



# **Lecture 1 : The Diamond- Mortensen- Pissarides framework**

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# Introduction

## Lecture contents :

1. The basic matching model
2. Endogenous job destruction
3. Alternative wage setting
4. The role of policy

# The basic matching model

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## A. Why should we consider search and matching on the labor market ?

New labor market theories to understand the foundations of equilibrium unemployment (see Pissarides (2000) Cahuc and Zylberberg (2004, MIT Press) or Ljungqvist and Sargent (2005)) Labor market is characterized by frictions :

- Labor market exchange is a time consuming activity
- There is both spatial mismatch and mismatch between skill requirements and human capital of workers
- Both unemployment and job vacancies exist = the so-called "Beveridge curve"



### Note

the theory of equilibrium unemployment has been jointly developed by Peter Diamond, Dale Mortensen and Chris Pissarides (Nobel prize 2010)

## B. Labor market flows and matching process

### 1. (1/3)

Continuous time model

Exogenous job destruction according to Poisson process with arrival rate  $\delta$

A matching function gives the number of hirings as a function of the number of unemployed workers  $u$  and vacant jobs  $v$  :  $m(u, v)$  is increasing and concave wrt both arguments with constant returns-to-scale

Jobs can be filled and unemployed find a job according to a random Poisson process whose rate depend on the matching function.

$$\theta = \frac{v}{u}$$

This rate is related to the market tightness

### 2. (2/3)

For the firms, the probability of filling a vacancy is given by

$$\frac{m(u, v)}{v} = m\left(\frac{1}{\theta}, 1\right) \equiv q(\theta)$$

from the properties of  $m(u, v)$  it

turns out that  $q'(\theta) < 0$

Similarly, for the workers, the probability to find a job is

$$\frac{m(u, v)}{u} = m(1, \theta) \equiv \theta q(\theta)$$

$$\lim_{\theta \rightarrow 0} [\theta q(\theta)] = \lim_{\theta \rightarrow +\infty} q(\theta) = 0 \text{ and}$$

$$\lim_{\theta \rightarrow +\infty} [\theta q(\theta)] = \lim_{\theta \rightarrow 0} q(\theta) = +\infty$$

There exists indeed matching externalities :

- positive "inter-group externalities" : an increase in  $u$  (or  $v$ ) increases  $q(\theta)$  (or  $\theta q(\theta)$ )
- negative "intra-group externalities" : an increase in  $v$  (or  $u$ ) decreases  $q(\theta)$  (or  $\theta q(\theta)$ )

This induces some externalities which may lead to inefficiencies

### 3. (3/3)

The dynamics of unemployment is driven by the difference between unemployment inflows and outflows :

$$\dot{u} = \delta(1 - u) - \theta q(\theta)u$$

In steady-state, we therefore find ;

$$u = \frac{\delta}{\delta + \theta q(\theta)}$$

This steady-state condition can also be written as

$$\delta(1 - u) = m(u, v)$$

This equation can be mapped in the plane  $(u, v)$  ; it is the so-called Beveridge curve (decreasing relationship between  $v$  and  $u$  from the properties of  $m(u, v)$ )

## C. Labor demand in the matching framework

### 1. (1/2)

In the matching framework, due to search frictions, labor demand is an intertemporal vacancy : firms post vacancy expecting the future value of the job at the time where the vacant job will be matched with an unemployed.

The intertemporal values of a job and a vacancy are respectively given by the following (steady-state) asset pricing equations :

$$\begin{aligned} rJ &= p - w - \delta(J - V) \\ rV &= -c + q(\theta)(J - V) \end{aligned}$$

with  $r$  the real interest rate and  $p$  the productivity



#### Note

if there was no exogenous shock the rent associated to the job would simply be  $J = (p - w)/r$

### 2. (2/2)

We consider that there does not exist any entry cost into the labor market, but a flow cost of recruiting a worker  $c$

A free entry condition gives the labor market tightness and the numbers of vacancies :

