1 Industry equilibrium

We develop an equilibrium model of an industry with many plants. The model is based on Hopenhayn (1992), and Hopenhayn and Rogerson (1993). The building blocks of the model are as follows:

- The industry is competitive and produces a homogeneous good. The industry is "small", so it takes the wage and the interest rate as given. The equilibrium of the industry determines the price and quantity of the good and the amount of labor hired in the industry.
- Instead of modelling a household sector explicitly, we'll assume that there is an exogenously given demand function for the good produced in the industry.
- Plants in the industry operate a decreasing returns to scale technology and are subject to productivity shocks. They hire labor as their only input. They pay a fixed operating cost.
- Every period incumbent firms choose whether to exit the market. There is free entry into the industry, subject to paying a fixed entry cost.

Jovanovic (1982) is another seminal paper on industry equilibrium which characterizes the evolution of an industry where costs are random and different among firms. Firms all have the same prior belief about costs when they enter the industry, and learn over time about their own true cost.

Applications of the model: One can use the model to study, for example:

- *Plant turnover* (in equilibrium some plants die, others enter) and *job turnover* from the growing firms (those with good shocks) to shrinking firms (those with bad shocks).
- The size distribution of firms: in the model, the large firms are those who had long sequences of good shocks.
- Effects of policies [e.g. a wage subsidy, a profit tax, a firing tax] on the size distribution, plant turnover, average profits, average productivity, etc...

- The diffusion of technologies across plants. If, in the model, the technology diffuses from young/small plants to old/large plants then we can replicate "S-shaped" diffusion curves.

- The role of financing constraints for firm's growth.

- Trade dynamics and export decisions of firms.

1.1 The Economy

Household sector: We represent the household sector simply through a demand function $D(p)$ where p is the price of the good produced by the plants in the industry and $D' < 0$.

Plant-level production technology: Each plant produces the homogenous good with technology

$$y = zf(n),$$

where $f' > 0$, $f'' < 0$ and $f(0) = 0$. The term $z \in Z$ denotes the plant-level idiosyncratic productivity shock which follows the continuous process $\Gamma(dz', z) = \Pr\{z_{t+1} \in dz' | z_t = z\}$.

Problem of an incumbent plant: Incumbent plants (i.e., those producing in the current period) incur in the per-period fixed cost ϕ in order to operate (e.g., they hire one unit of managerial time every period at cost ϕ). They hire labor at the wage ω.

Let the profits of the plant be denoted by $\pi(z)$. A plant that takes $\{p, \omega\}$ as given, solves:

$$\pi(z) = \max_n \{pzf(n) - \omega n - \phi\}. \tag{1}$$

Note that the profit function is increasing in z. Note also that $\omega = pzf_n(n)$. Competitive labor markets means that in equilibrium all plants face the same wage, so the more productive plants hire more labor. For example, suppose $f(n) = n^\alpha$. Then

$$pz\alpha n^{\alpha-1} = \omega \Rightarrow n(z;p) = \left(\frac{zp\alpha}{\omega}\right)^{\frac{1}{1-\alpha}}.$$

Therefore, take two plants with productivity z_i and z_j. The ratio between their size will be directly proportional to the productivity ratio, i.e.

$$\frac{n(z_j;p)}{n(z_i;p)} = \left(\frac{z_j}{z_i}\right)^{\frac{1}{1-\alpha}}.$$

So, once the decreasing return parameter α is known, by observing the size distribution one can fully recover the productivity distribution.

Exit decision: At the beginning of every period, before realizing the productivity shock z, the plant decides to exit. The value of exit is normalized to zero, i.e. the plant has no "scrap value". The value of an incumbent plant is therefore:

$$v(z;p) = \pi(z;p) + \frac{1}{1+r} \max\left\{ \int_Z v(z';p)\,\Gamma(dz', z), 0 \right\} \tag{2}$$

where the first argument of the max operator is the expected continuation value of the incumbent firm. Since profits are increasing in z and Γ is monotone, the value function v is also increasing in z. Let $\chi(z;p) = \{0,1\}$ be the exit decision. The exit decision involves a reservation rule. There exists a threshold productivity level $z^*(p)$ such that, for all $z < z^*(p)$, the firm will decide to exit. In equilibrium, incumbent firms may incur in negative profits temporarily and keep operating, if the shock is mean reverting.

Problem of an entrant plant: An entrant plant must pay the fixed cost κ for one period (e.g., hire the manager to set-up the plant) and then it draws its initial productivity z' from the distribution $G(dz')$. The value of an entrant plant is, therefore,

$$v^e(p) = -\kappa + \frac{1}{1+r} \int_Z v(z';p)\,G(dz'). \tag{3}$$

The entry decision is simple: a plant should enter as long as $v^e(p) \geq 0$. Free entry of plants will guarantee that, in equilibrium, $v^e(p) = 0$.[1] Let m denote the measure of entrant plants, i.e. those preparing to produce but not yet producing, hence not yet hiring labor.

1.2 Equilibrium

A *stationary recursive competitive equilibrium* for this industry is a list of: plants' decision rules $\{n, \chi\}$, value functions $\{v, v^e\}$, price $\{p\}$, an invariant measure of incumbent firms λ, and a measure of entrant plants m, such that:

- Given p, $n(z;p)$ solves the static hiring decision (1).

[1] Depending on the parameter values, in particular κ and ϕ, there may be equilibria where $v^e(p) < 0$, so there is no entry and no exit.

