Simon Fraser University

DEPARTMENT OF ECONOMICS

Comprehensive Examination in Macroeconomics

June 15, 2012

Committee: G. Dunbar (chair)
K. Kasa
P. Klein

The examination is four (4) hours long. Please write on one side only to facilitate photocopying. Indicate your pseudonym on each book and number all pages. The examination consists of three parts. They are worth 30, 60 and 60 points respectively. Allocate your time efficiently. Watch out for questions that continue on the next page. Good luck!

1 Answer <u>all three</u> of the following questions. (10 points each)

It is your reasoning that will be graded. You should be specific about the analytical framework(s) you are using by outlining the essential elements of any model and/or identifying the macroeconomic literature on which your analysis is based.

1. Compare and contrast the econometric identification strategies of Thomas Sargent and Christopher Sims. Discuss their relative strengths and weaknesses. How does each respond to the Lucas Critique?

2. In a real business cycle model with money introduced via a cash-in-advance constraint, does inflation reduce output by lowering the return to investment?

3. Is government debt neutral (*i.e.* has no effect on real allocations) in models with uninsured idiosyncratic risk?

2 Answer the following question (60 points)

Wage Dispersion in the Mortensen-Pissarides Model (Kasa). In the Mortensen-Pissarides model discussed in class, workers were identical, and there was no wage dispersion in equilibrium. This question asks you to extend the model in order to generate a simple theory of equilibrium wage dispersion.

Consider an economy consisting of a unit measure continuum of risk neutral, ex ante identical workers, and a larger measure continuum of ex ante identical firms. Time is discrete and infinite. Workers and firms share a common discount factor β. Workers can search freely, but firms must pay a vacancy cost of c while they are searching for workers. Matching takes place according to a standard constant returns matching function. Let $\mu(\theta_t)$ denote the probability that a worker meets a firm, and $\mu(\theta_t)/\theta_t$ be the probability that a firm meets a worker, where as usual, θ_t denotes labor market 'tightness' in period-t (ie, the measure of vacancies relative to unemployed workers). Assume $\mu(\theta)$ is continuous and twice differentiable, with $\mu''(\theta) < 0 < \mu'(\theta)$. Also assume $\mu(\theta) \leq \min\{\theta, 1\}$.

In contrast to the model discussed in class, assume now that when a worker and firm are matched, they draw a match-specific productivity, y, from a continuous distribution $F(y)$, with support $[\underline{y}, \overline{y}]$. Assume y is observed by both workers and firms, and is constant during a match. As usual, suppose matches are exogenously destroyed with probability s each period, and that workers and firms set wages according to Nash bargaining. Finally, assume that a worker gets b each period he is not matched.

(a) Write down the Bellman equations for an unemployed worker (U_t), an employed worker in a match with productivity y, ($V_t(y)$), a firm with an unfilled vacancy (W_t), and an operating firm with productivity y, ($J_t(y)$). Note that all value functions are evaluated at the beginning of the period, before matching and separation occur. Also, let $e_t(y) \in \{0, 1\}$ be an indicator variable denoting whether a job is created after observing y.

(b) Now focus on a steady state equilibrium. Write down the Nash bargaining conditions, assuming that all workers have bargaining power η. Show how a match surplus divided. Show that there is a threshold productivity, $\hat{y}$, such that $e(y) = 1$ iff $y \geq \hat{y}$. Derive an explicit expression for $\hat{y}$ in terms of b, β, and U.

(c) Derive an expression for equilibrium wages as a function of productivity, $w(y)$. Using this, characterize a steady state equilibrium by deriving explicit expressions for θ and $\hat{y}$.

(d) Finally, using the above expressions for $\hat{y}$ and $w(y)$, characterize the equilibrium wage distribution as a function of the productivity distribution $F(y)$.

