Duration dependence

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Outline

1. Importance of duration dependence
2. Basics of duration analysis
3. Non-parametric decomposition
4. Parametric models: Mixed proportional hazard model

Importance of duration dependence

Fast dynamics

- all workers find a job at rate f
- all worker lose their job at rate x
- evolution of unemployment rate: $\dot{u}(t) = (1 - u(t))x - u(t)f$
- fast dynamics
  - $u(t) = \exp(-(f + x)t)u_0 + (1 - \exp(-(f + x)t))\frac{x}{f+x}$
  - $\exp(-(f + x)t) = \frac{1}{2} \to t = \frac{2}{f+x} \approx 1$ month

Counterfactual implications

- empirically, unemployment is much more persistent
  - this reflects persistence in f_t and x_t, $u_t \approx x_t/(f_t + x_t)$
- empirically, individual unemployment is much more persistent
  - this can't be easily squared with the model

Krueger-Cramer-Cho (2014)

- three state model, E, U, O
- current duration predicts future employment status
- unemployment is very persistent in the data
- compare short-term and long-term unemployed, 2008–2013

		A^{15}	short-term	long-term
	E	0.527	0.495	0.359
month $t + 15$	U	0.051	0.233	0.304
	I	0.422	0.271	0.337

Duration analysis

Definition of hazard rate

- consider time to exit of a continuous variable
  - exit unemployment, exit employment, price change
- T – duration of stay in the state
  - restarted when a person enters the state; not a calendar time
  - T is a random variable
- $f(t), F(t)$ – pdf, cdf of T
- hazard rate
  - probability that a person who occupies the state at time t leaves within dt

$$h(t) = \lim_{dt \to 0} \frac{P(t \leq T < t + dt | T \geq t)}{dt}$$

- interpretation $f(t)$ versus $h(t)$
  - $f(45)$ – probability that a worker finds a job at age 45
  - $h(45)$ – probability that 45-year old worker finds a job

Useful formulas for hazard rate

- using Bayes rule

$$P(t \le T < t + dt | T \ge t) = \frac{P(t \le T < t + dt, T \ge t)}{P(T \ge t)} = \frac{P(t \le T < t + dt)}{P(T \ge t)}$$
$$= \frac{F(t + dt) - F(t)}{1 - F(t)}$$

- divide by dt, take the limit

$$h(t) = \frac{f(t)}{1 - F(t)} = \frac{f(t)}{S(t)} \tag{1}$$

- $S(t)$ – survivor function; prob. of surviving until t
- from (1), we get a differential equation

$$h(t) = -d \log(S(t))/dt \tag{2}$$

- solve differential equation (2) with boundary condition $S(0) = 1 - F(0) = 1$

$$S(t) = \exp\left(-\int_0^t h(s)\right) ds, \qquad f(t) = h(t) \exp\left(-\int_0^t h(s)\right) ds \tag{3}$$

Constant hazard: exponential distribution

- important distribution in duration analysis, often a benchmark
- no duration dependence

$$f(t;\theta) = \theta e^{-\theta t}, \quad F(t;\theta) = 1 - e^{-\theta t}$$

- constant hazard rate

$$h(t;\theta) = \theta$$

- economic examples: DMP search model, Calvo price setting

Non-constant hazard rates

- negative duration dependence : decreasing hazard rate
  - human capital depreciation
  - statistical discrimination from employers
- positive duration dependence : increasing hazard rate
  - menu cost models: hazard rate of adjusting price is increasing in time since the last price adjustment
- hump-shaped hazard rate
  - job separation in a model with learning about match quality
  - stopping time model with positive switching costs

Empirical hazard rate

- Kaplan-Meier nonparametric method
- N – sample size
- $t_1 \leq t_2 \leq \ldots \leq t_N$ – realized durations
- $n(t)$ – number of people at risk at duration t
- $d(t)$ – number of separations at duration t

