Notes

Aug 2022 Q3: How are two steady states possible?

In standard DMP, the wage curve is

$$w = (1-\gamma)b + \gamma(y + c\theta)$$

With additive labor taxes, we will have

$$w - \tau = (1-\gamma)b + \gamma(y - \tau + c\theta)$$

or

$$w = (1-\gamma)(b-\tau) + \gamma(y + c\theta)$$

$\frac{dw}{d\tau} < 0$ as taxes decrease match surplus

To get a wage curve in w, θ-space, use

- Balanced budget: $\tau = \tau(u)$ with $\tau' > 0$
- Steady state u + Beverigde: $u = u(\theta)$

Then compute the sign of w, w', w'' to find that $w(\theta)$ is increasing and convex, and also $\lim_{\theta\to 0} w(\theta) = \infty$ and $\lim_{\theta\to\infty} w(\theta) = \infty$.

Intuition: $\theta \to 0$, then $u \to 1$, then $\tau \to \infty$

Draw curve $\Rightarrow$ either 0, or two steady states

March 2022 Q2: How to use guess-and-verify?

With stochastic wage growth, the Bellman equation for employed is

$$rW(w) = w + \sigma(U - W(w)) + \lambda_w \int_{\mathcal{E}} (W(w(1+\epsilon)) - W(w))dG(\epsilon) \tag{1}$$

Guessing $W(w) = kw + m$, we have

$$(r + \sigma + \lambda_w)(kw + m) = w + \sigma U + \lambda_w \int_{\mathcal{E}} (kw(1 + \epsilon) + m) dG(\epsilon) \tag{2}$$

or

$$(r + \sigma + \lambda_w)(kw + m) = w + \sigma U + \lambda_w kw \left(1 + \int_{\mathcal{E}} \epsilon dG(\epsilon)\right) + \lambda_w m \tag{3}$$

or

$$(r + \sigma + \lambda_w)(kw + m) = w + \sigma U + \lambda_w kw \left(1 + \bar{e}\right) + \lambda_w m \tag{4}$$

or

$$(r + \sigma - \lambda_w \bar{\epsilon})kw + (r + \sigma)m = w + \sigma U \tag{5}$$

and we see that $k = \frac{1}{r+\sigma-\lambda_w \bar{\epsilon}}$ and $m = \frac{\sigma U}{r+\sigma}$ ("method of undetermined coefficients")

March 2022 Q1: How to solve and sketch IRFs?

Here, we guess that $C_t = \alpha + \delta K_t + \gamma \epsilon_t$.

Solution strategy: resource constraint will imply one equation relating K_{t+1} to K_t and ϵ_t, Euler equation another.

Resource constraint:

$$C_t + K_{t+1} = K_t + \epsilon_t + K_t$$

With our guess

$$\begin{aligned} & \alpha + \delta K_t + \gamma \epsilon_t + K_{t+1} = 2K_t + \epsilon_t \\ \Rightarrow \quad & K_{t+1} = -\alpha + (2 - \delta)K_t + (1 - \gamma)\epsilon_t \end{aligned}$$

Similar analysis of the Euler equation gives us

$$\Rightarrow \quad K_{t+1} = \frac{(2\beta - 1)(1 + 2\theta\alpha)}{4\beta\theta\delta} + \frac{1}{2\beta}K_t + \frac{\gamma(1 - 2\beta\rho)}{2\beta\delta}\epsilon_t$$

For both equations to hold, the coefficients must be the same, which implies that the the parameters α, δ, γ can only take certain values (just as in the previous question)

Once you have solved for these parameters, you have found the full solution! One linear function relating K_{t+1} to K_t and ϵ_t, another linear function relating C_t to K_t and ϵ_t.

Without log-linearizing (the are already linear!), you can directly draw to path of K_t and C_t using these equations.

It's just a matter of investigating which coefficients are positive, and which are negative.

March 2022 Q4: How to solve for changes in aggregate welfare?

In previous questions, you should have found that $c_{i1} = c_1$ for all i, and $c_{i2} = c_w^g$ for all households that experience shock $1 + \epsilon$

Therefore

$$\begin{aligned} W &= \int_{i=0}^{1} \log(c_{i1}) + \beta \log(c_{i2}) di \\ &= \log(c_1) + \beta \left(\frac{1}{2} \log(c_2^g) + \frac{1}{2} \log(c_2^b) \right) \end{aligned}$$

We have that

$$\frac{\partial W}{\partial K} = \frac{1}{c_1} \frac{\partial c_{i1}}{\partial K} + \beta \frac{1}{2} \left[\frac{1}{c_2^g} \frac{\partial c_2^g}{\partial K} + \frac{1}{c_2^b} \frac{\partial c_2^b}{\partial K} \right]$$

Solving this question boils down to solving for $c_1, c_2^g, c_2^b, \frac{\partial c_{i1}}{\partial K}, \frac{\partial c_2^g}{\partial K}, \frac{\partial c_2^b}{\partial K}$ at the equilibrium allocation.

For this, you use the equations that must be satisfied in equilibrium. STOP

Euler equation tells you how c_1 relate to c_2^g, c_2^b

Budget constraints tell you $\frac{\partial c_{i1}}{\partial K}, \frac{\partial c_2^g}{\partial K}, \frac{\partial c_2^b}{\partial K}$ given $\frac{\partial r}{\partial K}, \frac{\partial w}{\partial K}$

Firm optimality tells you $\frac{\partial r}{\partial K}, \frac{\partial w}{\partial K}$

March 2022 Q4: How to solve for the Hosios condition?

Private flow cost of opening a vacancy is $-c$.

Private gain: $\lambda_v(\theta) * J = \lambda_v(\theta)(1 - \gamma)S$

Social cost: $-c$

Social gain: $\frac{\partial M}{\partial v} S$