Aggregate Labor Market Dynamics
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Problem Set 4
Due on April 7, 2016
Suggested solution

The goal of this problem set is to work through wage setting mechanism as in Postel-Vinay and Robin (2002), and find the wage distribution for given parameters. We will then use this models to evaluate earnings losses of worker who lose jobs. The goal is to see if these model can generate magnitude of losses comparable to the ones measured in the data.

1 Wage distribution in a simplified model of Postel-Vinay and Robin (2002)

Consider a simplified version of the Postel-Vinay and Robin (2002) model, or of Jarosch (2014). The economy is populated by homogenous workers and heterogenous firms. The firm's productivity p is distributed according to $p \sim F(p)$on an interval $[\underline{p}, \bar{p}]$, which is given exogenously.

Workers are infinitely lived, risk-neutral and discount future at the rate r. Each worker can be either employed or unemployed, and in either case is looking for a job. Let λ_E and λ_U be the probability that a worker is contacted by a firm when employed or unemployed, respectively. The productivity of a contacting firm is drawn from $F(p)$. A match breaks exogenously at the rate δ. An unemployed worker receives unemployment benefits b.

The wage setting is given by sequential auction Postel-Vinay and Robin (2002). Firms make type- and state-contingent offers and counter-offers to workers. When an unemployed worker receives an offer, a firms chooses the wage so as to make the worker indifferent between taking the offer or not. If an employed worker with the current productivity p receives an outside offer with productivity p', the incumbent and poaching firm engage in a Bertrand competition. Once the worker takes the offer, the wage remains constant until the worker receives an outside offer.

If a worker who is currently employed in a firm with productivity p is contacted by a firm with productivity p', he takes the job if $p' > p$, otherwise he stays with the current employer.

The wage of the worker is determined by the second highest surplus. In particular, the wage of a worker is such that the *worker's surplus* equals the second highest match surplus.

A natural choice for state variable for worker's problem is his current firm's productivity p and the wage w. However, it will be more handy to use (p, p') as a state variable where p is the current productivity and p' is the productivity of the last poaching firm. The wage can be then expressed as a function $w(p, p')$. We will denote the wage of a worker who comes from unemployment as $w(p, u)$.

Let $S(p)$ be the surplus of a match, $W(p, p'), U$ be the worker's value of being employed and unemployed, respectively. Denote the wage as $w(p, p')$. We will need some more notation. Let's $M_1(p, p')$ be the set of productivities that a worker with state (p, p') does not take but which he uses to increase his wage. Let $M_2(p, p')$ be the set of productivities which a worker with state (p, p') accepts. Finally, let M_0 be the set of productivities that an unemployed worker accepts.

Question 1.1 Write down the value function for an employed worker, using (p, p') as a state variable.

Answer It is

$$\begin{aligned}
W\left(p, p'\right) &= w\left(p, p'\right) + \beta s U \\
&+ \beta(1-s)(1-\lambda_1) W\left(p, p'\right) \\
&+ \beta(1-s)\lambda_1 \left[\int_{p \in M_1(p,p')} W(p, x)\, dF(x)\right] \\
&+ \beta(1-s)\lambda_1 \left[\int_{p \in M_2(p,p')} W(x, p)\, dF(x)\right] \\
&+ \beta(1-s)\lambda_1 \left[\int_{p \notin M_1(p,p') \cup M_2(p,p')} W\left(p, p'\right) dF(x)\right]
\end{aligned}$$

■

Question 1.2 Formulate the value function for the surplus, $S(p)$. Explain why the formula is so simple and does not contain the term capturing search on the job. Solve for $S(p)$ in terms of parameters of the model.

Answer It is

$$\begin{aligned}
S(p) &= p - b + \beta(1-s) S(p) \\
S(p) &= \frac{p-b}{1-\beta(1-s)}
\end{aligned}$$

■

Question 1.3 Formulate the value function for being unemployed, U.

Answer It is

$$\begin{aligned} U &= b + \beta \left(1 - \lambda_0\right) U \\ &\quad + \beta \lambda_0 \int_{p \in M_0} W\left(x, u\right) dF\left(x\right) \\ &\quad + \beta \lambda_0 \int_{p \notin M_0} U dF\left(x\right) \end{aligned}$$

■

Question 1.4 When a firm hires a worker from unemployment, it extracts the entire surplus. Use this fact to solve for U in terms of parameters.

