Macroeconomics II, Lecture X: Diamond-Mortensen-Pissarides: Efficiency and Dynamics

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Recap and motivation

- DMP: A GE theory of unemployment
  - emphasizes vacancy creation and job destruction as key mechanisms
- Defining elements: Matching function + Wage-setting rule + Free-entry condition
- Last lecutre: steady state equilibrium and comparative statics
- Today: welfare and dynamics
- We'll focus on the vanilla DMP model with exogenous separations, but all results, if not highlighted, extends to the endogenous-separation model

Recap: equilibrium charachterization with exogenous separations

- Summary: the equilibrium steady state $\{w, \theta, u, v\}$ is characterized by

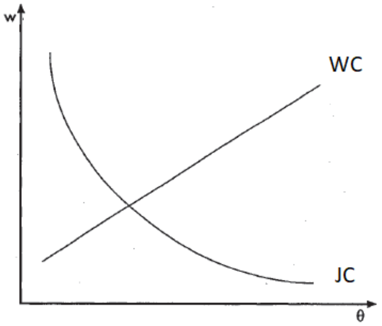
$$\begin{aligned}
\text{JC:} \quad & w = y - \frac{c(r+\sigma)}{\lambda_v(\theta)} \\
\text{WC:} \quad & w = (1-\gamma)b + \gamma(y + c\theta) \\
\text{BC:} \quad & u = \frac{\sigma}{\sigma + \lambda_u(\theta)} \\
\text{TD:} \quad & v = \theta u
\end{aligned}$$

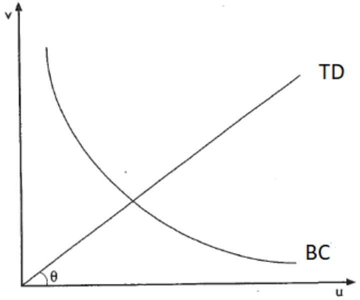
- Note: the system is *block recursive*, we can solve for θ through combining the first 2 equations

$$\frac{c}{\lambda_v(\theta)} = \frac{(1-\gamma)(y-b)}{r + \sigma + \lambda_u(\theta)\gamma}$$

- Given θ, we can solve for u and v from the last two equations

Recap: graphical view of equilibrium





Agenda

1. Efficiency
2. Dynamics
   - Steady-state approximation of dynamic equilibrium
   - Unemployment volatility puzzle
3. Summary and discussion

Efficiency

Efficiency

- Is the DMP equilibrium efficient?
  - Clearly not (in the usual sense)
  - An unconstrained social planner would ignore the search frictions and allocate all workers to a firm at all points in time
- Is the DMP equilibrium **constrained efficient**?
  - Constrained efficiency: would a social planner, who can pick the choices of all agents but still face the same frictions, make different choices than the agents in the decentralized equilibrium?
- Remember: the only choice in DMP is whether to post a vacany or not
- Reformulated question: would the creation of more vacancies increase or decrease welfare?
- In general, there are two externalities from a firm creating an additional vacancy
  - Positive *thick market externality*: workers will find jobs faster
  - Negative *congestion externality*: other firms will find workers slower
  - Note: these externalities arise due to the matching function

Social planner problem

- To find out, we need to compare the social planner solution to the decentralized solution

- Social planner problem (with equal Pareto weights):

$$\begin{aligned} \max_{v} \quad & \int_0^\infty e^{-rt}\left((1-u)y + ub - vc\right) dt \\ \text{s.t.} \quad & \dot{u} = \sigma(1-u) - \lambda_u\left(\frac{v}{u}\right)u \end{aligned}$$

  with u as the state variable and v as the control

- We can reformulate as

$$\begin{aligned} \max_{\theta} \quad & \int_0^\infty e^{-rt}\left((1-u)y + ub - \theta uc\right) dt \\ \text{s.t.} \quad & \dot{u} = \sigma(1-u) - \lambda_u(\theta)u \end{aligned}$$

  with u as the state variable and θ as the control

Social planner problem: solution

- Introduce the co-state μ and form the Hamiltonian:

$$H(u,\theta,\mu,t) = e^{-rt}\left[(y(1-u) + bu - \theta uc) + \mu(\sigma(1-u) - \lambda_u(\theta)u)\right]$$

