1 Transitional Dynamics in Bewley Models

So far, we focused on stationary equilibria of economies with heterogeneous agents and incomplete markets. However, certain policy questions that can be asked in this class of models are better addressed by computing the entire transition path across steady states. For example, Here we study the welfare effect of a tax reform, e.g., a permanent rise in the labor income tax rate from τ^* to τ^{**} with the tax revenues being used to finance lump sum government transfers ϕ.

Steady-state comparison– One way to evaluate the tax reform is computing a stationary RCE for these two levels of the tax rate and compare aggregate variables and welfare between the two steady-states. However, this approach is not fully satisfactory. It can only be used to assess whether a household would prefer to live in the stationary equilibrium of an economy with tax rate equal to τ^* or in the stationary equilibrium of an economy with tax rate equal to τ^{**}. For example, it is useful to answer the question whether an individual would prefer to live, forever, in a country with a given tax rate τ^* or in another country with tax rate τ^{**}.

Transition– A more interesting and relevant policy question is: consider a household living in the stationary equilibrium of an economy with initial tax rate τ^*. What is the welfare change (gain or loss) of the tax reform? To answer this question properly, one needs to compute the whole transition: the new policy will change the individual consumption/saving and, possibly, labor supply decision, hence aggregate prices and will induce dynamics away from the current steady-state towards the new one (assuming the system has stable dynamics). See Domeij and Heathcote (2003) for an example that applies these techniques.

How do we attack this problem? Since the transition is characterized by a sequence of aggregate prices and quantities, we need to modify appropriately the definition of recursive competitive equilibrium.

1.1 Definition of Equilibrium with Transition

First, let's define the household problem at time t still in recursive form

$$v_t(a, \varepsilon) = \max_{c_t, a_{t+1}} \left\{ u(c_t(a, \varepsilon)) + \beta \sum_{\varepsilon_{t+1} \in E} v_{t+1}(a_{t+1}(a, \varepsilon), \varepsilon_{t+1}) \pi(\varepsilon_{t+1}, \varepsilon) \right\} \quad (1)$$

$$s.t.$$

$$c_t(a, \varepsilon) + a_{t+1}(a, \varepsilon) = (1 + r_t) a + w_t (1 - \tau_t) \varepsilon + \phi_t$$

$$a_{t+1}(a, \varepsilon) \geq -b$$

Note now that value functions and policies are also a function of time since policies (τ_t, ϕ_t) and, hence, aggregate prices (r_t, w_t) are time-varying. It is important to understand that the transitional dynamics induced by the tax reform are *deterministic*. Since we know the exact future path for taxes, we know there will be a deterministic path for prices and for the distribution λ_t. Therefore, we do not need to keep track of the distribution as an additional state, as time is a sufficient statistic. We'll see that in presence of aggregate uncertainty, instead, the distribution λ becomes an aggregate state.

Let's denote the initial stationary distribution with λ^*. Given an initial distribution λ^*, and a sequence of tax rates $\{\tau_t\}_{t=0}^{\infty}$, a *recursive competitive equilibrium* is a sequence of value functions $\{v_t\}_{t=0}^{\infty}$ and decision rules for households $\{c_t, a_{t+1}\}_{t=0}^{\infty}$, firm choices $\{H_t, K_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, government transfers $\{\phi_t\}_{t=0}^{\infty}$ and distributions $\{\lambda_t\}_{t=0}^{\infty}$ such that, for all t:

- given prices $\{r_t, w_t\}$ and policies $\{\tau_t, \phi_t\}$, the decision rules $a_{t+1}(a, \varepsilon)$ and $c_t(a, \varepsilon)$ solve the household's problem (1) and $v_t(a, \varepsilon)$ is the associated value function,

- given prices $\{r_t, w_t\}$, the firm chooses optimally its capital K_t and its labor H_t, i.e. $r_t + \delta = F_K(K_t, H_t)$ and $w_t = F_H(K_t, H_t)$,

