

$$\frac{c}{\lambda_v(\theta)} = \frac{(1-\gamma)(y-b)}{r+b+\gamma\lambda_u(\theta)}$$

(4)

Use $\lambda_v(\theta) = \frac{\lambda_u(\theta)}{\theta}$

$$\Rightarrow \frac{c\theta}{\lambda_u(\theta)} = \frac{(1-\gamma)(y-b)}{r+b+\gamma\lambda_u(\theta)}$$

Total differentiation

$$\frac{c\lambda_u(\theta) - c\theta\lambda_u'(\theta)}{[\lambda_u(\theta)]^2} d\theta + 0 dy$$

$$= \frac{(1-\gamma)(y-b)}{[r+b+\gamma\lambda_u(\theta)]^2} (-1)\gamma\lambda_u(\theta) d\theta + \frac{(1-\gamma)}{r+b+\lambda_u(\theta)} dy$$

$$\Leftrightarrow \frac{c}{\lambda_u(\theta)} \left[1 - \frac{\theta\lambda_u'(\theta)}{\lambda_u(\theta)} \right] d\theta$$

$$= \frac{(1-\gamma)(y-b)}{r+b+\gamma\lambda_u(\theta)} \left[-\frac{\gamma\lambda_u'(\theta)}{r+b+\gamma\lambda_u(\theta)} d\theta + \frac{1}{y-b} dy \right]$$

$$\frac{c\theta}{\lambda_u(\theta)}$$

(5)

$$\Leftrightarrow \left[1 - \frac{\theta \lambda_u'(\theta)}{\lambda_u(\theta)} \right] d\theta$$

$$= - \frac{r \lambda_u(\theta)}{r + b + r \lambda_u(\theta)} \frac{\theta \lambda_u'(\theta)}{\lambda_u(\theta)} d\theta + \frac{\theta}{Y} \frac{y}{y-b} dy$$

Notice: $\frac{\theta \lambda_u'(\theta)}{\lambda_u(\theta)} = \frac{\partial \lambda_u}{\partial \theta} \cdot \frac{\theta}{\lambda_u} = \epsilon_{\lambda_u, \theta}$

$$\Leftrightarrow \left[1 - \epsilon_{\lambda_u, \theta} + \frac{r \lambda_u(\theta) \epsilon_{\lambda_u, \theta}}{r + b + r \lambda_u(\theta)} \right] \underbrace{\frac{y}{\theta} \frac{d\theta}{dy}}_{\epsilon_{\theta, y}} = \cancel{\frac{\theta}{Y} \frac{y}{y-b} dy}$$

ANS

$$\Leftrightarrow \frac{(1 - \epsilon_{\lambda_u, \theta})(r + b + r \lambda_u(\theta)) + r \lambda_u(\theta) \epsilon_{\lambda_u, \theta}}{r + b + r \lambda_u(\theta)} \epsilon_{\theta, y} = \frac{y}{y-b}$$

$$\Leftrightarrow \frac{(r + b)(1 - \epsilon_{\lambda_u, \theta}) + r \lambda_u(\theta)}{r + b + r \lambda_u(\theta)} \epsilon_{\theta, y} = \frac{y}{y-b}$$

$$\Leftrightarrow \epsilon_{\theta, y} = \frac{y}{y-b} \cdot \frac{r + b + r \lambda_u(\theta)}{(r + b)(1 - \epsilon_{\lambda_u, \theta}) + r \lambda_u(\theta)}$$