Search models with multi-worker firms

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Outline

1. bargaining games
   - Binmore, Rubinstein, Wolinsky (1986)
   - Stole and Zweibel (1996)
   - Brugemann, Gautier, Menzio (2015)
2. application
   - Elsby, Micheals (2013)

Introduction

Wage setting in multi-worker firms

- literature follows Stole-Zweibel (1996)
- bargaining protocol between a firm and n workers
- a unique subgame perfect equilibrium of this game
  - wages and firm profits coincide with Shapley values
  - Shapley values: cooperative game
  - important aspect: all workers are paid the same wage

Wage setting in multi-worker firms

- literature follows Stole-Zweibel (1996)
- bargaining protocol between a firm and *n* workers
- a unique subgame perfect equilibrium of this game
  - wages and firm profits coincide with Shapley values
  - Shapley values: cooperative game
  - important aspect: all workers are paid the same wage
- however, Brugemann, Gautier, Menzio (2015) show that
  - Stole-Zweibel (1996) have a mistake in their proof
  - SZ protocol leads to wages where each worker is paid a different wage
  - BGM propose a new bargaining protocol which leads to Shapley values
- existing applied literature is correct, only that it should say that they apply Rolodex game and not SZ

Bargaining games

Binmore, Rubinstein, Wolinsky (1986)

- alternating offer protocol
- session begins with worker making a wage offer to firm
- if firm accepts, session ends
- if firm rejects
  - negotiation breaks with probability p
  - negotiation continues with probability $1 - p$, firm makes a counteroffer
- if worker accepts the counteroffer – session ends
- if worker rejects the counteroffer, with prob p negotiation breaks down, $1 - p$ continues

Payoffs in the BRW game

- if they reach an agreement at the wage w
  - worker's payoff: w
  - firm's payoff: $y - w - t(w)$ where $t(w) = \alpha - \beta w$, $\beta \in [0, 1]$
- if negotiation breaks
  - worker's payoff: b
  - firm's payoff: z

Solution of the BRW game

- focus on subgame perfect equilibria
- if $y - z - b - t(b) < 0$
  - any SPE is such that a firm and a worker do not reach an agreement
  - no gains from trade
- if $y - z - b - t(b) \geq 0$
  - unique SPE
  - worker and firm immediately reach an agreement and the wage is

$$w = b + \frac{1}{(2-p)(1-\beta)} \left[y - z - b - t(b) \right]$$

- with $y - z - b - t(b) = 0$, multiple equilibria, all payoff equivalent

Comments

- $\beta = 0$ – perfectly transferable utility
  - one dollar increase in wage decreases firm's profit by one dollar
- $\beta > 0$ – imperfectly transferable utility
  - one dollar increase in wage decreases firm's profit by $1 - \beta$ dollar

Comments

- $\beta = 0$ – perfectly transferable utility
  - one dollar increase in wage decreases firm's profit by one dollar
- $\beta > 0$ – imperfectly transferable utility
  - one dollar increase in wage decreases firm's profit by $1 - \beta$ dollar
- gain from having a worker/ firm are

$$\begin{aligned} w - b &= \frac{1}{(2-p)(1-\beta)}\left[y - z - b - t(b)\right] \\ y - w - t(w) - z &= \frac{1-p}{2-p}\left[y - z - b - t(b)\right] \end{aligned}$$

- as $p \to 0$, the BRW solution coincides with Nash bargaining
- ratio of worker's and firm's gain is $1/(1-\beta)$ – increasing in β

Stole-Zweibel game

- players: firm and $n \geq 1$ workers
- firm employs $k \in \{0, 1, \ldots n\}$ workers at $w_1, w_2, \ldots w_k$ wages
- firms payoff: $y_k - w_1 - w_2 - \ldots w_k$
- y_k – output produced by k workers
- y_k strictly increasing, concave
- workers are ex-ante identical
- if a worker is hired at the wage w, his payoff is w
- worker who is not hired gets payoff $b \geq 0$

