# Exam Ph.D. Macroeconomics II

Department of Economics, Uppsala University

May 17, 2023

## Instructions

- Writing time: 5 hours.
- The exam is closed book.
- The exam has 76 points in total
- A passing grade requires a) at least 30 points on the exam, and b) 50 points in total for the course (incl the points you have from your problem sets).
- Start each question on a new paper. Write your anonymous code on all answer pages.
- You may write your solutions by pen or pencil; use your best handwriting.
- Answers shall be given in English.
- Motivate your answers carefully; if you think you need to make additional assumptions to answer the questions, state them.
- If you have any questions during the exam, you may call me (+46 730 606 796) at any time between 10 AM and noon.

# 1 Short Questions (4 points each)

Answer the question and provide a short explanation, *emphasizing economic intuition.*

1. The McCall model predicts that match quality decreases when the separation rate increases - True or False?

2. The basic Burdett-Mortensen model predicts that firm size is negatively correlated with the quit rate - True or False?

3. In the vanilla NK model, the equilibrium is generally inefficient. Why?

4. Consider the steady state of the vanilla RBC model. In $\{K, R\}$-space (where $K$ is the level of capital, and $R$ is the rental rate of capital), draw the demand and supply curve for capital. What is the elasticity of capital demand w.r.t. to the rental rate?

# 2 Cost-push shocks in the New-Keynesian Model (15 points)

Consider the vanilla New-Keynesian model that we have covered in class, but in which we now allow for shocks to firm optimal markups, usually referred to as "cost-push" shocks (such shocks can be micro-founded by, e.g., shocks to the elasticity of substitution in the demand function). The log-linearized equilibrium is characterized by the following set of equations:

$$\text{Intratemporal hh optimality:} \quad \hat{\omega}_t = \hat{c}_t + \varphi \hat{n}_t \tag{1}$$

$$\text{Intertemporal hh optimality:} \quad \hat{c}_t = -(\hat{i}_t - E_t \pi_{t+1}) + E_t \hat{c}_{t+1} \tag{2}$$

$$\text{Firm optimality:} \quad \pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t + \nu_t \tag{3}$$

$$\text{Marginal cost:} \quad \widehat{mc}_t = \hat{\omega}_t \tag{4}$$

$$\text{Goods clearing:} \quad \hat{c}_t = \hat{y}_t \tag{5}$$

$$\text{Labor clearing:} \quad \hat{y}_t = \hat{n}_t \tag{6}$$

$$\text{Policy:} \quad \hat{i}_t = \phi \pi_t \tag{7}$$

where $\lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}$, $\hat{\omega}_t = \hat{w}_t - p_t$ denotes log deivations in the real wage, and the shock process is AR(1): $\nu_t = \rho \nu_{t-1} + \epsilon$.

Figure 1 contains the IRFs to a positive cost push shock with $\rho = 0.5$. The other parameters take the same value as in class: $\varphi = 1, \beta = 0.99, \theta = 2/3, \phi = 1.5$.

