Problem set collection for Macro II: Part 2

July 11, 2018

McCall: Short questions

1. Consider the McCall model studied in class. Everything else equal, if the job-finding rate is reduced, then the wage dispersion is reduced in the model? T/F?
2. When reasonably calibrated, the McCall model can explain most of the residual wage dispersion seen in the data. T/F?
3. In thew basic McCall model, rasing unemployment benefits b increases observed match quality. T/F?
4. In the basic McCall model, rasing the unemployment rate without raising average unemployment duration is not possible. T/F?
5. Consider the basic McCall model with an ednogenous offer distribution. We assume that indetical firms post wages to maximize profits $p - w$, where p is the productivity of the worker. In equilibrium, the offer distribution is degenerate. T/F?
6. Explain the Daimond paradox.

McCall: Reservation wages and wage dispersion

You are now to investigate how an increase in wage dispersion affects reservation wages. Consider the McCall model stuided in class, with the reservation wage given by

$$w_R - b = \frac{\lambda_u}{r+\sigma} \int_{w \geq w_R} (w - w_R) dF(w)$$

1. Show that the reservation wage equation can be rewritten as

$$w_R - b = \frac{\lambda_u}{r+\sigma} \left[(Ew - w_r) - \int_{w < w_R} (w - w_R) dF(w) \right]$$

   where $Ew = \int_0^\infty w dF(w)$ is the expected wage of the next offer.

2. Using integration by parts, show that the previous equation can be rewritten

$$w_R - b = \frac{\lambda_u}{r+\sigma}\left[(Ew - w_r) + \int_{w<w_R} F(w)dw\right]$$

3. Using the last equation, what happens to w_r if there is a mean-preserving spread in the offer distribution F?

4. What is the economic intuition behind your answer in the previous question?

McCall: Reservation wages and minimum wages

Consider the McCall model studied in class, with the addition that there is a mandated minimum wage $\underline{w}$, such that workers are not allowed to accept offers with $w < \underline{w}$. Assume that model is parameterized such that the elasticity of the unemployment rate w.r.t. the benefit level b is $\epsilon > 0$. What is the elasticity of the unemployment rate w.r.t. the minimum wage $\underline{w}$?

DMP: Short questions

- Assume the aggregate matching function is Cobb-Douglas. Describe the data requirements and a method to estimate the parameters of the matching function.
- Consider the basic DMP model with productivity y. Suppose the wage level is set fixed to $w = y$. What is the unemployment rate in this model?
- Consider the basic DMP model studied in class. Increasing the unemployed utility flow b raises the unemployment rate u through rasing the reservation wage w_r. T/F?
- Consider the basic DMP model studied in class. Increasing the workers' bargaining power reduces the unemployment rate. T/F?
- Describe the difference between an efficient and a constrained efficient allocation.
- Consider the basic DMP model. Describe the two externalities from a firm creating an additional vacancy.
- Consider the basic DMP model. Under what condition is the equilibrium efficient?
- Briefly describe the Shimer puzzle.
- Consider the basic DMP model. Why is the steady state elasticity of tightness w.r.t. productivity informative about the model's capability to match unemployment fluctations in the data?

- Consider the basic DMP model. Replacing Nash Bargaining with a fixed wage resolves the Shimer puzzle, no matter if the fixed wage is high or low. T/F?
- Briefly describe how Hagedorn and Manovskii's alternative calibration of the DMP model resolves the Shimer puzzle.

DMP: Segmented markets for high and low-skilled

Consider the continuous-time DMP model studied in class but where a fraction ϕ of the workers are high-skilled, and a fraction $1-\phi$ are low-skilled. The difference between the two worker types is that when matched with a firm, the high-skilled workers produce y_h, whereas the low-skilled produce y_l, with $y_h > y_l$. We will study the determination of wages and unemployment under two different market structures.

- Market structure A: Unemployed workers cannot signal their productivity to potential employers. Accordingly, firms cannot direct their search and the probability that a worker match with an employer is $p(\theta)$, where $\theta = \frac{v}{u}$, $u = \alpha u_h + (1-\alpha)u_l$ and α is the share of high-skilled workers among all unemployed workers. The corresponding probability that an employer match with a worker is $q(\theta)$. The worker type is revealed to the employer upon a match, so this is known to both parties when entering Nash bargaining over wages.
- Market structure B: Unemployed workers can signal their productivity to potential employers. Accordingly, Firms can direct their search and the probability that a worker of type i match with an employer who search for workers of type i is $p(\theta_i)$, where $\theta_i = \frac{v_i}{u_i}$. The corresponding probability that an employer search for type i meets a worker of type i is $q(\theta_i)$.