- firms post vacancies up to a point where  $V = 0$
- if  $V > 0$  some firms post additional vacancies
- Since  $q'(\theta) < 0$ , when additional vacancies are posted the average expected recruitment cost( $c/q(\theta)$ ) is increasing up to a point where the zero-profit condition holds

## D. Labor market equilibrium with exogenous wages

If we take  $w$  as fixed, the steady-state equilibrium for vacancies and unemployment is unique and it is defined by the following system of equations :

$$\begin{aligned} q(\theta) &= \frac{(r + \delta)c}{p - w} \\ u &= \frac{\delta}{\delta + \theta q(\theta)} \end{aligned}$$

There is a unique solution for  $\theta$  from the first equation, implying that  $\theta$  is positively (negatively) related to  $p$  ( $w$  and  $c$ )

Then we get the unique solution for  $u$  from the second equation

## E. Nash bargaining of wages

### 1. (1/3)

Bargaining of wages is a "natural" assumption in a matching setting with (by definition) search frictions → the match creates a rent which is related to productivity of the job but also to the fact that some search costs are no longer supported.

Individual bargaining aims at sharing this rent. The usual assumption is that, at each period, the total intertemporal surplus is shared through a generalized bargaining process :

$$w = \operatorname{argmax} (W - U)^\beta (J - V)^{1-\beta}$$

where  $\beta$  is worker's bargaining power,  $W$  and  $U$  intertemporal values of employment and unemployment, respectively.

With risk-neutral firms and workers the solution of this program is exactly the same as an "ad-hoc" sharing rule :

$W - U = \beta S$  with  $S = W - U + J - V$  the total surplus generated by the match

### 2. (2/3)

Bellman value functions for the workers (= asset pricing equations) write as follows :

$$rU = b + \theta q(\theta)(W - U)$$

$$rW = w - \theta(W - U)$$

where  $b$  stands for home production/leisure gain

The first order condition for the Nash bargaining problem implies :

$$\begin{aligned} W - U &= \frac{\beta}{1-\beta}(J - V) \\ \iff (r + \delta)(W - U) &= (r + \delta)(J - V) \\ \iff (r + \delta)(w - b + \theta q(\theta)(W - U)) &= p - w \end{aligned}$$

### 3. (3/3)

From the free entry we have  $J = c/q(\theta)$ , the sharing rule implies  $W - U = \frac{\beta}{1-\beta} \frac{c}{q(\theta)}$

Substituting out this worker's surplus into the previous relationship leads to this wage setting equation :

$$w = (1 - \beta)b + \beta(p + c\theta)$$

if  $\beta = 1$ , real wage = productivity + average search cost ( $cv/u$ )

if  $\beta = 0$ , real wage = unemployed income (reservation wage)

## F. Labor market equilibrium with Nash bargaining

The LM equilibrium is defined by the intersection of a wage setting function ( $\approx$  labor supply curve) and the free entry condition ( $\approx$  labor demand curve) :

$$\begin{aligned} w &= (1 - \beta)b + \beta(p + c\theta) \\ (1 - \beta)(p - b) &= \frac{c}{q(\theta)} [\delta + r + \beta\theta q(\theta)] \end{aligned}$$

Then we can compute the steady state unemployment rate

This provides some theoretical grounds to equilibrium unemployment as a result of imperfect exchanges on the labor market and non-walrasian determination of wages

## G. Welfare and efficiency

### 1. (1/3)

The optimal allocation (first best efficiency) is determined by solving the problem of a planner

This problem consists in choosing  $\theta$  and  $u$  that maximize steady-state output net of turn-over cost (risk-neutral workers and no redistribution/inequity motives)

The problem writes :

$$\Omega = \int_0^{+\infty} e^{-rt} \{(1 - u)p + ub - c\theta u\} dt$$

subject to the constrain  $\dot{u} = \delta(1 - u) - \theta q(\theta)u$

The Hamiltonian therefore writes :

$$\mathcal{H} = e^{-rt}\{(1 - u)\rho + ub - c\theta u\} + \lambda[\delta(1 - u) - \theta q(\theta)u]$$

### 2. (2/3)

The Euler conditions are  $\frac{\partial \mathcal{H}}{\partial \theta} = 0$  and  $\frac{\partial \mathcal{H}}{\partial u} = -\dot{\lambda} + r\lambda$

$$ce^{-rt} + \lambda q(\theta)[1 - \eta(\theta)] = 0$$

$$(p - b + c\theta)e^{-rt} + \lambda[\delta + \theta q(\theta)] = \dot{\lambda}$$

Where  $\eta(\theta) = -q'(\theta)\theta/q(\theta)$  is the elasticity of the matching function with respect to unemployment.

