Price dispersion and wage posting models

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Outline

1. Overview
2. Burdett, Mortensen (1998)
3. Postel-Vinay, Robin (2002)
4. Lise and Robin (2013) (very briefly)

Overview of the literature

Introduction

- empirical observation: wages differ across observationally similar workers
- typical Mincer regression
$$w_i = X_i'\beta + \varepsilon_i$$
X_i: education, gender, job tenure, industry etc
- most variation in w_i comes from ε_i: *residual wage inequality*
- explanations
  - personal or firm effects (ability, productivity, etc)
  - compensating differentials: wages compensate for non-pecuniary characteristics of jobs (location, risk, amenities, etc.)

Abowd, Kramarz and Margolis (ECMA, 1999)

- first paper to include both firm and worker effects (data available)
- use match employer-employee data of 1 million French workers and 500,000 firms
- estimate

$$y_{i,t} = \mu_y + (x_{it} - \mu_x)\,\beta + \theta_i + \Psi_{J(i,t)} + \varepsilon_{it}$$

- where
  - $y_{i,t}$ - log-income of worker i time t
  - μ_y, μ_x - unconditional means of x, y
  - θ_i personal fixed effect
  - $\Psi_{J(i,t)}$ firm fixed effect, $J(i,t)$ is the firm in which worker i works at time t
- person fixed effects are found to be the most important source of variation
  - without θ_i: explain only 30 % of total variation
  - with θ_i: explain 80 % of total variation

Search frictions and dispersion

- Can search frictions help us understand the origin of residual wage inequality/variation in person effects?

  1. search is a natural environment to study price dispersion
     - arbitrage opportunities are limited by the fact that traders cannot contact trading partners directly
     - the *law of one price* does not necessarily hold: identical workers may have different wages
  2. search frictions - heterogenous outcomes for ex-ante homogenous workers
     - some lucky workers may find a job early, climb the wage ladder progressively; others may experience multiple unemployment spells and remain at the bottom of the wage distribution
     - this heterogeneity may explain part of the residual wage inequality

Hornstein, Krusell and Violante (AER,2011)

- examine whether standard search-and-matching models can generate a sizeable wage dispersion
- compare the *mean-min* ratio of residual wages for different models
- empirical estimates range between $[1.7, 1.9]$
  - McCall search model: $Mm = 1.05$
  - allowing for risk aversion, on-the-job search, directed search improves the fit but only modestly
  - job ladder models as Burdett and Judd (1983) or Burdett and Mortensen (1998): $Mm = 1.25$
  - "new generation" of models like Postel-Vinay and Robin (2002), Burdett and Coles (2003): $Mm = 2$

Origin of wage posting models

- Burdett (AER, 1978) "Job Search and Quit Rates"
  - extends the McCall model to allow for on-the-job search
  - employed workers look for jobs, but with a lower contact rate
  - wage offer distribution $F(w)$ is exogenous
- Burdett and Judd (ECMA, 1983) "Equilibrium Price Dispersion"
  - endogenizes the price distribution in the optimal sampling model of Stigler
  - result: dispersion can arise in a model with identical and fully rational agents on both sides of the market

Burdett, Mortensen (1998)

Overview

- McCall search model with *endogenous wage distribution* and on-the-job search
- workers take distribution of offers $F(w)$ as given, but then firms decide what wages to post
- Diamond paradox
  - difficult to rationalize the distribution function $F(w)$ as resulting from profit maximizing choices of firms
  - in the McCall model, $F(w)$ will be degenerate and firms will offer the lowest possible wage
  - more on the problem set
- key mechanism
  - both employed and unemployed workers sample job offers with some fixed probability
  - high-wage firms attract more workers and lose fewer workers to other firms, compensating for the higher cost of labor
  - firms are competing against other firms as they do not know outside option of a worker they meet

Setup

- continuous time, discount rate r
- continuum of measure 1 of employers, measure m of workers, all identical
- *one-to-many* matching: firms can employ more than one worker
- search
  - directed on the firm side: firms post wages, which affects the probability of match creation
  - random on the worker side: they meet firms randomly

