Macroeconomics II, Lecture XII: The Buffer-Stock Savings Model

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Introduction

- Last lecture: theoretical implications of uninsurable income risk for consumption-savings dynamics
  1. With incomplete markets, ex-ante homogeneous households will be **ex-post heterogeneous** in terms of $\{C, A, Y\} \Rightarrow$ no aggregation into representative agent
  2. With incomplete markets, consumption dynamics influenced by a **precautionary savings** motive

- Today: using incomplete-markets consumption-savings theory for quantitative empirical analysis
  - Research program pioneered by Deaton, Zeldes and Carroll
  - Builds on foundational work by Friedman, Modigliani, Hall and others

Agenda

1. Formulating and solving the canonical buffer-stock savings model
    - Recursive formulation
    - Solution algorithm
    - Calibration
2. Consumption-savings dynamics in the buffer-stock savings model
    - Consumption-savings dynamics without income risk
    - Consumption-savings dynamics with income risk
3. 2 famous applications
    1. Blundell-Preston-Pistaferri (AER 2008):
    2. Gourinchas-Parker (Ecmtra 2002): Consumption over the Life Cycle
    3. Kaplan-Violante (Ecmtra 2014): A Model of the Consumption Response to Fiscal Stimulus Payments

The canonical buffer-stock savings model

- Buffer-stock savings model = income-fluctuations problem with persistent shocks
  - Key papers: Zeldes (JPE 1989; QJE 1989), Deaton (Ecmta 1991, Book 1992), Carroll (QJE 1997)

- Household problem:

$$\begin{aligned} \max_{C_t, A_{t+1}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \\ \text{s.t.} \quad & C_t + A_{t+1} = Y_t + RA_t \\ & Y_t = P_t e^{\epsilon_t} \\ & P_t = P_{t-1} G e^{\nu_t} \\ & A_{t+1} \geq -\underline{A} \\ & C_t \geq 0 \end{aligned}$$

- Terminology:
  - P_t — permanent income
  - ν_t — permanent income shocks
  - ϵ_t — transitory income shocks

- We assume that ϵ_t, ν_t are known at the time of choosing C_t, A_{t+1}

Comments

- G is a constant for simplicity
  - It can be any deterministic function
  - In many applications, it is a function of age, education, ability etc.
- Why a permanent-transitory formulation of the income process?
  - In the data, we see that some changes to residualized income are very persistent, whereas others are very short-lived
  - Permanent-transitory formulation provides a parsimonious parameterization of household income process that captures these features
  - For some applications, estimating the persistence of income shocks can be important
    - E.g., business-cycle applications (typically quarterly models) with unemployment shocks
- CRRA utility: reasonable baseline + big tractability gains
  - allows us to normalize the problem w.r.t. to permanent income

Recursive formulation

- The model cannot be solved analytically
- To solve the problem, and to investigate the properties of the solution, we need to recast the problem on its recursive form
- Introduce cash on hand $M_t = Y_t + RA_t$
- Recursive formulation

$$
\begin{aligned}
V(\qquad) \quad &= \quad \max_{C,A'} U(C) + \beta E V(\qquad) \\
\text{s.t.} \quad & A' = M - C \\
& M' = P' e^{\epsilon'} + RA' \\
& P' = GPe^{\nu'} \\
& A' \geq -\underline{\mathrm{A}} \\
& C \geq 0
\end{aligned}
$$

- Which are the state variables?
  - What information, known at time t, is useful for the household in choosing C, A' and compute $U(C) + \beta EV'()$?

Recursive formulation

- The model cannot be solved analytically

- To solve the problem, and to investigate the properties of the solution, we need to recast the problem on its recursive form

- Introduce cash on hand $M_t = Y_t + RA_t$

- Recursive formulation

$$
\begin{aligned}
V(M,P) \quad &= \quad \max_{C,A'} U(C) + \beta E V(M',P') \\
\text{s.t.} \quad & A' = M - C \\
& M' = P' e^{\epsilon'} + RA' \\
& P' = GPe^{\nu'} \\
& A' \geq -\underline{A} \\
& C \geq 0
\end{aligned}
$$

- Which are the state variables?
  - What information, known at time t, is useful for the household in choosing C, A' and compute $U(C) + \beta E V'()$?

Normalization w.r.t. permanent income

- It appears that the state variables are cash on hand M and permanent income P
- With $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$ being CRRA, permanent income is actually not a state variable
- Define
  - $m = \frac{M}{P}$
  - $c = \frac{C}{P}$
  - $a' = \frac{A'}{P}$
  - $\underline{\mathrm{a}} = \frac{\underline{\mathrm{A}}}{P}$
  - $v(M, P) = \frac{V(M,P)}{P^{1-\sigma}}$

Normalization w.r.t. permanent income II

- Household problem

$$
\begin{aligned}
V(M,P) &= \max_{C,A'} \frac{C^{1-\sigma}}{1-\sigma} + \beta E V(M',P') \\
\text{s.t.} \quad & A' = M - C \\
& M' = P' e^{\epsilon'} + RA' \\
& P' = GPe^{\nu'} \\
& A' \geq -\underline{A} \\
& C \geq 0
\end{aligned}
$$

- Using our definitions

$$
\begin{aligned}
P^{1-\sigma} v(M,P) &= \max_{c,a'} \frac{(cP)^{1-\sigma}}{1-\sigma} + \beta E P'^{1-\sigma} v(M',P') \\
\text{s.t.} \quad & a'P = mP - cP \\
& m'P' = P' e^{\epsilon'} + Ra'P \\
& P' = GPe^{\nu'} \\
& a'P \geq -\underline{a}P \\
& cP' \geq 0
\end{aligned}
$$

Normalization w.r.t. permanent income III

- Dividing through

$$
\begin{aligned}
v(M,P) \quad &= \quad \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta E \left(\frac{P'}{P}\right)^{1-\sigma} v(M',P') \\
\text{s.t.} \quad & a' = m - c \\
& m' = e^{\epsilon'} + Ra'\frac{P}{P'} \\
& \frac{P'}{P} = Ge^{\nu'} \\
& a' \geq -\underline{a} \\
& c \geq 0
\end{aligned}
$$

- Substituting the law-of-motion for permanent income:

$$
\begin{aligned}
v(M,P) \quad &= \quad \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta E \left(Ge^{\nu'}\right)^{1-\sigma} v(M',P') \\
\text{s.t.} \quad & a' = m - c \\
& m' = e^{\epsilon'} + \frac{Ra'}{Ge^{\nu'}} \\
& a' \geq -\underline{a} \\
& c \geq 0
\end{aligned}
$$

Normalized recursive program

- Normalized household problem:

$$
\begin{aligned}
v(M,P) \quad &= \quad \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta E_t \left(Ge^{\nu'} \right)^{1-\sigma} v(M',P') \\
\text{s.t.} \quad & a' = m - c \\
& m' = e^{\epsilon'} + \frac{Ra'}{Ge^{\nu'}} \\
& a' \geq -\underline{a} \\
& c \geq 0
\end{aligned}
$$

- Which are the state variables?