Stole-Zweibel game – continued

- workers are placed in random but fixed order 1 to n
- sequence of bargaining session between firm and one of the workers
- start with a session between firm and the first worker in the order
- if a session ends with an agreement, move to the next worker in the order
- if breakdown
  - worker exits the game permanently
  - bargaining game starts over, all previous agreements are terminated
  - firm starts a session with the first worker in the order, among those who are still in the game
- the game ends when firm reaches agreement with all worker who are still in the game
- then wages are paid, production takes place
- each session follows a BRW protocol, with worker making first offer to the firms

SZ game with two workers

- $\Gamma_k^n(s)$ - subgame in which
  - there are n workers in the game
  - agreement was reached with $n-k$ workers, wages summing up to s
- denote $w_{k,i}^n(s)$ wage of i^{th} worker among k without agreement
- denote $t_k^n(s)$ the sum of wages of the k workers without agreement

SZ game with two workers

- start with $\Gamma_1^1(0)$: 1 worker in the game, breakdown with another worker
- if agreement at w: worker gets w, firm gets $y_1 - w$
- if breakdown: worker gets b, firm gets y_0
- when bargaining session ends, so does subgame
- this has a BRW structure, we can apply its solution
- assuming $y_1 - y_0 - b > 0$, agreement is reached immediately and

$$\begin{aligned} w_{1,1} &= b + \frac{1}{2-p}[y_1 - y_0 - b] \\ \pi_1 &= y_0 + \frac{1-p}{2-p}[y_1 - y_0 - b] \end{aligned}$$

- this is BRW with perfectly transferable utility

SZ game with two workers

- now consider $\Gamma_1^2(w_1)$: agreement with W1 at w_1, negotiation with W2
- if agreement at w_2, then firm's payoff $y_2 - w_1 - w_2$, W1 gets w_1, W2 gets w_2
- if breakdown, the second worker exits, agreement with W1 is terminated, start new negotiation with W1
  - in this case, payoffs are the same as in $\Gamma_1^1(0)$, and payoff to W2 is b
- overall, this is a game with BRW structure – apply BRW solution
- if $w_1 > y_2 - \pi_1 - b$: breakdown of negotiations with W2
- if $w_1 \leq y_2 - \pi_1 - b$: reach agreement immediately

$$w_{1,1}^2(w_1) = b + \frac{1}{2-p}[y_2 - w_1 - \pi_1 - b]$$

- in negotiations with the last worker, again perfectly transferable utility

SZ game with two workers

- notice that wage of W2 depends on w_1: $w^2_{1,1}(w_1)$
- w_1 does not affect firm's payoff if breakdown (renegotiation)
- w_1 negatively affects firm's payoff in case of agreement with W2 (here w_1 will be paid out)
- therefore w_2 depends on w_1
- we can write w_2 as

$$t^2_1(w_1) = \alpha_1 - \beta_1 w_1, \quad \beta_1 = \frac{1}{2-p}$$

- will be useful later

Solve $\Gamma_2^2(0)$

- if firm and W1 agree to $w_1 \leq y_2 - \pi_1 - b$, then
  - firm and W2 immediately agree to $w_{1,1}^2(w_1)$, game ends
  - firm gets $y_2 - w_1 - t_1^2(w_1)$, workers get w_1 and $w_{1,1}^2(w_1)$
- if firm and W1 agree to $w_1 > y_2 - \pi_1 - b$, then firm and W2 do not reach agreement
  - W1 and firm renegotiate, reach π_1 and w_1
- if firm and W1 do not agree, firm gets π_1, W1 gets b, and W2 gets $w_{1,1}$