- From Pontryagin', we have the optimaility conditions:

$$\begin{aligned} \frac{dH}{du} &= -\dot{\mu} \\ \frac{dH}{d\theta} &= 0 \end{aligned}$$

and thus

$$\begin{aligned} -y + b - \theta c - \mu(\sigma + \lambda_u(\theta)) &= -\dot{\mu} \\ -c - \mu\lambda_u'(\theta) &= 0 \end{aligned}$$

- $\mu =$ the value of having one more unemployed. $-\mu =$ equals the expected cost of matching that unemployed with a new vacancy:

$$-\mu = \frac{c}{\lambda_u'(\theta)}$$

Social planner problem solution II

- Evaluate in steady state $\dot{\theta} = 0$ and work out the algebra to find

$$\frac{c}{\lambda_v(\theta)} = \frac{(1 - \epsilon_{m,u}(\theta))(y - b)}{r + \sigma + \lambda_u(\theta)\epsilon_{m,u}(\theta)}$$

where

$$\epsilon_{m,u} = \frac{dM(u, v)}{du} \frac{u}{M(u, v)} = 1 - \frac{\theta \lambda_u'(\theta)}{\lambda_u(\theta)}$$

- Compare to decentralized solution

$$\frac{c}{\lambda_v(\theta)} = \frac{(1 - \gamma)(y - b)}{r + \sigma + \lambda_u(\theta)\gamma}$$

Is the decentralized equilibrium efficient?

- Hosios (REstud 1990): the decentralized equilibrium is efficient iff $\gamma = \epsilon_{m,u}(\theta)$!
- If $\epsilon_{m,u} < \gamma$, worker get too large share of the surplus leading to too little vacancy creation (thick market externality dominates and unemployment is too high)
- If $\epsilon_{m,u} > \gamma$, firms get too large share of the surplus and spend too much costly resources on creating vacancies (the congestion externality dominates and unemployment is too low)
- If $\epsilon_{m,u} = \gamma$, the social value of creating an additional vacancy equals the private gain to firm

Effiency: a comment

- In this model, there is no reason why we should expect $\epsilon_{m,u} = \gamma$
- Random matching implies generically inefficient equilibria, opening the room for welfare-improving government interventions
- Primary motivation for models with directed search: introduces competitive force that ensures an efficient benchmark (Moen, JPE 1997; Menzio and Shi, JPE 2011)

Dynamics

The question

- So far, our analysis has only concerned the steady state
- **Can DMP explain the cyclicality of unemployment?**
- This question refers to a dynamic response of unemployment rate to some underlying shock
- A natural candidate: shocks to productivity y

Strategy

- Strategy: Calibrate model primitives to match log-run moments, feed in shocks that fit detrended processes of TFP and separation rates, evalutate unemployment dynamics
  - Just as you would evaluate an RBC or a New-Keynesian model of business cycle dynamics in output
- A "proper" evaluation requires us to
  1. specify a process of the aggregate (productivty) shock, e.g, they hit with an arrival rate z
  2. describe how the value functions change with the aggregate state, e.g.,
  $$rU_y = b + \lambda_u(\theta_y)(W_y - U_y) + zE_{y'|y}(U_{y'} - U_y)$$
  3. solve the model numerically
- However, insights about the dynamics of DMP can be gained by simple comparative statics
- This is because the dynamics in DMP (and the data) is fast: with realistical job-finding rates, the model converges almost instantaneously to steady state

Fast dynamics: a steady-state approximation

- Consider the formula for steady state unemployment:

