Macroeconomics II, Lecture III:
RBC: Investment Dynamics

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Last time

- We worked harder on confronting the RBC model with data
- Learned the importance of generating a large fluctuations in efficiency and labor wedge $\Rightarrow$ led us to consider
  - Variable capacity utilization
  - Extensive-margin models of labor supply
  - GHH preferences/rigid wage contracts
- Investment wedge, however, small - does that mean basic RBC model has a fully satisfactory theory of investment?
  - BC accounting is just one (although a very nice one) measure of empirical fit
  - The basic RBC model had too little persistence, and one might think this has to do with the very jumpy response of investment
- Today, we'll dig deeper into the theory of investment

Agenda

1. RBC setup with firm ownership of capital
2. Neoclassical theory vs. Q theory of investment

RBC setup with firm ownership of capital

An alternative, but equivalent, setup

- In the basic RBC model we've studied, household owned and rented out the capital stock to the firm
  - Convenient because all dynamics of the model became encapsulated in household problem; firm problem was static
- To get us started thinking about investment, let's consider a more realistic setup where firms own the capital stock
- Households still own the firm equity, and therefore, indirectly, the firm capital stock
- Because there are no frictions, we'll see that the two setups are equivalent

Household problem

- Program of the representative household

$$\begin{aligned} \max_{\{C_t, N_t, B_{t+1}\}} \quad & E_O \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(N_t) \right] \\ \text{s.t} \quad & C_t + Q_t B_{t+1} \leq W_t N_t + B_t + T_t \\ & C_t, N_t, B_{t+1} \geq 0 \end{aligned}$$

- Note:
  - T_t are firm profits transfered back to the household (=0 in equilibrium)
  - Q_t is the price risk-free bonds that pay 1 unit of consumption goods in $t+1$ in terms of consumption goods in period t
  - $R_t = \frac{1}{Q_t}$ is the gross real return on bonds that pay in period $t+1$
  - In contrast to the $t+1$ return to capital investments in period t, R^r_{t+1}, R_t is known in period t

Household optimality conditions

- Set up the Langrangian, take the F.O.C. to find

$$\begin{aligned} U'(C_t)W_t &= V'(N_t) \\ U'(C_t) &= \beta \frac{1}{Q_t} E_t U'(C_{t+1}) \end{aligned}$$

  or we can write the second equation as

$$U'(C_t) = \beta R_t E_t U'(C_{t+1})$$

  - Note: in steady state $Q = \beta \Rightarrow R = \frac{1}{\beta}$

- Contrast with the optimality conditions in household-ownership setup

$$\begin{aligned} U'(C_t)W_t &= V'(N_t) \\ U'(C_t) &= \beta E_t(R^r_{t+1} + (1-\delta))U'(C_{t+1}) \end{aligned}$$

Asset pricing implications

- The Euler equation is also an asset valuation equation:

$$Q_t = \mathbb{E}_t \left[\frac{\beta U'(C_{t+1})}{U'(C_t)} \right]$$

- Define $Q_{t,t+s}$ as

$$\begin{aligned} Q_{t,t+s} &= Q_t \times Q_{t+1} \times .... \times Q_{t+s-1} \\ &= \mathbb{E}_t \left[\frac{\beta U'(C_{t+1})}{U'(C_t)} \right] \times \mathbb{E}_{t+1} \left[\frac{\beta U'(C_{t+2})}{U'(C_{t+1})} \right] \times .... \times \mathbb{E}_{t+s-1} \left[\frac{\beta U'(C_{t+s})}{U'(C_{t+s-1})} \right] \\ &= \mathbb{E}_t M_{t,t+s} \end{aligned}$$

where

$$M_{t,t+s} \equiv \beta^s \frac{U'(C_{t+s})}{U'(C_t)}$$

Asset pricing implications

- We label $M_{t,t+s}$ the stochastic discount factor

- The SDF measures the households' willingness to forego consumption in period t to have more consumption in a particular state in period $t + s$

- In asset market equilibrium, $M_{t,t+s}$ prices assets that pays off in a particular state in period $t + s$

