Directed search

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February 25, 2016

Outline

1. Overview
2. Moen (1997)
3. Menzio, Shi (2008)

Overview of the literature

Introduction

- DMP models (in general) lead to an inefficient allocation – unless Hosios condition holds
- in extended DMP models (risk aversion, ex-ante worker heterogeneity, non-CRS matching function) no such condition exists
- somewhat inconvenient: first welfare theorem does not apply
- welfare/policy analysis: no general efficient benchmark exists

Introduction

- directed search models provide such a (constrained) efficient benchmark
  - in a sense, DMP model is inefficient because agents are too passive
  - absence of competitive forces to bring the economy to its social optimum
  - directed search reintroduces some competitive forces by making firms and workers choose *where* to search
- our motivation
  - directed search models are efficient in general
  - provide another interesting and flexible wage setting mechanism (wage or contract posting)
  - useful property in dynamic settings: block recursivity(Menzio and Shi, 2010)

Moen (1997)

Setup

- continuous time
- measure 1 of homogenous risk-neutral workers
- continuum of risk neutral firms with free entry
- k – sunk cost of entering the market
- c – flow cost of posting a vacancy
- $y \sim F$ – productivity levels, discrete space $\{y_1, \ldots y_m\}$
- z – home production of unemployed workers
- one-to-one matching
- exogenous job destruction rate s

Labor market

- labor market is organized into submarkets
- vacancy comes with a wage offer w
  - wage w: same within a submarket, different across submarkets
- search friction arises at the level of a submarket
- once in a submarket, workers search randomly
  - workers can screen jobs based on wage, but not other observable dimensions (location, etc.)

Labor market

- $m \geq 1$ submarkets in equilibrium with $w_1 \leq w_2 \leq \ldots \leq w_m$
- CRS matching function in each submarket
  - number of matches $x(u_i, v_i)$
  - $p(\theta_i) = x(u_i, v_i) / u_i$
  - $q(\theta_i) = x(u_i, v_i) / v_i$
  - assume $\lim_{\theta \to 0} p(\theta) = \lim_{\theta \to \infty} q(\theta) = 0$
  - assume $\lim_{\theta \to \infty} p(\theta) = \lim_{\theta \to 0} q(\theta) = \infty$
- workers only visit one submarket at a time
- firms can only announce a single wage at a time

Worker's value functions

- value of unemployed worker searching on market i

$$rU_i = z + p(\theta_i)(E_i - U_i) \tag{1}$$

- value of employed worker on market i

$$rE_i = w_i - s(E_i - U_i) \tag{2}$$

- subtract and solve for U_i

$$E_i - U_i = \frac{w_i - z}{r + s + p(\theta_i)}$$

$$rU_i = \frac{(r+s)z + w_i p(\theta_i)}{r + s + p(\theta_i)} \tag{3}$$

- this holds if $w \geq z$, otherwise a worker does not search and gets z/r
- notice that we implicitly assume that the worker returns to the same market after losing a job; we'll that, in fact, he will be indifferent between all active markets

Worker's problem

- workers enter the submarket with the highest expected income

$$U = \max_i U_i$$

- thus in equilibrium, any active market must satisfy $U_i = U$, yielding

$$p(\theta_i) = (r+s)\frac{rU - z}{w_i - rU} \tag{4}$$

- indifference uniquely pins down the market tightness of active markets $\theta(w; U)$
- $\theta(w; U)$ decreases with w, increases with U
  - markets with low wages compensate with a high job finding probability
  - high value of unemployment reduces the surplus of a job, needs to be compensated with a higher job finding probability

Firm's problem

- $V(y_i, w, \theta)$ – value of a vacancy with w and y_i in a submarket with θ
- $J(y_i, w)$ – value of a filled job
- value function for $V(y_i, w, \theta)$

$$rV(y_i, w, \theta) = -c + q(\theta)\left[J(y_i, w) - V(y_i, w, \theta)\right]$$

