\alpha-1}}{(\alpha+(1-\alpha)(1+\epsilon))\,K^{\alpha}} - \frac{\alpha(1-\alpha)\epsilon K^{\alpha-1}}{(\alpha+(1-\alpha)(1-\epsilon))\,K^{\alpha}}\right] \\
&= \frac{\beta\alpha(1-\alpha)\epsilon}{2K}\left[\frac{1}{1+\epsilon(1-\alpha)} - \frac{1}{1-\epsilon(1-\alpha)}\right] \\
&= \frac{\beta\alpha(1-\alpha)\epsilon}{2K}\left[\frac{-2\epsilon(1-\alpha)}{1-\epsilon^2(1-\alpha)^2}\right] \\
&= -\frac{\beta\alpha(1-\alpha)^2\epsilon^2}{(1-\epsilon^2(1-\alpha)^2)K} \\
&< 0
\end{aligned}
$$

which shows the desired result. The intuition is that by lowering the capital stock, the return to capital increases and the return to labor decreases. This insures the households from realizing the bad productivity shock, as in that scenario, their savings will bring in more income relative to their labor income. This captures the notion of constrained efficiency: a planner who could force the households to save a little bit less while still letting all markets clear would raise welfare in this economy.

R

$\frac{1}{\beta} - (1-\delta)$ ————————————— ~~Dem~~ Supply

Demand

K

Hand-drawn Figure 1

c^R

Reservation cost equation

Beveridge curve

e

Hand-drawn Figure 2