Worker's and firm's problem

Worker's problem

- similar to the McCall model
- worker is
  - unemployed (status 0): utility flow b
  - employed (status 1): wage flow w; produces p
- workers receive job offers at Poisson rate λ_i, $i \in \{0, 1\}$
  - *on-the-job search*: workers keep receiving offers while employed
- workers draw offers from the endogenous distribution $F(w)$ of job offers
- jobs are destroyed at the exogenous rate δ
- **note**: the contact rates $\{\lambda_i\}$ are exogenous, there is no matching function (no crowding), but it is easy to introduce one (Mortensen, 2000)

Worker's value functions

- value of unemployment V_0

$$rV_0 = b + \lambda_0 \left[\int \max \left\{ V_0, V_1 \left(\tilde{w} \right) \right\} dF \left(\tilde{w} \right) - V_0 \right]$$

- value of employment $V_1(w)$

$$\begin{aligned} rV_1 \left(w \right) &= w + \delta \left[V_0 - V_1 \left(w \right) \right] \\ &\quad + \lambda_1 \left[\int \max \left\{ V_1 \left(w \right), V_1 \left(\tilde{w} \right) \right\} dF \left(\tilde{w} \right) - V_1 \left(w \right) \right] \end{aligned}$$

- equation for $V_1(w)$ defines a contraction in the space of continuous, increasing functions of w
- thus, $V_1 \left(w \right)$ is (strictly) increasing in w
- V_0 does not depend on w
- optimal strategy: a reservation wage R

Reservation wage

- R – reservation wage for unemployed workers: $V_1(R) = V_0$
- then

$$
\begin{aligned}
& R + \lambda_1 \left[\int \max \{ V_0, V_1(\tilde{w}) \} \, dF(\tilde{w}) - V_0 \right] \\
& = b + \lambda_0 \left[\int \max \{ V_0, V_1(\tilde{w}) \} \, dF(\tilde{w}) - V_0 \right] \\
& \Downarrow \\
& R - b = (\lambda_0 - \lambda_1) \int_R (V_1(\tilde{w}) - V_0) \, dF(\tilde{w}) \qquad (1)
\end{aligned}
$$

- interpretation
  - if $\lambda_0 > \lambda_1$, this equation has an *option value* interpretation: when accepting an offer, workers understand they will have a lower probability to find a job later on; this is a form of irreversibility $\Rightarrow$ firms have an option value of waiting captured by $R - b$
  - if $\lambda_0 < \lambda_1$, the opposite is true: workers are willing to accept job wages $w < b$ because they understand that having a job helps them climb the job ladder more easily

Reservation wage

A technical step:

$$(r+\delta)\,V_1(w) = w + \lambda_1\left[\int \max\{V_1(w), V_1(\tilde{w})\}\,dF(\tilde{w}) - V_1(w)\right] + \delta V_0$$

$$(r+\delta)\,V_1(w) = w + \lambda_1\int_w (V_1(\tilde{w}) - V_1(w))\,dF(\tilde{w}) + \delta V_0$$

$$\Downarrow$$

$$(r+\delta)\,V_1'(w) = 1 + \lambda_1\left[-\int_w V_1'(w)\,dF(\tilde{w}) - (V_1(w) - V_1(w))\,F'(w)\right]$$

$$\Rightarrow V_1'(w) = \frac{1}{r+\delta+\lambda_1(1-F(w))}$$

Reservation wage

We may now simplify equation (1) integrating by parts:

$$\int_R^{\infty} (V_1(\tilde{w}) - V_0)\, dF(\tilde{w})$$

$$= -\underbrace{\left[(V_1(\tilde{w}) - V_0)(1 - F(\tilde{w}))\right]_R^{\infty}}_{=0} + \int_R^{\infty} V_1'(\tilde{w})(1 - F(\tilde{w}))\, d\tilde{w}$$

The first term is 0 because V_1 has a bounded derivative and F will have bounded support.