1. Explain the sign of all responses for all variables displayed in Figure 1.

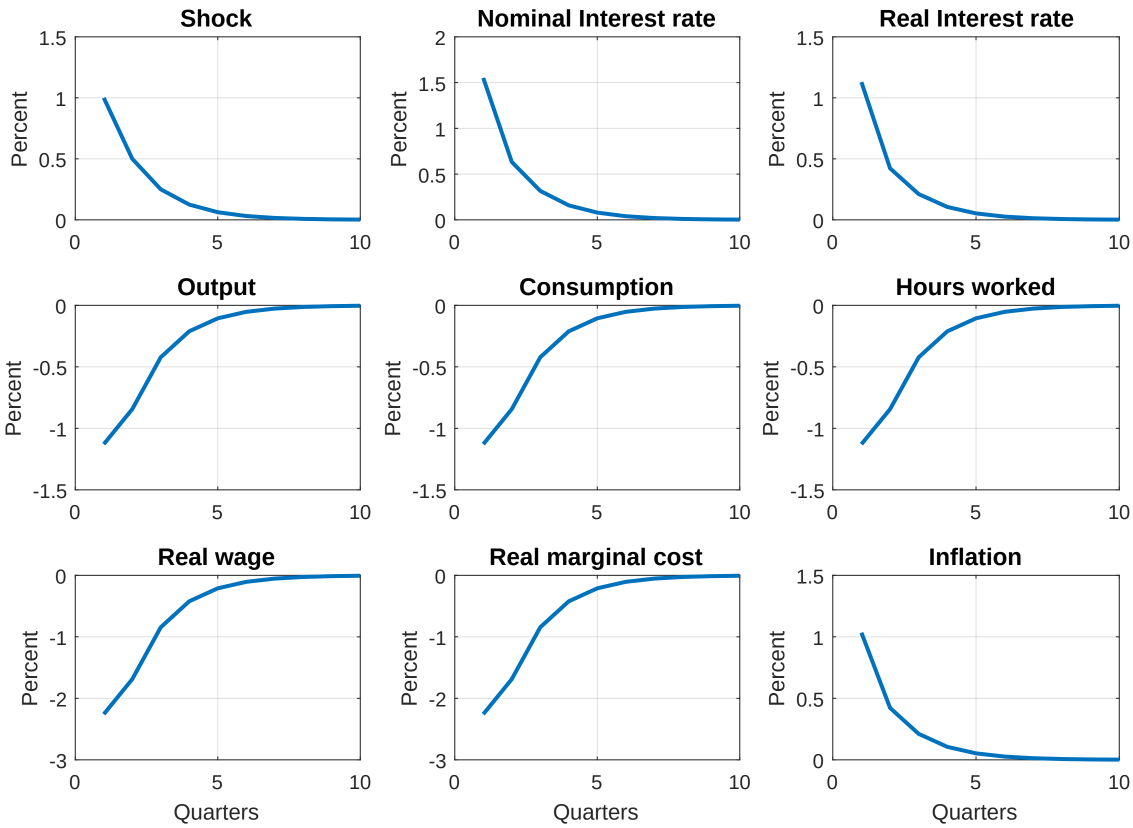


Figure 1: IRFs to a cost push shock in the vanilla NK model

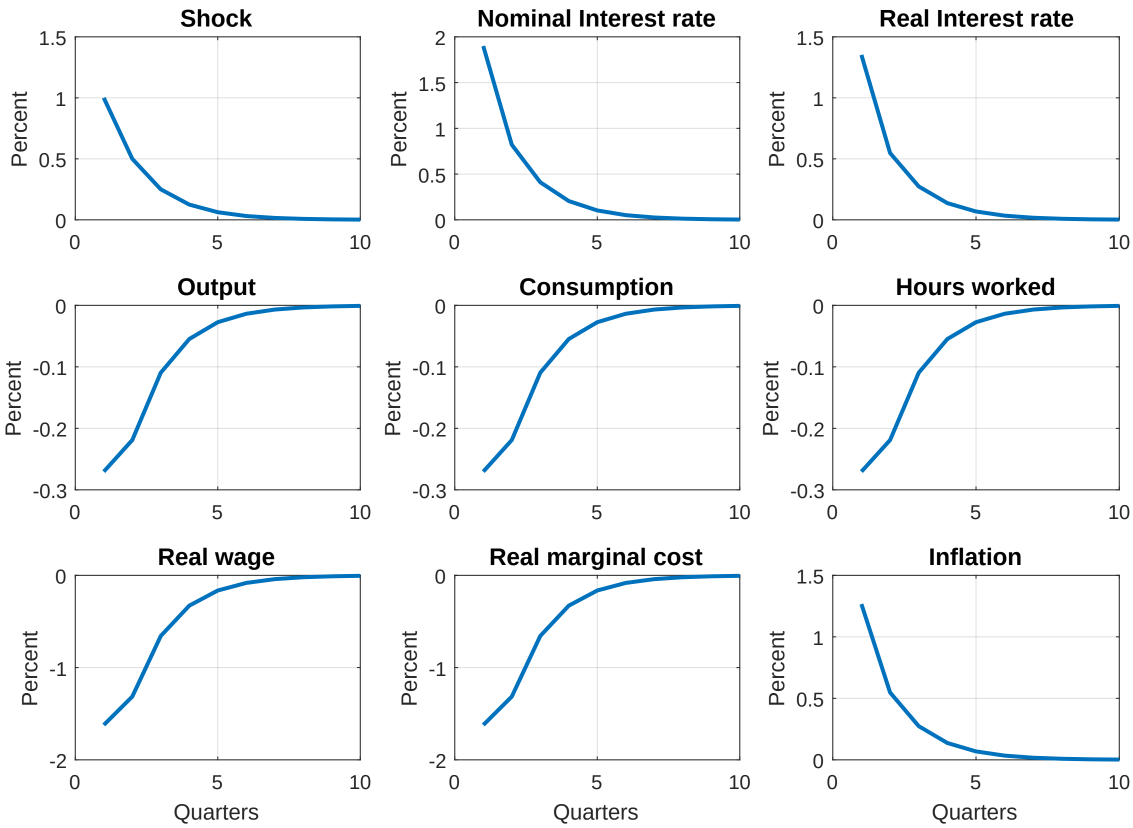


Figure 2: IRFs to a cost push shock in the NK model with $\sigma = 5$

2. Suppose that instead of having log utility in the household problem, we assume a CRRA utlity function $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$. How does the equilibrium characterization change in this case?

3. Figure 2 contains the IRFs to a positive cost push shock with the CRRA utility function and where we set $\sigma = 5$. Explain why the response of consumption, the real and nominal interest rates, real wages and inflation change in the way they do relative to Figure 1.

# 3 Unemployment on the Coconut Island (25 points)

Consider the following environment: Time is continuous. There is a continuum of agents of mass one on an island. The island is composed of a forest and a beach. The forest is where coconut trees grow, while the beach is where people meet. Coconuts are the only good in the economy. All coconuts are the same, and each agent gets utility $y$ from consuming a coconut. Agents discount utility at rate $r$.

Production is done by climbing at coconut trees, which are of various height. Agents without coconuts walk in the forest and look for trees. They are said to be unemployed. When they are unemployed, they randomly bump into coconut trees at exogenous rate $\alpha$. When climbing a tree, they incur a utility cost $c$, drawn from an i.i.d. distribution with CDF G(c), defined over $[\underline{c}, \bar{c}]$, with $\underline{c} > 0$, reflecting that higher trees are more costly to climb. One cannot collect or carry more than one coconut.