Normalized recursive program

- Normalized household problem:

$$\begin{aligned} v(m) &= \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta E_t \left(Ge^{\nu'}\right)^{1-\sigma} v(m') \\ \text{s.t.} \quad & a' = m - c \\ & m' = e^{\epsilon'} + \frac{Ra'}{Ge^{\nu'}} \\ & a' \geq -\underline{a} \\ & c \geq 0 \end{aligned}$$

- Which are the state variables?

What did we just learn?

- The sufficient state variable is $m = \frac{M}{P}$
- Economics:
  - Decision functions in normalized problem: $c = c(m), a' = a'(m)$
  - Un-normalized decision functions $C = Pc(m)$, $A' = Pa'(m)$
  - Implication: A permanent-income rich household will consume the same amount of an increase in his normalized cash-on-hand as permanent-income poor household
- Computations:
  - We have reduced a two-dimensional function equation to a one-dimensional functional equation
  - Big computational gain of dimension reduction when solving the problem numerically
- Key assumptions: CRRA utility (or, more generally, a power-function utility) and linear constraints

Properties of solution

- Solution given by a consumption function $c(m)$ and a savings function $a'(m) = m - c(m)$

- As usual, an interior solution $c(m)$ must satisfy the Euler equation

$$c^{-\sigma} = \beta R E \left(\left(G e^{\nu'} c' \right)^{-\sigma} \right)$$

- Else, the credit constraint is binding and $c(m) = m + \underline{a}$

- Given some parametric restrictions, the Bellman equation defines a contraction mapping, and we can solve the equation using value function iteration

Value function iteration

- Construct a discrete grid $\hat{m} = \{\hat{m}_1, \hat{m}_2, ..., \hat{m}_N\}$
- Guess a discrete value function $\hat{v}^0 = \{\hat{v}_1^0, \hat{v}_2^0, ..., \hat{v}_N^0\}$
- For each $\hat{m}_k$, $k = 1, ..., N$, solve the decision problem

$$\begin{aligned} \max_{c,a'} \quad & \frac{c^{1-\sigma}}{1-\sigma} + \beta \sum_i \sum_j \pi(\eta_i)\pi(\epsilon_j) \left(Ge^{\nu_i'}\right)^{1-\sigma} \hat{v}^0 \left(e^{\epsilon_j} + \frac{Ra'}{Ge^{\nu_i}}\right) \\ \text{s.t.} \quad & a' = \hat{m}_k - c \\ & a' \geq -\underline{a} \\ & c \geq 0 \end{aligned}$$

  using a standard numerical maximization method (e.g. Newton-Raphson)
- If $\hat{m}_k < e^{\epsilon_j} + \frac{Ra'}{Ge^{\nu_i}} < \hat{m}_{k+1}$ for some $k < N \Rightarrow$ interpolate v_k^0, v_{k+1}^0 to compute $\hat{v}^0 \left(e^{\epsilon_j} + \frac{Ra'}{Ge^{\nu_i}}\right)$
- The maximum of the objective function is your new guess $\hat{v}_k^1$
- Repeat until convergence

Parameterization

- Parameters: $\beta, \sigma, G, R, \underline{a}$ and the distribution of ϵ', ν'
- How to select parameter values?
- G and R can be directly observed in the data
- Data at hand: panels of income and consumption
  - Survey data, e.g., PSID (US)
  - Register data, e.g., Swedish income and wealth registers
- The distribution of ϵ', ν' can be estimated using panel data on household income
- Calibrate $\beta, \sigma, \underline{a}$ typically done by method of moments
  - Given preferences, our model defines a mapping from an income process to an allocation of assets and consumption at the household level
  - Populate an economy with, say 1000 households, and simulate the economy using the stochastic shock processes and the decision functions retrieved from solving the household problem
  - Set parameters so implied model moments fit data moments
    - Example moments: mean asset holdings, share with negative assets, consumption-income profiles...

Income process estimation

- Standard practice: parametric assumption + method of moments
- In logs ($\tilde{x} = \log X$), our process is

$$Y_t = P_t e^{\epsilon_t}, \qquad y_t = p_t + \epsilon_t$$
$$P_t = P_{t-1} G e^{\nu_t}, \quad p_t = p_{t-1} + g + \nu_t$$

which give us

$$\Delta y_t = g + \nu_t + \epsilon_t - \epsilon_{t-1}$$
$$\Delta y_{t-1} = g + \nu_{t-1} + \epsilon_{t-1} - \epsilon_{t-2}$$

which means that

$$Var(\Delta y_t) = Var(\nu_t) + 2Var(\epsilon_t)$$
$$CoV(\Delta y_t, \Delta \tilde{y}_{t-1}) = -Var(\epsilon_t)$$

- Typically, we apply this estimator to income growth residuals in household panel data
- This method can be generalized in several dimensions (and recent admin data has taught us a lot!):
  - Guvenen-Karahan-Ozkan-Song (Ecmtra 2021): income growth residuals exhibits fat tails and significant skewness (normality is a bad assumption)
  - Carter Braxton-Herkenhoff-Rothbaum-Schmidt (AER 2025): permanent earnings risk has evolved differently across the skill distribution over time
  - Harmenberg-Lizarraga (2025): a generalized square root process capture earnings dynamics at the top of the distribution very well

Consumption-savings dynamics in the buffer-stock savings model

- Now we know how to solve and parameterize our model
- Next step: look at how households behave in the model
- Recall:
  - state variable: $m = \frac{M}{P} = \frac{Y+RA}{P}$
  - law of motion: $m' = e^{\epsilon'} + \frac{Ra'(m)}{Ge^{\nu'}}$
  - decisions: $c = c(m)$ and $a' = a'(m) = m - c(m)$
  - unnormalized decisions: $C = c(m)P$ and $A' = a'(m)P$
- Defining features of the model: uninsurable income risk and potentially binding credit constraint
- To understand the implications of these features, we first remind ourselves about consumption-savings dynamics in a frictionless perfect-foresight model