$$R - b = (\lambda_0 - \lambda_1) \int_R \frac{1 - F(\tilde{w})}{r + \delta + \lambda_1 (1 - F(\tilde{w}))} d\tilde{w} \qquad (2)$$

Unemployment in the steady state

- unemployment evolves according to

$$\dot{u} = \delta\left(m - u\right) - \lambda_0 u \left(1 - F\left(R\right)\right)$$

- in the steady state

$$u = \frac{\delta m}{\delta + \lambda_0 \left(1 - F\left(R\right)\right)} = \frac{m}{1 + k_0 \left(1 - F\left(R\right)\right)} \tag{3}$$

where we let $k_i = \lambda_i / \delta, i \in \{0, 1\}$

Worker distribution

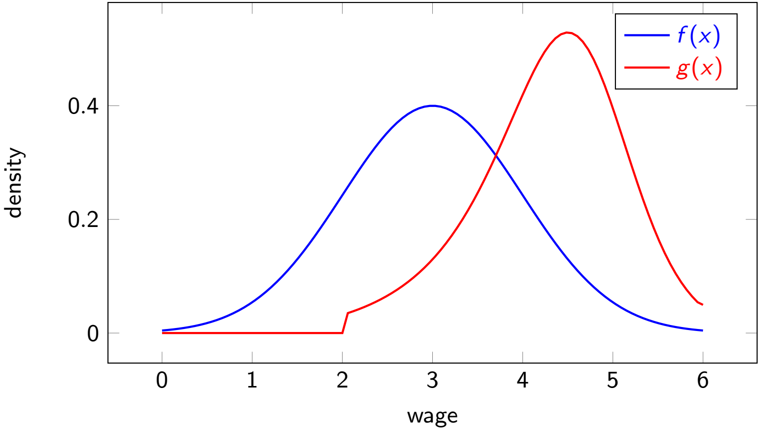
- $G(w)(m-u(t))$ – number of workers with wage $\leq w$
- using worker's optimal strategy, derive G from F
- evolution

$$\begin{aligned}\frac{dG(w,t)}{dt} &= \lambda_0 \max\{F(w)-F(R),0\}\,u(t) \\ &\quad -[\delta+\lambda_1(1-F(w))]\,G(w,t)(m-u(t))\end{aligned}$$

- steady state: $dG(w,t)/dt = 0$
- for $w \geq R$:

$$\begin{aligned}\lambda_0 F(w)\,u(t) &= (\delta+\lambda_1(1-F(w)))\,G(w)(m-u) \\ G(w) &= \frac{(F(w)-F(R))/(1-F(R))}{1+k_1(1-F(w))} \qquad (4)\end{aligned}$$

Worker distribution



- G is more skewed to the right because workers climb the job ladder

Employment

- focus on the steady state
- $(G(w) - G(w - \varepsilon))(m - u)$ – measure of workers earning a wage $\subset [w - \varepsilon, w]$
- $F(w) - F(w - \varepsilon)$ – measure of firms offering a wage $\subset [w - \varepsilon, w]$
- $l(w)$ – measure of workers per firm offering a wage w

$$l(w) \equiv \lim_{\varepsilon \to 0} \frac{G(w) - G(w - \varepsilon)}{F(w) - F(w - \varepsilon)}(m - u) \tag{5}$$

- $l(w)$ will be important for firm's decision
  - it specifies number of workers available to firms offering w

Employment

- use the expression for $G(w)$ and plug into (5) and take the limit
- the expression will contain $\lim_{\varepsilon \to 0} F(w-\varepsilon)$
- we consider a general case where $F(\cdot)$ can have mass points
- denoting the left limit as $F\left(w^{-}\right)$

$$\lim_{\varepsilon \to 0} F(w-\varepsilon) = F\left(w^{-}\right)$$

- after some algebra ...

$$l(w|R,F) = \frac{mk_0 \left[1 + k_1 (1 - F(R))\right] / \left[1 + k_0 (1 - F(R))\right]}{\left[1 + k_1 (1 - F(w))\right]\left[1 + k_1 (1 - F(w^{-}))\right]} \tag{6}$$

