Exam Ph.D. Macroeconomics II

Department of Economics, Uppsala University

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Instructions

- Writing time: 5 hours.
- The exam is open book. You may bring and consult any part of the course material, but you are not allowed to use any electronic devices.
- The exam has 76 points in total
- A passing grade requires a) at least 30 points on the exam, and b) 50 points in total for the course (incl the points you have from your problem sets).
- Start each question on a new paper. Write your anonymous code on all answer pages.
- You may write your solutions by pen or pencil; use your best handwriting.
- Answers shall be given in English.
- Motivate your answers carefully; if you think you need to make additional assumptions to answer the questions, state them.
- If you any questions during the exam, you may call me (+46 730 606 796) at any time between 3 PM and 5 PM.

A simplifed RBC model with additive technology shocks (20 points)

Consider an RBC economy consisting of a representative household that chooses consumption and investment to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t),$$

where $U(C_t) = C_t - \theta C_t^2$ and $\theta > 0$. For this question, assume that C_t is always in the range where $U'(C_t) > 0$. The depreciation rate in this economy is zero, so capital evolve according to

$$K_{t+1} = K_t + I_t.$$

The representative firm rents capital from the household and operates the production function

$$Y_t = K_t + \epsilon_t,$$

in a competitive market. We interpret ϵ_t as a technology shock. ϵ_t evolves according to

$$\epsilon_t = \rho \epsilon_{t-1} + \nu_t$$

where $\rho \in (0, 1)$ and ν_t are i.i.d. shocks with $E_t \nu_t = 0$ and $Var(\nu_t) = \sigma^2$.

1. (4 points) Set up the firm and household problem and define a competitive equilibrium for this economy
2. (4 points) This economy has no frictions and markets are complete. The first welfare theorem therefore applies and the allocation coincides with that of the social planner's problem. State the social planner problem for this economy.
3. (3 points) Compute the first order conditions of the Social Planner problem and find the Euler Equation.
4. (3 points) The simplified nature of this economy means that we can solve for the allocations analytically. Guess that consumption takes the form $C_t = \alpha + \delta K_t + \gamma \epsilon_t$. Given this guess and using the resource constraint, what is K_{t+1} as a function of K_t and ϵ_t?
5. (3 points) What values must the parameters α, δ, γ have for the Euler equation to be satisfied for all values of K_t and ϵ_t?
6. (3 points) Consider a one-time shock to ϵ at time t: $\nu_t > 0$. Sketch the impulse-respone functions for C_t and K_t.

A McCall model with on-the-job wage growth (20 points)

In this question we will study job search in partial equilibrium. Time is continuous. Workers' utility is linear and the discount rate is r. Workers can be employed or unemployed. Unemployed workers recieve flow utility b and get job offers at rate λ_u. When recieving a job offer, the wage of that offer is drawn from a distribution with cdf $F(w)$ with finite support $\mathcal{W} = [w_{min}, w_{max}]$, and the unemployed household decides whether to accept or reject. If accepting, the worker becomes employed and stick with the wage contract until he/she is separated, which happens at the exogenous rate σ.

1. (2 points) Denoting the value of employment with $W(w)$ and unemployment with U, write the Bellman equations for an employed and unemployed worker
2. (2 points) Find the reservation wage equation, which implicitly determines the reservation wage as function of primitives.

Now, we assume that employed workers can experience wage growth on the job. Specifically, while employed, workers experience wage changes at rate $\lambda_w > 0$. When the shock is realized, an employed worker with current wage w draws an ϵ from a distribution with cdf $G(\epsilon)$ with finite support $\mathcal{E} = [0, \epsilon_{max}]$ and mean value $\bar{\epsilon}$. The new wage w' is then $w' = w(1+\epsilon)$. All other assumptions of the model are unchanged.