- $\mathbb{E}M_{t,t+s}$ prices risk-free assets that pays off in period $t + s$

- $M_{t,t+s}$ is the key object of interest in much of macro finance

Firm problem

- A representative firm can choose investment and labor hirings, taking prices as given
- It can finance investment using internal funds (=equity) or risk-free debt
- The household owns the firm: firm therefore discounts future profits using household stochastic discount factor $M_{t,t+s}$
- Program

$$\begin{aligned} \max_{N_t, I_t, B_{t+1}, K_{t+1}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} \left(A_t F(K_t, N_t) - W_t N_t - I_t + Q_t B_{t+1} - B_t \right) \\ \text{s.t.} \quad & K_{t+1} \leq I_t + (1-\delta) K_t \end{aligned}$$

- Note:

$$V_0 = \sup \left[\mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t} \left(A_t F(K_t, N_t) - W_t N_t - I_t + Q_t B_{t+1} - B_t \right) \right]$$

is the value of the firm in period 0
  - Basic asset pricing result: value of firm = discounted NPV of future cash flows

Firm optimality conditions

- Set up the Lagrangian, take the F.O.C. to find: (Do on whiteboard)

$$\begin{aligned} W_t &= A_t F_N(K_t, N_t) && (1) \\ q_t &= 1 && (2) \\ q_t &= \mathbb{E}_t \left[M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta) q_{t+1} \right] \right] && (3) \\ Q_t &= \mathbb{E}_t M_{t,t+1} && (4) \end{aligned}$$

  where q_t is the Lagrange multiplier on the firm constraint

- Optimality conditions 2-3, together with definition of $M_{t,t+1}$, implies

$$U'(C_t) = \beta E_t (R^r_{t+1} + (1-\delta)) U'(C_{t+1})$$

  where $R^r_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1})$ - which we recognize!

- Note: Equation (4) is satisfied whenever households' are optimizing, what does this mean?

Firm optimality conditions

- Set up the Lagrangian, take the F.O.C. to find: (Do on whiteboard)

$$
\begin{aligned}
W_t &= A_t F_N(K_t, N_t) && (1)\\
q_t &= 1 && (2)\\
q_t &= \mathbb{E}_t \left[M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta) q_{t+1} \right] \right] && (3)\\
Q_t &= \mathbb{E}_t M_{t,t+1} && (4)
\end{aligned}
$$

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- Optimality conditions 2-3, together with definition of $M_{t,t+1}$, implies

$$U'(C_t) = \beta E_t (R^r_{t+1} + (1-\delta)) U'(C_{t+1})$$

  where $R^r_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1})$ - which we recognize!

- Note: Equation (4) is satisfied whenever households' are optimizing, what does this mean? Miller-Modgliani (AER 1963): the capital structure of the firm is irrelevant if the household can trade in the same assets as the firm

Equivalence

- Combining this with household optimaility conditions and resource constraints, the equilibrium is characterized by

$$\begin{aligned}
\text{HH intertemporal optimality:} \quad & U'(C_t) = \beta \frac{1}{Q_t} E_t U'(C_{t+1}) \\
\text{HH intratemporal optimality:} \quad & U'(C_t) W_t = V'(N_t) \\
\text{Firm optimality 1:} \quad & U'(C_t) = \beta E_t \left[(R^r_{t+1} + (1-\delta)) U'(C_{t+1}) \right] \\
\text{Resource constraint:} \quad & C_t + I_t = A_t F(K_t, N_t) \\
\text{Producation function:} \quad & Y_t = A_t F(K_t, N_t) \\
\text{Capital LOM:} \quad & K_{t+1} = (1-\delta) K_t + I_t \\
\text{Firm optimality 2:} \quad & R^r_t = A_t F_k(K_t, N_t) \\
\text{Firm optimality 3:} \quad & W_t = A_t F_n(K_t, N_t) \\
\text{TFP process:} \quad & A_t = A_{t-1}^{\rho_a} exp(\epsilon_t)
\end{aligned}$$

- Which, apart, from first equation is exactly the same set of equations characterizing the RBC model with household ownership of capital
  - One more unkown Q_t - one more equation

Firm optimality conditions: interpretation I

- Let's go back to the investment decision - firm optimality conditions:

$$
\begin{aligned}
W_t &= A_t F_N(K_t, N_t) \\
q_t &= 1 \\
q_t &= \mathbb{E}_t \left[M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta) q_{t+1} \right] \right] \\
Q_t &= \mathbb{E}_t M_{t,t+1}
\end{aligned}
$$