- $l(w) = 0$ for $w < R$

- properties of $l(w)$
  - increasing in w
  - continuous except where F has a mass point

Firm's problem

- p – output flow of an employed worker
- firms maximize the flow of profits from posting a wage w

$$\pi = \max_{w} \left(p - w\right) l\left(w\right) \tag{7}$$

- firms' trade-off
  - high wages induce large labor costs, but attract more workers

Definition of an equilibrium

- A steady-state equilibrium of this economy is a triple (R, F, π) such that
  - R satisfies (2)
  - π solves (7)
  - F is such that

$$\begin{aligned}(p-w)\,l(w) &= \pi \text{ for all } w \text{ in the support of } F \\ &\leq \pi \text{ otherwise}\end{aligned}$$

Solution

Uniqueness of the solution

1. F has a bounded support $[\underline{w}, \overline{w}]$
2. rule out non-continuous distributions F (non-degenerate, no mass points)
3. establish that $\underline{w} = R$
4. solve for the distribution of wages F
5. solve for the reservation wage R

Step 1: F has a bounded support

- no worker ever accepts an offer $w \leq R$, so $\underline{w} \geq R$
- no firm would ever make an offer above p, so $\overline{w} \leq p$
- the support of F is $[\underline{w}, \overline{w}] \subset [R, p]$

Step 2: F is continuous

- by contradiction, assume that F is discontinuous at point $\hat{w}$, i.e.,

$$F(\hat{w}) = F\left(\hat{w}^{-}\right) + v(\hat{w}), \text{ with } v(\hat{w}) > 0$$

- then there is a discrete jump in $l(w|R,F)$ at $\hat{w}$
- a firm offering $\hat{w} + \varepsilon$ for ε small would be better off ($\hat{w} < p$):

$$\pi(\hat{w} + \varepsilon) - \pi(\hat{w}) \approx -\underbrace{\varepsilon l(\hat{w})}_{\approx 0} + (p - \hat{w}) \underbrace{[l(\hat{w} + \varepsilon) - l(\hat{w})]}_{>0} > 0$$

- if $\hat{w} = \overline{w} = p$, then the firm would be better off by offering $\hat{w} - \varepsilon$
- hence, it is never optimal to choose a wage on a masspoint

Step 3: $\underline{w} = R$

- from Step 1, support $\subset [R, p]$ and $F(R) = 0$
- from Step 2, no mass points: $F(w^{-}) = F(w)$
- hence

$$l\,(w|R, F) = \frac{mk_0\,[1 + k_1]\,/\,[1 + k_0]}{[1 + k_1\,(1 - F\,(w))]^2}$$

- evaluating at $\underline{w}$

$$l\,(\underline{w}|R, F) = \frac{mk_0}{[1 + k_0]\,[1 + k_1]}$$

- does not depend on $\underline{w}$ (as long as $\underline{w} \geq R$)!
- hence a firm wants to offer the lowest possible wage: $\underline{w} = R$

Step 4: Solve for F

- in equilibrium, all wages in the support of $F(w)$ must yield the same profits:

$$\pi(w) = (p - w)\, l(w|R, F) = \pi(R) = (p - R)\frac{mk_0}{[1 + k_0][1 + k_1]}$$

- substitute for $l(w|R, F) = \frac{mk_0[1+k_1]/[1+k_0]}{[1+k_1(1-F(w))]^2}$ and solve for F:

$$F(w) = \frac{1 + k_1}{k_1}\left[1 - \left(\frac{p - w}{p - R}\right)^{\frac{1}{2}}\right]$$

- solve for $\overline{w}$ using $F(\overline{w}) = 1$:

$$\overline{w} = p - \frac{p - R}{(1 + k_1)^2}$$

Step 5: Solve for R

- use (1), plug in $F(x)$

$$\begin{aligned} R-b &= (k_0-k_1)\int_R \frac{1-F(\tilde{w})}{1+k_1(1-F(\tilde{w}))}d\tilde{w} \\ &= \frac{k_0-k_1}{k_1}\int_R \frac{k_1(1-F(\tilde{w}))}{1+k_1(1-F(\tilde{w}))}d\tilde{w} \\ &= \frac{k_0-k_1}{k_1}\int_R \frac{k_1\left(\frac{p-w}{p-R}\right)^{1/2}-1}{1+k_1\left(\frac{p-w}{p-R}\right)^{1/2}}d\tilde{w} \end{aligned}$$