- value function for $J(y_i, w)$

$$rJ(y_i, w) = y_i - w - sJ(y_i, w)$$

- after destruction of the job, the firm exits and goes back to 0 (not V)
- solving for V

$$(r + q(\theta))\, V(y_i, w, \theta) = q(\theta)\frac{y_i - w}{r + s} - c \tag{5}$$

Firm's problem

- firms take $\theta(w; U)$ of active submarkets as given
- they choose w to maximize

$$\hat{V}(y_i; U) = \max_w V(y_i, w, \theta(w; U)) \tag{6}$$

- problem is well defined: maximization of a continuous function
- solution must be in $[rU, y_i]$
  - a firm never offers more than y_i and make losses, because it can always exit and get 0
  - a firm never offers less than rU (flow value of unemployment), otherwise workers would be better off unemployed – such a market cannot be visited in equilibrium
- this determines $w^*(y_i, U)$ as a function of U
- we still need to determine U

Firm's problem

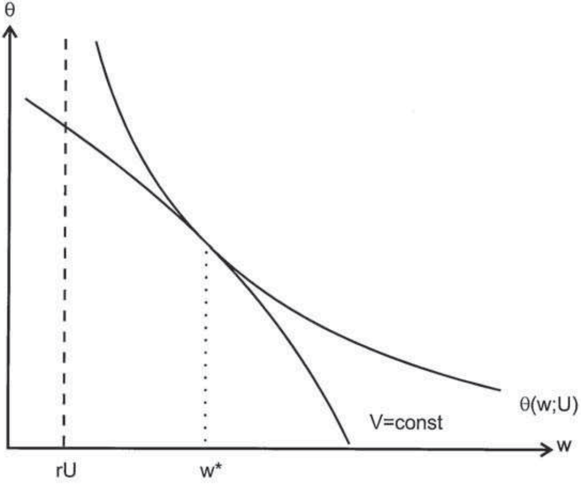

FIG. 2.—Equilibrium

Firm's problem

- for standard forms of matching functions, the wage decision is unique
- high productivity firms offer high wages
  - iso-profit curve is flatter for higher y
  - the opportunity cost of not filling the job is higher for high- productivity firms
  - more important to have a high job filling probability (high w, low θ)
- focus on equilibria in which $m = n$
  - firms with identical productivity post the same wage
  - ignore cases where firms are indifferent between posting different wages

Firm's problem

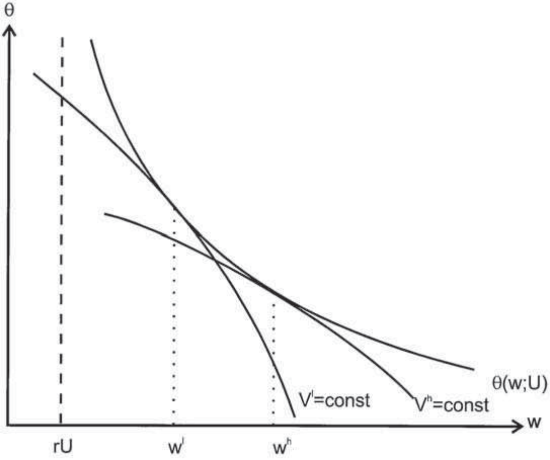

FIG. 4.—Equilibrium with heterogeneous firms and homogeneous workers. The figure shows the indifference curves for high- (h) and low- (l) productivity firms and the wages they are offering in equilibrium.