We focus on steady-state,  $\dot{\theta} = 0$ . Since  $\dot{\lambda} = -r\lambda$ , we get :

$$p - b = \frac{c}{q(\theta)} \left[ \frac{\delta + r + \theta q(\theta)\eta(\theta)}{1 - \eta(\theta)} \right]$$

### 3. (3/3)

Let compare this optimal condition with that at equilibrium with Nash bargaining of wages, it comes that the two solutions coincide if and only if :

$$\beta = \eta(\theta)$$

This has been showed by Hosios (1990) and Pissarides (2000) : *negative intra-group externalities and positive inter-group externalities* just offset one another in that case

Of course, there is no reason for  $\beta$  to be equal to  $\eta(\theta)$

The search-matching equilibrium is in general inefficient

When  $\beta$  is larger (lower) than  $\eta(\theta)$ , there is too much (not enough) unemployment, creating congestion in the matching process for the unemployed.

## H. Activity 1.1 : problem set 1

Consider a continuum of workers and firms. Each firm has only one job. The total mass of the population is normalized to one, which implies that the unemployment rate  $u$  is equal to the level of unemployment. Let  $v$  be the number of vacancies, the matching function is given by:

$$M(v, u) = v^{\frac{1}{3}} u^{\frac{2}{3}}$$

1. Define the transition rate for a worker from unemployment to employment as a function of the labor market tightness  $\theta \equiv v/u$ . Define also the transition rate for a vacancy position as a function of  $\theta$ .
2. Consider an exogenous rate of job destruction  $\delta$ , and compute the steady-state unemployment rate as a function of  $\delta$  and  $\theta$ .
3. Let denote  $w$  the wage,  $y$  productivity of a job-worker pair, and  $c$  the recruitment cost, by using Bellman equations, write down (i) the steady-state value of a firm with a filled position (denoted  $J$ ), (ii) the steady-state value of a vacant job (denoted  $V$ ).
4. Write the free entry condition which characterizes  $\theta$  as a function of  $y$ ,  $w$ ,  $r$ ,  $\delta$ ,  $c$ .

5. Consider now that the wage equation solves  $w = \frac{y+b}{2}$ , write down the two-dimensional system of equations which define steady-state equilibrium values of labor market tightness  $\theta^*$  and unemployment rate  $u^*$ .

6. The government introduces an employment subsidy paid to the employee such that wage equation is now defined by  $w = \frac{y+b-s}{2}$ . What is the impact of  $s$  on equilibrium labor market tightness and unemployment rate? Draw a picture of the equilibrium with and without the employment subsidy.

## I. Activity 1.2 : problem set 2

We consider the following matching model. The matching function is given by

$$m(u, v) = \frac{uv}{u + v}$$

$$W - U = \beta S$$

and

$$J - V = (1 - \beta)S$$

with the surplus  $S = W - U + J - V$  where  $W, U, J, V$  are the intertemporal values for employed, unemployed, filled job and vacant job. The job destruction rate is  $\delta$  and the interest rate is  $r$ :

1. Define the contact rate for firm,  $q(\theta)$ , with  $\theta = v/u$ . Check that  $m(u, v)$  has constant returns to scale and is concave in each of its arguments. What is the elasticity of  $m$  with respect to  $u$  as a function of  $\theta$ ?