$$\hat{S}(t) = \sum_{t_i \geq t} d(t_i)$$

$$\hat{h}(t) = \frac{d(t)}{\hat{S}(t)}$$

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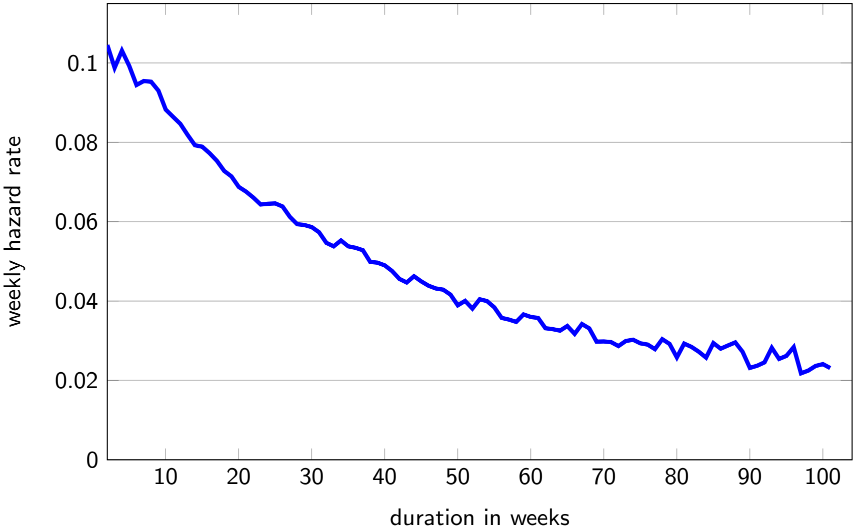
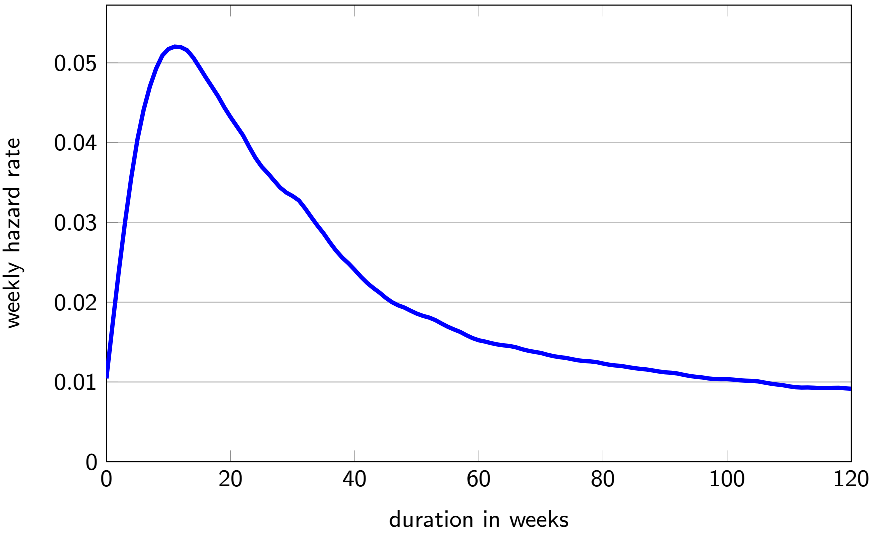
- typically, empirical hazard rate in economic applications is decreasing or hump-shaped

Price Change Hazard: Coffee



Non-employment Exit Hazard



Why is empirical hazard rate decreasing?

- structural negative duration dependence
- spurious negative duration dependence
  - aggregation of heterogenous population
  - heterogeneity ALWAYS biases empirical hazard rate down

Heterogeneity vs structural duration dependence

- $\theta \sim G(\theta)$ – (unobserved) individual heterogeneity
- $h(t;\theta)$ – hazard rate of individual θ at duration t
- $G(\theta; t)$ – distribution of types at duration t
- measured aggregate hazard rate

$$H(t) = \int h(t;\theta) dG(\theta; t)$$

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$$H(t) = \int h(t;\theta) dG(\theta;t)$$

- decreasing aggregate hazard rate
  - structural DD: $h(t;\theta)$ depends on t, $G(\theta)$ is degenerate
  - heterogeneity: $h(t;\theta) = \bar{h}(\theta)$, $G(\theta)$ is non-degenerate
  - combination of the two

Heterogenous population with constant hazards

- $\theta \sim G(\theta)$ – (unobserved) individual heterogeneity, non-degenerate
- $h(t;\theta) = \bar{h}(\theta)$ – hazard rate of individual θ at duration t
- $N(\theta;t)$ – number of workers of type θ at duration t
- it holds

$$\frac{dN(\theta;t)}{dt} = -h(t;\theta)N(\theta;t) \Rightarrow \frac{\dot{N}(\theta;t)}{N(\theta;t)} = -h(t;\theta)$$

- dynamic selection
  - at any duration, workers with a higher hazard are more likely to exit
  - number of these people among survivors declines faster
  - distribution of types shifts toward workers with a lower hazard
  - hence $H(t)$ declines with t

Non-parametric decomposition

Two sources of persistence

- long-term unemployed less likely to find jobs than short-term
  - empirical literature mostly focused on this
- newly employed more likely to lose jobs than long-term workers
  - also important

Two sources of duration dependence

- structural: declining hazard for each worker
- heterogeneity: dynamic selection of workers
- two main questions
    - how to decompose these two stories?
    - what is the source of structural duration dependence?