Answer We have

$$\begin{aligned} U &= b + \beta \left(1 - \lambda_0\right) U + \beta \lambda_0 U \\ U &= \frac{b}{1 - \beta} \end{aligned}$$

it is the present discounted value of being unemployed. ■

Question 1.5 We want to find a formula for wage $w(p, p')$ which we will do in steps. First notice that we know $S\left(p\right)$ and U. We will simplify value function $W\left(p, p'\right)$ using results from the wage setting. Observe that the wage setting implies the following:

$$\begin{aligned} W\left(p, u\right) - U &= 0 \\ W\left(p, p'\right) - U &= S\left(p'\right) \\ M_1\left(p, p'\right) &= \left[p', p\right] \\ M_2\left(p, p'\right) &= \left[p, \bar{p}\right] \end{aligned}$$

Make sure that you understand why.

Use the value function $W\left(p, p'\right)$ and the results above to find an equation for $w\left(p, p'\right)$ in terms of $S\left(p\right), U$ and parameters of the model. The idea is to eliminate $W\left(p, p'\right)$ from the value function using the relationship between $W\left(\cdot, \cdot\right) - U$ and $S\left(\cdot\right)$.

Answer We have

$$\begin{aligned}
W\left(p,p'\right) &= w\left(p,p'\right)+\beta sU+\beta\left(1-s\right)U \\
&\quad +\beta\left(1-s\right)\left(1-\lambda_1\right)\left(W\left(p,p'\right)-U\right) \\
&\quad +\beta\left(1-s\right)\lambda_1\left[\int_{p\in M_1(p,p')}\left(W\left(p,x\right)-U\right)dF\left(x\right)\right] \\
&\quad +\beta\left(1-s\right)\lambda_1\left[\int_{p\in M_2(p,p')}\left(W\left(x,p\right)-U\right)dF\left(x\right)\right] \\
&\quad +\beta\left(1-s\right)\lambda_1\left[\int_{p\notin M_1(p,p')\cup M_2(p,p')}\left(W\left(p,p'\right)-U\right)dF\left(x\right)\right]
\end{aligned}$$

Subtract U from both sides to be able to rewrite everything in terms of surplus $S\left(\cdot\right)$:

$$\begin{aligned}
W\left(p,p'\right)-U &= w\left(p,p'\right)-\left(1-\beta\right)U \\
&\quad +\beta\left(1-s\right)\left(1-\lambda_1\right)S\left(p'\right) \\
&\quad +\beta\left(1-s\right)\lambda_1\left[\int_{p'}^{p}S\left(x\right)dF\left(x\right)\right] \\
&\quad +\beta\left(1-s\right)\lambda_1 S\left(p\right)\left[\int_{p}^{\bar{p}}dF\left(x\right)\right] \\
&\quad +\beta\left(1-s\right)\lambda_1 S\left(p'\right)\left[\int_{\underline{p}}^{p'}dF\left(x\right)\right]
\end{aligned}$$

Hence the expression for wage is

$$\begin{aligned}
w\left(p,p'\right) &= S\left(p'\right)+\left(1-\beta\right)U \\
&\quad -\beta\left(1-s\right)\left(1-\lambda_1\left(1-F\left(p'\right)\right)\right)S\left(p'\right) \\
&\quad -\beta\left(1-s\right)\lambda_1 S\left(p\right)\left(1-F\left(p\right)\right) \\
&\quad -\beta\left(1-s\right)\lambda_1\left[\int_{p'}^{p}S\left(x\right)dF\left(x\right)\right]
\end{aligned}$$