Perfect-foresight model

- Set $\epsilon = \nu = 0$ and $\underline{a} = \underline{a}^{nbl}$, where $\underline{a}^{nbl}$ is the natural borrowing limit
- Perfect-foresight Bellman equation:

$$
\begin{aligned}
v(m) \quad &= \quad \max_{c,a'} \frac{c^{1-\sigma}}{1-\sigma} + \beta G^{1-\sigma} v(m') \\
\text{s.t.} \quad & a' = m - c \\
& m' = 1 + \frac{Ra'}{G} \\
& a' \geq -\underline{a}^{nlb} \\
& c \geq 0
\end{aligned}
$$

Perfect-foresight model

- As we will iterate on the budget constraint, I now reintroduce time subscripts
- As $\underline{a} = \underline{a}^{nbl}$, we will always have an interior solution
- Interior solution satisfies

$$c_t^{-\sigma} = \beta R \left(G c_{t+1}\right)^{-\sigma}$$

- For simplicity, set σ so that $(\beta R)^{-\frac{1}{\sigma}} G = 1 \Rightarrow$

$$c_t = c_{t+1}$$

Perfect-foresight model: solution

- To solve for $c = c(m)$, iterate on the budget constraint and the law-of-motion for m:

$$a_{t+1} = m_t - c_t$$
$$m_{t+1} = 1 + \frac{Ra_{t+1}}{G}$$

which gives us

$$
\begin{aligned}
& m_{t+1} = 1 + \frac{R}{G}(m_t - c_t) \\
\Rightarrow \quad & m_t = c_t - \frac{G}{R} + \frac{G}{R} m_{t+1} \\
\Rightarrow \quad & m_t = c_t - \frac{G}{R} + \frac{G}{R}\left(c_{t+1} - \frac{G}{R} + \frac{G}{R} m_{t+2}\right) \\
\dots \quad & \\
\Rightarrow \quad & m_t = \sum_{k=0}^{\infty}\left(\frac{G}{R}\right)^k c_{t+k} - \sum_{k=1}^{\infty}\left(\frac{G}{R}\right)^k + \lim_{T\to\infty}\left(\frac{G}{R}\right)^T m_{t+T} \\
\Rightarrow \quad & m_t = c_t \frac{1}{1-\frac{G}{R}} - \frac{G}{R}\frac{1}{1-\frac{G}{R}}
\end{aligned}
$$

where we have used that $c_{t+k} = c_t$ for all k and the transversality condition $\lim_{T\to\infty}\left(\frac{G}{R}\right)^T m_{t+T} = 0$

Perfect-foresight model: solution II

- Therefore, we get

$$c_t = \left(1 - \underbrace{\frac{G}{R}}_{\text{eff. disc. rate}}\right)\left(\underbrace{m_t}_{\text{c.o.h.}} + \underbrace{\frac{G}{R}\frac{1}{1-\frac{G}{R}}}_{\text{NPV future norm. inc.}}\right)$$

- $\Rightarrow$ household behave according to permanent-income hypothesis (PIH)

Perfect-foresight model: MPC out of transitory income shocks

- Consider a marginal change in M, holding P constant:
  - I.e., consider a transitory shock to current income

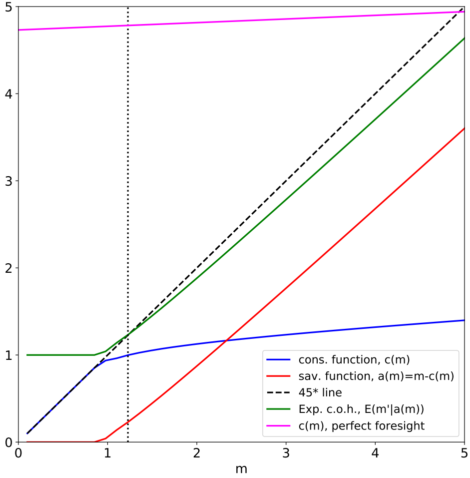
$$
\begin{aligned}
\frac{\partial C}{\partial M} &= \frac{\partial\left(c(m)P\right)}{\partial\left(mP\right)} \\
&= \frac{P\partial\left(c(m)\right)}{P\partial m} \\
&= \frac{\partial c(m)}{\partial m}
\end{aligned}
$$

which gives us

$$
\frac{\partial C}{\partial M} = \left(1 - \frac{G}{R}\right)
$$

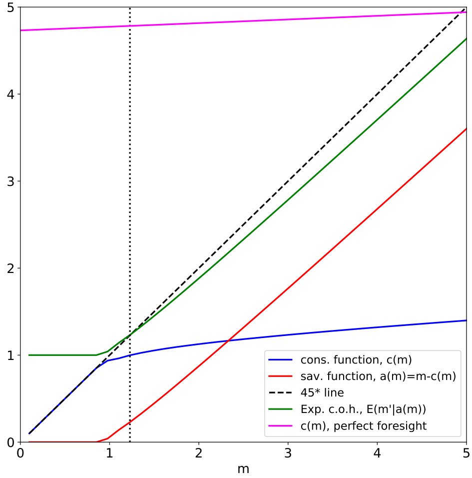
- Insights:
  - Marginal propensity to consume (MPC) out of current income is constant
  - For reasonable parameters: MPC out of current income is very small

Consumption dynamics in the buffer-stock savings model



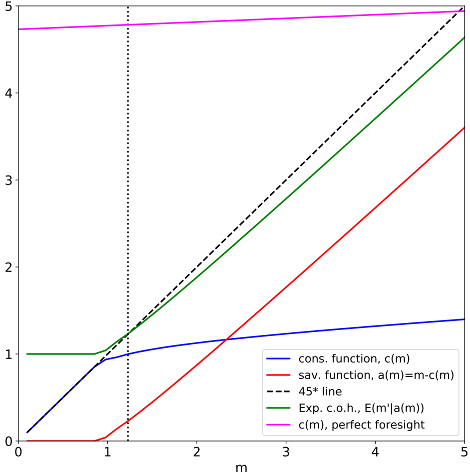
- Parameter values: $\beta = 0.95$. $\sigma = 1.5$, $R = 1.04$, $G = 1.03$, $\underline{a} = 0$, $\sigma_\epsilon, \sigma_\nu$ taken from Gourinchas-Parker (Ecmtra, 2002)
- Code available by email

