Planner's problem:

$$\max_{\{c_{it}(s^t)\}} \sum_i \sum_t \sum_{s^t} \alpha_i \beta^t \pi(s^t) u(c_{it}(s^t))$$

$$\text{s.t.} \quad \sum_i c_{it}(s^t) \le \sum_i y_{it}(s^t) \quad \forall t, s^t$$

F. O. C.

$$\frac{\alpha_i \beta^t \pi(s^t) u_c(c_{it}(s^t))}{\alpha_j \beta^t \pi(s^t) u_c(c_{jt}(s^t))} = \frac{\lambda_t(s^t)}{\lambda_t(s^t)}$$

$$\Rightarrow \quad \frac{u_c(c_{it}(s^t))}{u_c(c_{jt}(s^t))} = \frac{\alpha_j}{\alpha_i}$$

CRRA:

$$\frac{u_c(c_{it}(s^t))}{u_c(c_{jt}(s^t))} = \frac{c_{it}(s^t)^{-\sigma}}{c_{jt}(s^t)^{-\sigma}}$$

Full insurance implies

$$c_{it}(s^t) = \left(\frac{\alpha_j}{\alpha_i}\right)^{-\frac{1}{\sigma}} c_{jt}(s^t)$$

$$\Rightarrow \quad \sum_i c_{it}(s^t) = \sum_i \left(\frac{\alpha_j}{\alpha_i}\right)^{-\frac{1}{\sigma}} c_{jt}(s^t)$$

$$\Leftrightarrow \quad C_t(s^t) = \alpha_j^{-\frac{1}{\sigma}} c_{jt}(s^t) \cdot \sum_i \left(\frac{1}{\alpha_i}\right)^{-\frac{1}{\sigma}}$$

$$\Leftrightarrow \quad c_{jt}(s^t) = \underbrace{\frac{\alpha_j^{\frac{1}{\sigma}}}{\sum_i \alpha_i^{\frac{1}{\sigma}}}}_{\theta_j} C(s^t)$$

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$$\mathcal{L} = U(C) + \beta E V(M') - \lambda(M' - R(M - C) - Y') - \mu(C - M)$$

F.O.C.

$C: \; U_c(C) - R\lambda - \mu = 0$

$M': \; \beta E V_m(M') - \lambda = 0$

$\Rightarrow U_c(C) = \beta R E V_m(M') + \mu$

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At the optimum:

$C = C(M)$

$$\Rightarrow V(M) = U(C(M)) + \beta E V\left[R(M^{*} - C(M)) + Y'\right]$$

$$\Rightarrow V_m(M) = U_c(C(M))\, C_m(M)$$

$$+ \underbrace{\beta R E V_m[M']}_{U_c(C(M)) - \mu} \cancel{R} (1 - C_m(M))$$

$$\Rightarrow V_m(M) = U_c(C)\, C_m(M) + U_c(C)(1 - C_m(M))$$

$$- \mu (1 - C_m(M))$$

$$= U_c(C) - \mu (1 - C_m(M))$$

If cc binding: $C_m(M) = 1$

If not: $\mu = 0$

$$\Rightarrow V_m(M) = U_c(C)$$