If a worker comes from unemployment, her Bellman equation satisfies

$$\begin{aligned}
W\left(p,u\right) &= w\left(p,u\right)+\beta sU+\beta\left(1-s\right)U \\
&\quad +\beta\left(1-s\right)\lambda_1\left[\int_{p\in M_1(p,p')}\left(W\left(p,x\right)-U\right)dF\left(x\right)\right] \\
&\quad +\beta\left(1-s\right)\lambda_1\left[\int_{p\in M_2(p,p')}\left(W\left(x,p\right)-U\right)dF\left(x\right)\right]
\end{aligned}$$

$$
\begin{aligned}
w(p,u) &= (1-\beta)U \\
&\quad -\beta(1-s)\lambda_1\left[\int_{\underline{p}}^{p} S(x)\,dF(x)\right] \\
&\quad -\beta(1-s)\lambda_1(1-F(p))\,S(p)
\end{aligned}
$$

and so the value the wage is give by

$$
\begin{aligned}
w(p,u) &= (1-\beta)U \\
&\quad -\beta(1-s)\lambda_1 S(p)\left[\int_{p}^{\bar{p}} dF(x)\right] \\
&\quad -\beta(1-s)\lambda_1\left[\int_{0}^{p} S(x)\,dF(x)\right]
\end{aligned}
$$

You can evaluate everything on the RHS using the distribution $F(\cdot)$ and the surplus function, and hence get $w(p,p')$ for each combination of (p,p'), including the case when a worker comes from unemployment. ■

Question 1.6 We will now parameterize the model and solve it numerically. We know $S(p), U$, and we have an equation which determines the wage $w(p,p')$. Create a grid for p, and compute $w(p,p')$ for each combination of p and p' from the grid. We will use these values for simulations in the next section. Use the following values,

$$\lambda_0 = 0.1, \lambda_1 = \frac{2}{3}\lambda_0, \beta = 0.9958, s = 0.035, b = 0.5.$$

Assume that $p = 1+\varepsilon$ where ε is distributed according to a beta distribution with parameters (η,μ) where $\eta = 11.95$ and $\mu = 11.05$. This is only to bound p away from zero. The calibration is at a monthly frequency and is taken from Jarosch (2014).

Write down a code which computes $w(p,p')$ for each p,p' on the grid, and the case when $p' = u$.

2 Earnings losses from displacement

In this section, we will use the model to evaluate earnings losses from losing a job.

1. Simulated wage paths for a large number of workers, say $N = 10,000$, allowing for transitions to and from employment. Keep track of individual wages. Start with everybody being unemployed, and then throw away first 10,000 months. Keep the next 20 years of data, that is, $12 \cdot 20$ months.

2. In this new sample (after throwing away $10,000$ months), plot the wage distribution.

3. Define $t = 6$ as a displacement month. You can throw away all workers who are not employed at $t = 5$.

4. At time $t = 5$, you see a distribution of workers across states (p, p'). For any given (p, p'), you see some workers who lose a job in $t = 6$ and some who do not. We will refer to the first group as job separators, and the latter group as job stayers. For each (p, p'), compute average earnings of job separators in each period $t = 6, \dots T$. Do the same for job stayers. Use this to compute PDV of job separators and job stayers, and compare it. For each (p, p') you will have PDV of earnings loss. Plot the distribution of it.

5. Plot a time path of the earnings losses for the whole population. Do you get something similar to the data?

6. Notice that in the previous stage, we defined an exact counterfactual. This is a good environment to examine if the earnings losses computed this way will give you the same results as running a regression as in Davis, von Wachter (2011). Use your sample of simulated data to run a regression as in Davis, von Wachter, and compare the earnings losses recovered this way to the ones with an exact counterfactual. Did you get the same answer?

Discuss all these results.

First, you will notice that workers coming out of unemployment are earning negative wages. This is a general problem of this bargaining, and is not a consequence of our calibration. People proceed by dropping this wages, and we will follow that here too. This means, that we will set this wage to zero.

Figure 2 shows the wage distribution. The average wage in the economy around 1.4. There are two things to notice: first, the wage $w(p, u) < 0$ because workers are willing to sacrifice some income to have an option to climb the ladder. Second, workers coming from unemployment are heavily penalized in terms of wages, even after setting them to zero. This is a generic problem of this wage setting, and can be fixed with giving worker some bargaining power α, as we saw in class in Jarosch (2015). However, introducing a positive α compresses the wage distribution, which makes it harder to generate large earnings losses.

