Heterogeneous agents and inequality
Session 3
Aggregation and the representative consumer

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Macroeconomics II.2

Stockholm Doctoral Program in Economics 2017

Intro

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    1. **When is this neglect of heterogeneity justified?** When is the behaviour of aggregate equilibrium quantities and prices in the heterogeneous economy observationally equivalent to those in a representative agent economy?
    2. Consumption under uncertainty
    3. Empirical estimates of individual earnings risk

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  1. "Aggregation" (also "Gorman", or demand aggregation)
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- Two particular questions:
  1. "Aggregation" (also "Gorman", or demand aggregation)
     Under what conditions does **demand for goods** not depend on distribution of individual resources?
  2. Existence of a representative consumer
     Under what conditions do aggregate **equilibrium quantities** implied by individual optimality solve the problem of a (hypothetical) representative consumer?

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When is the neglect of heterogeneity justified?

- Discussion:
  1. For a given supply function, demand aggregation implies that equilibrium is independent of individual heterogeneity
  2. Demand aggregation does not necessarily imply that aggregate demand functions solve a representative consumer's problem. However, usually it implies a representative consumer with "aggregate" preferences of the same class as consumers.
  3. The equilibrium allocation may solve a representative consumer's problem **at equilibrium prices** even without demand aggregation.

Learning points

- Demand aggregation $\Leftrightarrow$ Quasi-homothetic preferences
- Complete markets $\Rightarrow$ Equilibrium quantities solve some RA problem
- The empirical implications of complete markets for consumption dynamics are usually rejected by the data

I. Gorman Aggregation

Aggregation: Definition

- An economy admits aggregation if aggregate quantities and prices are independent of the distribution of individual quantities across agents.

The Neoclassical Growth Model with heterogeneity

- $t = 1, 2, ...$
- Single consumption good
- **Agents**:
  - $i = 1, ..., N$ groups of consumers with mass μ_i, $\sum_{i=1}^{N} \mu_i = 1$
  - $j = 1, ..., M$ firms
- **Preferences**: consumers maximize time-separable utility

$$U_i = max_{\{c_{it}, a_{it+1}\}} E[\sum_{t=0}^{\infty} \beta^t u_i(c_{it})] \quad (1)$$

with $u_i' > 0$, $u_i'' \leq 0$

The Neoclassical Growth Model with heterogeneity

- **Endowments**: Consumer i starts with a_{i0}, receives exogenous stochastic labour efficiency units e_{it}, $t = 1, 2, ...$
- **Technology**: Firm j hires labour and capital to max profits from concave CRS technologies $z_j F_j(k_j, n_j)$
- **Competitive behaviour**: agents take prices as given

1. Aggregation with identical utility and production

- Assumptions:
  1. **A1** Equal endowments a_{i0}
  2. **A2** Identical, strictly concave preferences and technology
  3. **A3** No idiosyncratic or aggregate uncertainty
- Note: No assumption on market structure (assets available)

Result 1: Aggregation with identical utility and production

If every firm has the same CRS production function, consumers have the same initial endowments and same preferences and their utility function is <u>strictly</u> concave, then the neoclassical growth model admits a formulation with one representative firm and one representative household.

2. Relaxing **A1**: Aggregation with heterogeneous wealth levels

- Consider this in a **static** economy:
  - Populated by agents $i = 1, ..., N$, endowed with a_i, no production
  - u_i strictly increasing and concave over $\{c^1, ..., c^M\}$ with price vector p

$$C(p, \{a_i\}) = \sum_{i=1}^{N} \mu_i c_i (p, a_i),$$

- i.e. when is the consumtion of many different consumers equal to the consumtion of one representative consumer with wealth=their aggregate wealth?