Two sources of duration dependence

- structural: declining hazard for each worker
- heterogeneity: dynamic selection of workers
- two main questions
  - how to decompose these two stories?
  - what is the source of structural duration dependence?
- Alvarez, Borovičková, Shimer (2015): A Nonparametric Variance Decomposition Using Panel Data
  - how much can we say without a structural model?
- Alvarez, Borovičková, Shimer (2015): The Proportional Hazard Model: Estimation and Testing
  - leading model in duration analysis, we propose a test using multispell data

Notation

- workers $i = 1, \ldots I$
- distribution of duration of a spell is a random variable $F_i(t)$
  - individual mean $\mu_i = \int_0^\infty t f_i(t) dt$
  - individual variance $\sigma_i^2 = \int_0^\infty (t - \mu_i)^2 f_i(t) dt$
- suppose we knew F_i for all i
- could compute:
  - population mean duration: $\bar{\mu} = \frac{1}{I} \sum_i \mu_i$
  - population variance: $\bar{\sigma}^2 = \frac{1}{I} \sum_i \int_0^\infty (t - \bar{\mu}) f_i(t) dt$
  - within-worker variance: $\bar{\sigma}_w^2 = \frac{1}{I} \sum_i \sigma_i$
  - between-worker variance: $\bar{\sigma}_b^2 = \frac{1}{I} \sum_i (\mu_i - \bar{\mu})^2$
- exact variance decomposition, total $=$ within $+$ between

Estimating the decomposition

- suppose we only observe $n = 2$ spells per worker, durations $(t_{i,1}, t_{i,2})$
  - use this to perform the variance decomposition
- unbiased estimators
  - individual mean: $\hat{\mu}_i = \frac{1}{2}(t_{i,1} + t_{i,2})$
  - individual variance: $\hat{\sigma}_i^2 = \frac{1}{n-1}\sum_j (t_{i,j} - \hat{\mu}_i)^2 = \frac{1}{2}(t_{i,1} - t_{i,2})^2$
- compute
  - population mean: $\hat{\hat{\mu}} = \frac{1}{I}\sum_i \hat{\mu}_i$
  - population variance: $\hat{\hat{\sigma}}^2 = \frac{1}{2I-1}\sum_i \left((t_{i,1} - \hat{\hat{\mu}})^2 + (t_{i,2} - \hat{\hat{\mu}})^2\right)$
  - within-worker variance: $\hat{\hat{\sigma}}_w^2 = \frac{1}{I}\sum_i \hat{\sigma}_i^2$
  - between-worker variance: $\hat{\hat{\sigma}}_b^2 = \hat{\hat{\sigma}}^2 - \hat{\hat{\sigma}}_w^2$

Variance decomposition

- so far unconditional variance decomposition
- we can perform this decomposition conditional on duration in both spells exceeding $\underline{t}$

Interpretation of variance decomposition

Constant hazard benchmark

- constant hazard $h > 0$
- distribution of spells: $F_i(t) = 1 - e^{-ht}$ for all i and $t \geq 0$
  - mean duration of a spell: $\mu_i = \bar{\mu} = 1/h$
  - standard deviation: $\sigma_i = \bar{\sigma} = 1/h$
  - coefficient of variation: $\sigma_i/\mu_i = 1$
- result from statistics (for example, Barlow, Proschan, Hunter(1965))
- hazard rate $h(t)$: $h'(t) \leq 0$ and $h'(t) \neq 0$ for all t
  - $\sigma_i > \mu_i$
- given mean duration of a spell, a decreasing hazard rate boosts the variance of the spell
- opposite result for an increasing hazard rate

Excess within variance

- within variance

$$\bar{\sigma}_w^2 = \frac{1}{I}\sum_{i=1}^{I}\sigma_i^2$$

- define "constant-hazard within variance" – the within variance in the benchmark model (this is because $\sigma_i = \mu_i$)

$$\bar{\sigma}_c^2 = \frac{1}{I}\sum_{i=1}^{I}\mu_i^2$$

- define excess within variance

$$\bar{\sigma}_e^2 = \bar{\sigma}_w^2 - \bar{\sigma}_c^2 = \frac{1}{I}\sum_{i=1}^{I}(\sigma_i^2 - \mu_i^2)$$

- estimates

$$\hat{\bar{\sigma}}_c^2 = \frac{1}{I}\sum_{i=1}^{I}(\sigma_i^2 - \mu_i^2)$$

Excess within variance – continued

- constant hazard model: $\sigma_e^2 = 0$
- decreasing hazard model: $\sigma_e^2 > 0$
- increasing hazard model: $\sigma_e^2 < 0$