- Given p, $\chi(z;p)$ solves the exit decision (2) of the incumbent firm, and $v(z,p)$ is the associated value function

- Free entry of firms implies that $v^e(p) = 0$, i.e., at the equilibrium price p

$$\kappa = \frac{1}{1+r}\int_Z v(z';p)\,G(dz')$$

- The good market clears

$$D(p) = \int_Z zf(n(z;p))\,\lambda(dz;p) \tag{4}$$

- Let dz' be a generic set of the Borel sigma algebra on the state space Z. Then, the invariant measure of incumbent plants solves

$$\lambda(dz';p) = \int_Z \Gamma(dz',z)\,[1-\chi(z)]\,\lambda(dz;p) + mG(dz'). \tag{5}$$

  Note that the measure λ is linearly homogenous in m. To see this, suppose that Z is a discrete set with I values, then G is a vector and λ is a vector that satisfies

$$\underset{(I\times 1)}{\lambda} = \underset{(I\times I)}{\Gamma} \otimes \underset{(I\times I)}{X} \cdot \underset{(I\times 1)}{\lambda} + m \underset{(I\times 1)}{G}$$

  where Γ is the transition matrix for z, X is a matriz where each line is a collection of either 0 or 1 depending on the value of $1-\chi(z)$, and the symbol $\otimes$ denotes the element by element product. The above linear system of equations has solution

$$\lambda = m\left[\left(I-\tilde{\Gamma}\right)^{-1} G\right]$$

  where $\tilde{\Gamma} = \Gamma \otimes X$. This equation shows clearly that λ is homogeneous in m, the number of entrants.

1.3 Solution method

The key difference with respect to the computation of the equilibrium in Bewley models with heterogeneous households is that in models of industry equilibrium, we also need to determine the *number of plants,* which is endogenous. A fixed-point algorithm to solve for the equilibrium is as follows.

1. Guess a price p_0 and solve the static hiring problem (1) for the hiring decision $n(z, p_0)$. Let

$$\pi(z; p_0) = p_0 z f(n(z; p_0)) - \omega n(z; p_0) - \phi$$

2. Given the profit function $\pi(z; p_0)$, one can look for the fixed point of the Bellman equation

$$v(z; p_0) = \pi(z; p_0) + \frac{1}{1+r} \max\left\{\int_Z v(z'; p_0) \Gamma(dz', z), 0\right\}$$

and from this step, obtain $\{\chi(z; p_0), v(z; p_0)\}$. Note that $v(z; p_0)$ is increasing in p_0 since the flow profits are increasing in p_0.

3. Verify if the price p_0 satisfies the free-entry condition

$$\kappa = \frac{1}{1+r} \max\left\{\int_Z v(z'; p_0) G(dz'), 0\right\}$$

If, for example, the equation above holds with the $>$ sign, then guess a new value $p_1 > p_0$. Go back to step 1) with the new guess. Continue until we find a price that satisfies the free entry condition. Now that we solved for p, all we need to determine is the pair (λ, m).

4. Exploit the linear homogeneity of λ in m. Guess a value of entrant plants m_0 and from (5) compute, either by simulation or using the transition function, the invariant measure λ.

5. Compute total sales of the industry, the RHS of equation (4) and verify that, at the equilibrium price found in step 3), total sales equal the exogenously given aggregate demand. If, for example, total sales are below demand, update your guess of entrants to $m_1 > m_0$ and go back to step 4). However, now step 4) is much simpler because we don't need to recompute λ, we just rescale the invariant measure already obtained.

1.4 An industry with firing costs

Hopenhayn and Rogerson (1993) study the impact of firing restrictions on the average productivity of the industry. Suppose, for example, that the government imposes *firing costs* that can be summarized by the function

$$g(n, n_0) = \begin{cases} \chi(n_0 - n) & \text{if } n < n_0 \\ 0 & \text{if } n \geq n_0 \end{cases}$$

i.e., the government imposes a severance payment of size χ to the firm for every workers who is laid off.

The key novelty, with respect to the previous model, is that the employment choice is dynamic and past employment n_0 is a *state variable*. Plants need to keep track of their past employment to calculate the firing cost $g(\cdot)$ associated to their employment decision.

The hiring problem becomes similar to the investment problem (with or without adjustment costs) and the value function of the incumbent is:

$$v(z, n_0; p) = \max_n \left\langle pzf(n) - \omega n - \phi - g(n, n_0) + \frac{1}{1+r} \max \left\{ \int_Z v(z', n; p) \Gamma(dz', z), -g(0, n) \right\} \right\rangle$$

Results: A firing cost will reduce labor reallocation from the low-productivity firms (which should be shrinking by shedding workers) towards the high-productivity firms (which should be expanding by hiring workers). It also prevents inefficient firms from exiting because of the large exit cost associated to firing the entire workforce. Overall, it is easy to see that this policy reduces average productivity of the industry and labor turnover. It is a source of misallocation.

References

[1] Hopenhayn, Hugo (1992); Entry, Exit and Firm Dynamics in Long Run Industry equilibrium, *Econometrica*

[2] Hopenhayn, Hugo ad Richard Rogerson (1993); Job Turnover and Policy Evaluation: A General Equilibrium Analysis, *Journal of Political Economy*

[3] Jovanovic, Boyan (1982); Selection and Evolution of Industry, *Econometrica*