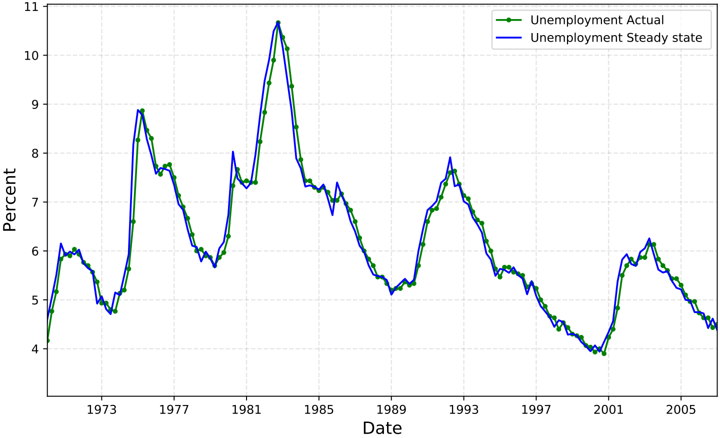
$$u_{ss} = \frac{\sigma}{\sigma + \lambda_{u,t}}$$

- At any given t, there exist a steady state unemployment rate $s_{ss,t}$ associated with the current flow rates

$$u_{ss,t} = \frac{\sigma_t}{\sigma_t + \lambda_{u,t}}$$

- How does $u_{ss,t}$ compare with actual unemployment u_t?

u_{ss} vs actual u_t



Own calculcations based on quarterly data. Unemployment data: FRED. Transition rate data: Shimer's webiste

Fast dynamics II

- What's going on?
- Consider the law of motion for unemployment

$$\dot{u} = \sigma(1 - u_t) - \lambda_u(\theta)u_t$$

- This is a first order linear ordinary differential equation
- General solution:

$$u_t = Ce^{-(\sigma+\lambda_u(\theta))t} + \frac{\sigma}{\sigma + \lambda_u(\theta)}$$

for some constant C

- For any positive σ, λ_u: $u_t \to u_{ss} = \frac{\sigma}{\sigma+\lambda_u(\theta)}$
- Larger $\sigma, \lambda_u \to$ faster convergence
- What is the half-life of **out-of-steady-state unemployment** ? i.e., which T solves the following equation?

$$\frac{u_T - u_{ss}}{u_0 - u_{ss}} = \frac{1}{2}$$

Fast dynamics III

- Steady state unemployment:

$$u_{ss} = \frac{\sigma}{\sigma + \lambda_u(\theta)}$$

- Half life given by

$$\begin{aligned} \frac{u_T - u_{ss}}{u_0 - u_{ss}} &= \frac{1}{2} \\ \frac{Ce^{-(\sigma+\lambda_u(\theta))T} + \frac{\sigma}{\sigma+\lambda_u(\theta)} - \frac{\sigma}{\sigma+\lambda_u(\theta)}}{Ce^{-(\sigma+\lambda_u(\theta))*0} + \frac{\sigma}{\sigma+\lambda_u(\theta)} - \frac{\sigma}{\sigma+\lambda_u(\theta)}} &= \frac{1}{2} \\ e^{-(\sigma+\lambda_u(\theta))T} &= \frac{1}{2} \end{aligned}$$

- That is, $T = -\frac{1}{\sigma+\lambda_u} \log \frac{1}{2}$

- US monthly transitition rates $\sigma \approx 0.03, \lambda_u \approx 0.6 \Rightarrow T = 1.1$ months

- Since convergence is so fast: $u_{ss}(\sigma_t, \lambda_{ut}) \approx u_t$

Solving dynamic DMP without simulations?

- Lesson: given sufficiently high transition rates, the labor market converges to steady state very fast

- Implication: Given that DMP is calibrated to match these high transition rates, the DMP equilibrium converges to steady state very fast

- Consider a dynamic DMP model in which a simulated sequence of productivty $\{y_0, y_1, y_2, ...\}$ maps into a sequence of unemployment $\{u_0, u_1, u_2, ...\}$

- Since convergence to steady state is almost immediate, we can approximate the unemployment sequence with $\{u_{ss}(y_0), u_{ss}(y_1), u_{ss}(y_2), ...\}$

- That is, we can simply do comparative statics of steady state unemployment u w.r.t. to y!