- substitute for the value of $\overline{w}$, we get a single solution:

$$R=\frac{(1+k_1)^2 b+(k_0-k_1)k_1 p}{(1+k_1)^2+(k_0-k_1)k_1}$$

- in general, $\underline{w}=R<\overline{w}<p$: distribution F is non-degenerate, despite identical agents!

Summary

- there is a unique non-degenerate solution, for which we have a closed-form solution
- too beautiful to be true ?
  - in practice, empirical distribution of wages is quite different from what the theory predicts
  - the model puts too much structure on equilibrium objects and leaves no degree of freedom to have a chance of fitting the data
- important theoretical contribution nonetheless
  - search frictions can generate price dispersion with ex-ante homogeneous agents

Wage posting/Job ladder models today

- Burdett et al.-type of model
  - Burdett and Menzio (2013): studies price-level dispersion in the product market along with menu costs
  - Burdett and Coles (2003): introduces wage/tenure contract in the BM98 model
  - Christensen et al. (2005): adds endogenous search effort
- Postel-Vinay and Robin (2002) - type model
  - new generation job ladder models
  - relaxes the assumption of wage posting
  - firms make *take-it-or-leave-it* offers but are allowed to make counteroffers

Postel-Vinay, Robin (2002)

Introduction

- estimate a structural search model of the labor market with firm and worker heterogeneity
  - more flexible model that allows a better fit with the data
  - find a much smaller person effect than Abowd, Kramarz and Margolis (1999)
  - search frictions explain the difference: heterogeneity in labor market experience create wage differentials
- another contribution of the paper is of interest
  - they use a clever trick
  - an alternative to block recursivity in a dynamic settings?
  - Lise and Robin (2013)

Setup

- measure M of atomistic workers, unit measure of firms
- workers differ in their ability $\varepsilon \sim$ cdf $H \subset [\varepsilon_{min}, \varepsilon_{max}]$ (constant)
- firms differ in their productivity $p \sim$ cdf $\Gamma \subset [p_{min}, p_{max}]$
- a match produces εp
- workers have home production εb
- exogenous job destruction rate δ
- unemployed workers receive offers at rate λ_0, employed at λ_1

Wage setting mechanism

- firms offer different wages depending on what workers they meet
- firms counteroffer offers received by their workers from competing firms
- firms make *take-it-or-leave-it* offers
- wages are long-term contracts – renegotiated only upon arrival of new offers

Wage setting mechanism

- firms offer different wages depending on what workers they meet
- firms counteroffer offers received by their workers from competing firms
- firms make *take-it-or-leave-it* offers
- wages are long-term contracts – renegotiated only upon arrival of new offers
- $V_0(\varepsilon)$ – value of unemployment
- $V(\varepsilon, w, p)$ – value of an employed worker in firm p earning wage w

Wage setting mechanism – part 1

- unemployed workers – indifferent between employment and unemployment, his wage $w_0(\varepsilon, p)$ is such that
$$V_0(\varepsilon) = V(\varepsilon, w_0(\varepsilon, p), p)$$
- if worker receives an offer from firm $p' > p$
  - the two firms compete through a Bertrand competition
  - the highest firm p is willing to bid $w = \varepsilon p$, providing worker with utility $V(\varepsilon, \varepsilon p, p)$
  - the worker moves to firm p' with wage $w(\varepsilon, p, p')$ such that
$$V(\varepsilon, \varepsilon p, p) = V\left(\varepsilon, w\left(\varepsilon, p, p'\right), p'\right)$$

Wage setting mechanism – part 2

- if worker receives an offer from $p' < p$
  - p' can never outbid p, the worker stays in the firm p
  - still these two firms engage in Bertrand competition
  - if current value of a worker is higher than $V(\varepsilon, \varepsilon p', p')$, nothing changes for the worker
  - otherwise, he can increase his wage to $w(\varepsilon, p', p)$ (if $> w$) such that
$$V\left(\varepsilon, \varepsilon p', p'\right) = V\left(\varepsilon, w\left(\varepsilon, p', p\right), p\right)$$

Why is this clever?