Excess within variance – continued

- constant hazard model: $\sigma_e^2 = 0$
- decreasing hazard model: $\sigma_e^2 > 0$
- increasing hazard model: $\sigma_e^2 < 0$
- we can do the decomposition of $\log(t)$
- all formulas the same, except for σ_c^2
- $Var[\log(t)] = \frac{\pi^2}{6}$, does not depend on h, and thus

$$\bar{\sigma}_c^2 = \frac{\pi^2}{6}, \quad \bar{\sigma}_e^2 = \bar{\sigma}_w^2 - \frac{\pi^2}{6}$$

Excess within variance – continued

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- calculation more complicated if we allow for heterogeneity in h but the conclusion above is still valid

Correlation of two spells

- ratio σ_b^2/σ^2 is the same as correlation of t_1 and t_2
  - it is enough to show that $cov(t_1, t_2) = \sigma_w^2$
- $corr(t_1, t_2) = \frac{\sigma_b^2}{\sigma^2} \geq 0$
  - implication: in the mixture model, the correlation cannot be negative

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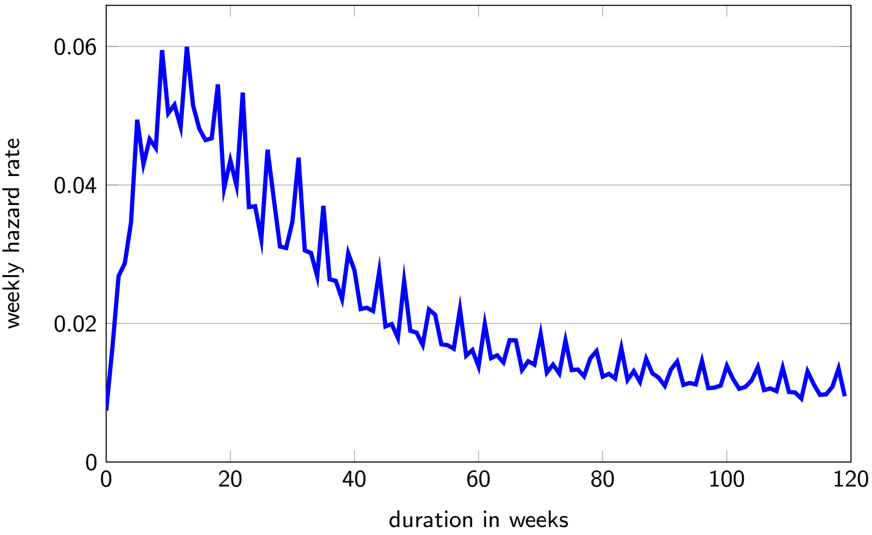
  - implication: in the mixture model, the correlation cannot be negative
- we assumed that the draws from spells are independent
- what drives the correlation? heterogeneity
- draws are independent conditional on type
- positive correlation is a sign of heterogeneity
- what if you measure a negative correlation in the data? then it cannot be a mixture model

Results

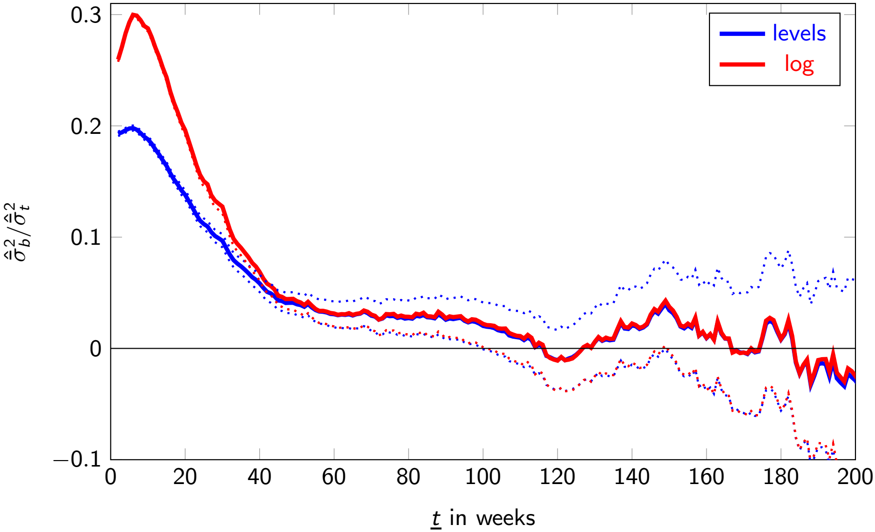
Austrian data

- universe of private sector employees, 1986–2007
  - observe employment, unemployment, retirement, maternity leave
  - full-time, part-time, "marginal" jobs
  - start and end date for each spell
- definition of non-employment spell:
  - end of one full-time job to start of next full-time job, in weeks
  - registered at Public Employment Service office for at least one day
- age criteria: no older than 45 in 1986, no younger than 40 in 2007, at least 25 at the beginning of spells