2. Show that the Beveridge curve has the following equation:

$$\theta = \frac{\delta(1 - u)}{u - \delta(1 - u)}$$

3. Write down the value functions  $J, W, U, V$  denoting  $y$  the productivity and  $b$  the unemployed income.

4. Show that the equilibrium value of  $\theta$  must satisfy:

$$\frac{y}{c} = \frac{r + \delta}{1 - \beta}(1 + \theta) + \frac{\beta}{1 - \beta}\theta$$

5. What is the steady-state unemployment rate  $u$ ? How does it depend on  $r, \delta, c, y$  and  $\beta$ ? Why?

6. Show that the following relationship exists between the wage  $w$  and  $\theta$ :

$$w = \frac{\beta}{1 - \beta}c[(r + \delta)(1 + \theta) + \theta]$$

7. How do wages depend on  $r, \delta, c, \beta$  and  $\theta$ ?

8. Consider  $r \rightarrow 0$ . Write down the central planner's problem, the Hamiltonian, and the corresponding first-order conditions.

9. Show that in steady state the optimal value of  $\theta$  must satisfy

$$\frac{y}{c} = (r + \delta)(1 + \theta)^2 + c\theta^2$$

10. What relationship must hold among the model's parameters for the optimal value of  $\theta$  to be the same as the equilibrium value of  $\theta$ ?

11. Check that this is equivalent to

$$\beta = \frac{\theta}{1 + \theta}$$

with  $\theta$  being the common solution to the two allocations.

12. How does this last condition relate to the Hosios efficiency condition?

# Endogenous job destruction

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## A. Modeling job destruction

Not only job creations but also job destructions are endogenous decisions  
This can occur by taking into account idiosyncratic productivity shocks (Mortensen and Pissarides [1994])

The productivity of the job is  $P + \varepsilon$  with  $\varepsilon$  a random variable

It is drawn from a distribution  $G(\varepsilon) \forall \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}]$

The job is created with the highest productivity (best technology available for new filled jobs)

At a Poisson rate  $\lambda$  the job is hit by a shock

Destruction if productivity is below a threshold  $\tilde{\varepsilon}$  ; destruction rate =  $\lambda G(\tilde{\varepsilon})$

## B. Value functions

The steady-state Bellman equations of workers are given by  
 $rU = b + \theta q(\theta)[W(\bar{\varepsilon}) - U]$

$$rW(\varepsilon) = w(\varepsilon) + \lambda \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} W(s)dG(s) + \lambda G(\tilde{\varepsilon})U - \lambda W(\varepsilon)$$

$$rJ(\varepsilon) = p + \varepsilon - w(\varepsilon) + \lambda \int_{\tilde{\varepsilon}}^{\varepsilon} J(s)dG(s) - \lambda J(\varepsilon)$$

$$rV = -c + q(\theta)[J(\bar{\varepsilon}) - V]$$

## C. Free-entry condition and labor demand

The free-entry condition now writes :

$$J(\bar{\varepsilon}) = \frac{c}{q(\theta)}$$

We have therefore the following decreasing relationship between labor market tightness and wages :

$$p + \bar{\varepsilon} - w(\bar{\varepsilon}) + \lambda \int_{\tilde{\varepsilon}}^{\varepsilon} J(s)dG(s) - \lambda J(\bar{\varepsilon}) = \frac{rc}{q(\theta)}$$

## D. Wage determination

The total surplus is now given by  $S = W(\varepsilon) - U + J(\varepsilon) - V$  and the result of the bargaining is :

$$W(\varepsilon) - U = \beta S \text{ and } J(\varepsilon) - V = (1 - \beta)S$$

This leads to :

$$(1 - \beta)[W(\varepsilon) - U] = \beta[J(\varepsilon) - V]$$

We obtain a standard wage equation, but which is now indexed on the productivity level  $\varepsilon$  :

$$w(\varepsilon) = (1 - \beta)b + \beta(p + \varepsilon + c\theta)$$

## E. The productivity reservation

### 1. (1/2)

To determine the reservation rule for productivity,  $\tilde{\varepsilon}$ , we have to solve the following equations

$J(\tilde{\varepsilon}) = 0$  ;  $W(\tilde{\varepsilon}) = U$  ; the job destruction is efficient both for the firm and the worker