There is a taboo on this island, according to which one cannot eat self-collected coconuts. Therefore, once a coconut is collected, an agent has to walk on the beach carrying her coconut, and she will randomly meet another agent with a coconut. When walking in search for a trade partner, agents are said to be employed. When a meeting happens, the pair of agents that meets exchange their coconuts (one against one) and eat them. We denote by $e$ (employment) the measure of agents that are holding a coconut and looking for a partner. $e \in [0, 1]$ is endogenous. Meeting someone else on the beach happens at rate $\beta$, which is a function of employment, $\beta = \beta(e)$. It is assumed that $\beta(e)$ is increasing and concave, and that $\beta(0) = 0$.

The life of an agent can therefore be described as follows: once agents have a coconut, they simply walk on the beach until they meet another agent with a coconut, and trade. Without a coconut, they walk in the forest until they run into the coconut tree, and then they decide whether to collect the coconut given the cost.

We will only be concerned with the steady state of this island economy.

1. The assumption that $\beta(e)$ is increasing implies a so called "thick market externality". Explain why, especially the "externality" part.

2. Let $V_E$ and $V_U$ denote the value of being employed and unemployed, respectively. The Bellman equation

for an employed agent is given by

$$rV_E(e) = \beta(e)\left(y + (V_U(e) - V_E(e))\right). \tag{8}$$

Explain in words what this equation says.

3. Argue that optimizing unemployed agents follow a *reservation cost strategy*: they will only climb the tree they bump into if the cost of doing so is smaller than some reservation cost $c_R(e)$. Why does the reservation cost depend on $e$?