Expected value of a vacancy

- $\hat{i}$ – lowest vacancy type in equilibrium
- that is, $\forall i \geq \hat{i}$, $\hat{V}(y_i; U) \geq 0$
- expected income of opening a vacancy

$$\bar{V}(U) = \sum_{i \geq \hat{i}} f(y_i) \hat{V}(y_i; U) \tag{7}$$

- $\bar{V}(U)$ is strictly decreasing in U
  - if U increases, $\theta(w; U)$ strictly increases for all $w > rU$
  - hence, the value of all vacancies falls and so does $\bar{V}(U)$
- free entry: firms enter until entry cost k exhaust expected profits

$$k = \bar{V}(U) \tag{8}$$

Unemployment dynamics

- steady state distribution of firm productivities

$$\tilde{f}_i = \frac{f_i}{1 - F_{\hat{i}}}, \quad \text{for } i \geq \hat{i}$$

- let a be the number of new entrants (net of those who exit immediately after drawing y)

- $\{u_1, \ldots, u_m\}$ – distribution of unemployed workers across submarkets

- in the steady-state, number of successful vacancies equals inflow of vacancies

$$v_i q(\theta_i) = \tilde{f}_i a$$

- it must bet hat $a = (1 - s) u$ to have balanced inflow and outflow of aggregate unemployment

- this yields

$$v_i q(\theta_i) = u_i p(\theta_i) = (1 - s) u_i \tilde{f}_i$$

- summing across all submarkets gives that inflow and outflow into unemployment is balanced

$$\sum_{i > \hat{\imath}} u_i p(\theta_i) = (1 - u) s, \quad \text{where } \sum_{i > \hat{\imath}} u_i = u$$

Definition of an equilibrium

An equilibrium is a value for unemployment U, a value for vacancies V, a set of submarkets with wages $\{w_1, \ldots, w_n\}$ and tightness $\{\theta_1, \ldots, \theta_n\}$, a threshold $\hat{i}$, a distribution of searching workers $\{u_1, \ldots, u_n\}$ and beliefs $\theta(w; U)$ such that:

1. $rU = \frac{(r+s)z + p(\theta_i)w_i}{r+s+p(\theta_i)}, \quad i \geq \hat{i}$

2. $\max_w V(y_i, w, \theta(w; U)) \geq 0 \Leftrightarrow i \geq \hat{i}$ and w_i is the maximizer

3. $\overline{V}(U) = k$

4. $u_i p(\theta_i) = \tilde{f}_i (1-u) s$

5. $\sum_{i \geq \hat{i}} u_i = u$

6. $\forall i, \theta_i = \theta(w_i; U)$ and $\theta(w; U) = p^{-1}\left((r+s)\frac{rU - z}{w_i - rU}\right)$

Discussion

- U is the key variable, determine in 1.
- given U and using 6., 2. determine wages in each submarket
- then 3. pins down values of θ
- finally 4. and 5. determine u_i

Existence of equilibrium, uniqueness of U

- equilibrium exists if

$$\sum_{i=1}^{n} f_i \frac{\max\left[y_i - z, 0\right]}{r+s} > k$$

- the value of U is unique
  - $\bar{V}(U)$ is continuous and strictly increasing in U
  - also, $\bar{V}(U) = 0$ if $rU > y_n$
  - it is thus sufficient to show that $\exists U$ s.t. $\bar{V}(U) > k$
  - we will show that $\lim_{rU \to z} \bar{V}(U) > k$
  - from (4), $\lim_{rU \to z} p(\theta) = 0$ for all $w > rU$, and hence $\theta \to 0$ as well
  - value of a vacancy with $y > z$ thus converges to $(y - z)/(r+s)$
  - from (7)

$$\lim_{rU \to z} \bar{V}(U) = \sum_{i=1}^{n} f_i \frac{\max\left[y_i - z, 0\right]}{r+s}$$

- equilibrium is not necessarily unique as some firms can be indifferent between different submarket; we however ignore this possibility here

Efficiency

- similar idea as in the DMP
  - derive conditions which describe equilibrium
  - identify these conditions with the planner's allocation
- denote $\eta = -\theta q'(\theta)/q(\theta)$ the elasticity of the matching function

Decentralized equilibrium

- value of the firm from (5)

$$\left(r+q\left(\theta_i\right)\right)V\left(y_i,w,\theta\right)=q\left(\theta_i\right)\frac{y_i-w}{r+s}-c$$