The wage determination through bargaining (sharing rule) makes these two conditions identical

To compute this threshold, first note that by substituting the wage equation into  $J(\varepsilon)$ , we have :

$$(r + \lambda)J(\tilde{\varepsilon}) = (1 - \beta)(p + \tilde{\varepsilon} - b) - \beta c \theta + \lambda \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} J(s)dG(s)$$

## 2. (2/2)

Now, compute  $J(\varepsilon) - J(\tilde{\varepsilon}) = 0$ , and use  $J(\tilde{\varepsilon}) = 0$ , we get :

$$(r + \lambda)J(\varepsilon) = (1 - \beta)(\varepsilon - \tilde{\varepsilon})$$

Replacing this value into the  $J(s)$  yields :

$$(r + \lambda)J(\varepsilon) = (1 - \beta)(p + \varepsilon - b) - \beta c\theta + \frac{\lambda(1 - \beta)}{r + \lambda} \int_{\tilde{\varepsilon}}^{\varepsilon} (s - \tilde{\varepsilon})dG(s)$$

Evaluate at  $\varepsilon = \tilde{\varepsilon}$  and use the reservation rule  $J(\tilde{\varepsilon}) = 0$ , we get :

$$\tilde{\varepsilon} - \frac{b}{p} - \frac{\beta}{1 - \beta}c\theta + \frac{\lambda}{r + \lambda} \int_{\tilde{\varepsilon}}^{\varepsilon} (s - \tilde{\varepsilon})dG(s) = 0$$

## F. The job creation condition

Consider  $\varepsilon = \bar{\varepsilon}$ , we obtain :

$$(r + \lambda)J(\bar{\varepsilon}) = (1 - \beta)(\bar{\varepsilon} - \tilde{\varepsilon})$$

This allows to rewrite the job creation condition :

$$(1 - \beta) \frac{\bar{\varepsilon} - \tilde{\varepsilon}}{(r + \lambda)} = \frac{c}{q(\theta)}$$

## G. Unemployment determination

The number of workers who enter unemployment is  $\lambda G(\tilde{\varepsilon})(1 - u)$  and the number of workers who leave unemployment is  $\theta q(\theta)u$

The evolution of unemployment is thus given by the difference between these two flows,

$$\dot{u} = \lambda G(\tilde{\varepsilon})(1 - u) - \theta q(\theta)u$$

where  $\dot{u}$  is the variation of unemployment with respect to time.

In steady state, the rate of unemployment is constant and therefore these two flows are equal :

$$u = \frac{\lambda G(\tilde{\varepsilon})}{\lambda G(\tilde{\varepsilon}) + \theta q(\theta)}$$

## H. Equilibrium characterization

The labor market equilibrium determines  $u$ ,  $\tilde{\varepsilon}$  and  $\theta$  that satisfies the three following equations :

$$\begin{aligned} \tilde{\varepsilon} &= \frac{b}{p} + \frac{\beta}{1 - \beta}c\theta - \frac{\lambda}{r + \lambda} \int_{\tilde{\varepsilon}}^{\varepsilon} (s - \tilde{\varepsilon})dG(s) \\ \frac{c}{q(\theta)} &= (1 - \beta) \frac{(\bar{\varepsilon} - \tilde{\varepsilon})}{(r + \lambda)} \\ u &= \frac{\lambda G(\tilde{\varepsilon})}{\lambda G(\tilde{\varepsilon}) + \theta q(\theta)} \end{aligned}$$

## I. Efficiency

The welfare function is still given by  $\Omega$  and the constraints include (where  $\widehat{x}$  stands for optimal values) :

$$\dot{y} = p\widehat{\theta}q(\widehat{\theta})u + \lambda(1-u) \int_{\widehat{\varepsilon}}^{\varepsilon} (p+s)dG(s) - \lambda y$$

which is the dynamics equation of the productivity.