Consider first the market structure A.

1. Show that in steady state, $\alpha = \phi$.
2. Write down all the relevant value functions.
3. Solve for the equilibrium unemployment rate and wages for the two different worker types. Which worker type has the higher wage? Which has the higher unemployment rate? Comment on your results.

Consider now the market structure B.

4. Write down all the relevant value functions.
5. Solve for the equilibrium unemployment rate and wages for the two different worker types. Which worker type has the higher wage? Which has the higher unemployment rate? Compare with your results in subquestion 3.

DMP: Endogenous separations

Consider the DMP model with Cobb-Douglas matching function studied in class but instead of Nash bargaining, simply assume that all wages are set equal to the outside option: $w = b < y$, and instead of assuming that matches are separated at constant rate σ, assume the following: when a firm is matched with a worker, the production is initially y. From that moment and onwards, at every instant there is probability σ_ϵ that firms are hit with an i.i.d. productivity shock ϵ drawn from the distribution F with support $[\underline{\epsilon}, \bar{\epsilon}]$ and expected value $E(\epsilon) = 1$. Given productivity ϵ, firm production is $y\epsilon$. After the shock is realized, the firm decides whether to keep with the going match, or to desolve the match and open a new vacancy instead. The firm value functions are therefore

$$\begin{aligned} rV &= -c + q(\theta)(J(1) - V) \\ rJ(\epsilon) &= \epsilon y - b + \sigma_\epsilon \left[\int_{\underline{\epsilon}}^{\bar{\epsilon}} \max\{J(x), V\} dF(x) - J(\epsilon) \right] \end{aligned}$$

1. Argue that firms follow a reservation productivity strategy, in which all jobs with productivity $\epsilon < \epsilon_r$ are desolved. (Hint: Show that $\frac{\partial J(\epsilon)}{\partial \epsilon} > 0$)

2. What is $J(\epsilon_r)$?

3. Solve for $J(\epsilon)$ in closed form as function of ϵ_r.

4. Show that that the reservation productivity ϵ_r satisifies

$$\epsilon_r = \frac{b(r + \sigma_\epsilon)}{yr} - \frac{(r + \sigma_\epsilon)}{r} \left[1 + \int_{x < \epsilon_r} F(x) dx \right]$$

   (Hint: use integration by parts)

5. How does a mean-preserving spread of the productivity distribution F affect ϵ_r?

6. Solve for the steady state unemployment rate as a function of ϵ_r and θ and draw the Beveridge curve. What happens to Beveridge curve if there is a mean-preserving spread of the productivity distribution F?

7. How does θ respond to the mean-preserving spread? What happens to the equilibrium unemployment rate?

DMP: balanced-budget taxation

Consider the basic continuous-time DMP model studied in class. There is a conintuum of workers with mass 1, and large mass of firm who decides whether to post a vacancy or not. Posting a vacancy cost c

at every instant. The job-finding and job-filing rates are given by the aggregate matching function, which is $m(u, v) = Au^{\alpha}v^{1-\alpha}$. Jobs desolve at exogenous rate σ. The discount rate is r. Upon a match between a worker and firm they produce y. The wage level is determined by Nash bargaining, in which the worker bargaining power is γ.

The unemployed workers retrieve utility b. We interpret b as an unemployment benefit provided by the government with b exogenously fixed. This benefit is financed by taxing all employed workers with a fixed tax τ, which is allowed to vary to keep the government's budget is balanced. A balanced budget means that the total pax income equals the total benefint payments at every instant.

1. Write the government's budget constraint and show that τ is increasing in u.
2. Derive a relation between v and u in steady state (a Beveridge curve).
3. Write down the value functions of a firm with an open vacancy and of a form with a filled job.
4. What is the value of an open vacancy in equilibrium? Using this and the firm value functions, derive the job-creation curve in $\{w.\theta\}$-space.
5. Write down the value functions of an unemployed and employed worker
6. Use the value functions and the solution to the Nash bargaining game to derive the wage curve. Is the wage level increasing or decreasing in τ? What is the intuition?
7. Use the government's budget constraint and the beveridge curve to write the wage curve in terms of θ, w and exogenous parameters.
8. Describe the shape of the wage curve and plot it in a graph together with the job-creation curve. How many equilibria does the model have? Do your answer depend on the parameters of the model? Discuss the intuition behind your answer.
9. What is the effect of increasing b on the wage curve? Is it clear what happens to equilibrium unemployment rate u?