Figure 2 depicts earning losses of separated workers, in levels. The black line corresponds to constructing an exact counterpart to a job loser, while the blue line shows the losses estimated from regression. I constructed the first one as follows: for each (p, p'), I selected workers employment at $t = 5$ and split them into groups: those who lose a job in $t = 6$ and those who don't. I compute the average earnings for each group and each month $t = 1, \dots 240$, and subtract. The black line is then an average of earnings losses for each (p, p'), weighted by number of (p, p') job losers. The blue line shows earnings losses as coefficients δ_k from the regression introduced in class, except that here I only control for worker fixed effects. We see that these lines do not exactly coincide, the reason is that they use different counterfactual earnings path. There is a large loss on impact, but workers recover from the job loss very quickly, basically within 5 years. This is contrary to the data where earnings losses were very persistent. With the exact counterfactual, earnings before the separation are similar in the control and treatment group. This is not the case when running a regression with workers fixed effects, as I discussed in class.

Figure 2 depicts earnings losses as Figure 2 , but here I normalize by the average pre-separation wage of separated workers. Depending on which method I use, the earnings losses on impact are 100 and 50 percent.

Finally, I plot the distribution of PDV of earnings losses in Figure 2. It happens to be the case that some job separators have actually a higher earnings than non-separators. This is rather a consequence of a small number of simulations. Losses are concentrated close to the 100 percent, with mean 97 percent. This is way too large compared to the data.

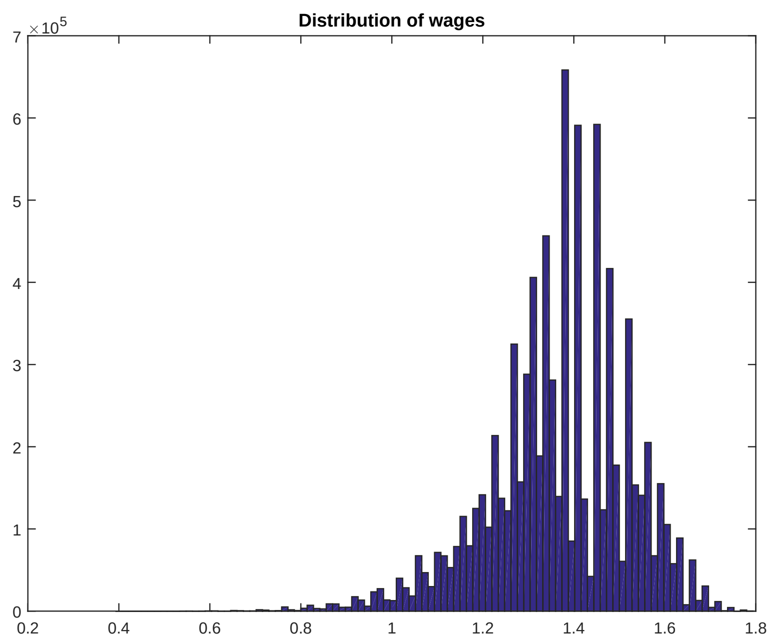

Figure 1: Steady state wage distribution.