- take derivatives with respect to w and set $V'=0$

$$q'\left(\theta_i\right)\frac{d\theta}{dw}V\left(y_i,w_i,\theta_i\right)=q'\left(\theta_i\right)\frac{d\theta}{dw}\frac{y_i-w_i}{r+s}-q\left(\theta_i\right)\frac{1}{r+s}$$

- this gives

$$q'\left(\theta_i\right)\frac{d\theta}{dw}\left[\frac{y_i-w_i}{r+s}-V\left(y_i,w_i,\theta_i\right)\right]=\frac{q\left(\theta_i\right)}{r+s} \tag{9}$$

Decentralized equilibrium

- recall definition of $\theta(w, U)$, from (4)

$$p(\theta_i) = \theta_i q(\theta_i) = (r+s)\frac{rU - z}{w_i - rU}$$

- take derivatives

$$q(\theta_i)\left(1 + \frac{\theta_i q'(\theta_i)}{q(\theta_i)}\right)\frac{d\theta_i}{dw} = -(r+s)\frac{rU - z}{(w_i - rU)^2} = -\frac{\theta_i q(\theta_i)}{w_i - rU}$$

$$(1 - \eta_i)\frac{d\theta_i}{dw} = -\frac{\theta_i}{w_i - rU}$$

- plug into (9),

$$\frac{\eta_i}{1 - \eta_i} = \frac{w_i - rU}{y_i - w_i - (r+s)V(y_i, w_i, \theta_i)} = \frac{E_i - U}{J_i - V_i} \tag{10}$$

Decentralized equilibrium

- we have obtained a surplus sharing rule that corresponds to a Nash bargaining with bargaining power $\gamma_i = \eta_i$ (Hosios):

$$\frac{\eta_i}{1-\eta_i} = \frac{E_i - U}{J_i - V_i}$$

- we can also define a match surplus as in the DMP model ($V_i \neq 0$),

$$\begin{aligned} S_i &= E_i - U + J_i - V_i \\ &= \frac{y_i - z - (r+s)\,V_i}{r + s + \eta_i p\,(\theta_i)} \end{aligned}$$

  and an analog of the free-entry condition

$$rV_i = -c + q\,(\theta_i)\,(1-\eta_i)\,S_i$$

- in a sense, in the directed search model, firms self-select into a collection of separated, but efficient DMP models

Social planner

- consider constrained planner's problem
- let a be the gross flow of new vacancies (entrants)
- let N_i be the number of employees working in a firm of type y_i
- maximization problem

$$
\begin{aligned}
&\max_{a,u_i,\hat{\imath}} \int_0^\infty e^{-rt}\left[\sum_{i=\hat{\imath}}^{n}\left(N_i y_i + z u_i - c v_i\right) - ak\right] dt \\
&\text{s.t. } \dot{N}_i = v_i q\left(\frac{v_i}{u_i}\right) - sN_i, \quad i \geq \hat{\imath} \\
&\dot{v}_i = a f_i - v_i q\left(\frac{v_i}{u_i}\right), \quad i \geq \hat{\imath} \\
&\sum_{i=1}^{n}\left(u_i + N_i\right) = 1
\end{aligned}
$$

Social planner

- present value Hamiltonian

$$H = \sum_{i=\hat{\imath}}^{n} (N_i y_i + z u_i - c v_i) - ak + \sum_{i=\hat{\imath}}^{n} \lambda_i \left[v_i q\left(\frac{v_i}{u_i}\right) - sN_i \right]$$
$$+ \sum_{i=\hat{\imath}}^{n} \gamma_i \left[af_i - v_i q\left(\frac{v_i}{u_i}\right)\right] + \alpha \left[1 - \sum_{i=1}^{n} (u_i + N_i)\right]$$