- all labor market transitions are efficient (always move to the most productive firm)
  - the physical allocation is trivial and can be computed without knowing wages (even without solving for the value functions...)
- more importantly, the value functions that arise from this problem are independent from aggregate labor market conditions
  - Postel-Vinay and Robin (2002) didn't notice that
  - Lise and Robin (2013) only recently noticed how powerful this trick is for dynamic settings
  - why?

Lise and Robin (2013) (simplified)

- value of unemployment

$$\rho V_0(\varepsilon) = \varepsilon b + \lambda_0 \int \underbrace{\left[V(\varepsilon, w_0(\varepsilon, p), p) - V_0(\varepsilon)\right]}_{=0} dF(p)$$

- value of employment

$$\rho V(\varepsilon, w, p) = w + \lambda_1 \left[\int^{p} V\left(\varepsilon, w\left(\varepsilon, p', p\right), p\right) dF\left(p'\right) + \int_{p} V(\varepsilon, \varepsilon p, p)\, dF\left(p'\right) \right] - (\lambda_1 + \delta) V(\varepsilon, w, p) + \delta V_0(\varepsilon)$$

Lise and Robin (2013) (simplified)

- value of job:

$$\rho J(\varepsilon, w, p) = \varepsilon p - w + \lambda_1 \left[\int^{p} J\left(\varepsilon, w\left(\varepsilon, p', p\right), p\right) dF\left(p'\right) \right.$$

$$\left. + \int_{p} \underbrace{J(\varepsilon, \varepsilon p, p)}_{=0} dF\left(p'\right) \right] - (\lambda_1 + \delta) J(\varepsilon, w, p)$$

- match surplus $S(\varepsilon, p) = V(\varepsilon, w, p) + J(\varepsilon, w, p) - V_0(\varepsilon)$, independent of the wage:

$$\rho S(\varepsilon, p) = \varepsilon (p - b) + \lambda_1 \left[\int^{p} \underbrace{\left(J\left(\varepsilon, w\left(\varepsilon, p', p\right), p\right) + V\left(\varepsilon, w\left(\varepsilon, p', p\right), p\right) - V_0(\varepsilon)\right)}_{=S(\varepsilon, p)} dF\left(p'\right) \right.$$

$$\left. + \int_{p} \underbrace{(J(\varepsilon, \varepsilon p, p) + V(\varepsilon, \varepsilon p, p)) - V_0(\varepsilon)}_{=S(\varepsilon, p)} dF\left(p'\right) \right] - (\lambda_1 + \delta) S(\varepsilon, p)$$

Lise and Robin (2013) (simplified)

- easy solution

$$V_0(\varepsilon) = \varepsilon b \text{ and } S(\varepsilon, p) = \frac{\varepsilon(p - b)}{\rho + \delta}$$

- independent from aggregate labor market conditions or a distribution of workers across firms

- value functions can be easily computed without solving any complicated high-dimensional fixed point problem

- this trick becomes interesting in dynamic settings if we endogenize $\lambda_0 = p(\theta)$ and $\lambda_1 = \lambda p(\theta)$

  - θ can be easily computed ex-post along the dynamics as a function of the match surplus S

Some predictions of Postel-Vinay, Robin (2002)

- features similar to Burdett, Mortensen (1998)
  - distribution of wages is continuous
  - larger firms tend to pay higher wages
  - wages tend to increase with experience
  - negative relationship between quits and wage
- new features
  - there is a within-firm wage dispersion
  - wages tend to increase with tenure
  - workers sometimes accept wage cuts
- job ladder models are used to study wage dispersion rather than unemployment