- the labor market clears: $H_t = \int_{A \times E} \varepsilon d\lambda_t = H$,

- the asset market clears: $K_{t+1} = \int_{A \times E} a_{t+1}(a, \varepsilon) d\lambda_t$,

- the goods market clears: $\int_{A \times E} c_t(a, \varepsilon) d\lambda_t + K_{t+1} - (1 - \delta) K_t = F(K_t, H_t)$,

- the government budget constraint is balanced: $\phi_t = \tau_t w_t H$,

- for all $(\mathcal{A} \times \mathcal{E}) \in \mathcal{S}$, the probability measure λ_{t+1} satisfies

$$\lambda_{t+1}(\mathcal{A} \times \mathcal{E}) = \int_{A \times E} Q_t((a, \varepsilon), \mathcal{A} \times \mathcal{E}) \, d\lambda_t,$$

where Q_t is the transition function defined as

$$Q_t((a, \varepsilon), \mathcal{A} \times \mathcal{E}) = I_{\{a_{t+1}(a, \varepsilon) \in \mathcal{A}\}} \sum_{\varepsilon_{t+1} \in \mathcal{E}} \pi(\varepsilon_{t+1}, \varepsilon). \tag{2}$$

1.2 Numerical Computation of the Transition Path

The economy at time $t = 0$ is in steady-state with stationary distribution $\lambda_0 = \lambda^*$ over assets and individual productivities and tax rate τ^*. At the end of period $t = 0$, the government makes the surprise announcement that from $t = 1$ onward the tax policy will change to $\tau^{**} > \tau^*$ and that the additional revenues will augment the lump-sum transfer ϕ_t. Hence, the relevant tax sequence needed to compute the equilibrium is

$$\tau_t = \begin{cases} \tau^*, & \text{for } t = 0 \\ \tau^{**}, & \text{for } t \geq 1. \end{cases}$$

Next, we will assume that after T periods, with T arbitrarily large but finite, the economy will settle to the final steady-state. This assumption is helpful because it allows us to guess a finite sequence of aggregate capital stocks and use backward induction for the solution of the household problem.

To compute the equilibrium follows these steps:

1. Fix T (say $T = 200$).

2. Compute the initial steady state objects $\{v^*, c^*, a^*, K^*\}$ corresponding to $\tau = \tau^*$ and the final steady state objects $\{v^{**}, c^{**}, a^{**}, K^{**}\}$ corresponding to $\tau = \tau^{**}$.

3. Guess a sequence of aggregate capital stocks $\left\{\hat{K}_t\right\}_{t=1}^{T}$ of length T such that $\hat{K}_1 = K^*$ (capital at time 1 is predetermined at time $t = 0$ which is a steady-state) and $\hat{K}_T = K^{**}$. For example, you can make a guess based on the representative agent equivalent of your economy. Note that $H_t = H$ (i.e. constant) for every t in absence of endogenous labor supply. Hence, it is easy to determine, for each t,

$$\begin{aligned} \hat{w}_t &= F_H(\hat{K}_t, H), \\ \hat{r}_t &= F_K(\hat{K}_t, H), \\ \hat{\phi}_t &= \tau_t \hat{w}_t H, \end{aligned}$$

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which are all the elements we need in the budget constraint of the household to solve the household problem at time t.[1]

4. Since we know that $c_T(a, \varepsilon) = c^{**}(a, \varepsilon)$, we can solve the household problem *by backward induction* and derive $\{\hat{c}_t(a, \varepsilon)\}_{t=1}^{T-1}$ from (1) and the associated policy functions $\{\hat{a}_{t+1}(a, \varepsilon)\}_{t=1}^{T-1}$ by iterating over the Euler equation

$$u_c\left(R_t a + (1 - \tau_t) w_t \varepsilon - a_{t+1}(a, \varepsilon)\right) \geq$$

$$\beta R_{t+1} \sum_{\varepsilon'} u_c\left(R_{t+1} a_{t+1}(a, \varepsilon) + (1 - \tau_{t+1}) w_{t+1} \varepsilon_{t+1} - a_{t+2}(a_{t+}\right.$$

and note that the function $a_{t+2}(\cdot)$ is always known.