Consumption dynamics in the buffer-stock savings model



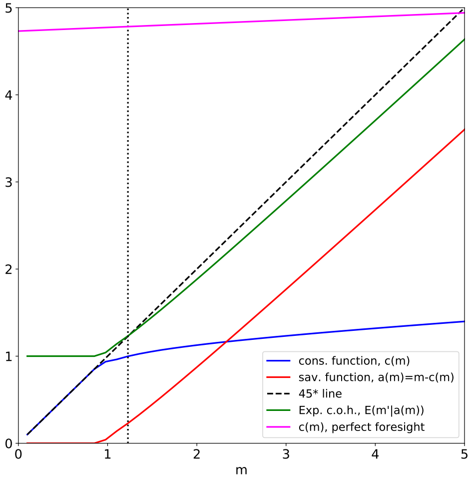
- Result 1: with income risk, households consume less than if no income risk
  - with income risk, having assets have insurance value
  - without income risk, household run down current assets quickly due to impatience

Consumption dynamics in the buffer-stock savings model



- Result 2: with income risk, consumption function is concave
  - Equivalently: with income risk, MPC out of current income is larger and decreasing
  - As household approaches constraint, MPC out of current income grows to 1
  - With a lot of assets, households behaves as if no income risk and $MPC \rightarrow \left(1 - \frac{G}{R}\right)$

Consumption dynamics in the buffer-stock savings model



- Result 3: because of concavity, there exists a target buffer stock of assets
  - To insure themselves, households seek to hold a certain amount of money relative to their permanent income
  - With a lot of assets, households dissave

Using the buffer-stock savings model

- Now we understand the basic properties of the buffer-stock savings model
- Move on to analyze how to use to model to interpret the data
- Early literature (Zeldes, Deaton, Carroll and others) focused on testing buffer-stock model against PIH model (description does not fit all papers)
  - Perhaps ex post unsurprisingly, PIH was rejected in several dimensions
  - In late 80's/early 90's, numerical solutions were still difficult $\Rightarrow$ calibrated/estimated models were not used much
- We will focus on second-generation literature
- Two influential applications:
  1. Gourinchas-Parker (Ecmtra 2002): Consumption over the Life Cycle
  2. Blundell-Pistaferri-Preston (AER 2008): Consumption Inequality and Partial Insurance
  3. Kaplan-Violante (Ecmtra 2014): A Model of the Consumption Response to Fiscal Stimulus Payments
- I use my notation and will make some simplifications when discussing these papers

Gourinchas-Parker: overview

- Q: how does consumption-savings dynamics evolve over the life-cycle?
  - Does buffer-stock savings behavior or permament-income hypothesis provide better fit of the data? Does it depend on age?
- Method: Estimate structural life-cycle model with idiosyncratic income risk
  - Estimate income processes in PSID data
  - Construct consumption-income age profiles using CEX data
  - Estimate model to match profiles
  - Decompose savings behavior over the life cycle using estimated model

Gourinchas-Parker: Model

- Life-cycle version of the model we have described:

$$\begin{aligned} \max_{C_t, A_{t+1}} \quad & E_0 \sum_{t=0}^{T} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \\ \text{s.t.} \quad & C_t + A_{t+1} = Y_t + RA_t \\ & A_{t+1} \geq 0 \\ & C_t \geq 0 \end{aligned}$$

- For $0 \leq t \leq N$, households are working and income process is

$$\begin{aligned} Y_t &= P_t e^{\epsilon_t} \\ P_t &= P_{t-1} G_t e^{\nu_t} \end{aligned}$$

note that G_t has t subscript

- For $N+1 \leq t \leq T$, households are retired and Y_t is deterministic

Gourinchas-Parker: Calibration

- $R = 1.034$ taken from average return on safe bond assets
- For other parameters, use micro data
  - PSID (panel with rich info on income but small sample)
  - CEX (rich consumption data and larger sample, but weak panel dimension)
- Income process:
  - Estimate age-income profile G_t by regressing income on age dummies with various controls
  - Estimate permanent-transitory income shocks from income growth residuals
- Consumption-income profiles
  - Estimate consumption-income age profiles in the data
  - Set β and σ so that model mimics (as close as possible) this profile
- GP do this procedure for 16 different educational-occupational groups, but we only focus on the results for the total population here

Gourinchas-Parker: Estimated consumption-income profile

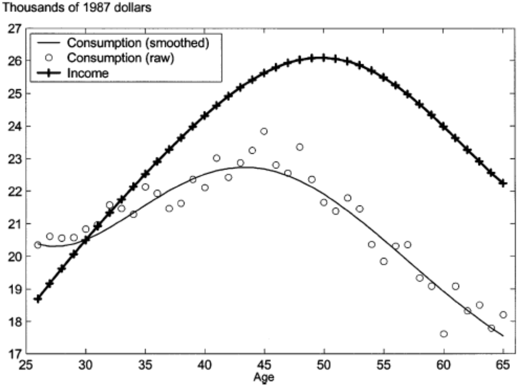
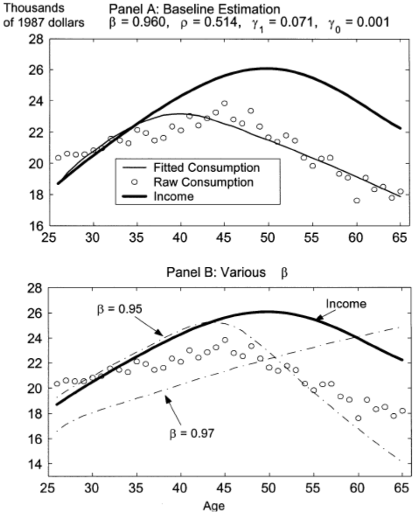


FIGURE 2.—Household consumption and income over the life cycle.

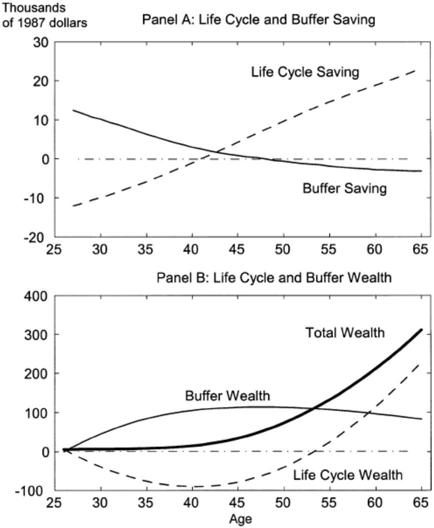
- Hump-shape in consumption directly rejects that working-age hosueholds have full insurance - why?