- λ_i and γ_i are the costates

- optimality conditions

$$\frac{\partial H}{\partial u_i} = 0 \qquad \Rightarrow \qquad z - \theta_i^2 q'(\theta_i)(\lambda_i - \gamma_i) = \alpha$$
$$\frac{\partial H}{\partial a} = 0 \qquad \Rightarrow \qquad k = \sum \gamma_i f_i$$
$$\frac{\partial H}{\partial N_i} = r\lambda_i \qquad \Rightarrow \qquad y - s\lambda_i - \alpha = r\lambda_i \Rightarrow \lambda_i = \frac{y-\alpha}{r+s}$$
$$\frac{\partial H}{\partial v_i} = r\gamma_i \qquad \Rightarrow \qquad r\gamma_i = -c + (\lambda_i - \gamma_i) q(\theta_i)\left[1 + \theta_i \frac{q'(\theta_i)}{q(\theta_i)}\right]$$

Social planner

- summarize our optimality conditions

$$\begin{aligned}
\alpha &= z + p(\theta_i)\,\eta_i\,(\lambda_i - \gamma_i) \\
\lambda_i &= \tfrac{y-\alpha}{r+s} \\
r\gamma_i &= -c + q(\theta_i)\,(1-\eta_i)\,(\lambda_i - \gamma_i) \\
k &= \sum f_i \gamma_i
\end{aligned}$$

- identifying $\alpha \leftrightarrow rU$, $\lambda_i \leftrightarrow J_i$ and $\gamma_i \leftrightarrow V_i$ (so that $S_i = \lambda_i - \gamma_i$), we conclude:
- The competitive allocation is efficient.

Directed search and efficiency

- Why do we get efficiency?
- when choosing their wage, firms face the equilibrium tightness schedule $\theta(w; U)$, i.e., they internalize the effect that their wage choice has on tightness in equilibrium
- this is what makes them internalize the crowding out externality and set a wage that splits the surplus according to η

A note on the equilibrium

- we have implicitly used a very particular equilibrium notion
- recall the firm's problem

$$\hat{V}(y_i; U) = \max_w V(y_i, w, \theta(w; U))$$

- it is very important for this equilibrium to be (trembling-hand) stable that the beliefs for *out-of-equilibrium* submarkets are pinned down by the function $\theta(w; U)$
- in this case, even if a positive mass of firms deviates, workers are indifferent and the whole equilibrium is unaffected
- note however that other unstable rational expectation equilibria may exist

Menzio and Shi (2008) - working paper version

Overview

- working paper from 2008 is easier and clearer than the JPE version
- directed search with
  - aggregate shocks
  - search on the job
- they obtain so called block recursivity
  - workers' and firms' decision do not depend on aggregate variables
- key ingredients: directed search and free-entry condition

Setup

- discrete time, infinite horizon, discount factor β
- measure one of workers,
- continuum of firms with positive measure
- workers and firms are risk-neutral
- labor market is organized in submarkets indexed by $x \in \mathbb{R}$
- vacancy-searching workers ratio is $\theta(x) \in \mathbb{R}_+$

Setup

- productivity: $y + z$, y is aggregate, z idiosyncratic
- aggregate state: $\psi \equiv (y, u, g) \in \Psi$
- aggregate productivity: $y \in Y = \left\{y_1, \ldots, y_{N_y}\right\}, N_y \geq 2$
- unemployed workers: $u \in [0, 1]$
- idiosyncratic productivity: $z \in Z = \{z_1, \ldots z_{N_z}\}, N_z \geq 2$
- measure of workers employed in jobs with z: $g : Z \to [0, 1]$
- it holds: $u + \sum_z g(z) = 1$

Each period has 4 stages

1. separation
2. search
3. matching
4. production

Separation stage

- employed worker becomes unemployed with prob. $\tau \in [\delta, 1]$
- τ – specified in worker's contract
- δ – exogenous separation prob.

