Macroeconomics II: Problem set 6

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Send your solutions to Andrii by **March 27, 12.00** at the latest.

Exercise 1: Unconstrained versus constrained investment

Consider the RBC model in lecture 3, where the firm faces a potentially binding collateral constraint. Specifically, we assume that the representative firm can borrow up within-period debt up to $\xi q_t K_t$, at 0 interest rate, such that

$$\begin{aligned} \max_{N_t, I_t, K_{t+1}} \quad & E_O \sum_{t=0}^{\infty} M_{0,t} \left(A_t F(K_t, N_t) - W_t N_t - I_t - C(I_t, K_t) \right) \\ \text{s.t.} \quad & K_{t+1} \leq I_t + (1-\delta) K_t \\ & I_t \leq \xi q_t K_t \end{aligned}$$

1. Interpret $q_t K_t$ - What is this?
2. The collateral constraint may or may not bind the solution to the firm problem. Compute the F.O.C. and the complementary slackness conditions.
3. Show that for the constraint to be binding in steady state, the parameters must satisfy $\xi < \delta$
4. Assume that $\xi = \frac{1}{2} * \delta$, write down the equilibrium characterization of the full model, log-linearize, and compute the IRF to a TFP shock, using the same parameter values in class.
5. Assume that $\xi > \delta$, write down the equilibrium characterization of the full model, log-linearize, and solve for the IRF to a TFP shock, using the same parameter values in class.
6. Explain why the response of consumption, investment and the price of capital q_t is different in your two IRFs.

Exercise 2: Solving the NK model with the method of undetermined coefficients

Consider the vanilla NK model studied in lecture 4, written on the 3-equation form:

$$\begin{aligned}
\text{DIS curve:} \quad & \hat{y}_t = -(\hat{i}_t - E_t\pi_{t+1}) + E_t\hat{y}_{t+1} \\
\text{Phillips curve:} \quad & \pi_t = \beta E_t\pi_{t+1} + \kappa\hat{y}_t \\
\text{Policy rule:} \quad & \hat{i}_t = \phi\pi_t + \nu_t
\end{aligned}$$

where we assume the monetary policy shocks follows an AR(1) process $\nu_t = \rho\nu_{t-1} + \epsilon_t$, and ϵ_t is an i.i.d. disturbance.

Because this model have no state variable, we can actually solve it analytically*. Let's compute the policy functions to a single disturbance in preiod t, i.e., $\epsilon_t > 0$ and $\epsilon_{t+s} = 0$ for all $s > 1$.

1. Since this is a linear model, a policy function for any variable $x_t \in \{\hat{y}_t, \pi_t, \hat{i}_t\}$ is some linear combination of past shocks

   $$x_t = \sum_{s=0}^{t} a^x_{t-s}\nu_s.$$

   where the coefficients a^x_{t-s} depends on the parameters of the model. Argue that $a_{t-s} = 0$ for all $s < t$.

2. Insert the three policy functions into the the three equilibrium conditions. You should now have a system of the three equations in three unknowns: $\alpha^y_0, \alpha^\pi_0, \alpha^i_0$.

3. Solve the system for the values of a^y_0, a^π_0, a^i_0.

4. Using your policy functions, plot the IRF to a monetary policy shock, using the same parameter values as in class. Verify that the IRFs are the same as in the lecture notes.

*This is not strictly true, there exist a variant of this method that also applies to models with one endogeneous state variable, see Roulleau-Pasdeloup (JEDC 2023)

Exercise 3: Identification of Taylor Rules

A big literature has, in various ways, estimated time-series regressions of nominal interest rates on inflation (and other variables), with the purpose of retrieving the coefficients of the Taylor rule, i.e., regressions of the form:

$$i_t = \alpha + \beta\pi_t + \zeta_t \tag{1}$$

The perhaps most famous example is Clarida-Gali-Gertler (QJE 2000). Let's explore how we can interpret such regressions, focusing on the case with flexible price setting.

The flexible-price model studied in the class is summarized by

$$\begin{aligned} \text{DIS curve:} \quad & i_t = r + E_t \pi_{t+1} \\ \text{Policy rule:} \quad & i_t = r + \phi \pi_t + \nu_t \end{aligned}$$

where r is the log of the steady state real interest rate and the monetary policy shock is assumed to follow $\nu_t = \rho \nu_{t-1} + \epsilon_t$, where ϵ_t is an i.i.d. disturbance. Note, the system is not written in terms of log deviations from steady state, but just in terms of logs.

1. Assume that $\phi > 1$. Show that the unique bounded solution is given by

$$\pi_t = -\frac{\nu_t}{\phi - \rho}$$

2. Show that, in equilibrium, inflation follows

$$i_t = r + \rho \pi_t. \tag{2}$$

3. Why does this equilibrium condition (2) not contradict the assumed policy rule $i_t = r + \phi \pi_t + \nu_t$?

4. Would you interpret the time-series estimate of β in Equation (1) as the structural Taylor rule coefficient ϕ? Explain in the words what the identification problem is.

5. Bonus question (not graded): Show that the same identification problem arises also with sticky price setting. You may wanna use the solutions you retrieved in Exercise 2.