Problem Set 5

Due: before the last TA session

15th May 2017

DMP Model

Assume linear utility and no costs of searching on the worker side. The vacancy posting cost is c per period. The output of a match is z_t and unemployment benefit is b. The exogenous separation rate is σ.

Labor market tightness is $\theta_t \equiv V_t/U_t$. The surplus for a worker of being employed is denoted by H_t. The surplus for a firm of being matched with a worker is denoted by J_t.

The matching function has Cobb-Douglas form:

$$M_t = \gamma V_t^{1-\lambda} U_t^{\lambda},\ 0 < \lambda < 1$$

where γ represents the matching efficiency.

The job finding probability f_t and vacancy filling probability q_t are defined as in the lecture notes. Wages are flexible and determined by Nash bargaining. Workers have bargaining strength φ.

All settings are the same as in the notes for lecture #4. Please refer to these notes if something is missing.

The equations defining the market equilibrium of the core part of the model are:

$$\begin{aligned}
J_t &= z_t - w_t + \beta (1-\sigma) E_t J_{t+1} \\
H_t &= w_t - b + \beta [(1-\sigma) - f(\theta_t)] E_t H_{t+1} \\
c &= q(\theta_t) \beta E_t J_{t+1} \\
H_t &= \varphi (H_t + J_t)
\end{aligned}$$

Find analytical solutions for question 1-2. Please motivate your answer clearly.

1. Solve for the steady state competitive equilibrium of the model:

   (a) Find the implicit function for the steady state expression for vacancy creation (as a function of parameters as well as of $q(\theta)$ and $f(\theta)$).

(b) Find the implicit function for the steady state unemployment rate.

2. Consider the social planner's problem.

$$\max_{\{V_t, N_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t E_t \left((z_t - b)N_t - cV_t\right)$$

s.t.

$$N_{t+1} = (1-\sigma) N_t + M_t$$
$$U_{t+1} = 1 - N_{t+1}$$
$$M_t = \gamma V_t^{1-\lambda} U_t^{\lambda}$$

(a) Find the socially optimal vacancy creation and unemployment rate.

(b) Compare the socially optimal allocation with the competitive equilibrium result. Explain the difference.

3. Use Dynare to solve for the dynamics of the competitive equilibrium.

Use log-linear approximations (write variables as exp(variable)). A Dynare example code is on Mondo.

z_t is a stochastic process with the form:

$$\log(z_{t+1}) = \rho \log(z_t) + e_t$$

where

$$e_t \sim \mathcal{N}(0, 0.009)$$

In addition to the core equations above, you might want to use

$$\begin{aligned} M_t &= \gamma V_t^{1-\lambda} U_t^{\lambda} \\ N_t &= (1-\sigma) N_{t-1} + M_{t-1} \end{aligned}$$

Note how in Dynare the time-subscript denotes the period in which a variable is determined, therefore e.g. N_t and not N_{t+1}. You can add definitions of auxiliary variables as needed. See the lecture notes for inspiration.

Use the following parameters:

$\beta = 0.998, \rho = 0.975, \sigma = 0.032, b = 0.71, \varphi = 0.5, \lambda = 0.5, c = 0.47$

Choose γ to get $U = 5\%$.

The frequency of the model is monthly.

(a) Compute the standard deviations of all variables, including a measure of average labor productivity (as in Shimer, 2005). Plot impulse response functions. Comment on the relative volatility of θ_t and average labor productivity (according to Shimer the former has 19 times larger standard deviation in the data).

(b) Consider an alternative specification where there are no vacancy costs by removing the equation $c = q(\theta_t)\beta E_t J_{t+1}$. Instead there are quadratic hiring costs, so that the marginal cost of hiring is cM_t. This implies $cM_t = \beta E_t J_{t+1}$. Re-calibrate c to $c_{new} = c_{old}\frac{1}{M_{ss,old}}$ to keep the steady state costs of hiring equal to the steady state costs of vacancy posting in the previous specification. Recompute the volatilities for this specification and plot impulse response functions. Compare to the standard DMP model with vacancy costs (in a) above. Point out any quantitative drawback of the hiring costs model compared to the DMP model.