Gourinchas-Parker: Matching model to estimated consumption-income profile



- Matching consumption-income profile pins down β and σ

Gourinchas-Parker: Results



- By turning on and off income risk in the model, GP decompose households savings behavior into its *life-cycle* and *precautionary* components

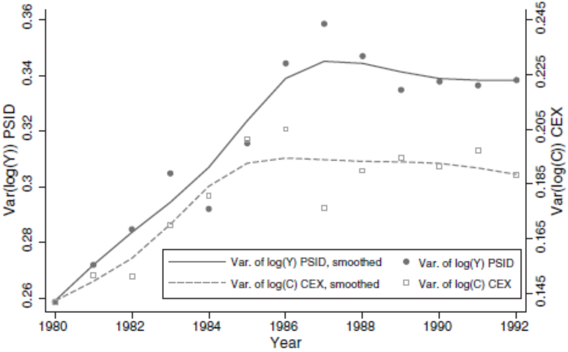
Gourinchas-Parker: Results

- Young households are buffer-stock savers, old households are similar to PIH savers
- Young households: start with no buffer and and retirement is far away $\Rightarrow$ saving primarily reflect precautionary behavior
- Middle-age housholds: already have a buffer and retirement is near $\Rightarrow$ saving primarily reflect consumption-smoothing w.r.t. retirement

Gourinchas-Parker: Discussion

- Early paper estimating a consumption-savings model to match key moments in micro data
- Large literature has followed
- Two semi-recent (job market) papers:
  - Boar (ReStud 2021): Estimate OLG-version of the model using PSID data
    - Finds that significant part of household buffers is explained by childrens' income risk
  - Druedahl and Martinello (ReStat 2020): Use Swedish register data to estimate consumption response to exogenous bequest shocks
    - Find that households quickly consume these bequests $\Rightarrow$ suggests low β
    - Still, these household typically have sizable wealth holdings.
    - Matching these moments suggests that risk aversion is much higher than previously estimated

Blundell-Pisaterri-Preston (AER 2008): The question



- In the 80's, US income inequality grew a lot
- Consumption inequality also grew, but at lower pace and the gap between consumption and income inequality increased significantly in the late 80's/early 90's
- What can explain this divergence?

BPP: The idea

- Buffer-stock savings theory teaches us that consumption responds very differently to permanent vs. transitory income shocks

- With micro panel data on consumption and income, we can estimate
    1. The evolution of permanent income risk/inequality
    2. The evolution of transitory income risk/inequality
    3. The evolution of the pass-through of permanent and transitory income to consumption (the extent of insurance)

- For the third part, BPP suggest to estimate the effect using a linear consumption rule
    - People sometimes say this is a "semi-structural" approach, I don't know what "semi" means other than "linear"

- The paper makes two contributions
    1. Show how to impute consumption in PSID (panel data on income) using a food Engel curves
    2. Use consumption-augmented PSID to estimate to evolution of permanent/transitory income risk and pass-through the consumption

- I focus on the second contribution here

BPP: Setup

- Suppose we have panel data on log consumption and log income $\{c_{it}, y_{it}\}$
- First, take out deterministic component by regressing c_{it}, y_{it} on various demographic variables
- Second, assume residualized income evolves according to (slightly more general process in the paper, but it doesn't make any difference)

$$y_{it} = p_{it} + \epsilon_{it}$$
$$p_{it} = p_{it-1} + \nu_{it}$$

- Third, assume consumption responds to shocks according to

$$\Delta c_{it} = \phi_t \nu_{it} + \psi_t \epsilon_{it} + \xi_{it}$$

  - Note 1: treatment effect is time-varying
  - Note 2: this consumption rule is postulated, not derived

BPP: Strategy

- With the postulated consumption rule, we have

$$Var(\Delta c_{it}) = \phi_t^2 Var(\nu_{it}) + \psi_t^2 Var(\epsilon_{it}) + Var(\xi_{it})$$

  Assuming that the error term is stationary, consumption growth inequality may grow because
  1. Inequality $Var(\nu_{it}), Var(\epsilon_{it})$ increase, or
  2. Pass-through parameters ϕ_t, ψ_t increase

- Goal: estimate the time series of $Var(\nu_{it}), Var(\epsilon_{it}), \phi_t, \psi_t$

- Method: Estimation by method of moments (we need at least four moments)

BPP: Identification

- Income process implies

$$Var(\Delta y_{it}) = Var(\nu_{it}) + 2Var(\epsilon_{it})$$
$$CoV(\Delta y_{it}, \Delta y_{it-1}) = -Var(\epsilon_{it})$$

- $\Rightarrow$ A rise in income growth inequality without a corresponding decline in the autocovariance means that permanent, and not transitory, income variance has increased

- We also have that

$$\begin{aligned} CoV(\Delta c_{it}, \Delta y_{it}) &= CoV(\phi_t \nu_{it} + \psi_t \epsilon_{it} + \xi_{it}, \nu_{it} + \epsilon_{it} - \epsilon_{it-1}) \\ &= \phi_t Var(\nu_{it}) + \psi_t Var(\epsilon_{it}) \end{aligned}$$

  which, together with

$$\Delta Var(\Delta c_{it}) = Var(\nu_{it})\Delta\phi_t^2 + \phi_{t-1}^2 \Delta Var(\nu_{it}) + Var(\epsilon_{it})\Delta\psi_t^2 + \psi_t^2 \Delta Var(\epsilon_{it})$$