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$$C\left(p, \{a_i\}\right) = \sum_{i=1}^{N} \mu_i c_i\left(p, a_i\right),$$

- Question: When can we write $C\left(p, \{a_i\}\right) = C\left(p, \sum_{i=1}^{N} \mu_i a_i\right)$?
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- Question: When can we write $C\left(p, \{a_i\}\right) = C\left(p, \sum_{i=1}^{N} \mu_i a_i\right)$?
- Answer: When $\sum_{i=1}^{N} \mu_i c_i(p, a_i) = C(p, \sum_{i=1}^{N} a_i)\ \forall p, a_i$; need constant & equal MPC out of wealth (linear Engle curves)

$$\frac{\partial c_i\left(p, a_i\right)}{\partial a_i} = \frac{\partial c_j\left(p, a_j\right)}{\partial a_j} \text{ for all } (i, j)$$

Relaxing **A1**: Aggregation with heterogeneous wealth

Constant, equal MPC out of wealth implies Marshallian demand function of form

$$c_i\left(p, a_i\right) = \kappa_i^0\left(p\right) + \kappa\left(p\right) a_i \tag{2}$$

Roy's Identity: Implies indirect utility function:

$$v_i\left(p, a_i\right) = \alpha_i\left(p\right) + \beta\left(p\right) a_i \tag{3}$$

Result 1.1: Gorman Aggregation

If and only if agents' indirect utility functions can be represented as $v_i\left(p, a_i\right) = \alpha_i\left(p\right) + \beta\left(p\right) a_i$, *then aggregate consumption can be expressed as the choice of a representative agent with indirect utility* $v\left(p, A\right) = \alpha\left(p\right) + \beta\left(p\right) A$ *where*
$\alpha\left(p\right) = \sum_{i=1}^{I} \mu_i \alpha_i\left(p\right)$, and
$A = \sum_{i=1}^{I} \mu_i a_i\left(p\right)$.

Gorman Aggregation: Examples

- Quasi-linear utility

  $U_i = g_i\left(c_{i1}\right) + \beta c_{i2}$

- Cobb-Douglas Utility with individual bliss points

  $U_i = (c_1 - \overline{c_{i1}})^{\alpha}(c_{i2} - \overline{c_{i2}})^{1-\alpha}$

Relaxing other assumptions of homogeneous model

1. **A2**: Identical preferences
   - Gorman form of preferences allows heterogeneity in level of Engle curves, not slopes (as long as all consumers consume strictly positive amounts)
   - Requires identical discount factors AND curvature parameter σ in dynamic economy
2. **A3**: No aggregate or idiosyncratic uncertainty
   - Aggregate uncertainty in returns OK (Rubinstein 1974).
   - But: idiosyncratic risk requires assumptions about market structure

Gorman Aggregation: Discussion

- No restrictions on technology, markets
- Result relies entirely on preferences
- Preferences of Gorman form imply aggregation AND representative agent with 'aggregate' preferences

II. Conditions for the existence of a (hypothetical) representative consumer

- First Welfare Theorem
- Complete Markets

- Constantinides (1982) showed how to re-formulate the problem as repr. agent with an appropriately defined $U(C_t)$ leading to the same aggregate K and C

II. Conditions for the existence of a (hypothetical) representative consumer

- First Welfare Theorem
- Complete Markets
- We know that with complete markets, the competitive equilibrum allocation $\{c_{it}^*, k_{it}\}^*_{i=1,..,N;t=1,2,...}$ of a standard dynamic economy is the solution to an appropriate planning problem.
- Constantinides (1982) showed how to re-formulate the problem as repr. agent with an appropriately defined $U(C_t)$ leading to the same aggregate K and C

Existence of Representative Consumer with Complete Markets - Discussion

- Constantinides (1982) is a general result: holds with differences in preferences, idiosyncratic risks, etc
- But: requires complete markets
- And:
  - does NOT imply demand aggregation: preferences of representative agent depend on planner weights $\{\alpha_i\}$ that capture initial wealth differences.
  - These wealth differences depend on relative quantities AND prices of endowments.

    $\Rightarrow$ Aggregate quantities solve representative agent problem only at equilibrium prices.