- How to interpret q_t?
  - Lagrange multiplier = shadow value of relaxing constraint = shadow value of having one more unit of installed capital K_{t+1}
  - Supposed the firm has optimized and then, out of the sky, it gains some extra ∂K_{t+1} - what will it do?
  - Optimal choice of K_{t+1} has not changed, so it just lowers investment by ∂K_{t+1} and uses proceeds to increase current profits
  - Along the optimal path, we therefore have $q_t = \frac{\partial V_t}{\partial K_{t+1}}$ (recall envelope theorem)
  - Implication 1: q_t is the price of capital in terms of goods - why?
  - Implication 2: $q_t = 1$ - why?

Firm optimality conditions: interpretation II

- Firm optimality conditions:

$$
\begin{aligned}
W_t &= A_t F_N(K_t, N_t) \\
q_t &= 1 \\
q_t &= \mathbb{E}_t \left[M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta) q_{t+1} \right] \right] \\
Q_t &= \mathbb{E}_t M_{t,t+1}
\end{aligned}
$$

- Optimality condition 3 can be iterated forward:

$$
q_t = \frac{1}{1-\delta} \mathbb{E}_t \sum_{s=1}^{\infty} M_{t,t+s} (1-\delta)^s \left(A_{t+s} F_K(K_{t+s}, N_{t+s}) \right)
$$

  - RHS = marginal benefit of having one more unit of installed capital K_{t+1}
  - LHS = price of having one more unit of installed capital

Firm optimality conditions: interpretation III

- Firm optimality conditions:

$$\begin{aligned} W_t &= A_t F_N(K_t, N_t) \\ q_t &= 1 \\ q_t &= \mathbb{E}_t \left[M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) + (1-\delta) q_{t+1} \right] \right] \\ Q_t &= \mathbb{E}_t M_{t,t+1} \end{aligned}$$

- With $q_t = 1$, optimality condition 3 can be rewritten

$$1 = (1-\delta)\mathbb{E}_t M_{t,t+1} + \mathbb{E}_t M_{t,t+1} \mathbb{E}_t MPK_{t+1} + CoV(MPK_{t+1}, M_{t,t+1})$$

where $MPK_{t+1} = A_{t+1} F_K(K_{t+1}, N_{t+1})$

- To a first order, we thus have

$$r_t + \delta = E_t A_{t+1} F_K(K_{t+1}, N_{t+1})$$

where

$$r_t = R_t - 1 = \frac{1}{Q_t} - 1$$

The neoclassical theory of investment

- The vanilla RBC model embeds the neoclassical theory of investment
  - Pioneered by Jorgenson (AER 1963); Hall-Jorgenson (AER 1967)
- Key idea: firms should invest until $E_t MPK_{t+1}$ equals user cost
- User cost $=$ alternative cost r_t $+$ direct cost δ
- Leaving the first-order approximation, the user cost also reflects investment risk (measured in terms of the covariance between the payoff and the discount factor)
- Very intuitive, but this hinges on that the real price of capital goods is always 1

Equilibrium characterization including q_t

- Adding the price of capital q to the model means that we split the firm optimality condition 1 into two pieces:

$$
\begin{aligned}
\text{HH intertemporal optimality:} \quad & U'(C_t) = \beta \frac{1}{Q_t} E_t U'(C_{t+1}) \\
\text{HH intratemporal optimality:} \quad & U'(C_t) W_t = V'(N_t) \\
\text{Firm optimality 1:} \quad & q_t = E_t \frac{\beta U'(C_{t+1})}{U'(C_t)} [R^r_{t+1} + (1-\delta) q_{t+1}] \\
\text{Firm optimality 2:} \quad & q_t = 1 \\
\text{Resource constraint:} \quad & C_t + I_t = A_t F(K_t, N_t) \\
\text{Producation function:} \quad & Y_t = A_t F(K_t, N_t) \\
\text{Capital LOM:} \quad & K_{t+1} = (1-\delta) K_t + I_t \\
\text{Firm optimality 3:} \quad & R^r_t = A_t F_k(K_t, N_t) \\
\text{Firm optimality 4:} \quad & W_t = A_t F_n(K_t, N_t) \\
\text{TFP process:} \quad & A_t = A_{t-1}^{\rho_a} exp(\epsilon_t)
\end{aligned}
$$