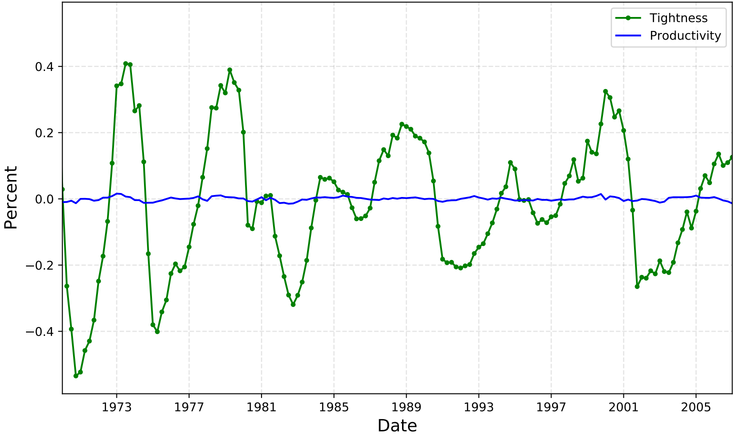
Recap: steady state equilibrium

- Recall: the steady state equilibrium is given by

$$\frac{c}{\lambda_v(\theta)} = \frac{(1-\gamma)(y-b)}{r+\sigma+\lambda_u(\theta)\gamma}$$
$$u = \frac{\sigma}{\sigma+\lambda_u(\theta)}$$

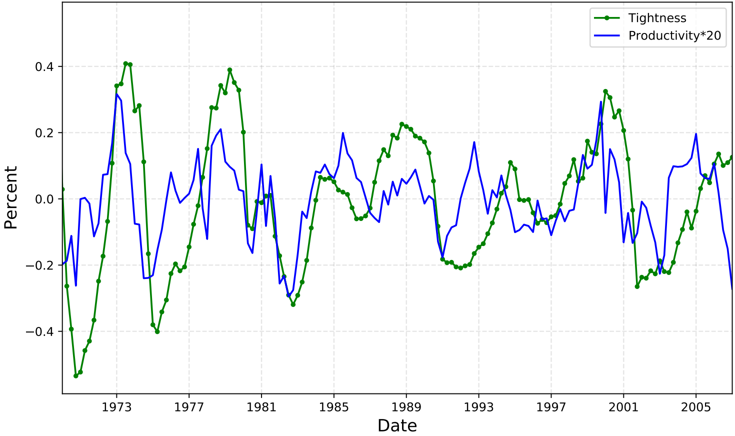
- Given that the matching function is (approximately) correct, the model response of u to y shock matches the data if the elasticity of θ w.r.t. y matches the data

Tightness-productivity elasticity in the data



Own calculations using detrended quarterly data using hp-filter. OECD Labor productivity measure downloaded from FRED. Tightness constructed using BLS unemployment from FRED and vacancy data (Help-wanted index) from Regis Barnichon's website.

Tightness-productivity elasticity in the data, rescaled



Own calculations using detrended quarterly data using hp-filter. OECD Labor productivity measure downloaded from FRED. Tightness constructed using BLS unemployment from FRED and vacancy data (Help-wanted index) from Regis Barnichon's website.

Can DMP explain unemployment volatility? Shimer (AER 2005)

- Empirical elasticity $\epsilon_{\theta,y} \approx 20$
- DMP elasiticty $\epsilon_{\theta,y}$?
- Job creation curve in $\{y, \theta\}$-space (using $\lambda_u(\theta) = \theta\lambda_v(\theta)$):

$$\frac{c}{\lambda_v(\theta)} = \frac{(1-\gamma)(y-b)}{r+\sigma+\lambda_u(\theta)\gamma}$$

- Total differentiation to find (Do on whiteboard):

$$\epsilon_{\theta,y} \equiv \frac{y}{\theta}\frac{\partial\theta}{\partial y} = \frac{y}{y-b}\frac{r+\sigma+\gamma\lambda_u(\theta)}{(r+\sigma)(1-\epsilon_{\lambda_u,\theta})+\gamma\lambda_u(\theta)}$$

where $\epsilon_{\lambda_u,\theta} \equiv \frac{\theta}{\lambda_u(\theta)}\lambda_u'(\theta)$ is the elasticity of $\lambda_u(\theta)$ w.r.t. θ

Can DMP explain unemployment volatility? Shimer (AER 2005)

$$\epsilon_{\theta,y} = \frac{y}{y-b}\frac{r+\sigma+\gamma\lambda_u(\theta)}{(r+\sigma)(1-\epsilon_{\lambda_u,\theta})+\gamma\lambda_u(\theta)}$$