BM: Short questions

- Augmenting the McCall model with on-the-job-search can resolve the Daimond Paradox. T/F?
- Consider the basic Burdett-Mortensen model. Why most the equilibrium wage distribution be continuous?

- Why does the basic Burdett-Mortensen model generate more wage dispersion than the basic McCall model?
- Consider the basic Burdett-Mortensen model. How is it possible that although all firms face an identical optimization problem, the equilibrium wage distribution is non-degenerate?
- The basic Burdett-Mortensen model predicts that tenure is negatively correlated with wage. T/F?
- The basic Burdett-Mortensen model predicts that firm size is positvely correlated with wage. T/F?
- The basic Burdett-Mortensen model predicts that firm size is negatively correlated with quit rate. T/F?
- Consider the model of Postel-Vinay and Robin (Econmetrica, 2002). In this model, workers may accept a job offer where the wage is lower than their current job. T/F?
- Consider the model of Postel-Vinay and Robin (Econmetrica, 2002). Name the two sources of *within-firm* wage dispersion in the model.

BM: Long questions

Consider the basic Burdett-Mortensen model.

1. State the worker value function equations
2. Argue that the optmial acceptance rule of employed workers implies that their value function equation can be written
$$rW(w) \quad = \quad w + \lambda_e \int_{w' \geq w} (W(w') - W(w))dF(w') + \sigma(U - W(w))$$
3. Argue that the optmial acceptance rule of unemployed workers implies that their value function equation can be written
$$rU \quad = \quad b + \lambda_u \int_{w \geq w_R} (W(w) - U)dF(w)$$
4. Show that the reservation equation satisfies
$$w_R - b \quad = \quad (\lambda_u - \lambda_e) \int_{w \geq w_R} \frac{1 - F(w)}{r + \sigma + \lambda_e(1 - F(w))} dw$$
5. Show that, using the facts that the equilibrium offer distribution satisfies $F(w_R) = 0$ and is continuous and differentiable, that
$$\frac{\partial w_R}{\partial b} \quad = \quad \frac{r + \sigma + \lambda_e}{r + \sigma + \lambda_u}$$
6. Why is $\frac{\partial w_R}{\partial b}$ decreasing in λ_u and increasing in λ_e? Provide some intuition.

ICM in PE: Short questions

1. What are the two sources of a precuationary savings motive in the standard incomplete-markets model?

2. Under what condition on the household's utility function does her preference exhibit prudence?

3. Why cannot precuationary savings arise in a two-period model with quadratic preferences?

4. Consider an infinetly lived household with standard preferences that earns deterministic income stream $\{y_t\}_0^\infty$, who can borrow/save in a risk-free bond a_{t+1} at interest rate r_t subject to the borrowing constraint $a_{t+1} \geq 0$. Suppose $\beta R > 1$. The the solution of the household problem entails $c_{t+1} > c_t$ for all t. T/F?

5. Consider an infinetly lived household with standard preferences that earns deterministic income stream $\{y_t\}_0^\infty$, who can borrow/save in a risk-free bond a_{t+1} at interest rate r_t subject to the borrowing constraint $a_{t+1} \geq 0$. Suppose $\beta R < 1$. Then, the solution of the household problem entails $c_{t+1} < c_t$ for all t. T/F?

6. Consider an infinetly lived household with prudent preferences that earns stochastic income stream $\{y_t\}_0^\infty$, who can borrow/save in a risk-free bond a_{t+1} at interest rate r_t subject to the natural borrowing constraint. Suppose $\beta R = 1$. Then, the optimal asset sequence $\{a_{t+1}, a_{t+2}, ...\}$ is bounded from above. T/F?

7. Consider an infinetly lived household with prudent preferences that earns deterministic income stream $\{y_t\}_0^\infty$, who can borrow/save in a risk-free bond a_{t+1} at interest rate r_t subject to the natural borrowing constraint. Suppose $\beta R = 1$. Then, the optimal asset sequence $\{a_{t+1}, a_{t+2}, ...\}$ is unbounded. T/F?

The natural borrowing limit

What is the maxmimum amount of debt that a household can purchase without risking default? Consider an infinetly lived household that earns income stream $\{y_t\}_0^\infty$, retrieves utility from consumption, which is not allowed to be negative, and who can, in each period t, borrow/save in a risk-free bond a_{t+1} that pays of $(1+r)a_{t+1}$ in period $t+1$.

1. Write the budget constraint of the household.

2. Suppose that the income stream $\{y_t\}_0^\infty$ is deterministic. Show that the household can repay its debt a_{t+1}if and only if:

$$a_{t+1} \geq -\sum_{k=0}^{\infty} \frac{y_{t+k+1}}{(1+r)^{k+1}}$$

3. What is economic interpretation of the right-hand side of this equation?

4. Suppose $\{y_t\}_0^\infty$ is stochastic: $y_t \sim F$ where F has support $[y_{min}, y_{max}]$. What is the maximum amount of debt that the household can repay?