3 Answer <u>two</u> of the following three questions (30 points each)

1. Competitive equilibrium (Klein)
Consider an environment where a representative agent has preferences represented by

$$\sum_{t=0}^{\infty} \beta^t [\ln c_t - \frac{1}{2} h_t^2].$$

The resource constraint is given by

$$c_t + g_t = h_t; \ t = 0, 1, \ldots$$

where $\{g_t\}_{t=0}^{\infty}$ is an exogenously given sequence. Additional constraints are $c_t \geq 0$ and $h_t \geq 0$. The government can raise revenue in two ways only: it can issue one-period bonds or it can tax labour income proportionally at rate τ_t. Initial government debt b_0 is zero.

(a) Show that any allocation satisfying the resource constraint and the equation

$$\sum_{t=0}^{\infty} \beta^t h_t^2 = \frac{1}{1-\beta}$$

is part of a competitive equilibrium profile.
Hint: First state what is meant by this precisely.

(b) Suppose you are given a competitive equilibrium allocation. How would you go about finding the sequence of government debt issues b_t, $t = 1, 2, \ldots$?

(c) Let p_t denote the relative price of consumption in period t in terms of consumption in period 0. Show that in any competitive equilibrium, $p_t b_t \to 0$ as $t \to \infty$. Hint: You may invoke without proof the converse of what you showed in (a).

2. Overlapping generations (Klein)
Consider an overlapping generations environment where agents live for two periods. Each generation t has infinitely many small members of total mass $N(t)$. Population growth is constant and given by n. More explicitly,

$$N(t+1) = nN(t).$$

Preferences are represented by

$$u_t^h = \ln c_t^h(t) + \ln c_t^h(t+1).$$

Endowments are $\omega_t^h = [2, 1]$.

(a) Find the competitive equilibrium. Notice that it is independent of n.

(b) Suppose $n = 1$. Show that the competitive equilibrium allocation is not Pareto optimal.

(c) Suppose $n = 1/3$. Is the competitive equilibrium Pareto optimal?

3. International Business Cycles (Dunbar)

This questions studies the role of international investment in business cycles. Suppose there are two countries, indexed $i = 1, 2$. Country i specializes in the production of good i at time t, y_{it}, which can be consumed or invested in either country. Output is produced using capital, k_{ijt}, where i refers to the country of origin, j refers to the country of production and t indexes time. The production possibility frontiers are described by:

$$c_{11t} + c_{12t} + k_{11t+1} + k_{12t+1} = y_{1t} = z_{1t} k_{11t}^{\alpha} k_{21t}^{\alpha}$$

$$c_{21t} + c_{22t} + k_{21t+1} + k_{22t+1} = y_{2t} = z_{1t} k_{12t}^{\alpha} k_{22t}^{\alpha}.$$

In the above equations, c_{ijt} refers to consumption in country j of output produced in country i at time t. z_{it} is a country-specific technology shock in country i at time t. Assume that $[z_{1t} z_{2t}]^T]$ evolves according to a Markov chain. Note that there is complete depreciation of capital.

Agents in both countries have identical preferences:

$$U_i = \mathsf{E}_0 \sum_{t=0}^{\infty} \beta^t (\ln c_{i1t} + \ln c_{i2t}), \quad \beta \in (0,1), i = 1, 2.$$

(a) Define the "equal weights" social welfare function

$$W = \mathsf{E}_0[U_1 + U_2]$$

Write down a Bellman equation which represents the dynamic programming problem of maximizing this social welfare function subject to the world-wide feasibility constraints.

(b) Show that the solution to the problem in (1) satisfies:

$$k_{i,j,t} = \mathsf{A} y_{i,t}$$

and

$$c_{i,j,t} = \frac{1 - 2\mathsf{A}}{2} y_{i,t}$$

where A is a constant. Solve for A as a function of the parameters of the problem.

(c) Write down a system of equations giving output in each country (y_{1t} and y_{2t}) as functions of lagged output. Using this system, describe how a country-specific technology shock affects output in the two countries over time.

(d) Is the solution to this problem a competitive equilibrium allocation? Explain.