- Shimer's (standard) calibration, quarterly frequency:
  - Normalize $y = 1$
  - Take outside estimates $\sigma = 0.1, r = 0.012, b = 0.4$
  - Estimate cobb-Douglass matching function with $A = 1.34, \alpha = 0.72$, $\epsilon_{\lambda_u,\theta} = 1 - \alpha$
  - Set $c = 0.2$ to match steady state tightness θ

- If $\gamma = \alpha$ (Hosios condition): $\lambda_u(\theta) = 2.13$ and

$$\epsilon_{\theta,y} = \frac{y}{y-b} \times \frac{r+\sigma+\alpha\lambda_u(\theta)}{(r+\sigma)(1-\epsilon_{\lambda_u,\theta})+\alpha\lambda_u(\theta)} = 1.7$$

- If $\gamma = 0$:

$$\epsilon_{\theta,y} = \frac{y}{y-b} \times \frac{1}{1-\epsilon_{\lambda_u,\theta}} = 3.5$$

- The model comes no way near the data

Shimer (AER 2005): intuition

- Why does DMP fail in generating unemployment volatility?

- Shimer suggests intuition:
  - $y \uparrow \Rightarrow$ Value of vacancy $\uparrow$
  - Free entry and $v \uparrow \Rightarrow$ Tightness $\uparrow$
  - $\lambda_u(\theta) \uparrow \Rightarrow$ Value of unemployment $\uparrow$
  - Worker's outside option $\uparrow \Rightarrow$ Wages $\uparrow$
  - Value of vacancy $\downarrow \Rightarrow$ Equilibrium response small

- Summarizing: The equilibrium response of wages via Nash bargaining inhibits firms' incentive to create vacancies

The Shimer puzzle

- Result: a reasonable calibration of the basic DMP model cannot explain observed unemployment volatility

- What about endogenous separations? Same problem, and actually creates another one:
  - Consider a positive separation shock (as implied by a negative productivity shock)
  - Higher u, but also lower θ
  - $\Rightarrow$ induces more vacancy creation
  - $\Rightarrow$ positive correlation between vacancies and unemployment!

- This is not quite correct, as we will see soon

The Shimer puzzle: resolutions

- Hall (AER 2005): Ad hoc wage rigidity
- Hagedorn-Manovskii (AER 2008): alternative calibration
- Hall (AER 2017): discount factor shocks
- Coles-Kelishomi (AEJmacro 2018): sluggish vacancy creation

The Shimer puzzle: resolutions

- Hall (AER 2005): Ad hoc wage rigidity
- **Hagedorn-Manovskii (AER 2008): alternative calibration**
- Hall (AER 2017): discount factor shocks
- **Coles-Kelishomi (AEJmacro 2018): separation shocks and sluggish vacancy creation**

Hagedorn-Manovskii (2008)

- Hagedorn and Manovskii argue: the previous literature have calibrated the DMP model wrongly; if you do it right, there is no puzzle
- Again, tightness-productivity elasticity with Nash bargaining:

$$\epsilon_{\theta,y} = \frac{y}{y-b}\frac{r+\sigma+\gamma\lambda_u(\theta)}{(r+\sigma)(1-\epsilon_{\lambda_u,\theta})+\gamma\lambda_u(\theta)}$$

- If b is high relative to y, the model can generate substantial unemployment fluctations
- Why is b the key parameter? Let's investigate

Hagedorn and Manovskii's argument

- Intuition: To generate a strong vacancy creation response to a productivity shock, two criteria need to be satisfied
  1. The productivity increase must not be fully absorbed by an increase in wages
  2. The initial level of profits need to be small

- Why does the initial level of profits matter?
  - The elasticity of vacancy creation w.r.t. to productivity creation depends on the elasticity of profits w.r.t. productivity
  - Suppose wages are fixed and study a 1 percent productivity increase; $y_0 = 1 \rightarrow y_1 = 1.01$
  - Suppose $w = 0.99$:

$$\epsilon_{\pi,y} = 100 * \frac{(1.01 - 0.99) - (1 - 0.99)}{(1 - 0.99)} = 100\%$$

  - Suppose $w = 0.01$:

$$\epsilon_{\pi,y} = 100 * \frac{(1.01 - 0.01) - (1 - 0.01)}{1 - 0.01} = 1\%$$

Hagedorn and Manovskii's argument II

- When is initial level of profits (wages) small (large) with Nash Bargaining?