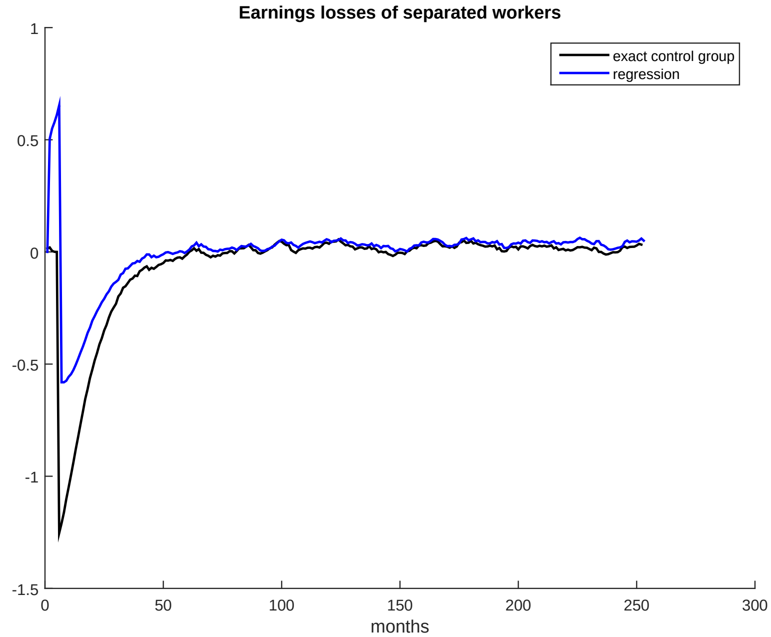

Figure 2: Earnings losses of workers who separate from their job, in levels.

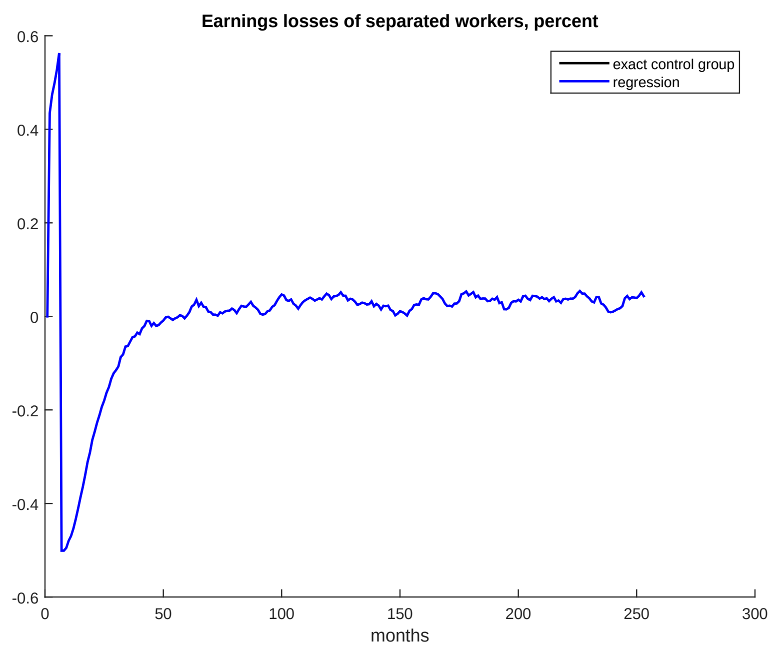

Figure 3: Earnings losses of workers who separate from their job, as percent of pre-separation average income of job separators.

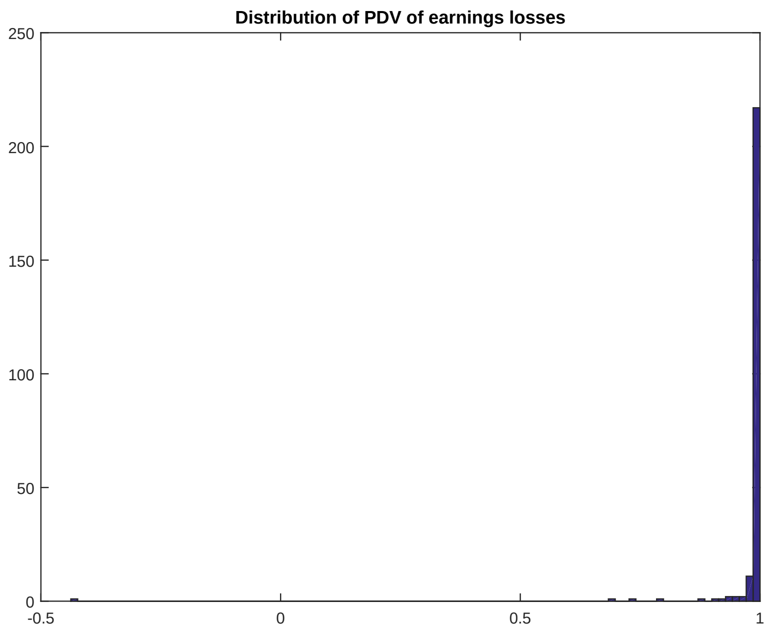

Figure 4: Distribution of PDV of earnings losses.