- Not very interesting, but allows better comparison to the next model that we introduce

Neoclassical theory vs. Q theory of investment

The Q theory of investment

- The neoclassical theory of investment comes with the prediction that the price of capital is constant
- This is very much at oods with the data
- A more reasonable theory of investment has
  - Fluctuations in the price of capital, and
  - that fluctuations in the price of capital matter for investment decisions
- Tobin (JMCB 1969): given fluctuations in the price of capital, a reasonable theory of investment would say: invest if

$$\frac{\text{Market value of firm capital}}{\text{Replacement cost of capital}} > 1$$

  - Th left-hand side ratio is called Tobin's Q
  - This idea has guided much empirical research on investment
- Note, in the notation of our firm problem:

$$\text{Tobin's Q} = \frac{V}{k} \quad \text{while} \quad q_t = \frac{\partial V_t}{\partial k_{t+1}}$$

- Refining Tobin's intuition: what ought to matter for an optimizing firm is the *marginal value of firm capital*, i.e, q_t

Operationalizing the Q theory of investment

- Some version of the (marginal) Q theory naturally comes out when adding investment costs to the firm problem
- Such costs are also very plausible - think about installing a new machine, building a new plant etc.
- Under some conditions of the investment cost function, the Q theory comes out exactly
- Suppose that the firm faces investment costs of the form

$$C = C(I_t, K_t)$$

- Q theory arises with the following assumptions
  1. $C(\cdot)$ is convex in investment size, i.e., $C_I(I_t, K_t) \geq 0$, $C_{II}(I_t, K_t) \geq 0$
  2. $C(\cdot)$ is homogeneous of degree 1
  3. $C(\delta K_t, K_t) = 0$
  4. $C_I(\delta K_t, K_t) = 0$
  5. $C_K(I_t, K_t) < 0$

Firm problem

- Popular cost function that satisfies these assumptions

$$C(I_t, K_t) = \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$$

- Consider a firm problem that faces such a cost function

$$\max_{N_t, I_t, K_{t+1}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} M_{0,t}\left(A_t F(K_t, N_t) - W_t N_t - I_t - C(I_t, K_t)\right)$$
$$\text{s.t.} \quad K_{t+1} \leq I_t + (1-\delta)K_t$$

- I've taken out debt financing B_{t+1} since it doesn't matter anyway

- Note: if $\phi = 0 \Rightarrow$ we're back to vanilla RBC

Firm optimality conditions

- Set up the Lagrangian, take the F.O.C. to find:

$$
\begin{aligned}
W_t &= A_t F_N(K_t, N_t) \\
q_t &= 1 + C_I(I_t, K_t) \\
q_t &= \mathbb{E}_t M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) - C_K(I_{t+1}, K_{t+1}) + (1-\delta) q_{t+1} \right]
\end{aligned}
$$

  where q_t is, again, the Lagrange multiplier on the firm constraint

- Observations:
  - $q_t \geq 1$ - why?
  - As before, we can iterate on third condition to find

$$
q_t = \frac{1}{1-\delta} \sum_{s=1}^{\infty} M_{t,t+s} (1-\delta)^s \left(A_{t+s} F_K(K_{t+s}, N_{t+s}) - C_K(I_{t+s}, K_{t+s}) \right)
$$

q and investment

- With our functional form $C(I_t, K_t) = \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$, optimality condition 2 becomes

$$\begin{aligned} q_t &= 1 + C_I(I_t, K_t) \\ &= 1 + \phi\left(\frac{I_t}{K_t} - \delta\right) \end{aligned}$$

or

$$\frac{I_t}{K_t} = \frac{1}{\phi}(q_t - 1) + \delta$$

- Predictions:
  1. investment rate > 1 if $q_t > 1$
  2. q_t is a sufficienct statistic for investment