5. Suppose $y_{min} = 0$. What is the maximum amount of debt that the household can repay?

6. Suppose a household faces the borrowing constraint derived in question 4. Under what (standard) condition on the household's utility function u does the borrowing constraint never bind in the solution to the household's problem?

ICM in PE: CARA utility (partly from Werning)

In this problem, we will closely study the nature of precautionary savings when households have CARA utility. Consider an infinetly lived household that solves

$$\max_{a_{t+1}, c_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$
$$\text{s.t.} \quad c_t + \frac{1}{1+r} a_{t+1} = y_t + a_t$$

and a No-Ponzi constraint. We assume $y_t = \bar{y} + \epsilon_t$ where ϵ_t is i.i.d. and $E_t \epsilon_{t+1} = 0$.

Part 1

1. Assume households have quadratic utility: $u(c) = \bar{c} - c^2$. Do these preferences exhibit prudence?

2. Construct the Lagrangian, take the F.O.C.s and show us the Euler equation.

3. By iterating on the budget constraint, show that
$$\sum_{k=0}^{\infty} \frac{1}{(1+r)^k} c_{t+k} = \sum_{k=0}^{\infty} \frac{1}{(1+r)^k} y_{t+k} + a_t$$

4. By taking expectations over the previous equation and invoking the Euler equation, show that if $\beta(1+r) = 1$, then
$$c_t = \frac{r}{1+r}(y_t + a_t + \frac{1}{r}\bar{y})$$

5. Interpret this equation

6. Again assuming $\beta(1+r) = 1$, show that
$$\Delta c_t = \frac{r}{1+r}(y_t - \bar{y})$$

7. Interpret this equation

Part 2

1. Assume households have CARA utility: $u(c) = -\frac{1}{\gamma}e^{-\gamma c}$. Do these preferences exhibit prudence?

2. Show that, if the consumption function is

$$c_t = \frac{r}{1+r}(y_t + a_t + \frac{1}{r}\bar{y}) - \pi$$

where π is some constant that depend on the parameters of the household problem ($\{\beta, \gamma, r\}$ and the dsitribution of ϵ), then

$$\Delta c_t = \frac{r}{1+r}(y_t - \bar{y}) + r\pi$$

3. Use the previous result to show that the postulated consumption function is optimal for a particular value of π. (Hint: Construct the Euler equation, and show that it is satisfied with the postulated consumption function for a particular value of π)

4. Assume, for this particular subquestion, that $\epsilon \sim N(0, \sigma)$ (which allows you to compute $E_t e^{-\epsilon_{t+1}}$). Is c decreasing or increasing in σ?

5. Show that if $\beta(1 + r) = 1$, then $\pi > 0$. Compare the consumption function to that with quadratic utility. Explain the underlying reason for the difference in the two types of consumption behaviour.

6. Now assume that there is an economy populated by ontinuum (measure 1) of households solving the same problem. Argue that for the aggregate levels of consumption and assets to be constant, the interest rate must be such that $\pi = 0$. Is this interest rate higher or lower than $\frac{1}{\beta} - 1$?

7. Denote the long-run aggregate level of assets by $A(r)$. Is $A(r)$ continuous in the vicinity of r^* implicitly defined by $\pi(r^*) = 0$?

ICM in GE: Short questions

1. Consider two Ayiagari economies of the sort studied in class: one with low idiosyncratic income risk and one with high idiosyncratic income risk. In all other aspects, they are identical. Which economy has a higher capital stock?

2. Consider an Ayigari economy in which households have quadratic preferences. The equilibrium level of capital in this economy is the same as in a corresponding representative agent economy. T/F?

3. The equilibrium level of capital in an Huggett economy is always higher than in an Ayiagari economy. T/F?

4. Name three amendments to the standard Aiyagari model that has been proposed to improve its fit of the US wealth distribution.

5. The first-best allocation in a standard Ayiagari model is equivalent to the market outcome in the same model without idiosyncratic income risk. T/F?

6. The equilibrium level of output in the standard Ayiagari model is always lower than in the unconstrained Planner's solution. T/F?

7. The equilibrium level of output in the standard Ayiagari model is always higher than in the constrained Planner's solution. T/F?

8. Consider a standard Ayiagari model with exogenous labor supply, as studied in class. Adding a progressive income tax raises welfare. T/F?