- Look at wage curve:

$$\begin{aligned} w &= (1-\gamma)b + \gamma(y + c\theta) \\ &= b + \gamma(y-b) + \gamma c\theta \end{aligned}$$

  Either γ or b must be high

- Why then was it only b that seemed to have a large impact on tightness-productivity elastcity?

$$\epsilon_{\theta,y} = \frac{y}{y-b} \frac{r+\sigma+\gamma\lambda_u(\theta)}{(r+\sigma)(1-\epsilon_{\lambda_u,\theta}) + \gamma\lambda_u(\theta)}$$

- High γ also implies volatile wages w.r.t to $y \rightarrow$ level and volatility effect on vacancy elasticity approximately cancel each other out

Hagedorn and Manovskii's argument III

- Main question: is data supportive of high b?
- Shimer did not set his parameters to be consistent with the cyclicality of wages? Instead, he just set $b = 0.4$, $\gamma = \alpha$, and adjust the vacancy posting cost to match steady state tightness
- H&M stresses:
  - In the data, wages are moderately procyclical, $\epsilon_{w,y} = 0.45$
  - taking both labor and capital cost of vacancy posting into account, $c \approx 0.6$
- For a given b and vacancy post cost c, $\epsilon_{w,y}$ pins down γ
- H&M matches this moment, and find a much lower $\gamma = 0.05$
- With lower γ, b must be higher to match the same steady state tightness θ
- H&M find $b = 0.95$

Hagedorn and Manovskii's calibration

- With these numbers, the tightness-productvity elasticity becomes:

$$\epsilon_{\theta,y} = \frac{y}{y-b}\frac{r+\sigma+\gamma\lambda_u(\theta)}{(r+\sigma)(1-\epsilon_{\lambda_u,\theta})+\gamma\lambda_u(\theta)} = 20.6$$

- Does this make economic sense? H&M: yes!
- b is really the total utility flow of being unemployed: unemployment benefit value, leisure value, home production value etc.
- A larger model of household labor decisions will equate the marginal value of search for job with the marginal cost of giving up time for nonmarket activities.
- If the value of unemployment is not close to that of being employed, it is difficult to explain why unemployed people don't search harder for jobs
- This is a controversial argument, if the welfare loss of being unemployed is so small, why do we seemingly care so much about it?

Comment: wage levels and wage rigidity

- Shimer's intuition was the failure to match unemployment volatility reflected the flexibility of wages in the model

- H&M showed that this intuition is partly wrong, or at least, incomplete
  - Yes, the productivity increase must not be fully absorbed by an increase in wages in response to the shock
  - but, given that it is not fully absorbed, the important thing is whether the steady state level of profits/wages is high or small relative to productivity
  - Eloborated in Ljungqvist-Sargent (AER 2017)

- Nevertheless, much of the literature has focused on wage rigidity
  - Hall (AER 2005): DMP with an ad hoc fully rigid wage, shows that model matches unemployment cyclicality if steady state wage is high enough
  - Pissarides (Ecmtra 2009): it is only the wage of new hires that is allocative in the model, wage rigidity among imcumbents unimportant
  - Mixed evidence on the extent of wage rigidity for new hires in the literature, see e.g., Kudlyak (JME 2014), Grigsby-Hurst-Yildirmaz (AER 2021), Hazell-Taska (AER 2024)
  - Carlsson-Westermark (AEJmacro 2022): with endogeneous separations, rigidity among incumbents also matters

Coles-Kelishomi (2018)

