Retake Exam Ph.D. Macroeconomics II

Department of Economics, Uppsala University

August 16, 2022

Instructions

- Writing time: 5 hours.

- The exam is open book. You may bring and consult any part of the course material, but you are not allowed to use any electronic devices.

- The exam has 76 points in total

- A passing grade requires a) at least 30 points on the exam, and b) 50 points in total for the course (incl the points you have from your problem sets).

- Start each question on a new paper. Write your anonymous code on all answer pages.

- You may write your solutions by pen or pencil; use your best handwriting.

- Answers shall be given in English.

- Motivate your answers carefully; if you think you need to make additional assumptions to answer the questions, state them.

- If you any questions during the exam, you may call me (+46 730 606 796) at any time between 9 AM and 11 AM.

The Social Planner problem in the vanilla RBC model (15 points)

Consider the vanilla RBC model discussed in lecture 1. Households have standard separable preferences

$$U_0 = E_O \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(N_t) \right],$$

where $U(\cdot)$ and $V(\cdot)$ satisfies the usual regularity conditions. The Production technology is $Y_t = A_t F(K_t, N_t)$ where $F(\cdot)$ is CRS, and the capital depreciation rate is δ.

1. What is the aggregate resource constraint and what is the capital law of motion in this economy? (3 points)

2. The social planner seeks to maximize household utility subject to the resource constraint and the capital law of motion. State the social planner problem of this economy. (3 points)

3. Set up the Lagrangian, and compute the optimaility conditions to the social planner problem. (3 points)

4. Interpret the optimality conditions in terms of *marginal rate of substitutions* and *marginal rate of transformations*. (3 points)

5. Explain how we can tell that the solution to the social planner problem coincides with the allocation in the decentralized competitive equilibrium of the economy. Explain why the two allocations will coincide. (3 points)

Identification of the Phillips curve (20 points)

An opinion piece in the Financial Times on Jul 27, 2022, contained Figure 1, and argued that since the 70's, US data show little empirical support for a positive comovement between unemployment/output and inflation, and, therefore, that the concept of a Phillips curve is dead. Let's examine the argument from the point of view of the New Keynesian model.

Consider the dynamics of the following version of the linearized NK model:

$$
\begin{aligned}
\text{DIS curve:} \quad & \hat{y}_t = -(\hat{i}_t - E_t \pi_{t+1}) + E_t \hat{y}_{t+1} + \nu_t \\
\text{Phillips curve:} \quad & \pi_t = \beta E_t \pi_{t+1} + \kappa \hat{y}_t + \xi_t \\
\text{Policy rule:} \quad & \hat{i}_t = \phi E_t \pi_{t+1}
\end{aligned}
$$

where ν_t is a "demand shock" and ξ_t is a "supply shock". There are no "policy shocks", and to simplify some algebra, we assume that monetary policy responds to expected inflation rather than current inflation.

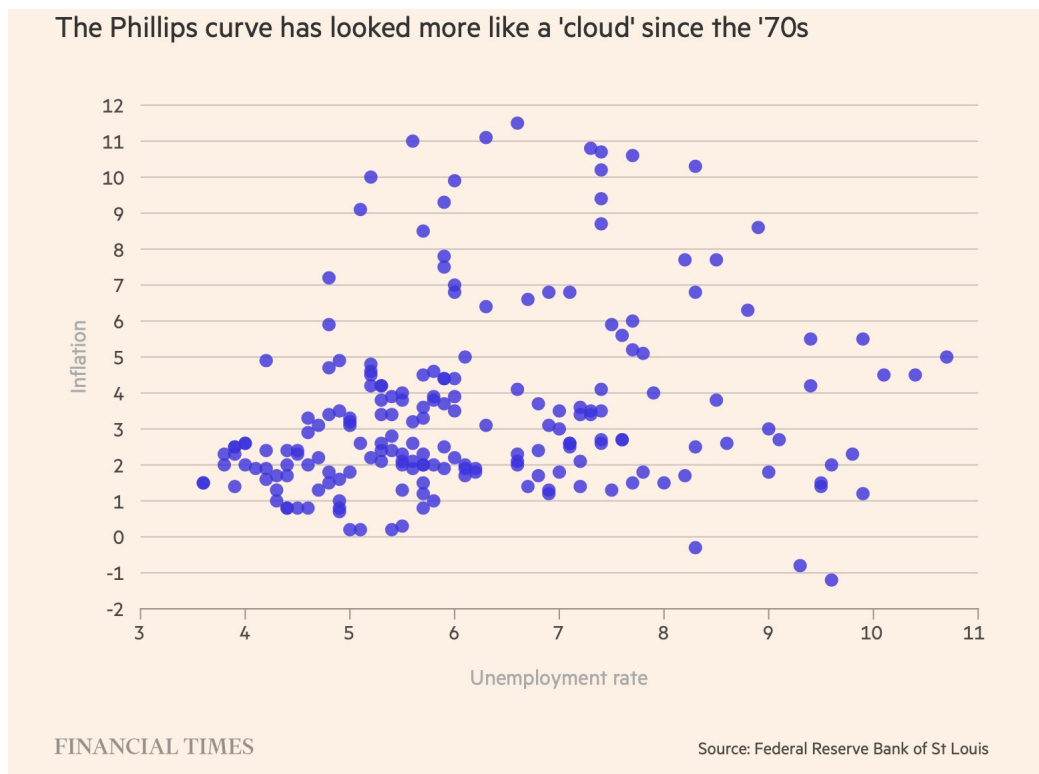The Phillips curve has looked more like a 'cloud' since the '70s