9. In general, solving the Ayiagari model exactly with aggregate shocks requires keeping track of the infinite-dimensional distribution of households' assets and income. T/F?

10. In general, solving the Ayiagari model exactly with aggregate shocks is difficult due to the assumption of rational expectations. T/F?

ICM in GE: The real interest rate in a Huggett economy with a tight borrowing constraint

Consider an infinite-horizon economy with a continuum (measure 1) of ex-ante identical households each having efficiency units of labor ϵ_{it}, drawn from distribution F with finite support $[\epsilon_{min}, \epsilon_{max}]$ and mean 1, i.i.d. across households and time. Consumers can trade a non-contingent bond but cannot borrow. Each household i solves

$$\begin{aligned} \max_{a_{it+1}, c_{it}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}) \\ \text{s.t.} \quad & c_{it} + a_{it+1} \leq \epsilon_{it} w_t + (1 + r_t) a_{it} \\ & a_{it+1} \geq 0 \end{aligned}$$

where u satisfies standard conditions. A representative firm employs production function $Y_t = L_t$, where L_t is the aggregate labor endowment. There is no goverment and assets are in zero net supply.

1. Define a competitive equilibrium

2. Set up the Lagrangian to the household problem and compute the First order conditions

3. From the first order condtions, retrieve the Euler equation

$$u_c(c_{it}) \geq \beta(1+r_t)E_t u_c(c_{it+1})$$

4. Argue that the equilibrium allocation coincides with autarky, i.e., that $c_{it} = \epsilon_{it} w_t$ for all i, t.

5. Argue that the equilibrium real interest rate satisfies

$$1 + r_t \leq \frac{1}{\beta} \frac{u_c(\epsilon_{max})}{E_t u_c(\epsilon_{it+1})}$$

6. In particular, verify that there is an equilibrium in which the interest rate satisfies

$$1 + r_t = \frac{1}{\beta} \frac{u_c(\epsilon_{max})}{E_t u_c(\epsilon_{it+1})}$$

7. Why does the curvature of the utility function affect the equilibrium real interest rate, but the not equilibrium allocation, in this economy?

8. Is the equilibrium allocation efficient?

9. Is the equilibrium allocation constrained efficient?

ICM in GE: Constant savings rate I (From Perri)

Consider an economy with a continuum (measure 1) of ex-ante identical consumers each having efficiency units of labor ϵ_{it} which is i.i.d. over time and across agents, has non-negative support and mean 1. Consumers can trade a non contingent bond but cannot borrow ($a_{it+1} \geq 0$). Let's ignore micro-foundations and simply assume that the households save their labor income at constant rate γ:

$$a_{it+1} = (1+r)a_{it} + \gamma \epsilon_{it} w_t,$$

where r_t, w_t are the equilibrium interest and wage rates. Firms employ a Cobb-Douglas productions function, $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$, renting capital and labor at prices r_t, w_t and capital depreciates at geometric rate δ.

1. Show that a stationary asset distribution cannot exist if $r \geq 0$

2. For $r < 0$, solve for the aggregate supply of assets $A(r)$ in the stationary distribution and the demand for assets, $K(r)$. Draw the two functions in a graph.

3. Solve for r_t and w_t in the stationary equilibrium

4. Show and discuss what happens to long-run values of production Y_t, interest rate r_t and wages w_t if the savings rate γ increases or productivity Z_t decreases permanently.

ICM in GE: Constant savings rate II

Consider an economy with a continuum (measure 1) of ex-ante identical consumers each having efficiency units of labor ϵ_{it} which is i.i.d. over time and across agents, has non-negative support and mean 1. Consumers can trade a non contingent bond but cannot borrow ($a_{it+1} \geq 0$). Firms employ a Cobb-Douglas productions function, $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$, renting capital and labor at prices r_t, w_t and capital depreciates at geometric rate δ.

1. Let's ignore micro-foundations and simply assume that the households save their labor income at constant rate γ:

   $$a_{it+1} = (1+r)a_{it} + \gamma \epsilon_{it} w_t.$$

   Show that a stationary asset distribution cannot exist if $r \geq 0$

2. Now assume that the households save their total available resources at constant rate γ:

   $$a_{it+1} = \gamma \left[(1+r)a_{it} + \epsilon_{it} w_t\right].$$

   Show that a stationary asset distribution cannot exist if $r \geq \frac{1}{\gamma} - 1$

3. In each of the two cases, solve for the aggregate supply of assets $A(r)$ in the stationary distribution. Draw the two supply functions in a graph together with the demand for assets, $K(r)$. Under which savings behavior does the equilibrium entail a higher level of capital?