Taking the Q-theory to the data

- In the data, it is easy to observe the *average* $q = \frac{V}{K}$
  - V could be stock market valuation of firm
  - K is the net worth on the firm balance sheet
- The model tells us we should relate investment to *marginal* $q = \frac{\partial V}{\partial K}$
- Hayashi (Ecmtra, 1982): if both $C(\cdot)$ and $F(\cdot)$ are homogeneous of degree 1, then average q = marginal q
  - Take-home exercise: show that this is true with our quadratic $C(\cdot)$-function and Cobb-Douglas $F(\cdot)$!
- Hayashi's theorem provides rationale for estimating regression

$$\frac{I_{it}}{K_{it}} = \alpha + \beta(\text{Average Q}_{it} - 1) + \sum \gamma_k X_{kit} + \epsilon_{it}$$

  using firm level micro data
- A few key papers: Summers (BPEA, 1981); Fazzari-Hubbard-Petersen (BPEA, 1988); Cummins-Hassett-Hubbard (BPEA 1994); Kaplan-Zingales (QJE 1997)

Cummins-Hassett-Hubbard (BPEA 1994): Compustat data 1962-1988

Table 3. Basic Investment Equations: Tax-Adjusted q Model[a]

Model feature	*OLS*		*GMM*		*OLS*[b]		*GMM*[b]	
Independent variable								
$Q_{i,t}$	0.025 (0.001)	0.019 (0.001)	0.019 (0.003)	0.015 (0.003)	0.040 (0.001)	0.028 (0.001)	0.057 (0.002)	0.044 (0.002)
Cash flow $(CF/K)_{i,t}$	. . .	0.164 (0.005)	. . .	0.154 (0.026)	. . .	0.193 (0.006)	. . .	0.344 (0.013)
Instrumental variables	. . .	. . .	$Q_{i,t-2,\ t-3}$ $(I/K)_{i,t-2,\ t-3}$ $(CF/K)_{i,t-2,\ t-3}$		. . .	. . .	$QT_{i,t}$, $Q_{i,t-2,\ t-3}$ $(I/K)_{i,t-2,\ t-3}$ $(CF/K)_{i,t-2,\ t-3}$	
Fixed firm effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed year effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.039	0.049	. . .	. . .	0.068	0.127	. . .	. . .
$\chi^2_{(n-p)}$ (p-value)	. . .	. . .	13.18 (0.022)	11.75 (0.019)	. . .	. . .	500.46 (7×10^{-105})	448.98 (8×10^{-95})
Number of observations	19,855	19,855	18,729	18,399	18,168	18,168	18,129	17,973

Source: Authors' calculations using Compustat data.

- Estimating the equation reduced-form tends to produce small coefficients (implying unreasonbly large adjustment costs)
- Treatment effect of firm cash flow is seemingly much larger

Cummins-Hassett-Hubbard (BPEA 1994): Compustat data 1962-1988

Table 4. Basic Investment Equations: Tax-Adjusted q Model (Focusing on Tax Variation)[a]

Model feature	OLS All years	OLS 1962	OLS 1972	OLS 1981	OLS 1986	GMM All years	GMM 1962	GMM 1972	GMM 1981	GMM 1986
Independent variable										
$QT_{i,t}$	0.083 (0.006)	0.554 (0.165)	0.198 (0.067)	0.299 (0.091)	0.178 (0.083)	0.063 (0.006)	0.585 (0.161)	0.136 (0.065)	0.262 (0.090)	0.245 (0.085)
Instrumental variables	...	...	...	...	...			$QT_{i,t}$ $Q_{i,t-2,\,t-3}$ $(I/K)_{i,t-2,\,t-3}$ $(CF/K)_{i,t-2,\,t-3}$		
Second differences	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed year effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$\bar{R}^2$	0.011	0.041	0.015	0.012	0.010	...	...	...	...	...
χ^2_6 (p-value)	...	...	...	...	...	523.32 (8×10^{-110})	6.31 (0.390)	13.40 (0.037)	32.60 (1×10^{-5})	27.63 (1×10^{-4})
Number of observations	18,168	267	572	861	892	17,632	266	555	860	890

Source: Authors' calculations using Compustat data.