5. Given the policy functions, we can reconstruct the sequence of transition functions $\left\{\hat{Q}_t\right\}_{t=1}^{T}$ and, since we know that $\lambda_0 = \lambda^*$, we can recover the whole sequence of measures $\left\{\hat{\lambda}_t(a, \varepsilon)\right\}_{t=1}^{T}$ and calculate

$$\hat{A}_{t+1} = \int_{A \times E} \hat{a}_{t+1}(a, \varepsilon) d\hat{\lambda}_t.$$

To compute this integral, we use simulation techniques. We simulate histories of length T for N workers (say $N = 10,000$) starting from the steady-state distribution at $t = 1$ (distribution that we also obtain by simulation). Note that when computing the optimal consumption and saving choices of each of the N individuals in our sample at time t, we must use the time t decision rules computed in the previous step.

6. Check market clearing in the asset market in every period t, i.e. check if the guess of equilibrium capital stocks $\left\{\hat{K}_t\right\}_{t=1}^{T}$ is consistent with aggregate wealth $\left\{\hat{A}_t\right\}_{t=1}^{T}$ that households would accumulate when facing the sequence of prices induced by the proposed sequence of aggregate capital. In other words, choose a convergence criterion η and check whether

$$\max_{1 \leq t \leq T} |\hat{A}_t - \hat{K}_t| < \eta. \tag{3}$$

Note that if $|\hat{A}_T - K^{**}| < \eta$ is satisfied, we have implicitly also checked that T is large enough.

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[1]In this step, one can equally guess a path for the interest rate or for wages. Then, from the FOC's of the firm, one would recover aggregate capital at each t.

7. If inequality (3) is not satisfied at every t, we need a new guess of the capital stock, for example

$$\hat{K}_t^{new} = 0.5 \left(\hat{K}_t^{old} + \hat{A}_t \right),$$

and go back to step 3. with this new guess.

1.3 Results on the Monotonicity of Transitional Dynamics

Consider first an economy with heterogeneity in individual endowments and a borrowing constraint, but no idiosyncratic risk. Then, the Euler equation for every agent in the economy is

$$u'(c_t) \geq \beta R_{t+1} u'(c_{t+1}),$$

where there is no expectation on the RHS because of the lack of risk. Let $R_{t+1} = 1 + f'(K_{t+1} - \delta) \equiv F'(K_{t+1})$. Assume that the aggregate capital stock is below the steady-state level, so that

$$\beta R_{t+1} = \beta F'(K_{t+1}) > \beta F'(K^*) = \beta R^* = 1,$$

since we know that in this economy in steady-state the borrowing constraints do not bind and therefore $\beta R^* = 1$. Then, we have

$$u'(c_t) \geq \beta R_{t+1} u'(c_{t+1}) > u'(c_{t+1})$$

which implies that $c_{t+1} > c_t$. Consumption grows for every agent in the economy, hence aggregate consumption grows. One can show that also the capital stock is increasing monotonically towards K^*. For more details, see Hernandez (1991).

Consider now the neoclassical growth model with idiosyncratic risk (and borrowing constraints). Assume $u' > 0, u'' < 0, u''' > 0$ and the Inada condition $\lim_{c \to \infty} u'(c) = 0$. Huggett (1997) proves the following theorem.

Theorem: If $\beta F'(K_t) > 1$ and $K_t > 0$, then $K_{t+1} > K_t > K_{t-1}$.