Matching stage

- unemployed at beginning of the period can search w/ prob λ_u
- employed worker can search w/ prob. λ_e
- worker separated in the separation stage cannot search

Matching stage

- unemployed at beginning of the period can search w/ prob λ_u
- employed worker can search w/ prob. λ_e
- worker separated in the separation stage cannot search
- searchers choose a submarket x where to search
- firms choose how many vacancies to post, and in which submarket
- cost of maintaining a vacancy $k > 0$
- firms and workers take $\theta(x)$ parametrically

Matching stage

- matching process in each submarket x
- worker meets a vacancy with probability $p(\theta(x))$
- vacancy meets a worker w/ $q(\theta(x))$
- p, q twice continuously differentiable

Matching stage

- matching process in each submarket x
- worker meets a vacancy with probability $p(\theta(x))$
- vacancy meets a worker w/ $q(\theta(x))$
- p, q twice continuously differentiable
- job in submarket x gives a worker a lifetime utility x
- if a worker accepts a new job, he gives up his current job
- if a worker rejects a new job, he goes back to his old job
- if a new job, they draw idiosyncratic productivity $\tilde{z} \in Z$
- $\tilde{z}$ has a density $f : Z \to [0, 1]$

Production stage

- unemployed worker consumes b
- employed worker produces $y + z$
- employed worker gets wage w, specified in his contract
- at the end of the period, a new aggregate productivity is drawn w/ $\phi(\hat{y}|y)$

Contractual environment

- complete contract; it prescribes
  - wage, separation strategy, worker's on-the-job strategy as a function of entire match history
- history of a match: $(z; y^t) \in Z \times Y^t$
- $y^t = \{y_1, y_2, \ldots y_t\}$
- employment contract $\underline{a} \in A^{N_z}$ is allocation $\{w_t, \tau_t, n_t\}_{t=0}^{\infty}$
  - wages as a function of tenure and history: $w_t : Z \times Y^t \to \mathbb{R}$
  - separation prob. as a function of tenure and history: $\tau_t : Z \times Y^t \to \mathbb{R}$
  - submarket where to search while on the job: : $n_t : Z \times Y^t \to \mathbb{R}$
- notice:

$$\underline{a}\left(z; y^t\right) = \left\{w_t\left(z; y^t\right), \tau_t\left(z; y^t\right), n_t\left(z; y^t\right)\right\} \cup \underline{a}\left(z; y^t, \hat{y}\right)$$

Worker's value of searching

- utilities are measured at the beginning of the production stage
- aggregate state $\psi = (y, u, a)$
- $W(z; y|a), U(y), J(z; y|a)$ – lifetime utility of emp. and unemp. worker, and firm
- employed worker searching in submarket x
  - finds a job with prob $p(\theta(x))$, receives life-time utility x
  - does not find a job with prob $1 - p(\theta(x))$, receives life-time utility v
  - v is worker's current utility position
  - his expected utility : $v + p(\theta(x))(x - v)$
- conditionally on choosing x optimally, workers expected utility is $v + D(v; y)$

$$D(v; y) = \max_x p(\theta(x; y))(x - v) \tag{11}$$

- let $m(v; y)$ be the solution for x

Worker's value of unemployment

- value of being unemployed

$$U(y) = b + \beta \mathbb{E}\left[U(\hat{y}) + \lambda_u D(U(\hat{y}); \hat{y})\right] \tag{12}$$

- recall: measurement at the beginning of the production stage so a worker can search only tomorrow
- expectation is over $\hat{y}$ tomorrow

Worker and firm values

- consider a matched pair of a firm and a worker (beginning of a production stage)
- history $\left\{z, y^t\right\}$, contract $a = \{w, \tau, n\} \cup \hat{a}$
- worker's value

$$\begin{aligned} W(z; y|a) &= w + \beta\mathbb{E}\left[\tau(\hat{y})\, U(\hat{y})\right] \\ &\quad + \beta\mathbb{E}\left[(1-\tau(\hat{y}))\, W(z; \hat{y}|\hat{a}(\hat{y}))\right] \\ &\quad + \beta\mathbb{E}\left[(1-\tau(\hat{y}))\, \lambda_e p(\theta(n(\hat{y}); \hat{y}))\left[n(\hat{y}) - W(z; \hat{y}|\hat{a}(\hat{y}))\right]\right] \end{aligned} \tag{13}$$