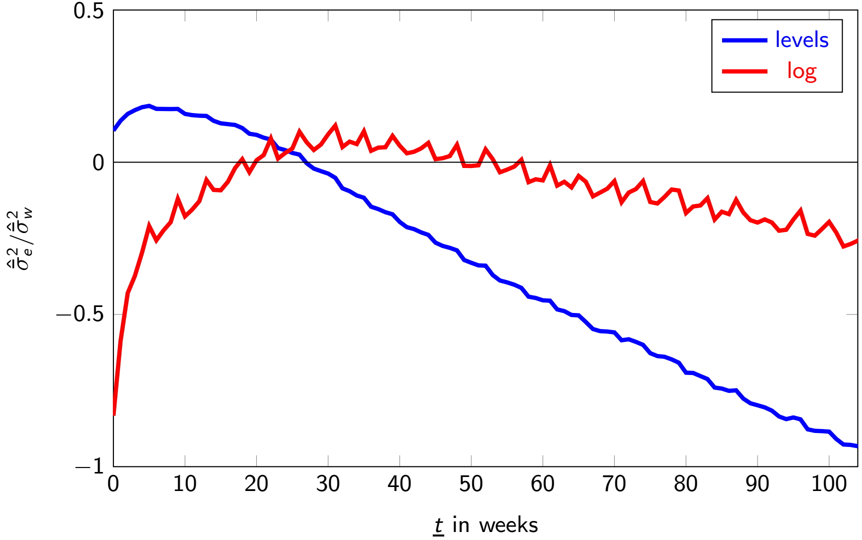
Non-employment exit hazard rate



Variance decomposition: between-total variance ratio



Excess variance: excess-within variance ratio



Extension

- we can decompose t^k for $k \neq 1$
  - larger k focuses on longer durations
  - $k \to 0$ is log duration
- within-between decomposition is unchanged
- for $k > \frac{1}{2}$, constant hazard gives within variance $C(k)$ time mean-squared

$$C(k) = \frac{\Gamma(2k+1)}{\Gamma(k+1)^2} - 1$$

Generalized decomposition

- mean: $\hat{\hat{\mu}} = \frac{1}{2I} \sum_i \left(t_{i,1}^k + t_{i,2}^k \right)$
- population variance: $\hat{\hat{\sigma}}^2 = \frac{1}{2I-1} \sum_i \left(t_{i,1}^{2k} + t_{i,2}^{2k} - 2\hat{\hat{\mu}}^2 \right)$
- within variance: $\hat{\hat{\sigma}}_w^2 = \frac{1}{2I} \sum_i \left(t_{i,1}^k - t_{i,2}^k \right)^2$
- between variance: $\hat{\hat{\sigma}}_b^2 = \hat{\hat{\sigma}}^2 - \hat{\hat{\sigma}}_w^2$
- constant hazard within variance: $\hat{\hat{\sigma}}_c^2 = \frac{C(k)}{n} \sum_i t_{i,1}^k t_{i,2}^k$
- excess within variance: $\hat{\hat{\sigma}}_e^2 = \hat{\hat{\sigma}}_w^2 - \hat{\hat{\sigma}}_c^2$

Interpretation

- our intuition (not proven though!)
- constant hazard: $\hat{\hat{\sigma}}_e^2 = 0$ for all k
- decreasing hazard: $\hat{\hat{\sigma}}_e^2 > 0$ for all k
- increasing hazard: $\hat{\hat{\sigma}}_e^2 < 0$ for all k
- hump-shaped hazard: there is k for which $\hat{\hat{\sigma}}_e^2$ changes the sign

Proportional hazard model

Proportional hazard model

- goal: what is the role of structural duration dependence and heterogeneity
- answer this in a non-structural way
- often used in the literature
  - job-finding rate (many papers...)
  - price changes (Nakamura and Steinsson, 2010)

Model structure

- population of measure 1
- assume $h(t;\theta,x)=\theta\psi(x)h(t)$
  - unemployment exit hazard of an individual (θ,x)
  - $h(t)$ is the unknown baseline hazard rate
  - $\psi(x)$ is an unknown function of observable characteristics x
  - θ unobservable characteristics, unknown density g