Recall that the steady state of this model features $\beta R^* = \beta F'(K^*) < 1$, so the Theorem says that, when the capital stock is sufficiently below the steady-state, the convergence will be monotonic. The proof is by contradiction. Assume that $\{K_t\}$ is decreasing (by Lemma 2 in Hugget's paper it turns out that it's not restrictive). Then,

the sequence $\{\beta R_\tau\}_{\tau=t}^\infty$ will be increasing and always above 1 from date t onward. The Euler equation for the individual is

$$u'\left(c_t\left(a_t, \varepsilon_t\right)\right) \geq \beta R_{t+1} E_t\left[u'\left(c_{t+1}\left(g_t\left(a_t, \varepsilon_t\right), \varepsilon_{t+1}\right)\right)\right].$$

Integrate both sides of the expression wrt λ_t and obtain

$$\int_{A\times E} u'\left(c_t\left(a_t, \varepsilon_t\right)\right) d\lambda_t \geq \beta R_{t+1}\int_{A\times E} E_t\left[u'\left(c_{t+1}\left(g_t\left(a_t, \varepsilon_t\right), \varepsilon_{t+1}\right)\right)\right] d\lambda_t.$$

From Theorem 8.3 in SLP, we know that for a continuous function h:

$$\int\left(Th\right)\left(z\right)\lambda\left(dz\right) = \int h\left(z'\right)\left(T^*\lambda\right)\left(dz'\right)$$

which implies

$$\int_{A\times E} E_t\left[u'\left(c_{t+1}\left(g_t\left(a_t, \varepsilon_t\right), \varepsilon_{t+1}\right)\right)\right] d\lambda_t = \int_{A\times E} u'\left(c_{t+1}\left(a_{t+1}, \varepsilon_{t+1}\right)\right) d\lambda_{t+1}$$

since both integrals express the expected value of h next period: it does not matter the order in which the integration is performed. Put it differently, the LHS integrates wrt ε_{t+1} through the E operator and then integrates over all the pairs (a, ε) the function $g_t\left(a, \varepsilon\right)$ which is equivalent to integrating over a_{t+1}. So, both integrals are with respect to $\left(a_{t+1}, \varepsilon_{t+1}\right)$. Therefore we obtain

$$\int_{A\times E} u'\left(c_t\left(a_t, \varepsilon_t\right)\right) d\lambda_t \geq \beta R_{t+1}\int_{A\times E} u'\left(c_{t+1}\left(a_{t+1}, \varepsilon_{t+1}\right)\right) d\lambda_{t+1}$$

And repeating this argument, we get

$$\int_{A\times E} u'\left(c_t\left(a_t, \varepsilon_t\right)\right) d\lambda_t \geq \beta^n\left(R_{t+1}\cdot R_{t+2}\cdot...\cdot R_{t+n}\right)\int_{A\times E}\left[u'\left(c_{t+n}\left(a_{t+n}, \varepsilon_{t+n}\right)\right)\right] d\lambda_{t+n}$$

The first term of the RHS goes to infinity because $\beta R_\tau > 1$ for every $\tau > t$. Therefore, $\int_{A\times E}\left[u'\left(c_{t+n}\left(a_{t+n}, \varepsilon_{t+n}\right)\right)\right] d\lambda_{t+n}$ must converge to zero because the LHS is finite. Given any $\eta > 0$, let n such that

$$\eta > \int_{A\times E}\left[u'\left(c_{t+n}\left(a_{t+n}, \varepsilon_{t+n}\right)\right)\right] d\lambda_{t+n} > u'\left(\int_{A\times E} c_{t+n}\left(a_{t+n}, \varepsilon_{t+n}\right) d\lambda_{t+n}\right) = u'\left(C_{t+n}\right)$$

where the second inequality requires $u''' > 0$ and Jensen's inequality. This second inequality implies aggregate consumption C_{t+n} goes to infinity by the Inada condition, which violates the resource constraint, since the aggregate capital stock is assumed to be a decreasing sequence. This cannot be an equilibrium, and we found a contradiction.