Solve $\Gamma_2^2(0)$

- bargaining game between firm and W1 does not have BRW payoff structure
  - wage they agree upon does not affect their payoffs if negotiation breaks down with W2
- additional assumptions
  - whenever indifferent, firm rejects any wage from W1 that would lead to a breakdown with W2
  - similarly, firm chooses not to make wage offer which would lead to a breakdown with W2
- under these assumptions, outcome of the game is the same as BRW
- if $y_2 - b - t_1^2(b) - \pi_1 > 0$, then firm and W1 agree upon wage

$$w_{2,1} = b + \frac{1}{1-p}\left[y_2 - \pi_1 - b - t_1^2(w_{2,1})\right]$$

- point: this is BRW game with imperfectly transferable utility
  - increasing wage of W1 by 1 dollar increases payoff of W1 by 1
  - marginal costs to firm is $(1-p)/(2-p)$ because increasing wage of W1 reduces gains from trade with W2

Solution

- solution

$$
\begin{aligned}
w_{2,1} &= b + \frac{1}{2-p}\left[y_2 - \pi_1 - 2b\right] \\
w_{2,2} &= b + \frac{1-p}{(2-p)^2}\left[y_2 - \pi_1 - 2b\right]
\end{aligned}
$$

- W1 has a higher wage than W2: $w_{2,1} > w_{2,2}$
- if W1 gets an extra 1 dollar, firms looses 1 dollar in profits, but also decreases the wage of W2 by 50 cents
- paying W2 an extra dollar decreases firms payoff by 1 dollar
- utility between F and W1 is transferred at the rate 1 to 2, between F and W2 at 1 to 1
- therefore, W1 captures twice as much as W2
- this solution cannot coincide with Shapley values where every worker earns the same

Rolodex game

- almost the same as SZ game, with one twist
- consider the situation where a firm makes an offer which worker rejects and negotiation does not break down
- in SZ game, bargaining between W and F continues
- in Rolodex game, worker takes the last place in the order of workers who have not reached an agreement yet, and firm bargains with the next worker in line
- in SZ game, rejecting the offer does not change worker's position – can take advantage of the fact that his wage lowers wages of others' – strategic position
- this is not the case is Rolodex game
- advantage: every worker ends up with the same wage

Rolodex game with 2 workers – only intuition

- solution to $\Gamma_1^1(0)$ and $\Gamma_1^2(w_1)$ is the same as in SZ since there is only one worker
- key part is the first stage $\Gamma_2^2(0)$, negotiation with 2 workers
- why it works here
  - every worker earns the same as the last worker in line
  - if a worker rejects firm's offer, he becomes last in line
  - hence, this worker has the same outside option and earns the same wage as the last in line
- gains from trade to the last worker are the same as to the firm (he does not affect wage of anybody else)
- hence, everybody is "marginal"

Elsby, Michaels (2013)

Introduction

- introducing the notion of firm size in a DMP model
- the model matches
  - distribution of employer size
  - employment growth across establishments
  - amplitude and propagation of cyclical fluctuations in worker flows
  - negative co-movement of unemployment and vacancies
  - dynamics of the distribution of firm size across business cycle

Model

- measure 1 of firms: fixed, no free entry
- measure L of workers
- matching function: $M = M(U, V)$
  - CRS: $q(\theta), f(\theta)$

Firm's problem

- firm's output: $y = pxF(n)$
- p aggregate
- x idiosyncratic, transition $G(x'|x)$
- $F'(n) > 0, F''(n) \leq 0$
- labor demand decision after observing x
  - fire workers at no costs
  - hire workers by posting vacancies, flow cost c per vacancy

Firm's problem

- value function

$$\Pi\left(n_{-1},x\right)=\max_{n,v}\left\{pxF\left(n\right)-w\left(n,x\right)-cv+\beta\int\Pi\left(n,x'\right)dG\left(x'|x\right)\right\}$$

where $w\left(n,x\right)$ is bargained wage

- number of hires: $\Delta n1^{+}=qv$ where 1^{+} indicates that firm is hiring

- hence

$$\Pi\left(n_{-1},x\right)=\max_{n,v}\left\{pxF\left(n\right)-w\left(n,x\right)-\frac{c}{q}\Delta n1^{+}+\beta\int\Pi\left(n,x'\right)dG\left(x'|x\right)\right\}$$