- recall: if a worker loses a job in stage 1, he cannot search ; if he stays employed, he can search on the job w/ λ_e
- firm's value

$$\begin{aligned} J(z; y|a) &= y + z - w \\ &\quad + \beta\mathbb{E}\left[(1-\tau(\hat{y}))\,(1-\lambda_e p(\theta(n(\hat{y}); \hat{y})))\, J(z; \hat{y}|\hat{a}(\hat{y}))\right] \end{aligned} \tag{14}$$

Joint value of a match

- consider a hypothetical problem of choosing allocation a to maximize $J + W$
- the maximized joint value is

$$
\begin{aligned}
V(z;y) \quad = \quad & \max_{w,\tau,n} y + z + \beta \mathbb{E}\left[\tau(\hat{y}) U(\hat{y}) + (1-\tau(\hat{y})) V(z;\hat{y})\right] \\
& + \beta \lambda_e \mathbb{E}\left[(1-\tau(\hat{y})) p(\theta(n(\hat{y});\hat{y}))\left[n(\hat{y}) - V(z;\hat{y})\right]\right] \\
& w \in \mathbb{R}, \quad \tau : Y \to [\delta, 1], \quad n : Y \to \mathbb{R}
\end{aligned}
\tag{15}
$$

- objective is linear in $\tau \Rightarrow$ corner solution
- collect terms containing $\tau(\hat{y})$
- optimal to set $\tau^*_{t-1}\left(y^t\right) = 1$ iff

$$
U(y_t) > V(z;y_t) + \lambda_e D(V(z;y_t), y_t)
$$

- set $\tau^*_{t-1}\left(y^t\right) = \delta$ otherwise

Joint value of a match

- look at terms with n
- it gives that $n^*_{t-1}\left(y^t\right) = m\left(V\left(z; y_t\right); y_t\right)$
- there is no wage w in the objective function
- wage is a transfer between firm and worker, both of them are risk-neutral
- optimal allocation $a^*\left(z; y\right)$ can specify any $\{w^*_t\}_{t=0}^{\infty}$; hence it can attain any division of the joint surplus

Firm's value of a meeting

- firm chooses a contract which maximizes its expected profit subject to providing x to the worker
- formally

$$\max_{\underline{a} \in A^{N_z}} \sum_i J(z_i; y | \underline{a}(z_i)) f(z_i) \tag{16}$$
$$\text{s.t.} \sum_i W(z_i; y | \underline{a}(z_i)) f(z_i) = x$$

- what is the solution?
- for any contract $\underline{a}$, a firm cannot get more than $\sum_i V(z_i; y) f(z_i) - x$
- hence, consider $\underline{a}^* = \{a^*(z_i; y)\}_i$
- for a particular realization of z, the firm gets $J(z; y | a^*(z; y))$, the worker gets $W(z; y | a^*(z; y))$
- for the appropriate selection of wages, worker gets expected utility x, a firm gets $\sum_i V(z_i; y) f(z_i) - x$

Optimal contract

- The optimal contract is given by the following.

1. The firm's value from meeting a worker in a submarket x is $\sum V(z_i; y) f(z_i) - x$.
2. Any employment contract which solves (16) is given by
   2.1 $n_{t-1}(z; y^t) = m(V(z; y_t); y_t)$
   2.2 $\tau_{t-1}(z; y^t) = d(z; y_t)$, where

$$
\begin{aligned}
d(z; y_t) &= 1 \text{ iff } U(y) > V(z; y) + \lambda_e D(V(z; y); y) \\
d(z; y_t) &= \delta \text{ otherwise}
\end{aligned}
$$

- In what follows, we will characterize contracts by $\{d(z; y), m(v; y)\}$ instead of $\{\tau_t, n_t\}_{t=0}^{\infty}$.