- Using tax reforms with heterogeneous treatment effect as instrumental variable: estimates much more reasonable
- Still, financial variables, e.g., firm cash flow tend to show up as large and significant $\Rightarrow$ sufficient statistic hypothesis rejected
- This motivates introducing *financial frictions* in firm investment decisions

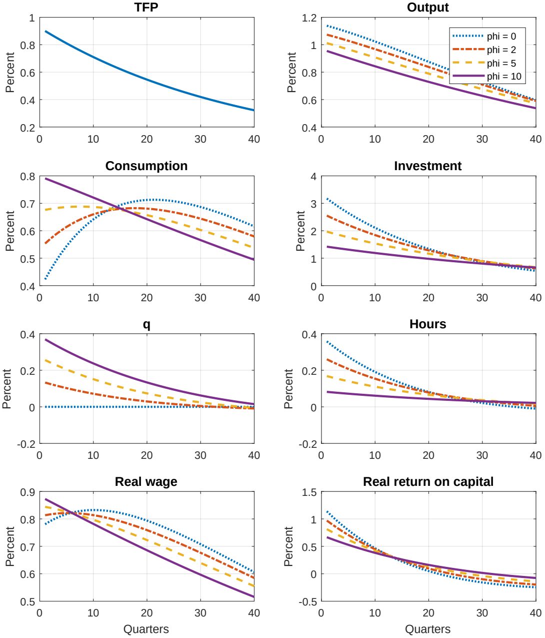
Integrating the Q theory in our RBC model

- Replacing the firm optimality conditions, and also the resource constraint in our equilibrium characterization, we have

$$
\begin{aligned}
\text{HH intertemporal optimality:} &\quad U'(C_t) = \beta \frac{1}{Q_t} E_t U'(C_{t+1}) \\
\text{HH intratemporal optimality:} &\quad U'(C_t) W_t = V'(N_t) \\
\text{Firm optimality 1:} &\quad q_t = E_t \frac{\beta U'(C_{t+1})}{U'(C_t)} \left[R^r_{t+1} + (1-\delta) q_{t+1} \right] \\
\text{Firm optimality 2:} &\quad q_t = 1 + C_I(I_t, K_t) \\
\text{Resource constraint:} &\quad C_t + I_t + C(I_t, K_t) = A_t F(K_t, N_t) \\
\text{Producation function:} &\quad Y_t = A_t F(K_t, N_t) \\
\text{Capital LOM:} &\quad K_{t+1} = (1-\delta) K_t + I_t \\
\text{Firm optimality 3:} &\quad R^r_t = A_t F_k(K_t, N_t) - C_K(I_t, K_t) \\
\text{Firm optimality 4:} &\quad W_t = A_t F_n(K_t, N_t) \\
\text{TFP process:} &\quad A_t = A_{t-1}^{\rho_a} exp(\epsilon_t)
\end{aligned}
$$

- $\Rightarrow$ Log-linearize and Dynare it

IRF to TFP shock



Mechanism

- TFP up $\Rightarrow$ q_t up $\Rightarrow$ investment up
- With adjustment costs, large investment jumps are especially costly
- Investment response therefore more smooth $\Rightarrow$ increases persistence
  - To get persistence of both investment and overall GDP right, you typically want to include inbestment adjustment costs
- Flip side: on impact, consumption jump larger, wage jump smaller $\Rightarrow$ labor supply response smaller

Comment: Non-convex adjustment costs

- Quadratic adjustment cost implies that investment dynamics is smooth
- However, it is clearly seen in micro data that investment is *lumpy*: sometimes you invest nothing, sometimes you invest a lot
- Lumpy investment dynamics follow from *non-convex adjustment costs*, e.g., fixed costs:

$$C(I_t, K_t) = \begin{cases} 0 & \text{if } I_t = \delta K_t \\ \zeta & \text{otherwise} \end{cases}$$

- Non-convex decision problems $\rightarrow$ optimum cannot be solved with F.O.C.s, we need a computer to characterize firm problem
- Models that seriously try to get micro-level dynamics right typically find that both convex and non-convex adjustment costs are needed, see, e.g., Ottonello-Winberry (Ecmtra 2020)
- Type of cost matters a lot for some macro applications (e.g. uncertainaty shocks, see Bloom Ecmtra 2007), little for others