First order condition writes :

$$\frac{c}{q(\widehat{\theta})} = [1 - \eta(\theta)] \frac{(\bar{\varepsilon} - \widehat{\varepsilon})}{(r + \lambda)}$$

$$\widehat{\varepsilon} = \frac{b}{p} + \frac{\eta(\theta)}{1 - \eta(\theta)} c\theta - \frac{\lambda}{r + \lambda} \int_{\widehat{\varepsilon}}^{\varepsilon} (s - \widehat{\varepsilon}) dG(s) = 0$$

Yet, the condition  $\beta = \eta(\theta)$  insures that, not only the labor tightness, but also the reservation productivity  $\tilde{\varepsilon}$  is at its optimal level  $\widehat{\varepsilon}$

## J. Activity 1.3 : reading 1



### *Complement : Reading 1*

- Job Creation and Job Destruction in the Theory of Unemployment (PDF) (see )

# Alternative wage setting

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How robust is (in)efficiency result of equilibrium unemployment to the wage setting process ?

- Monopoly union
- Efficiency wage
- Training costs and insider wage

## A. Monopoly union

### 1. (1/3)

In many labor markets terms of employment are determined by collective bargaining agreements.

- Monopoly union = standard approach to modeling wage formation
- Stackelberg game of employment and wage determination : the union first sets the wage and then employer responds by determining employment
- The union chooses the shares of the match surplus obtained by workers

### 2. (2/3)

This share is at its efficient Hosios' value when all members of the union are unemployed (Pissarides [1990]).

Otherwise, *ie.* if the union acts in the interest of employed workers, the share exceeds the social optimum, but still the union does not fully exploit the market power by appropriating the entire match surplus.

Insider-outsider conflict exists.

If union were to represent unemployed workers, it would choose  $\beta$  to maximize the equilibrium return to search which can be write as

$$rU = b + \frac{\beta}{1-\beta} c\theta$$

subject to the definition of the labor market tightness as given by the free entry condition (from backward induction).

The optimal solution of this problem is then given by  $\beta = \eta(\theta)$



### Note

This shows a trade-off between wage gains and expected unemployment duration.

## B. Efficiency wage

### 1. (1/2)

The wage can also be set to motivate worker effort (Shapiro and Stiglitz [1984])

- Job destruction is excessive
- Assumptions :
  - workers would rather take leisure on the job than supply effort
  - an employer fires any worker found shirking
  - monitoring is imperfect

### 2. (2/2)

To motivate effort, the employer pays an efficiency wage that equates the expected loss in future worker income if caught shirking to the value of leisure enjoyed while shirking :

$$\phi(W - U) = b$$

where  $\phi$  is the probability to be caught when shirking (monitoring frequency)

Let define the following value functions :

$$rW = w - \delta G(\tilde{\varepsilon})(W - U)$$

$$rU = b + \frac{b}{\phi}[r + \delta G(\tilde{\varepsilon}) + \theta q(\theta)]$$

## C. Rent sharing with turn over costs

### 1. (1/3)

Consider "Training Costs" implying that once a job has been created a firm has to pay the flow cost  $C$ , and a firing tax  $F$

In the context of Nash bargaining two situations can emerge :

- worker and employer are able to pre-commit on an enforceable wage



- because the second tier is generally higher than the first tier a worker, once inside, has an incentive to default on the original agreement  $\iff$  hold-up problem

## 2. (2/3)

If enforceable contracts exist, initial and subsequent (after each shock) Nash bargaining problems write

$$w_0 = \operatorname{argmax} (W_0 - U)^\beta (J_0 - C - V)^{1-\beta}$$

$$w_1(\varepsilon) = \operatorname{argmax} (W(\varepsilon) - U)^\beta (J(\varepsilon) + F)^{1-\beta}$$

In that context, Hosios condition is sufficient to achieve efficiency and the wage structure is given by :

$$w_0 = \beta [p - (r + \delta)C - \delta F] + (1 - \beta)rU$$

$$w_1(\varepsilon) = \beta [p\varepsilon + rF] + (1 - \beta)rU$$

$$rU = b + \frac{\beta}{1 - \beta}c\theta$$

where, as before, we have

## 3. (3/3)

Otherwise, hold-up problem arises in the absence of a two-tier wage structure

The second tier wage applies initially  $\iff$  the initial wage is determined by the continuing wage bargaining outcome