- kink at $n=n_{-1}$ due to irreversibility of hiring decision

- take FOC for the case $\Delta n\neq 0$

$$\begin{aligned}pxF'\left(n\right)-w\left(n,x\right)-w_{n}\left(n,x\right)n-\frac{c}{q}1^{+}+\beta D\left(n,x\right) &= 0\\ D\left(n,x\right) &= \int\Pi_{n}\left(n,x'\right)dG\left(x'|x\right)\end{aligned}$$

Value functions

- denote $J(n, x)$ marginal value of labor to the firm

$$J\left(n,x\right)=pxF'\left(x\right)-w\left(n,x\right)-w_{n}\left(n,x\right)+\beta D\left(n,x\right)$$

- denote $W(n, x)$ value of employment in a firm of size n

$$W\left(n,x\right)=w\left(n,x\right)+\beta E\left[s\Upsilon'+\left(1-s\right)W\left(n',x'\right)|n,x\right]$$

  where s is an endogenous probability of losing a job
- denote Υ value of being unemployed

$$\Upsilon=b+\beta E\left[\left(1-f\right)\Upsilon'+fW\left(n',x'\right)\right]$$

Wage setting

- bargaining solution of Stole and Zwiebel – or the Rolodex game?
- wage is an outcome of bargaining over *marginal surplus*
- wage are an outcome of Nash bargaining, with worker's bargaining power η

$$(1-\eta)\left[W(n,x)-\Upsilon\right]=\eta J(n,x)$$

- wage $w(n,x)$ solves the differential equation

$$w(n,x)=\eta\left[pxF'(n)-w_n(n,x)\,n+\beta f\frac{c}{q}\right]+(1-\eta)\,b$$

- MAGIC! a lot of work goes into deriving this expression

Wage setting

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- wage is an outcome of bargaining over *marginal surplus*
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$$(1-\eta)\left[W(n,x)-\Upsilon\right]=\eta J(n,x)$$
- wage $w(n,x)$ solves the differential equation
$$w(n,x)=\eta\left[pxF'(n)-w_n(n,x)\,n+\beta f\frac{c}{q}\right]+(1-\eta)\,b$$
- MAGIC! a lot of work goes into deriving this expression
- one additional term compared to DMP: $w_n(n,x)\,n$
    - if negotiation breaks down, firm has to pay remaining workers a higher wage
    - fewer workers – higher marginal product
- assume $F(n)=n^{\alpha}$, then
$$w(n,x)=\eta\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}+\beta f\frac{c}{q}\right]+(1-\eta)\,b$$

Firm's optimal employment policy

- combine FOC and wage to get

$$(1-\eta)\left[\frac{px\alpha n^{\alpha-1}}{1-\eta(1-\alpha)}-b\right]-\eta\beta f\frac{c}{q}-\frac{c}{q}\Delta n1^{+}+\beta D(n,x)=0$$

- optimal hiring policy is given by two thresholds, $R_v(n_{-1}), R(n_{-1})$ so that

$$n(n_{-1},x)=\begin{cases} R_v^{-1}(x) & \text{if } x>R_v(n_{-1}) \\ n_{-1} & \text{if } x\in[R(n_{-1}),R_v(n_{-1})] \\ R^{-1}(x) & \text{if } x<R(n_{-1}) \end{cases}$$

- thresholds satisfy

$$\begin{aligned} J(n,R_v(n)) &= \frac{c}{q} \\ J(n,R(n)) &= 0 \end{aligned}$$

- everything else is derived in the notes

Comparative statics

- suppose the process for x is as follows

$$x' = \begin{cases} x & \text{with prob } 1-\lambda \\ \tilde{x} \sim \tilde{G}(\cdot) & \text{with prob } \lambda \end{cases}$$