Bottom line: Complete markets assumption needed for aggregate dynamics

- To model aggregate dynamics the assumption of complete markets is needed to "justify" representative agents assumption
- BUT, the empirical implications of complete markets for consumption dynamics are usually rejected by the data
- In practice: HUGE majority of business cycle literature abstracts from heterogeneity
  - Practical computational considerations (laziness)
    - Trade-off between allowing for many aggregate state variables and capturing effects of heterogeneity
  - Appears that for many applications **aggregate** quantitative effects of (incorrect) complete markets assumption are small

Summary

- Demand aggregation $\Leftrightarrow$ Quasi-homothetic preferences
- Complete markets $\Rightarrow$ Equilibrium quantities solve some RA problem

Consumption under uncertainty - Warm-up to Aiyagari model

- Hows does the presence of (uninsurable) uncertainty affect hhs savings choices and equilbrium prices?
- Setting: incomplete markets, only financial asset is a risk-free one-period bond
- Definitions and temporary assumption
  - Assume: $\beta(1 + r_t) = 1$
  - Absolut risk-aversion: $-U_{cc}/U_c$
  - Relative risk-aversion: $-U_{cc}/U_c * c$

Consumption under uncertainty - hhs problem

$$\max_{\{c_t, a_{t+1}\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$s.t.$$

$$a_{t+1} + c_t = w_t + (1 + r_t) a_t$$

- Euler equation becomes: $U_{c_t} = \beta E_t[U_{c_{t+1}}(1 + r_t)]$

Consumption under uncertainty - precautionary savings

Euler equation: $U_{c_t} = \beta E_t[U_{c_{t+1}}(1 + r_t)]$

- Sidepoint: Without uncertainty $U_{c_t} = \beta U_{c_{t+1}}(1 + r_t)$
  $\Rightarrow$ consumption constant
- Standard (reasonable) utility assumption is convex marginal utility, $U''' > 0$
  - All DARA utility functions, such as *CRRA*, have $U''' > 0$
- Implies: $E_t U_{c_{t+1}} > U_{E_t c_{t+1}} \Rightarrow U_{c_t} > U_{E_t c_{t+1}} \Rightarrow c_t < E_t c_{t+1}$
  - This is **precautionary savings**: the extra savings generated by income uncertainty
  - a hint that $\beta(1 + r_t) = 1$ probably is not an equlibrium

Certainty equivalence - no precautionary savings

- In some settings common to assume quadratic utility of consumption, $U(c) = bc - ac^2$
  - $U''' = 0$
- Only advantage is convenience - yields closed form solution for the consumption function
- Many unattractive feature of this utility assumption
  - Allows for negative consumption levels
  - Marginal utility negative for large consumption levels
  - Implies increasing absolute and relative risk-aversion
  - No precautionary motive
- Quadratic utility implies **certainty equivalence**:
  - Solution unaffected by the presence of uncertainty

Earnings processes

Earnings processes

Standard: "Restricted Income Profiles"

$$
\begin{aligned}
y_{it} &= g(t, observables) + y_{it}^{p} + u_{it}, \\
y_{it}^{p} &= \rho y_{it-1}^{p} + v_{it}.
\end{aligned}
$$

- Can be estimated easily on panel data set of household earnings
  - σ_u, σ_v, ρ
- Interpret time-variation in incomes as resulting from shocks that are found to be very persistent ($\rho \approx 0.99$).
- Look at comovement of income and consumption to test models of risk-sharing

An alternative view of earnings processes

Alternative: "Heterogeneous Income Profiles"

$y_{it} = g(t, observables) + [\alpha_i + \beta_i t] + y^p_{it} + u_{it}$

- Include individual-specific trend and level
- Assume advance knowledge of α_i, β_i
- Implications:
  - Majority of earnings inequality of middle-aged explained by ex ante "income profile"
  - Estimated shock persistence much lower, $\rho \approx 0.8$

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Extra material - Gorman

Gorman Aggregation in a dynamic economy

- Consider the neoclassical growth model
- Replace **A1** by heterogeneity in initial wealth a_{i1}.
- Assume HARA "quasi-homothetic" preferences

$$u(c) = \left\{ \begin{array}{ccc} \ln(\bar{c}+c) & \text{with } \bar{c}+c>0, & \bar{c}\leq 0 \\ \frac{(\bar{c}+c)^{1-\sigma}}{1-\sigma} & \text{with } \bar{c}+c>0, & \bar{c}\leq 0 \\ -\bar{c}\exp(-\sigma c) & \text{with } \bar{c}>0 & \end{array} \right\}$$