1.4 Computing the Welfare Change from the Tax Reform

The crucial question to ask, from a policy perspective, is: how much agents gain/lose from the tax reform? In the first steady-state, an agent with initial individual state (a, ε) has expected lifetime utility associated with the stationary decision rule $c^* (a, \varepsilon)$ given by

$$v^* (a, \varepsilon) = E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(c^* (a_t, \varepsilon_t) \right) | (a_0 = a, \varepsilon_0 = \varepsilon) \right], \tag{4}$$

where the conditional expectation E_0 is taken over histories of the shocks conditional on a time-zero realization of the shock equal to ε (as made clear by the second equality) and conditional to an agent's wealth level equal to a. Note that, we can also define $v^* (a, \varepsilon)$ as the fixed point of

$$v_{n+1}^* (a, \varepsilon) = u \left(c^* (a, \varepsilon) \right) + \beta \sum_{\varepsilon'} v_n^* \left(a' (a, \varepsilon), \varepsilon' \right) \pi (\varepsilon', \varepsilon), \tag{5}$$

where the subscript n denotes the iteration, which is a contraction mapping. These two expressions gives us two ways to compute v^*. First, inspired by (4), by simulating S histories of an agent with initial conditions (a, ε), computing discounted utility for every history and the averaging across the S simulations. Second, by iterating over the contraction mapping (5). We set a grid over (a, ε), we guess a matrix v_n^* over the grid and use the steady-state decision rules to compute v_{n+1}^*, and continue until convergence. To evaluate the function v_n^* outside grid point, we need standard interpolation methods.

Let's turn to the transition. Define the expected discounted utility of an agent with initial state (a, ε) at date $t = 0$ going through the transition as

$$\tilde{v}_0 (a, \varepsilon) = E_0 \left[\sum_{t=0}^{\infty} \beta^t u \left(\tilde{c}_t (a_t, \varepsilon_t) \right) | (a_0 = a, \varepsilon_0 = \varepsilon) \right],$$

where $\tilde{v}$ differs from v^* because it is computed using the sequence of decision rules $\{\tilde{c}_t, \tilde{a}_{t+1}\}_{t=0}^{\infty}$ along the transition path.

How do we compute this value function? We can always do it by brute force, by simulation. But, it is more efficient to do it by backward induction. We know that, for

an agent going through the transition:

$$\tilde{v}_0(a, \varepsilon) = u(c^*(a, \varepsilon)) + \beta \sum_{\varepsilon'} \tilde{v}_1(a^*(a, \varepsilon), \varepsilon') \pi(\varepsilon', \varepsilon)$$

$$...$$

$$\tilde{v}_t(a, \varepsilon) = u(\tilde{c}_t(a, \varepsilon)) + \beta \sum_{\varepsilon'} \tilde{v}_{t+1}(\tilde{a}_{t+1}(a, \varepsilon), \varepsilon') \pi(\varepsilon', \varepsilon)$$

$$...$$

$$\tilde{v}_{T-1}(a, \varepsilon) = u(\tilde{c}_{T-1}(a, \varepsilon)) + \beta \sum_{\varepsilon'} v^{**}(a^{**}(a, \varepsilon), \varepsilon') \pi(\varepsilon', \varepsilon)$$

and using this recursion, we can construct $\tilde{v}_0(a, \varepsilon)$ by backward induction. Note that at every date t we don't need to iterate, we can construct $\tilde{v}_t$ from $\tilde{v}_{t+1}$ directly, given that we know $\tilde{v}_{t+1}$ and the decision rules.

Conditional welfare change– The first question we can ask is: how much would an agent with initial state (a, ε) gain, in percentage terms of lifetime consumption, if he went through the transition induced by the policy reform, compared to the no-reform scenario where he lives in the initial steady-state forever? So, welfare changes are expressed in terms of *consumption-equivalent variation*. Precisely, we ask: "how much do we need to change consumption of the agent in every state in the stationary equilibrium so that he'd be indifferent between living through the tax reform and living in the pre-reform economy?".