- Proposition: if n is sufficiently large then

$$\frac{\partial R_v}{\partial p} < 0, \quad \frac{\partial R}{\partial p} < 0, \quad \frac{\partial R_v}{\partial \theta} > 0, \quad \frac{\partial R}{\partial \theta} > 0$$

- if p is larger, workers are more productive, hence for given x, firms are more likely to hire and less likely to shed worker
- higher $\theta \Rightarrow$lower $q(\theta)$, hence higher c/q and higher wages – this reduces hiring
- effect of higher θ on firing less clear: on one hand, wages are higher (want to shed workers), but it is harder to rehire workers (c/q is high) – which effect dominates depend on firm size

Aggregation

- law of motion for distribution

$$\Delta H(n) = \lambda \tilde{G}(R(n))(1 - H(n)) - \lambda \left(1 - \tilde{G}(R_v(n))\right) H(n)$$

- steady state distribution of employment across firms is

$$H(n) = \frac{\tilde{G}(R(n))}{1 - \tilde{G}(R_v(n)) + \tilde{G}(R(n))}$$

- steady state employment

$$N = \int n dH(n)$$

- steady state separations and hires

$$\begin{aligned} S &= \lambda \int [1 - H(n)] \tilde{G}(R(n)) dn \\ M &= \lambda \int H(n) \left[1 - \tilde{G}(R_v(n))\right] dn \\ S &= M \end{aligned}$$

- steady state unemployment

$$U = L - N$$

Aggregate shocks

- no trivial to introduce aggregate shocks
- firms need to forecast wages, hence (from wage equation) they need to forecast θ, hence the evolution of $H(n)$
- follow Krusell and Smith
- aggregate shocks follow a random walk

$$p' = \begin{cases} p + \sigma_p & \text{with prob } 1/2 \\ p - \sigma_p & \text{with prob } 1/2 \end{cases}$$

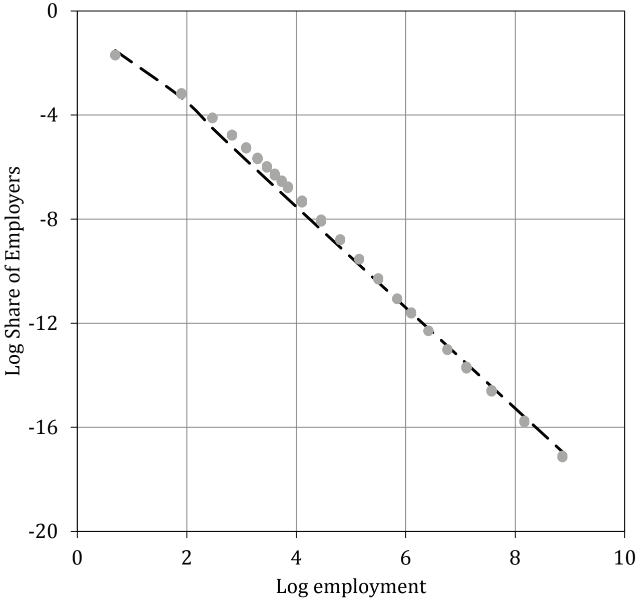
- conjecture that only mean of $H(n)$ is needed to forecast θ
- for $\sigma_p \approx 0$, approximate N and θ as

$$\begin{aligned} N' &\approx N^* + v_N (N - N^*) + v_p (p' - p) \\ \theta' &\approx \theta^* + \theta_N (N - N^*) + \theta_p (p' - p) \end{aligned}$$