4. Write the Bellman equation for an unemployed agent.

5. Show that $c_R(e)$ satisfies

$$\beta(e)y - (\beta(e) + r)c_R(e) = rV_U(e). \tag{9}$$

6. Using the previous results, solve for an equation that implicitly solves for $c^R(e)$. Let's name this equation the *reservation cost equation*.

7. Using the reservation cost equation, show that $c^R(e)$ is increasing and concave in $e$ whenever $y > c_R(e)$, and that $c_R(0) = 0$. What is the intuition for the "increasing" part?

   For answering this question you might find Leibniz' rule useful. Recall that this rule says that if the functions $f(x,t), \alpha(t), \beta(t)$ are differentiable in $t$, the function

$$\phi(t) = \int_{\alpha(t)}^{\beta(t)} f(x,t)dg(x)$$

   is differentiable, and

$$\phi'(t) = f(\beta(t),t)\frac{dg(\beta(t))}{dt} - f(\alpha(t),t)\frac{dg(\alpha(t))}{dt} + \int_{\alpha(t)}^{\beta(t)} f_t(x,t)dg(x).$$

8. Write the law of motion for employment in this economy, and solve for the steady state value of employment in terms of $\alpha$ and $\beta(e)$. Let's name this steady state relation to the "Beveridge curve".

9. The "Beveridge curve" implies yet another relation between $c^R$ and $e$, which is increasing and convex. By drawing an appropriate graph, show that the model either has zero or two steady state equilibria; in the latter case, one equilibrium is associated with low emplyment and the other with high employment. Explain how this can be the case.

## A two-period Aiyagari model (20 points)

Consider an economy with a continuum (measure 1) of ex-ante identical households, each living for two periods. Each household $i$ has utility given by

$$\log(c_{i1}) + \beta E \log(c_{i2}) \tag{10}$$

where $c_{i1}, c_{i2}$ are period 1 and 2 consumption, $\beta$ is the discount factor and $E$ is the expected value operator. In period 1, each household is endowed with $y_1$ units of output that can either be consumed, $c_{i1}$, or invested, $k_i$. In period 2, households recieve income from the capital they saved in period 1 and from wages earned from supplying $l_i$ efficiency units of labor. $l_i$ is a random variable, i.i.d. across households and equals $1+\epsilon$ with probability 1/2 and $1-\epsilon$ with probability 1/2, with $0 < \epsilon < 1$. The law of large numbers implies that the aggregate efficiency units of labor supply $L = 1$. In period 2, output is produced by a competitive representative firm which operate a Cobb-Douglas production function $K^\alpha L^{1-\alpha}$, renting capital and labor services from the households at rate $r$ and $w$, respectively.

1. Write the household and firm problems and define a competitive equilibrium for this economy.
2. For the case $\epsilon = 0$ (no uncertainty), solve for the equilibrium level of capital and interest rate.
3. For the case $\epsilon > 0$, solve for the equilibrium level of capital and interest rate and show that the interest rate is decreasing in $\epsilon$.
4. Explain the intuition for why the interest rate is decreasing in $\epsilon$.
5. For the case $\epsilon > 0$, define individual consumption risk as the ratio between individual period 2 consumption in the "good" state and the "bad" state. Show that a) household $i$ percieves that its individual consumption risk is lower if it chooses a higher $k_i$ and b) that, in equilibrium, household $i$'s individual consumption risk is not affected by its choice of $k_i$. Explain how this can be case.
6. For the case $\epsilon > 0$, define the ex-ante social welfare function as

$$W = \int_{i=0}^{1} \log(c_{i1}) + \beta \log(c_{i2}) di$$

Show that, when evaluating $W$ at the decentralized equilibrium allocation, $W$ would increase if all households where to save a little bit less (while maintaining that markets clear). Explain the intuition for this result and how it relates to the notion of "constrained efficiency". Hint: compute $\frac{\partial W}{\partial K}$ using the fact the households behave individually optimally at the equilibrium allocation.