3. (4 points) Write the new Bellman equation for an employed worker.
4. (4 points) Guess that a solution to the worker problem has an employment value $W(w)$ that is linear in w: $W(w) = kw + m$. What are the values of k and m if the Bellman equation of the employed worker is to be satisfied for all wages w?
5. (4 points) Using the guess, find the reservation wage equation. Is the reservation wage in this model higher or lower compared to the model without stochastic wage growth?
6. (4 points) In the model without stochatic wage growth, the mean-min ratio of observed wages is given by

$$Mm = \frac{\frac{\lambda}{r+\sigma} + 1}{\frac{\lambda}{r+\sigma} + \rho}$$

   where λ is the job-finding rate and ρ is the average replacement rate (ρ is defined by $b = \rho\bar{w}$, where $\bar{w}$ is the average observed wage). If matched to the same data on the discount rate, separation rate, job-finding rate and average replacement rate, would you expect the model with stochastic wage growth to predict more or less residual wage dispersion compared to the model with constant wages on the job? Would it make a big difference?

The Hosios condition (16 points)

In class, I claimed that the vanilla DMP model is constrained efficient if and only if

$$\gamma = \epsilon_{M,\theta}$$

where γ is worker bargaining power and $\epsilon_{M,\theta} = 1 - \frac{\theta\lambda_u'(\theta)}{\lambda_u(\theta)}$ is the elasticity of matches M w.r.t. to tightness θ (and $\lambda_u(\theta)$ is the job-finding rate). In this question, I will ask you to prove this condition in steady state.

1. (5p) Consider a firm that is considering opening a vacancy: What is private expected gain to the firm from doing so? Write this gain in terms of the job-filling rate, worker bargaining power and total surplus S. What is the private cost? (Hint: the expected gain must equal the probability of the firm matching with a worker times the gain from matching)

2. (5p) Now consider a hypothetical social planner: what is the expected social gain from a firm opening a vacancy? Write this gain in terms of the marginal impact of vacancy creation of matching, $\frac{\partial M}{\partial v}$, and total surplus S. What is the social cost?

3. (6p) Prove the Hosios condition by equating the private and social *net gains* from opening a vacancy.

Constrained efficiency in a two-period Aiyagari model (20 points)

Consider an economy with a continuum (measure 1) of ex-ante identical households, each living for two periods. Each household i has utility given by

$$\log(c_{i1}) + \beta E \log(c_{i2})$$

where c_{i1}, c_{i2} are period 1 and 2 consumption, β is the discount factor and E is the expected value operator. In period 1, each household is endowed with y_1 units of output that can either be consumed, c_{i1}, or invested, k_i. In period 2, households recieve income from the capital they saved in period 1 and from wages earned from supplying l_i efficiency units of labor. l_i is a random variable, i.i.d. across households and equals $1+\epsilon$ with probability 1/2 and $1-\epsilon$ with probability 1/2, with $0 < \epsilon < 1$. The Law of large numbers imply that the aggregate efficiency units of labor supply is $L = 1$. In period 2, output is produced by a competitive representative firm which operate a Cobb-Douglas production function $K^\alpha L^{1-\alpha}$, renting capital and labor services from the households at rate r and w, respectively.

1. (2p) Write the household and firm problems and define a competitive equilibrium for this economy.

2. (5p) Solve for the equilibrium level of capital and interest rate and show that the interest rate is decreasing in ϵ.

3. (3p) Explain the intuition for why the interest rate is decreasing in ϵ.

4. (5p) Define individual consumption risk as the ratio between individual period 2 consumption in the "good" state and the "bad" state. Show that a) in partial equilibrium, household i percieves that its individual consumption risk is lower if it chooses a higher k_i and b) that, in general equilibrium, if all households choose a higher k_i, their individual consumption risk is unaffected. Explain how this can be the case.

5. (5p) Define the ex-ante social welfare function as

$$W = \int_{i=0}^{1} \log(c_{i1}) + \beta \log(c_{i2}) di$$

Show that, when evaluating W at the decentralized equilibrium allocation, W would increase if all households where to save a little bit less (while maintaining that markets clear). Explain the intuition for this result and how it relates to the notion of "constrained efficiency". Hint: compute $\frac{\partial W}{\partial K}$ using the fact the households behave optimally at the equilibrium allocation.