Problem with identification

- suppose there are no observables: $h(t; \theta, x) = \theta h(t)$
- data: individual unemployment duration
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- model 1: $h(t) = 1$ for all t, $\theta \sim g$
  - CDF of spells: $F(t; \theta) = 1 - e^{-\theta t}$
  - observe: $F(t) = \int (1 - exp(-\theta t)) \, g(\theta) d\theta$
  - empirical hazard rate:

$$h^{emp}(t) = \frac{f(t)}{1 - F(t)} = \frac{\int \theta exp(-\theta t) g(\theta) d\theta}{\int exp(-\theta t) g(\theta) d\theta}$$

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$$h^{emp}(t)=\frac{f(t)}{1-F(t)}=\frac{\int\theta exp(-\theta t)g(\theta)d\theta}{\int exp(-\theta t)g(\theta)d\theta}$$

- model 2: no heterogeneity, $h(t)=h^{emp}(t)$
- no way you can tell these models apart with one spell

Identification of the proportional hazard model

- how to estimate the model?
  1. parametric assumptions on $g(\theta)$
  2. parametric assumptions on $F_i(t)$
  3. observables x (Elbers-Ridders(1982))
  4. data on two spells (Honore(1993))
- Heckman-Singer (1984)
  - assumptions on g affect shape of $h(t)$

Identification with multiple spells

Identification with multiple spells

- based on Honoré (*REStud* 1993)
- no observables: $h(t;\theta) = \theta h(t)$
- $\theta \sim G$
- probability that a spell ends before t

$$F(t;\theta) \equiv 1 - e^{-\theta \int_0^t h(\tau)d\tau}$$

- t_1, t_2 – two spells for each individual
- individual: (t_1, t_2) independent draws from $F(\cdot;\theta)$

Survivor function

- survivor function

$$\Phi(t_1, t_2) = \int e^{-\theta(Z(t_1)+Z(t_2))} dG(\theta)$$

- $Z(t)$ is the *integrated* baseline hazard

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$$Z(t) \equiv \int_0^t h(\tau) d\tau$$

- differentiate with respect to t_1 and t_2

$$\begin{aligned}
\Phi_1(t_1, t_2) &= -h(t_1) \int \theta e^{-\theta(Z(t_1)+Z(t_2))} dG(\theta) \\
\Phi_2(t_1, t_2) &= -h(t_2) \int \theta e^{-\theta(Z(t_1)+Z(t_2))} dG(\theta)
\end{aligned}$$

Nonparametric identification

- take the ratio

$$\frac{h(t_1)}{h(t_2)} = \frac{\Phi_1(t_1, t_2)}{\Phi_2(t_1, t_2)}$$

- RHS is the data
- this equation identifies $h(t)$ up to scale
  - normalization: $h(0) = 1$ or $\int \theta g(\theta) d\theta = 1$

Nonparametric identification – continued

- how to get the distribution g?
- definition of a Laplace transform for function f

$$L_f(s) = \int_0^{\infty} e^{-st} f(t) dt$$

- survivor function is a Laplace transform of g

$$\Phi(t_1, t_2) = \int e^{-\theta(Z(t_1)+Z(t_2))} dG(\theta) = L_g(Z(t_1) + Z(t_2))$$

- you can get g by inverting Laplace transform

Nonparametric identification – continued

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- you can get g by inverting Laplace transform
- does not work well in practice, inverting Laplace transform is not numerically stable, it oscillates

Identification with observables

Identification with observables

- now assume $h(t;\theta,x) = \theta\psi(x)h(t)$
- let $S(t,x)$ denote the share of individuals for whom
  - first spell lasts at least t periods
  - observable characteristic is x

$$S(t,x) = \int \exp\left(-\theta\psi(x)\int_0^t h(s)ds\right) g(\theta)d\theta$$

- differentiate with respect to t

$$S_t(t,x) = \psi(x)h(t)\int \theta \exp\left(-\theta\psi(x)\int_0^t h(s)ds\right) g(\theta)d\theta$$

- evaluate at $t = 0$:

$$S_t(0,x) = -\psi(x)h(t)\int \theta g(\theta)d\theta$$

- normalize: $h(0) = \int \theta g(\theta)d\theta = 1$ to identify $\psi(x)$

Identification with observables

- differentiate with respect to x

$$S_x(t,x) = -\psi'(x)\int_0^t h(s)ds \int \theta \exp\left(-\theta\psi(x)\int_0^t h(s)ds\right) g(\theta)d\theta$$

- take ratio

$$\frac{S_t(t,x)}{S_x(t,x)} = \frac{\psi(x)h(t)}{\psi'(x)\int_0^t h(s)ds}$$

- ordinary differential equation for $y(t) \equiv \int_0^t h(s)ds$

$$\frac{S_t(t,x)}{S_x(t,x)}\frac{\psi'(x)}{\psi(x)} = \frac{y'(t)}{y(t)}$$

- identification strategy from Elbers-Ridder(1982), and Heckman-Singer(1984)