The answer to this question is a function $\omega(a, \varepsilon)$ that solves the equation

$$E_0 \sum_{t=0}^{\infty} \beta^t u([1 + \omega(a, \varepsilon)] c_t^*) = E_0 \sum_{t=0}^{\infty} \beta^t u(\tilde{c}_t).$$

For the case of power-utility this calculation is really easy to make. When $u(c) = c^{1-\sigma}$, we can exploit the homogeneity of the utility function and the equation above becomes

$$[1 + \omega(a, \varepsilon)]^{1-\sigma} v^*(a, \varepsilon) = \tilde{v}_0(a, \varepsilon),$$

$$\omega(a, \varepsilon) = \left[\frac{\tilde{v}_0(a, \varepsilon)}{v^*(a, \varepsilon)}\right]^{\frac{1}{1-\sigma}} - 1. \tag{6}$$

This welfare change is called *conditional welfare change*, because it is computed for an individual that is in a particular state (a, ε). Thus, we can compute the welfare change for the rich household, the poor household, the productive household, the unproductive household, etc... Moreover, we can compute the entire distribution of welfare changes

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and study whether the reform would be politically feasible, e.g. whether the majority of agents have positive welfare gains, hence they would support the reform.

Utilitarian social welfare change– The second type of welfare calculation is based on a Benthamian social welfare function that puts equal weight to every household in the initial steady-state (which is also the initial period of the reform), i.e. it uses the weighting criterion $\lambda^*(a, \varepsilon)$. The solution to this welfare calculation, for the power utility case is *one number only*, ω^U that solves

$$\omega^U = \left[\frac{\int_{A \times E} \tilde{v}_0(a, \varepsilon) \, d\lambda^*}{\int_{A \times E} v^*(a, \varepsilon) \, d\lambda^*} \right]^{\frac{1}{1-\sigma}} - 1.$$

So, ω^U computes the welfare change for "society", where every agent in society is given equal weight: some will lose, some will gain and we average across everyone with equal weights.

An alternative interpretation of this welfare criterion is that of an *ex-ante* welfare gain, or welfare gain *under the veil of ignorance.* In other words, $\int_{A \times E} \tilde{v}_0(a, \varepsilon) \, d\lambda^*(a, \varepsilon)$ represents the expected discounted utility of a newborn agent who is dropped at random in the first steady-state without knowing at which point in the distribution she will be, i.e. under the veil of ignorance.

1.5   A Welfare Change Decomposition

Sources of changes in social welfare– The typical utilitarian equal-weight social welfare function is:

$$\int_{A \times E} v(a, \varepsilon) \, d\lambda = \int_{A \times E} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \, d\lambda. \tag{7}$$

Welfare can increase for three reasons:

1. If average consumption $E(c_t)$ increases, since utility is monotone. If the tax reform generates additional resources, at least one individual can be made better off, while the others receive the same utility. We can call this effect, the *level effect.* For example, by reallocating resources more efficiently a tax reform can increase average consumption.

2. If the uncertainty/volatility of each individual consumption path $\{c_t\}_{t=0}^{\infty}$ is reduced, since agents are risk averse. We can call this the *uncertainty effect.* For example, by

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redistributing from the lucky to the unlucky, the tax reform can provide additional insurance.

3. If inequality across individuals at any point in time is reduced, since the value function is concave. This is easily seen by just applying Jensen's inequality to the left hand side of (7). We can call this effect, the *egalitarian effect*. Note the key difference between 2) and 3): even when there is no uncertainty in consumption sequences, a policy that redistributes initial wealth more equally across agents would achieve a welfare gain, under this social welfare function. This makes the social welfare function not a fully desirable criterion when studying the welfare implication of a policy reform because it *mixes concern for risk/uncertainty with concern for interpersonal equality.*

Conditional welfare is preferable– For this reason, conditional welfare is a somewhat more satisfactory welfare criterion because only the level and uncertainty effect play a role. We now show that, starting from the conditional welfare criterion, the welfare change $\omega(a, \varepsilon)$ can be decomposed additively into 1 and 2. This is a useful result because, when evaluating the welfare implication of a policy reform, one can compute separately the welfare change due to the fact that the policy is 1) generating more/less aggregate consumption, and 2) increasing/decreasing consumption insurance.