- Result: *With HARA utility, heterogeneous initial wealth and free intertemporal trade, the NCGM with* **A2** *and* **A3** *admits a single-agent representation.*

Gorman Aggregation in a dynamic economy: example

- Consider the NGCM when HH own shares of the firm that owns capital and solves

$$A_t = \max_{\{I_\tau\}} \sum_{\tau=t}^{\infty} \left(\frac{p_\tau}{p_t} \right) [f(K_\tau) - I_\tau] \quad (4)$$

$$s.t.$$

$$K_{\tau+1} = (1-\delta) K_\tau + I_\tau,$$

- HH problem

$$\max_{\{c_\tau^i\}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} u\left(c_\tau^i\right)$$

$$s.t.$$

$$\sum_{\tau=t}^{\infty} p_\tau c_\tau^i \leq p_t a_t^i \quad (5)$$

Gorman Aggregation in a dynamic economy: example

- HH Decision Rule becomes

$$c_t^i = \Theta\left(p^t, \bar{c}\right) + (1-\beta)\, a_t^i, \tag{6}$$

- So aggregate dynamics are independent of a_i

$$C_t = \Theta\left(p^t, \bar{c}\right) + (1-\beta)\, A_t. \tag{7}$$

- which is the solution to the RA problem

$$\max_{\{C_\tau\}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} u\left(C_\tau\right) \tag{8}$$

$$s.t. \; \sum_{\tau=t}^{\infty} p_\tau C_\tau \leq p_t A_t \tag{9}$$

Gorman Aggregation in a dynamic economy: example

- Aggregate dynamics (and SS) independent of wealth distribution
- But inverse not true: Can show that wealth share $s_{it} = \frac{a_{it}}{A_t}$ satisfies:

$$\frac{s^i_{t+1}}{s^i_t} > 1 \quad \Leftrightarrow \quad \Theta\left(p^t, \bar{c}\right)\left(a^i_t - A_t\right) > 0,$$

- Moreover, $\Theta\left(p^t, \bar{c}\right) > (<)0$ if $K_t < K^\star$
- So poorer agents' wealth share declines in an economy growing towards steady state, as they pay proportionally more to cover subsistance consumption. But no change in ranking of households.
- And: SS wealth distribution depends on initial conditions. Continua of wealth distributions compatible with the same SS.

Gorman Aggregation in a dynamic economy

Rubinstein (1974): Extends the result to an environment with risky assets (but no production).

Existence of a representative consumer - Planner's problem

Consider planner weights $\{\alpha^i\}$ that implements $\{c_{it}^*, k_{it}\}_{i=1,..,N;t=1,2,...}^*$ as solution to

$$\begin{aligned}
& \max_{\{c_t^i, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \sum_{i=1}^{N} \mu_i \alpha^i u\left(c_t^i\right) \\
& s.t. \\
K_{t+1} + C_t &= f\left(K_t\right) + (1-\delta) K_t \\
C_t &= \sum_{i=1}^{N} \mu_i c_t^i
\end{aligned}$$

Define $\{K_t^* = \sum_i \mu_i k_{it}^*, C_t^* = \sum_i \mu_i c_{it}^*\}$

Planner's problem split in 2 stages

P1 $U(C_t) = \max_{\{c_t^i\}} \sum_{i=1}^{N} \alpha^i u\left(c_t^i\right)$

s.t. $C_t = \sum_{i=1}^{N} \mu_i c_t^i$ for all t

P2 $\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$

s.t. $K_{t+1} + C_t = f(K_t) + (1-\delta) K_t$

K_0 given

Constantinides (1982):

1. $\{K_t^*, C_t^*\}$ solve P2
2. $U(C_t)$ is increasing and concave
3. $\{K_t^*, C_t^*\}$ solve the problem of a representative consumer endowed with the sum of endowments and $U(C_t)$

Cross-sectional variance of consumption and income over life-cyle

Figure 1: Within-Cohort Inequality Over Life Cycle