Remarks

- identification with two spells works the same way in discrete time
- identification with observables only works in continuous time

Nonparametric testing with multiple spells

Nonparametric testing

- we derived
$$\frac{h(t_1)}{h(t_2)} = \frac{\Phi_1(t_1, t_2)}{\Phi_2(t_1, t_2)}$$
- testing: multiple ways of recovering $h(t)$
- ratio of above expressions evaluated at (t_1', t_2) and (t_1, t_2)
$$\Psi(t_1, t_1'; t_2) \equiv \frac{\Phi_1(t_1, t_2)\Phi_2(t_1', t_2)}{\Phi_2(t_1, t_2)\Phi_1(t_1', t_2)} = \frac{h(t_1)}{h(t_1')}$$
- the right hand-side does not depend on t_2
- we use this to test the model

Hazard rate dominance

- consider a population whose first spell lasts exactly t_1
- denote the hazard rate at duration t_2 their second spell by $H(t_2|t_1)$
- Theorem: In the MPH, $H(t_2|t_1)$ is decreasing in t_1.
  - the baseline hazard in the first and second spell do not have have to be the same
  - result is driven by separability between θ and $h(t)$, and dynamic selection of θ
- easy to investigate in multi-spell data

Sampling framework

- need survivor data $\Phi(t_1, t_2)$
- measure hazard rate till some pre-specified T

Sampling framework

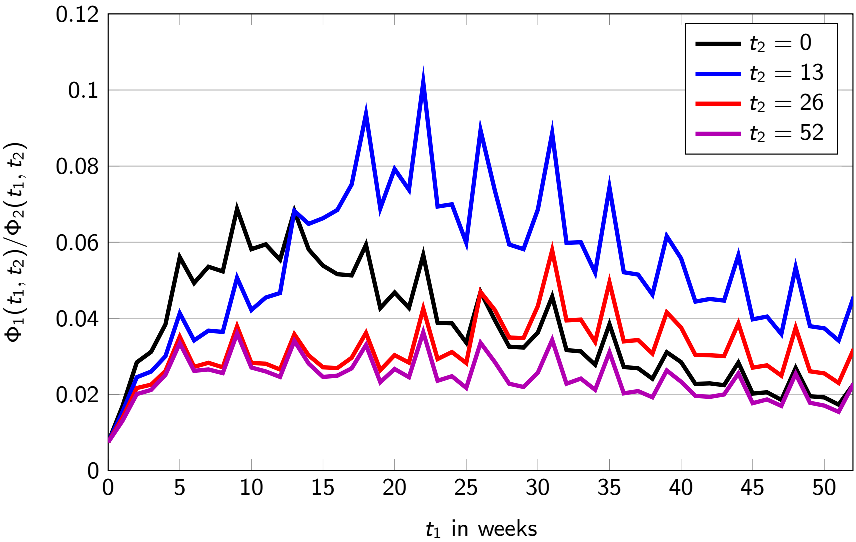
- need survivor data $\Phi(t_1, t_2)$
- measure hazard rate till some pre-specified T
- start with all products in the dataset for at least $2T - 1$ periods
- t_1 duration of the first spell, top-coded at T
  - if $t_1 < T$, then t_2 is the second spell top-coded at T
    - $n(t_1, t_2)$ – number of products with duration (t_1, t_2)
  - if $t_1 \geq T$
    - $n(T, \cdot)$ – number of products with first spell lasting at least T

Sampling framework - continued

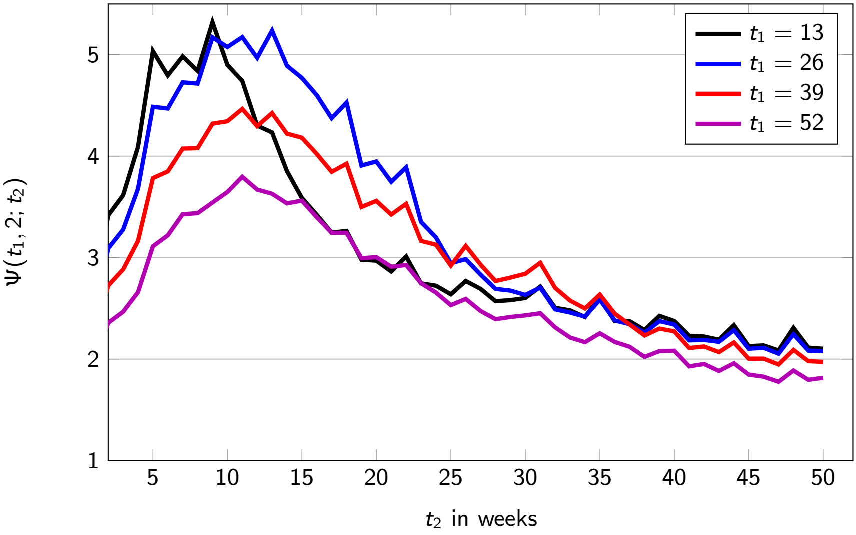
- our measure of number of spells $N(t_1, t_2)$ for all $(t_1, t_2) \in \{1, 2, \dots T\}^2$
- use the symmetry
  - $t_1 < T, t_2 < T$: $N(t_1, t_2) = (n(t_1, t_2) + n(t_2, t_1))/2$
  - $t < T$: $N(t, T) = N(T, t) = n(t, T)$
  - at T: $N(T, T) = n(T, \cdot) - \sum_{t<T} n(t, T)$
- define survivor function as