  can be used to estimate ϕ_t, ψ_t given the income variances

BPP: Income moments in the data

Table 3—The Autocovariance Matrix of Income Growth

Year	$\text{var}(\Delta y_t)$	$\text{cov}(\Delta y_{t+1}, \Delta y_t)$	$\text{cov}(\Delta y_{t+2}, \Delta y_t)$
1980	0.0832	−0.0196	−0.0018
	(0.0089)	(0.0035)	(0.0032)
1981	0.0717	−0.0220	−0.0074
	(0.0075)	(0.0034)	(0.0037)
1982	0.0718	−0.0226	−0.0081
	(0.0051)	(0.0035)	(0.0026)
1983	0.0783	−0.0209	−0.0094
	(0.0066)	(0.0034)	(0.0042)
1984	0.0805	−0.0288	−0.0034
	(0.0055)	(0.0036)	(0.0032)
1985	0.1090	−0.0379	−0.0019
	(0.0180)	(0.0074)	(0.0038)
1986	0.1023	−0.0354	−0.0115
	(0.0077)	(0.0054)	(0.0038)
1987	0.1116	−0.0375	0.0016
	(0.0097)	(0.0051)	(0.0046)
1988	0.0925	−0.0313	−0.0021
	(0.0080)	(0.0042)	(0.0032)
1989	0.0883	−0.0280	−0.0035
	(0.0067)	(0.0059)	(0.0034)
1990	0.0924	−0.0296	−0.0067
	(0.0095)	(0.0049)	(0.0050)
1991	0.0818	−0.0299	NA
	(0.0059)	(0.0040)	
1992	0.1177	NA	NA
	(0.0079)		

- Both variance and autocovariance of income growth grew during the eigthies, but autocovariance grew relatively more

BPP: Consumption moments in the data

Table 4—The Autocovariance Matrix of Consumption Growth

Year	$\text{var}(\Delta c_t)$	$\text{cov}(\Delta c_{t+1}, \Delta c_t)$	$\text{cov}(\Delta c_{t+2}, \Delta c_t)$
1980	0.1275	−0.0526	0.0022
	(0.0097)	(0.0076)	(0.0056)
1981	0.1197	−0.0573	0.0025
	(0.0116)	(0.0084)	(0.0043)
1982	0.1322	−0.0641	0.0006
	(0.0110)	(0.0087)	(0.0060)
1983	0.1532	−0.0691	−0.0056
	(0.0159)	(0.0100)	(0.0067)
1984	0.1869	−0.1003	−0.0131
	(0.0173)	(0.0163)	(0.0089)
1985	0.2019	−0.0872	NA
	(0.0244)	(0.0194)	
1986	0.1628	NA	NA
	(0.0184)		
1987	NA	NA	NA
1988	NA	NA	NA
1989	NA	NA	NA
1990	0.1751	−0.0602	−0.0057
	(0.0221)	(0.0062)	(0.0067)
1991	0.1646	−0.0696	NA
	(0.0142)	(0.0100)	
1992	0.1467	NA	NA
	(0.0130)		

- Variance of consumption growth grew in the early eigthies, declined thereafter

BPP: Consumption-income moments in the data

TABLE 5—THE CONSUMPTION-INCOME GROWTH COVARIANCE MATRIX

Year	$\text{cov}(\Delta y_t, \Delta c_t)$	$\text{cov}(\Delta y_{t+1}, \Delta c_t)$	$\text{cov}(\Delta y_t, \Delta c_{t+1})$
1980	0.0040	0.0013	0.0053
	(0.0041)	(0.0039)	(0.0037)
1981	0.0116	−0.0056	−0.0043
	(0.0036)	(0.0032)	(0.0036)
1982	0.0165	−0.0064	−0.0006
	(0.0036)	(0.0031)	(0.0039)
1983	0.0215	−0.0085	−0.0075
	(0.0045)	(0.0049)	(0.0043)
1984	0.0230	−0.0030	−0.0119
	(0.0052)	(0.0043)	(0.0050)
1985	0.0197	−0.0035	−0.0035
	(0.0068)	(0.0047)	(0.0065)
1986	0.0179	−0.0015	NA
	(0.0048)	(0.0052)	
1987	NA	NA	NA
1988	NA	NA	NA
1989	NA	NA	0.0030
			(0.0040)
1990	0.0077	0.0045	−0.0016
	(0.0045)	(0.0065)	(0.0042)
1991	0.0112	0.0011	−0.0071
	(0.0044)	(0.0049)	(0.0042)
1992	0.0082	NA	NA
	(0.0048)		

- No trend in consumption-income correlation

BPP: Estimated evolution of permanent-income inequality

TABLE 6—MINIMUM-DISTANCE PARTIAL INSURANCE AND VARIANCE ESTIMATES

		Whole sample	No college	College	Born 1940s	Born 1930s
σ_ζ^2	1979–81	0.0102	0.0067	0.0099	0.0074	0.0057
(Variance perm. shock)		(0.0035)	(0.0037)	(0.0053)	(0.0035)	(0.0072)
	1982	0.0207	0.0154	0.0252	0.0210	0.0166
		(0.0041)	(0.0053)	(0.0060)	(0.0061)	(0.0075)
	1983	0.0301	0.0317	0.0233	0.0184	0.0246
		(0.0057)	(0.0075)	(0.0089)	(0.0058)	(0.0086)
	1984	0.0274	0.0333	0.0176	0.0219	0.0224
		(0.0049)	(0.0074)	(0.0060)	(0.0077)	(0.0102)
	1985	0.0293	0.0287	0.0204	0.0187	0.0333
		(0.0096)	(0.0073)	(0.0151)	(0.0066)	(0.0225)
	1986	0.0222	0.0173	0.0312	0.0222	0.0111
		(0.0060)	(0.0068)	(0.0101)	(0.0077)	(0.0114)
	1987	0.0289	0.0202	0.0354	0.0307	0.0079
		(0.0063)	(0.0073)	(0.0098)	(0.0080)	(0.0111)
	1988	0.0157	0.0117	0.0183	0.0155	0.0007
		(0.0069)	(0.0079)	(0.0110)	(0.0076)	(0.0099)
	1989	0.0185	0.0107	0.0274	0.0176	0.0217
		(0.0059)	(0.0101)	(0.0061)	(0.0082)	(0.0182)
	1990–92	0.0134	0.0092	0.0216	0.0081	0.0063
		(0.0042)	(0.0045)	(0.0065)	(0.0059)	(0.0091)