Market tightness

- a firm chooses how many vacancies and where to post them
- cost of creating a vacancy is k
- if costs $>$ benefits: create no vacancies in x
- if costs $<$ benefits: create infinitely many vacancies in x
- in any submarket which is visited in equilibrium it must be

$$q\left(\theta\left(x;y\right)\right)\left[\sum_i V\left(z_i;y\right)f\left(z_i\right)-x\right]\leq k \tag{17}$$

  and $\theta\left(x;y\right)\geq 0$ with complementary slackness
- we assume that (17) holds even in market which are not visited by worker in equilibrium

Law of motions

- $\hat{u}$ - measure of unemployed at the end of the period

$$\hat{u} = u\left(1 - \lambda_e p\left(\theta_u\left(y\right)\right)\right) + \sum_i d\left(z_i\right) g\left(z_i\right) \tag{18}$$

- $\hat{g}$ - measure of workers employed at z at the end of the period

$$\hat{g}\left(z\right) = h\left(\psi\right) f\left(z\right) + \left(1 - d\left(z; y\right)\right)\left(1 - \lambda_e p\left(\theta_z\left(z; y\right)\right)\right) g\left(z\right) \tag{19}$$

- $h\left(\psi\right)$ – measure of workers hired during the matching stage

$$h\left(\psi\right) = u\lambda_u p\left(\theta_u\left(y\right)\right) + \sum_i \left(1 - d\left(z_i; y\right)\right) \lambda_e p\left(\theta_z\left(z_i; y\right)\right) g\left(z_i\right)$$

Definition of equilibrium

A block recursive equilibrium (BRE) consists of a mkt tightness θ^*, search value function D^*, policy function m^*, unemployment value function U^*, match value function V^*, separation function d^*, and the laws of motion $\hat{u}^*, \hat{g}^*$. These functions satisfy

- For all $x \in R$ and $\psi \in \Psi$, θ^* satisfies the functional equation (17).
- For all $x \in R$ and $\psi \in \Psi$, D^* satisfies the functional equation (11), and m^* is the associated optimal policy function.
- For all $\psi \in \Psi$, U^* satisfies the functional equation (12).
- For all $z \in Z$ and $\psi \in \Psi$, V^* satisfies the functional equation (16), and d^* is the associated optimal policy function.
- For all $\psi \in \Psi$, $\hat{u}^*$ and $\hat{g}^*$ satisfies the functional equation (18) and (19).

Some properties of an equilibrium

- workers of different types search in different submarkets
- worker in a low-value employment (unemployed or low z) searches
  - in a market where $p(\theta)$ is high and x is relatively low
- $\theta_z^*(z; y)$ is decreasing in z
- no analytical results for relationship between $\{d^*, \theta_u^*, \theta_e^*\}$ and y

Efficiency

- BRE is efficient: same as in Moen (1997)
- complete contracts
- when an employed worker makes a decision, he takes into account impact on his current employer
- directed search
- complete contracts are crucial for efficiency but not for existence of BRE

Why do we get BRE?

- in general, workers would need to know distribution of workers across firms to forecast wages and market tightness
- this is problematic for computation
- here, policy functions do not depend on aggregate state or distribution
- Schaal (2015) has a nice description of the intuition
- key ingredients: directed search and free entry condition

Importance of directed search for BRE

- directed search
  - different workers search in different submarkets
  - firms know that by entering x they meet only one type of a worker
  - value of meeting a worker in a submarket x does not depend on the entire distribution of workers
- free entry
  - guarantees that the prob of meeting a worker in a submarket x does not depend on the distribution
  - cost of posting a vacancy is same for each market – free entry gives $\theta(x)$ as a function of a value of a job
  - as argued above, value of a new job does depend on the distribution

Random search

- with random search, we cannot have BRE
- workers in high- and low- value employment search in one market
- employment contract depends on the worker's type
- thus firm's expected value of meeting a worker depends on the distribution of types
- then market tightness depends on the distribution
- and hence worker's decision depends on the distribution