The insider equilibrium wage contract satisfies

$$w(\varepsilon) = \beta[p\varepsilon + c\theta + rF + (C+F)\theta q(\theta)] + (1 - \beta)b$$

By forcing the employer to bear the whole of the job creation and job destruction costs, the worker earns more at a given level of market tightness

The Hosios no longer achieves efficiency

## D. Activity 1.4 : reading 2



### *Complement : Reading 2*

- Efficient v.s. equilibrium unemployment with match-specific costs by A. Cheron (see )

# Labor policy

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## A. Labor policy issue

If the Hosios condition is not satisfied ( $\eta(\theta) \neq \beta$ ) of hold-up problems arise, there is a room for labor policy to increase welfare ; so does also in a second best perspective i.e. in the context of distortions related to the welfare state

In addition to payroll taxes, firing taxes and/or hiring subsidies (=  $C < 0$ ) are natural candidates in the job creation-job destruction framework.

## B. Equilibrium with labor policy

### 1. (1/2)

Nash bargaining problems write

$$(1 + \tau)(1 - \beta)(W_0 - U) = \beta(J_0 - C - V)$$

$$(1 + \tau)(1 - \beta)W(\varepsilon) - U = \beta(J(\varepsilon) + T)$$

where  $\tau$  is the payroll tax rate (labor cost is  $(1 + \tau)w$ )

The modified job destruction rule shows that the higher the firing tax, the lower the reservation productivity  $\varepsilon$ .

The productivity threshold indeed solves now :

$$J(\tilde{\varepsilon}) = -F \iff p\left(\tilde{\varepsilon} + \frac{\delta}{r + \delta} \int_{\tilde{\varepsilon}}^{\varepsilon} (s - \tilde{\varepsilon}) dG(s)\right) + rF = rU$$

If  $r = 0$  the firing tax does not reduces job destructions (only reduces job creation)  
 $\iff$  due to infinite-lived context, the firm knows she will pay the tax at a moment in time ; so if there is no discounting the tax has no effect

The extensive definition of the equilibrium therefore solves :

$$\begin{aligned}\frac{c}{q(\theta)} &= (1 - \beta) \left[ p \left( \frac{1 - \tilde{\varepsilon}}{r + \delta} \right) - F - C \right] \\ (1 + \tau) \left[ b + \frac{\beta}{1 - \beta} c \theta \right] &= p \left( \tilde{\varepsilon} + \frac{\delta}{r + \delta} \int_{\tilde{\varepsilon}}^{\bar{\varepsilon}} (s - \tilde{\varepsilon}) dG(s) \right) + rF\end{aligned}$$

The impact of policy parameters on  $\theta$ ,  $\tilde{\varepsilon}$  hence unemployment are not clear cut

If  $\beta = 0$ , provided that  $r > 0$ , an increase in  $F$  implies a decrease in both  $\tilde{\varepsilon}$  (fewer job destructions) and  $\theta$  (fewer job creations). An increase in  $C$  reduces  $\theta$  but leaves unchanged  $\tilde{\varepsilon}$

More generally, the firing tax can reduce both job creations and job destructions → see **numerical example**

## C. Activity 1.5 : reading 3



### *Complement : Reading 3*

- Taxes, Subsidies and Equilibrium Labor Market Outcomes (see )

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The benchmark DMP model is particularly well-suited to analyse job creations, job destructions and therefore the determinants of unemployment ( $\iff$  a theory of equilibrium unemployment).

It provides some theoretical grounds to address efficiency issues : externalities, congestion effects and hold-up problem can lead to inefficient labor market outcomes.

It also gives a benchmark to deal with the design of labor market policy, not only from a first best perspective but also with the second one.

Further line of extensions are of particular interest, and are discussed in the next lecture : (i) finite-lived agents and life cycle labor market flows, (ii) human capital issues.