- CKs argument: Unemployment variability can be explained with separation shocks if relaxing free entry
- Free entry is a reasonable in the long run, but in the short run, there are all kinds of barriers
- Extend the vanilla DMP model with stochastic entry costs κ

$$\kappa \sim H(\cdot)$$

- $\Rightarrow$ all firms drawing $\kappa < V_t$ create a vacancy. Vacancy creation rate: $\iota_t = H(V_t)$
- With $H(x) = x^{\xi}$, vacancy creation ι_t has a constant elasticity w.r.t. vacancy values:

$$\iota_t = V_t^{\xi}$$

- Note: Free entry means $\xi \to \infty$

Coles-Kelishomi (2018)

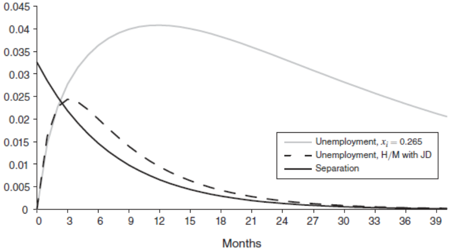


FIGURE 2. IMPULSE RESPONSE OF UNEMPLOYMENT TO A SEPARATION SHOCK

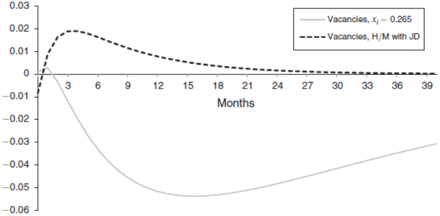


FIGURE 3. IMPULSE RESPONSE OF VACANCIES TO A SEPARATION SHOCK

Coles-Kelishomi (2018)

- With sluggish vacancy creation, newly separated deplete the current vacancy stock
  - Negative correlation between vacancy stock and unemployment, as in the data
  - $\Rightarrow$ Increase in separations depresses the job-finding rate, double effect on unemployment
  - Same logic with endogenous separations and productivity shocks
- Growing literature on implications, microfoundations and empirics:
  - Schoefer-Mercan-Sedlacek (AEJmacro 2023): sluggish entry due to imperfect substitability
  - Engbom (2021): sluggish entry due to compositional effect over the cycle
  - Broer-Druedahl-Harmenberg-Öberg (2025): implications for unemployment-risk dynamics
  - Cederlöf-Engbom-Nybom-Öberg-Tuominen (in progress): evidence from local Swedish labor markets

Summing up

- DMP: A GE theory of unemployment levels and dynamics
  - emphasizes vacancy creation as the key economic mechanism
- Defining elements: Matching function + Wage-setting rule + Free-entry condition
- Generates reasonable predictions, explored by growing empirical literature
- Key question: can DMP explain unemployment volatility?
- Maybe, but still not fully understood

Search models: what we have not covered

- The theory of money
  - Goods markets are charachterized by the absence of *double coincidence of wants*
  - Search process without mediating transaction technology is very costly
  - Useless assets, such as fiat money, can have positive value beacuse it mediates trade
  - See Williamson and Wright (Handbook ME 2010)

- Liquidity in financial markets
  - Some finanical markets are close to the Walrasian ideal. Others, such as the market for mortage-backed securities, operate with *over-the-counter* trading
  - Search theory charachterize volume of trading, bid-ask spreads, how the supply of buyers might collapse during crises
  - See Lagos and Rocheteau (Ecmtra 2009)

- Family economics
  - Marriage is a matching process under costly search
  - See Shimer and Smith (Ecmtra 2000)

Search models: what we have not covered: Epidemilogy

- The baseline model of disease spread is a random matching model - the SIR model
- Given a number of Susceptibles S and a number of infected I, assume that the number of newly infected is given by matching function $M = \beta SI$
- Law of motions

$$\begin{aligned} \dot{S} &= -\beta SI \\ \dot{I} &= \beta SI - \gamma I \\ \dot{R} &= \gamma I \end{aligned}$$

  where R is the number of dead/recovered

- SIR offers a way to forecast disease spread and study lockdown policies
- In epi literature, there is little treatment of optimizing behaviour and incentives
  - In other words, β is taken as a parameter and not an endogeneous variable
- Eichenbaum-Rebelo-Trabant (RFS 2021): $\beta = \beta(C_t, H_t)$, integrate SIR with standard RBC economy
  - A lot of papers currently extendning this analysis in many interesting ways