$$\Phi(t_1, t_2) = \frac{\sum_{\tau_1 \geq t_1, \tau_2 \geq t_2} N(\tau_1, \tau_2)}{\sum_{\tau_1 \geq 1, \tau_2 \geq 1} N(\tau_1, \tau_2)}$$

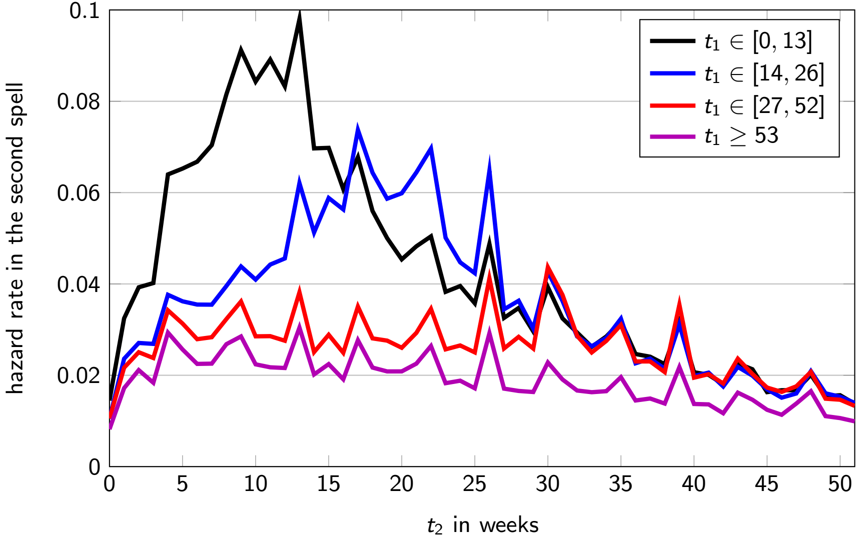
Non-employment exit rate – Hazard rates



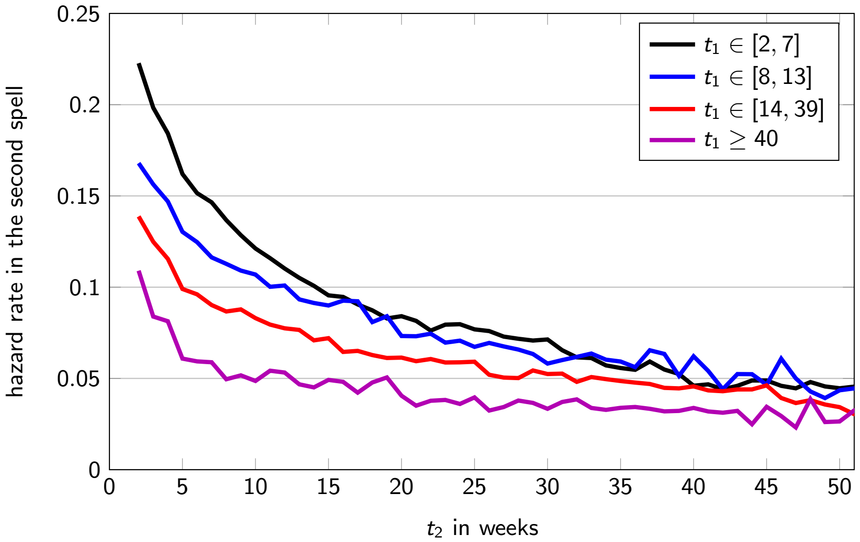
Non-employment Exit Rate – Hazard ratio



Hazard for second spell conditional on first – non-employment exit



Hazard for second spell conditional on first – price changes



Joint density of two spells