- Permanent-income inequality grew a lot in the eigthies, declined thereafter

BPP: Estimated evolution of transitory-income inequality

σ_ε^2	1979	0.0415	0.0465	0.0302	0.0314	0.0342
(Variance trans. shock)		(0.0059)	(0.0096)	(0.0056)	(0.0054)	(0.0070)
	1980	0.0318	0.0330	0.0284	0.0269	0.0306
		(0.0039)	(0.0053)	(0.0059)	(0.0056)	(0.0072)
	1981	0.0372	0.0364	0.0253	0.0319	0.0267
		(0.0035)	(0.0053)	(0.0046)	(0.0058)	(0.0064)
	1982	0.0286	0.0376	0.0214	0.0264	0.0342
		(0.0039)	(0.0063)	(0.0042)	(0.0049)	(0.0078)
	1983	0.0286	0.0372	0.0186	0.0190	0.0284
		(0.0037)	(0.0063)	(0.0037)	(0.0045)	(0.0077)
	1984	0.0351	0.0405	0.0305	0.0223	0.0453
		(0.0039)	(0.0059)	(0.0051)	(0.0047)	(0.0100)
	1985	0.0380	0.0356	0.0496	0.0280	0.0504
		(0.0075)	(0.0056)	(0.0130)	(0.0062)	(0.0115)
	1986	0.0544	0.0474	0.0452	0.0261	0.0672
		(0.0058)	(0.0076)	(0.0085)	(0.0060)	(0.0153)
	1987	0.0480	0.0520	0.0421	0.0440	0.0499
		(0.0054)	(0.0082)	(0.0071)	(0.0093)	(0.0095)
	1988	0.0383	0.0472	0.0343	0.0386	0.0543
		(0.0047)	(0.0074)	(0.0060)	(0.0068)	(0.0148)
	1989	0.0369	0.0539	0.0219	0.0360	0.0493
		(0.0068)	(0.0126)	(0.0051)	(0.0070)	(0.0132)
	1990–92	0.0506	0.0536	0.0345	0.0429	0.0753
		(0.0040)	(0.0062)	(0.0049)	(0.0060)	(0.0127)

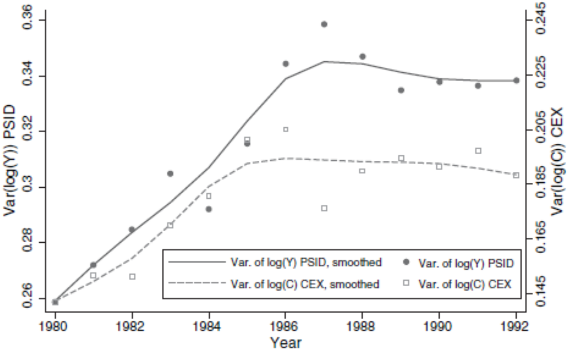
- Transitory-income inequality declined in the early eigthies, grew from mid-eighties and onwards

BPP: Estimated insurance

ϕ	0.6423	0.9439	0.4194	0.7928	0.6889
(Partial insurance perm. shock)	(0.0945)	(0.1783)	(0.0924)	(0.1848)	(0.2393)
ψ	0.0533	0.0768	0.0273	0.0675	−0.0381
(Partial insurance trans. shock)	(0.0435)	(0.0602)	(0.0550)	(0.0705)	(0.0737)
p-value test of equal ϕ	23%	99%	8%	81%	18%
p-value test of equal ψ	75%	33%	29%	76%	4%

- Consumption responds a lot more to permanent income than to transitory income shocks, but no evidence of change over time

BPP: Back to the question



- In the early 80's, US consumption inequality tracked income inequality because income inequality growth was driven by permanent-income inequality growth
- The series diverged as from the mid 80's and onwards, income inequality growth stem trom transitory inequality growth, against which households are much more insured (meaning that pass-through to consumption is low)

BPP: Discussion

- Why do we care?
  - Important to understand the nature of income inequality in order to assess welfare implications
  - An increase in income inequality driven by transitory-income shocks is much less severe compared to it being driven by permanent-income shocks
- See Heathcote-Storesletten-Violante (RED 2010) for documentation of similar facts for following decades
- Semi-recent (job market) papers:
  - Commault (AEJmacro 2021): Natural experiment evidence suggest much less insurance to transitory income shocks, how does this square with BPP and other semi-structural estimation approaches?
    - Shows that BPP's postulated consumption rule is not consistent with buffer-stock savings model
    - Using the model-derived rule and a similar identification strategy, she finds much less insurance to transitory income shocks
  - Straub (2019): Augmented BPP-like estimation procedure suggest consumption does not respond linearly to permanent-income shocks
    - Poor household respond more, rich less
    - Shows that a calibrated buffer-stock model can account for this if adding non-homothetic preferences
    - Implication 1: the rise in income inequality can explain the secular decline in the risk-free interest rate
    - Implication 2: Secular increase in debt may imply secular decline in aggregate consumption demand, causing "secular stagnation" (Mian-Straub-Sufi, QJE 2021)

Kaplan-Violante (Ecmtra 2014): Motivation

- Natural experiments suggest that the within-a-quarter *aggregate* MPC out of transitory income shocks is quite large
  - In 2001 and 2007-2008, US government provided cash transfers to households as part of stimulus program, with random timing of the transfer
  - Johnson-Parker-Souleles (AER 2006) and Parker-Souleles-Johnson-McClelland (AER 2011) exploits random timing to estimate MPCs of approximately 0.25
  - See also Misra-Surico (AEJmacro, 2014) and Jappelli-Pistaferri (AEJmacro, 2014)
- We know from buffer-stock theory that individual MPC can be large among wealth-poor households
- However, households with little net assets account for small share of total consumption $\Rightarrow$ cannot explain why aggregate MPC is large
- How to make sense of this?
- Aggregate MPC is a key moment for understanding business cycles and the macroeconomic effect of stablization policies

Kaplan-Violante: Overview

- Idea: even though many households have substantial wealth holdings, much of this wealth is *illiquid*
  - Housing wealth, retirement accounts...
- It is primarily the access to liquid wealth that should determine the ability of households to insure against transitory income shocks
- Introduce the concept of "wealthy hand-to-mouth" (wealthy HtM) households: households with a lot of assets but little liquid assets
- Two contributions:
  1. Document that up to 1/4 of US households can be classified as wealthy HtM households
  2. Extend buffer-stock savings model to have both liquid and illiquid savings
     - Calibrate to match documented evidence
     - Assess whether model can explain quasi-experimental evidence on aggregate MPC

Kaplan-Violante: Descriptive evidence on asset holdings

	Median (\$2001)	Mean (\$2001)	Fraction Positive	Return (%)
Earnings plus benefits (age 22–59)	41,000	52,745	–	–
Net worth	62,442	150,411	0.90	1.7
Net liquid wealth	2,629	31,001	0.77	−1.5
Cash, checking, saving, MM accounts	2,858	12,642	0.92	−2.2
Directly held MF, stocks, bonds, T-Bills	0	19,920	0.29	1.7
Revolving credit card debt	0	1,575	0.41	–
Net illiquid wealth	54,600	119,409	0.93	2.3
Housing net of mortgages	31,000	72,592	0.68	2.0
Retirement accounts	950	34,455	0.53	3.5
Life insurance	0	7,740	0.27	0.1
Certificates of deposit	0	3,807	0.14	0.9
Saving bonds	0	815	0.17	0.1

[a] Authors' calculations based on the 2001 Survey of Consumer Finances (SCF). The return reported in the last column is the real after-tax risk-adjusted return. MM: money market; MF: mutual funds. See Appendix B.1 for additional details.