Figure 1: From FT, Jul 27 2022

1. Suppose there are no supply shocks, $\xi_t = 0$, but only AR(1) demand shocks: $\nu_t = \rho\nu_{t-1} + \epsilon_t$, where ϵ_t is exogenous. Guess that in a solution to a positive demand shock ϵ_t, output and inflation responds linearly to ν_t:

$$\hat{y}_t = \theta_y \nu_t$$

$$\pi_t = \theta_\pi \nu_t$$

By using this guess, solve for θ_y, θ_π in terms of the model parameters (and thereby verify that the guess is correct). Is there a positive correlation between $\hat{y}_t$ and π_t? (5 points)

2. Now, suppose there are no demand shocks, $\nu_t = 0$, but only AR(1) supply shocks: $\xi_t = \rho\xi_{t-1} + \epsilon_t$, where ϵ_t is exogenous. Analogously, guess that in the solution to a positive supply shock ϵ_t:

$$\hat{y}_t = \theta_y \xi_t$$

$$\pi_t = \theta_\pi \xi_t$$

Solve for θ_y, θ_π in this case too. Is there a positive correlation between $\hat{y}_t$ and π_t? (5 points)

3. Much research have showed in the late 70's, US monetary policy shifted from being "passive" to "active". Put differently, since the late 70's, the Fed has tried to raise the real interest rate in response to inflation rising above target (and lowering rates in response to inflation below target). We can interpret this in our model as if ϕ shifted from $\phi < 1$ to $\phi > 1$. Ignoring determinacy issues, how does this parameter shift affect the correlation between inflation and output in response to demand and supply shocks? (5 points)

4. In general, do you think that we should expect a positive correlation between inflation and real activity in the data? Is the reduced-form relationship in Figure 1 a valid way to estimate the slope of the Phillips curve? Use your findings from subquestions 1-3 to motivate your answer. (5 points)

Taxes and Government Spending in the DMP Model (25 points)

Consider the basic continuous-time DMP model studied in class. There is a conintuum of workers with mass 1, and large mass of firm who decides whether to post a vacancy or not, with free entry. Denote the unemployment and vacancy rate with u, v repsectively. The job-finding and job-filing rates are given by the aggregate matching function, which is $m(u, v) = Au^\alpha v^{1-\alpha}$. The vacancy posting cost is c. Jobs desolve at exogenous rate σ. The discount rate is r. Upon a match, the worker-firm pair produces y. The wage level is determined by Nash bargaining, in which the worker bargaining power is γ.

The unemployed workers retrieve utility b. We interpret b as an unemployment benefit provided by the government with its level being exogenously fixed. This benefit is financed by taxing all employed workers

with a lump-sum tax τ, which is allowed to vary to keep the government's budget is balanced. A balanced budget means that total tax income equals total benefit payments at every instant.

Put together, the Bellman equations for the vacancy value V, Job value J, unemployment value U and employment value E are, respectively, given by

$$
\begin{aligned}
rV &= -c + q(\theta)(J - V), \\
rJ &= y - w + \sigma(V - J), \\
rU &= b + p(\theta)(W - U), \\
rW &= w - \tau + \sigma(U - W),
\end{aligned}
$$

where $q(\theta), p(\theta)$ are the job-filling and job-finding rates implied by the matching function.

1. Write the government's balanced budget constraint and show that τ is increasing in u. (2 points)

2. Derive a relation between v and u in steady state (a Beveridge curve). (2 points)

3. Derive the job-creation curve in $\{w.\theta\}$-space. (3 points)

4. Using the results above, derive the wage curve. Is the wage level increasing or decreasing in τ? What is the intuition? (6 points)

5. Use the government's budget constraint and the Beveridge curve to write the wage curve in terms of θ, w and exogenous parameters only. (4 points)

6. Describe the shape of the wage curve and draw it in a graph together with the job-creation curve. Argue that under certain conditions, the model can have two steady state equilibria. Describe the intuition as to how this can be possible. (6 points)

7. Is it clear that increasing b increases the equilibrium unemployment rate u in this model? (2 points)

An Ayiagari model with an exogenous savings rule (16 points)

Consider a stationary economy with a continuum (measure 1) of ex-ante identical households each having efficiency units of labor ϵ which is drawn from a discrete distribution with PDF $\pi(\epsilon)$, i.i.d. over time and across households. The distribution has non-negative support and mean 1. Households can trade a risk-free asset a but cannot borrow $a \geq 0$. Assume that the households' decision rule for savings a' has this form

$$
a' = (1 + r)a + \phi w \epsilon,
$$

where r, w are the interest rate and the wage rate, respectively, and $0 < \phi < 1$. That is, in each period, they add a constant fraction ϕ of their current-period labor income to their savings account. In the economy, there

are also competitive firms which hire labor and capital at prices w and r and operate a standard production function $K^\alpha L^{1-\alpha}$, where $0 < \delta < 1$ is the depreciation of capital. The economy is closed, and in equilibrium, the sum of household asset holdings must equal the capital stock.

1. Set up the firm problem and show the first order conditions. (2 Points)

2. Show that a stationary distribution of a cannot exist if $r \geq 0$. (Hint: study the evolution of aggregate asset supply $A' = \int_i a'_i di$ where i denotes an individual household) (4 Points)

3. For $r < 0$, solve for the long-run aggregate asset supply $A(r)$. Draw a graph of $A(r)$ together with capital demand $K(r)$. (5 Points)

4. Discuss what happens to output, the interest rate and the wage level if the savings rate ϕ increases in the stationary state of this economy. (3 Points)