A couple of practice questions for the final

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Question 1: Consider the following economy

- Each period a continuum of agents of mass $(1 - \delta)$ is born

- These agents discount at rate β, and survive from one period to the next with constant probability δ

- Each agent born in period t receives an endowment of labor income y_t in the first period of life. From the second period of life on, agents are retired and consume only out of asset income.

- At each age, agents can buy stocks (ex dividend) at price p_t. The total mass of shares is assumed to be equal to one.

- Stocks provide a stream of dividends d_t

- y_t and d_t follow arbitrary stochastic processes

- There are perfect annuity markets that redistribute the shareholdings of those who die to existing shareholders, in proportion to the amount of shares they hold - thus if an agent of age a at date t purchases x shares and survives to the next period, $s_{t+1}^{a+1} = \frac{x}{\delta}$

- Adopt the notation convention that agents are born at age $a = 1$, let c_t^a denote the consumption at date t of an agent of age a, let s_t^a denote the share holdings at the start of date t of an agent of age a (chosen at $t-1$). Note that $s_t^1 = 0$.

1. Define an agent's maximization problem in this economy

2. Define a sequence of markets equilibrium

   Suppose the period utility function is $u(c) = \ln(c)$. Conjecture that every agent in the economy optimally consumes a fraction $(1 - \beta\delta)$ of the total present value of their wealth in each period, i.e.,

   $$
   \begin{aligned}
   c_t^1 &= (1 - \beta\delta)y_t \\
   c_t^a &= (1 - \beta\delta)(s_t^a(p_t + d_t)) \qquad a > 1
   \end{aligned}
   $$

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3. Given this conjecture, solve for the price of stocks, p_t. Given some intuition for this expression

4. Verify that given this price, the conjectured decision rule for consumption satisfies all agents' first order conditions for stock purchases

Question 2: Consider a two-country version of the standard stochastic real business cycle economy. The two economies produce the same good, and the representative agent on each economy has the same preferences. The only difference between the two countries is that firms receive country-specific shocks to TFP. The production technology in country i is given by

$$I_i = z_i K_i^\theta N_i^{1-\theta}$$

$$K_{t+1} = (1 - \delta)K_t + I_t - C_t$$

Suppose that output is initially in the form of new capital (I_t), and that new or existing capital can be freely converted into consumption. This is just a way to say that there are two sorts of things in this economy: hours and capital. Hours can be used to produce more capital (via production) or to produce utility (as leisure). Capital can be used to produce more capital (via production) or to produce utility (as consumption). 'Investment' and 'Consumption' are labels used for capital when it is (1) the output of the production technology, and (2) the input to the utility technology.

Assume preferences for the representative agent in country i have the form

$$E\left[\sum_{t=0}^{\infty} \beta^t u(c_{i,t}, n_{i,t})\right]$$

$$u(c_i, n_i) = v_1(c_i) + v_2(1 - n_i)$$

We won't make any specific assumptions on the joint process for z_1 and z_2, other than to assume that the two shocks are imperfectly correlated, and that the process is symmetric.

Consider the following alternative versions of the model.

1. Neither capital nor labor can be shifted between countries

2. Capital is internationally mobile, but labor is not

3. Labor is mobile, but capital is not

4. Capital and labor are both mobile internationally

In each case, if a factor is mobile, assume it can be freely reallocated across countries after observing (z_{it}, z_{2t}) within the period. In each case, assume that there are no restrictions on the set of financial assets that may be traded internationally, and that the world is symmetric at the start of time ($K_{0,1} = K_{0,2}, z_{0,1} = z_{0,2}$). Thus we can in principle think about allocations being determined by a planner.

1. Is it really sensible to talk about markets being complete in all of these economies?

2. For each economy, characterize as many properties of the equilibrium as you can, without making further assumptions.