- Define HtM household as a household with liquid assets $< 1/2$ of monthly earnings
- Define wealthy HtM as a HtM households with net illiquid assets $> x$ dollars ($x = 0, 1000, 3000$)
- Depending on x and the definition of illiquid assets, you find that wealthy HtM households make up $7 - 24\%$ of the population

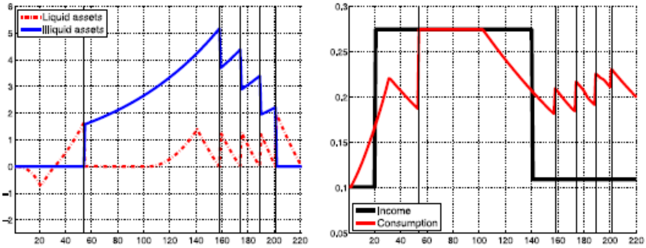
Kaplan-Violante: (simplified) Model

- A simplified version of KV's life-cycle model

$$
\begin{aligned}
\max_{C_t, A_{t+1}} \quad & E_0 \sum_{t=0}^{T} \beta^t U(c_t) \\
\text{s.t.} \quad & C_t + A^{\text{liq}}_{t+1} + A^{\text{ill}}_{t+1} = Y_t + R^{\text{liq}} A^{\text{liq}}_t + R^{\text{ill}} A^{\text{ill}}_t - AC(A^{\text{ill}}_{t+1}, A^{\text{ill}}_t) \\
& AC(A^{\text{ill}}_{t+1}, A^{\text{ill}}_t) = \begin{cases} 0 & \text{if } A^{\text{ill}}_{t+1} = RA^{\text{ill}}_t \\ \kappa & \text{if } A^{\text{ill}}_{t+1} \neq RA^{\text{ill}}_t \end{cases} \\
& Y_t = P_t e^{\epsilon_t} \\
& P_t = P_{t-1} G e^{\nu_t} \\
& A^{\text{ill}}_{t+1}, C_t \geq 0 \\
& A^{\text{liq}}_{t+1} \geq -\bar{A}
\end{aligned}
$$

- Their full model also include
  - Utility flow from illiquid assets (think housing)
  - Different interest rate on borrowing compared to saving
  - Taxes and transfers
  - Richer income process
- When taking model to the data, the key difficulty lies in calibrating κ

Kaplan-Violante: Behavior of a wealthy HtM household

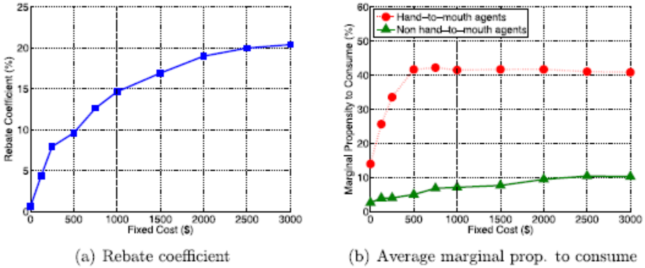


(a) Life-cycle asset accumulation (b) Life-cycle income and consumption path

FIGURE 2.—Example of life-cycle of a wealthy hand-to-mouth agent in the model.

- A model period is a quarter, period 0 is age 20.
- Since $R^{\text{ill}} > R^{\text{liq}}$, saving in illiquid assets maximizes life-time consumption
- Fixed cost κ means that you want to make as few transactions in illiquid assets as possible $\Rightarrow$ illiquid assets provide poor insurance
- If return difference is large enough, household put in all cash-on-hand into illiquid asset account when investing, becoming liquidity constrained for several periods thereafter

Kaplan-Violante: Implications for the aggregate MPC



(a) Rebate coefficient

(b) Average marginal prop. to consume

FIGURE 5.—Rebate coefficient and marginal propensity to consume, by transaction cost.

- Matching average returns on liquid and illiquid savings, KV finds that even with modest fixed cost, the model can explain high aggregate MPC
- KV also show that this model can match cross-sectional evidence on the distribution of MPC across income and wealth

Kaplan-Violante: Discussion

- In conventional (rep-agent) macro models, MPC is low and fluctuations in income therefore plays little role for fluctuations in aggregate demand
  - This is counterintuitive and at odds with micro-level evidence
  - KV provides a theory for why income fluctuations may matter
  - Important for the development of Heterogeneous-Agent New-Keynesian (HANK) models (more about this later...)
- An alternative theory to explain high aggregate MPC: some households are just not very optimizing and consume directly what they get independently of their financial situation
  - See, e.g., Campbell-Mankiw (NBER annual, 1989)
  - Fagereng-Holm-Natvik (AEJmacro 2021): Using Norwegian register data on consumption response to lottery winnings, they show aggregate MPC is smoothly declining with time
  - Auclert-Rognlie-Straub (JPE 2025): Alternative behavioral theory has a hard time explaining the time pattern in MPC, but KV's two-asset model can
- Recent explorative approaches to understanding heterogeneity in MPC
  - Aguiar-Bils-Boar (REStud 2025), Colarieti-Mei-Stantcheva (2025), Carlsson-D'Amico-Öberg-Skans-Walentin (2025)

Summing up

- Buffer-stock savings model provides a powerful framework for quantitative and empirical research of consumption-savings dynamics
- Big literature - we've only glanced at some applications today.
- Accompied with big literature on estimating household earnings dynamics
- Research in this area operates at the intersection of theory and micro data, often using both structural and reduced-form approaches
- So far, our investigation has been focused on microeconomics: how households behave taking prices, shocks and policy as given
- Next lecture: how does incomplete-market households interact with the aggregate economy?
  - We need to develop a general equilibrium model