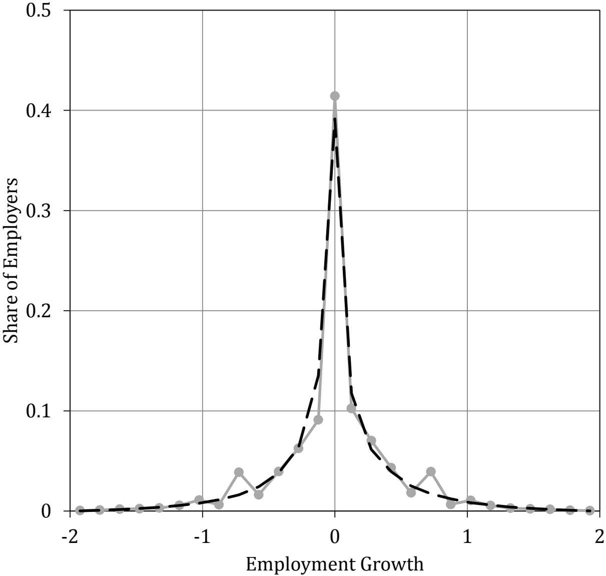
- here N^*, θ^* are values which would be realized if p stays at its current value
- this can be used to approximate hiring policy

$$D(n, x; N, p, \sigma_p) \approx D(n, x; N^*, p, 0) + D_N^* (N - N^*)$$

Size distribution



Growth distribution

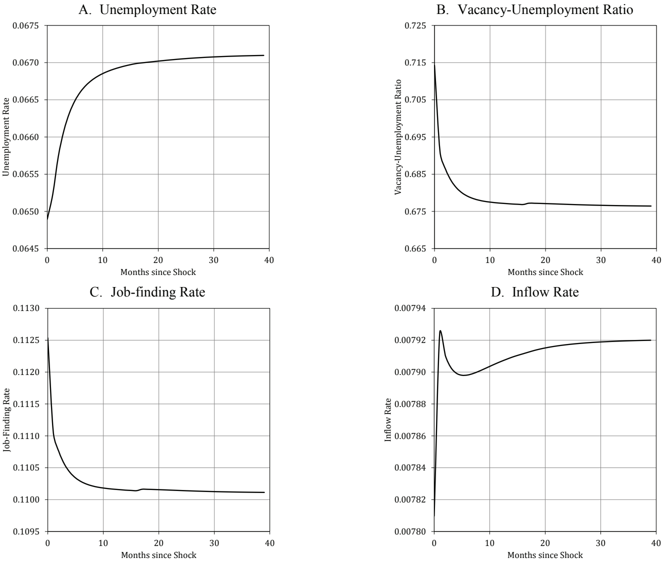


Elasticities

Model / Outcome	**Mean Level**		**Elasticity w.r.t. output per worker**	
	Data	*Model*	*Data*	*Model*
A. Generalized				
Job Finding Rate, f	[0.1125]	[0.1125]	2.65	2.55
Inflow Rate, s	[0.0078]	[0.0078]	-1.89	-1.64
Vacancies, V	--	--	2.91	2.47
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	6.37
B. MP (i)				
Job Finding Rate, f	[0.1125]	[0.1125]	2.65	0.91
Inflow Rate, s	[0.0078]	[0.0078]	-1.89	[-1.64]
Vacancies, V	--	--	2.91	-0.32
Tightness, $\theta = V/U$	[0.72]	[0.72]	6.44	2.28

Response to a permanent productivity shock

Figure 6. Model Impulse Responses to a Permanent 1-percent Decline in Aggregate Labor Productivity, *p*



Additional results

- size distribution is achieved through the assumption of Pareto distribution of fixed productivity
- slower propagation of shocks: it takes time until the size distribution adjusts
  - inaction region where firms don't adjust employment
- higher elasticity: average versus marginal surplus

$$\frac{d \log \theta}{d \log p} \approx \frac{(1-\eta)\, p}{\phi\left[(1-\eta)(p-b)-\eta\beta c\theta\right]+\eta\beta c\theta}$$

where ϕ is elasticity of the matching function