

Problem Set 8
Due in Lecture Thursday Oct. 30

1 A Warm-up Exercise

The basic model of consumption under uncertainty (with quadratic utility, and uncertainty only about labor income) predicts that: (check all that applies)

1. The change in income will not be predictable on the basis of past changes in consumption.
2. The change in consumption will not be predictable on the basis of past changes in income.
3. The change in consumption will not be correlated with the current change in income.
4. The change in consumption will depend on the revision in expectations about future income.
5. The change in consumption will depend on the revision in expectations about current income.
6. (1) and (2)
7. (1) and (3)
8. (2) and (3)
9. (2) and (4)
10. (3) and (5)
11. (4) and (5)
12. all of the above

2 Another Warm-up Exercise

Consider a consumer maximizing $u(c_1) + u(c_2)$, with $u'(\cdot) > 0$, $u''(\cdot) < 0$, and $u'''(\cdot) > 0$, who can save or borrow at a real interest rate of zero ($R = 1$). Then, letting $E[\cdot]$ denote expectations conditional on period-1 information, if the consumer is optimizing:

1. $E[c_2] < c_1$ and $E[u'(c_2)] < u'(c_1)$.
2. $E[c_2] < c_1$ and $E[u'(c_2)] = u'(c_1)$.
3. $E[c_2] < c_1$ and $E[u'(c_2)] > u'(c_1)$.
4. $E[c_2] = c_1$ and $E[u'(c_2)] < u'(c_1)$.
5. $E[c_2] = c_1$ and $E[u'(c_2)] = u'(c_1)$.
6. $E[c_2] = c_1$ and $E[u'(c_2)] > u'(c_1)$.
7. $E[c_2] > c_1$ and $E[u'(c_2)] < u'(c_1)$.
8. $E[c_2] > c_1$ and $E[u'(c_2)] = u'(c_1)$.
9. $E[c_2] > c_1$ and $E[u'(c_2)] > u'(c_1)$.
10. because the consumer can borrow at zero interest rate, it will choose to make c_1 and c_2 arbitrarily large

3 Saving and Interest Rates

Consider the canonical model of consumption under certainty with T periods. Suppose instantaneous preferences take the form $u(c) = \ln(c)$ and that the discount rate is $\beta = 1$. Suppose the gross real interest rate R is constant, that income grows at a constant growth rate $G \equiv y_{t+1}/y_t$, assumed smaller than the interest rate: $G < R$. Initial assets are denoted a_0 .

1. Calculate initial consumption c_0 . Will c_0 rise or fall when R increases? Explain your results in terms of the substitution, income and wealth effects.
2. How would your answer change if (a) preferences are Leontief, i.e. the intertemporal elasticity of substitution is zero; (b) the horizon is infinite and (c) there are no initial assets ($a_0 = 0$)? Does your answer depend on G and if so, why?

4 Consumption Smoothness in the CEQ model

Consider the assumptions of the Certainty Equivalent Model (CEQ): the horizon is infinite; The income process $\{\tilde{y}_t\}$ is exogenous and stochastic; Preferences follow $U = E_0[\sum_{t=0}^{\infty} \beta^t u(c_t)]$ where $u(c) = \alpha c - 1/2\gamma c^2$ is quadratic with $\alpha, \gamma > 0$; The gross real interest rate R is constant and satisfies $\beta R = 1$.

1. Show briefly that consumption follows a random walk: $c_t = E_t[c_{t+1}]$.
2. Using the intertemporal budget constraint, show that consumption satisfies:

$$c_0 = (1 - \beta)(a_0 + \sum_{t=0}^{\infty} R^{-t} E_0[\tilde{y}_t])$$

3. Using the previous expression, show that the innovation in consumption at time t , defined as $\epsilon_{t+1} = c_{t+1} - E_t[c_{t+1}]$ satisfies:

$$\epsilon_{t+1} = (R - 1) \sum_{s=t+1}^{\infty} R^{-(s-t)} (E_{t+1}\tilde{y}_s - E_t\tilde{y}_s)$$

Interpret.

4. Suppose the income process satisfies $\tilde{y}_{t+1} = \rho\tilde{y}_t + \eta_{t+1}$ where $0 \leq \rho < 1$ and η_{t+1} is a white noise process (i.e. a sequence of identically distributed independent variables with zero mean and finite variance). Derive a relationship between ϵ_{t+1} and η_{t+1} .
5. Which has the higher variance: the innovation to income η_{t+1} or the innovation to consumption ϵ_{t+1} ? Explain.
6. Suppose instead that $\eta_{t+1} = \mu_{t+1} + \delta\mu_t$ where $\delta > 0$ and μ_t is white noise (in the time series lingo, one would say that η follows a moving average process of order 1). What does this imply for the relationship between ϵ_{t+1} and μ_{t+1} ? Is it possible for the variance of consumption innovations to exceed that of income innovations?
7. Deaton (1987) argued that $\rho = 1$ and $\delta > 0$ is a good characterization of the income process. He also found that consumption innovations appear *less* volatile than income innovations. Based on your answers to the previous questions, explain whether this is consistent with the Certainty Equivalent model. [Caveat: Some of you may notice that the method you used to answer question 6 above is not valid anymore when $\rho = 1$ because income is not stationary in that case. But it turns out the correct method yields the *same formula*, a result first demonstrated by Hansen and Sargent (1981). I am not asking you to re-derive the formula when $\rho = 1$, simply to use it.]

5 Romer 8.5.