Comment: Financial frictions

- Micro evidence: firm financial variables (like cash flow) predict investment
- Macro evidence: many severe crisis epsiodes linked to Financial shocks (e.g. the Great Depression and the Great Recession)
- Big literature on the effect of financial frictions for firm investment decisions
- Applied macro reseach intertwined with micro-theory research: Financial frictions always originate from an agency problem, i.e., for some reason, a creditor cannot trust that a debtor will repay his/her debt
- Canonical models: Bernanke-Gertler (AER 1989); Kiyotaki-Moore (JPE 1997)
- KM (1997) show that collaterized debt can overcome a problem of limited enforcement
- A reduced-form collateral constraint in our firm problem (with within-period debt):

$$
\begin{aligned}
\max_{N_t, I_t, K_{t+1}} \quad & E_O \sum_{t=0}^{\infty} M_{0,t} \left(A_t F(K_t, N_t) - W_t N_t - I_t - C(I_t, K_t) \right) \\
\text{s.t.} \quad & K_{t+1} \leq I_t + (1-\delta) K_t \\
& I_t \leq \xi q_t K_t
\end{aligned}
$$

Optimality

- Lagrangian

$$L = E_O \sum_{t=0}^{\infty} \quad M_{0,t}\Big[(A_t F(K_t, N_t) - W_t N_t - I_t - C(I_t, K_t))$$
$$+ q_t \left(I_t + (1-\delta)K_t - K_{t+1} \right) + \mu_t \left(\xi q_t K_t - I_t \right) \Big]$$

- F.O.C.

$$\begin{aligned} W_t &= A_t F_N(K_t, N_t) \\ q_t &= 1 + C_I(I_t, K_t) + \mu_t \\ q_t &= E_t M_{t,t+1} \left[A_{t+1} F_K(K_{t+1}, N_{t+1}) - C_K(I_{t+1}, K_{t+1}) + \left(\mu_{t+1}\xi + (1-\delta) \right) q_{t+1} \right] \end{aligned}$$

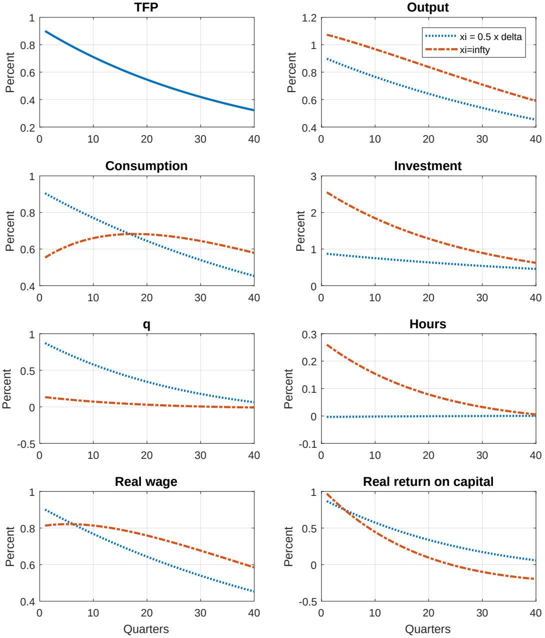
- Complementary slackness

$$\begin{aligned} \mu_t &\geq 0 \\ I_t &= \xi q_t K_t \ \text{ iff } \mu > 0 \end{aligned}$$

  - If constraint lax $\Rightarrow \mu_t = 0$, back to standard model
  - It constraint binds $\Rightarrow I_t = \xi q_t K_t$ and $\mu_t > 0$

- One can show that (see your problem set): constraint binds in steady state iff $\xi < \delta$

IRF to TFP shock with $\phi = 2$



Summing up

- Without any market frictions - capital ownership doesn't matter for business cycle dynamics
- Vanilla RBC predicts a constant price of capital
- Investment adjustment costs predicts procyclical price of capital, and a "Q theory of invesment"
- This closes our investigation of the RBC approach to studying business cycle dynamics
- Next up: Nominal rigidities and monetary policy (the "New-Keynesian" model)

RBC: what have we not covered?

- Other shocks
  - Investment-specific shocks: see, e.g., Greenwood-Hersowitz-Krusell (EER 2000)
  - Uncertainty shocks: see, e.g., Bloom (Ecmtra 2007)
  - News shocks: see, e.g., Beaudry-Portier (AER 2006; Koslyk 2023)
- International business cycle models
  - see, e.g., Backus-Kehoe-Kydland (JPE 1992); Baxter (HBintecon 1995); Schmitt-Grohe-Uribe (Book 2017)