Decomposition– For simplicity, we focus on a welfare comparison between steady-states, but the methodology can be extended to transitions. Let the ex-ante welfare change between economy A (e.g. the initial steady state) and economy B (e.g. the final steady-state) be

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left((1+\omega) c_t^A\right) = E_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t^B\right), \tag{8}$$

where E_0 is the expectation taken at date $t = 0$ conditional on an initial value for the pair (a, ε). To simplify the notation, we have omitted the dependence of ω from the pair (a, ε).

Let C^j denote the average consumption in economy $j = A, B$, i.e.

$$C^j = \int c^j(a, \varepsilon) \, d\lambda^j, \text{ with } j = A, B.$$

Then the *welfare gain of increased consumption levels* between A and B ω^{lev} is defined by

$$\left(1 + \omega^{lev}\right) C^A \equiv C^B. \tag{9}$$

Next, let the certainty equivalent consumption bundle be defined by $\bar{C}^j$ that solves

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t^j\right) = \sum_{t=0}^{\infty} \beta^t u \left(\bar{C}^j\right). \tag{10}$$

Then, we can define the cost of uncertainty p^j as

$$\sum_{t=0}^{\infty} \beta^t u \left(\left(1 - p^j\right) C^j\right) \equiv \sum_{t=0}^{\infty} \beta^t u \left(\bar{C}^j\right), \tag{11}$$

which is the fraction of average consumption that an individual in economy j would be willing to give up to avoid all the risk associated to productivity fluctuations.

Then, the *welfare gain of reduced uncertainty* between economy A and economy B is

$$\omega^{unc} \equiv \frac{1 - p^B}{1 - p^A} - 1. \tag{12}$$

We are now ready to state:

Proposition (Floden, 2001): Assume that $u\left(xc\right)$ is "homogenous" in the sense that $u\left(xc\right) = g\left(x\right) u\left(c\right)$, then

$$1 + \omega = \left(1 + \omega^{unc}\right) \left(1 + \omega^{lev}\right) \Rightarrow \omega \simeq \omega^{unc} + \omega^{lev}.$$

Proof: The total welfare change is given by that value for ω that solves (8). Consider the expected utility in economy B, i.e. the R.H.S. of equation (8):

$$E_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t^B\right) = \sum_{t=0}^{\infty} \beta^t u \left(\bar{C}^B\right) = \sum_{t=0}^{\infty} \beta^t u \left(\left(1 - p^B\right) C^B\right) = g \left(1 - p^B\right) \sum_{t=0}^{\infty} \beta^t u \left(C^B\right),$$

where the first equality follows from (10), the second from (11) and the third from the homogeneity assumption. Then,

$$g \left(1 - p^B\right) \sum_{t=0}^{\infty} \beta^t u \left(C^B\right) = g \left(1 - p^B\right) \sum_{t=0}^{\infty} \beta^t u \left(\left(1 + \omega^{lev}\right) C^A\right),$$

where the equality follows from definition (9). Next,

$$
\begin{aligned}
g\left(1-p^{B}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(\left(1+\omega^{lev}\right)C^{A}\right) &= g\left(1-p^{B}\right)g\left(1+\omega^{lev}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(C^{A}\right) \\
&= g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)g\left(1-p^{A}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(C^{A}\right) \\
&= g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(\left(1-p^{A}\right)C^{A}\right)
\end{aligned}
$$

The line above follows from the homogeneity assumption. Using definition (11),

$$
g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(\left(1-p^{A}\right)C^{A}\right) = g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(\bar{C}^{A}\right).
$$

From the definition of certainty equivalent consumption

$$
g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)\sum_{t=0}^{\infty}\beta^{t}u\left(\bar{C}^{A}\right) = g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)E_{0}\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t}^{A}\right).
$$

And, finally, from homogeneity

$$
g\left(\frac{1-p^{B}}{1-p^{A}}\right)g\left(1+\omega^{lev}\right)E_{0}\sum_{t=0}^{\infty}\beta^{t}u\left(c_{t}^{A}\right) = E_{0}\sum_{t=0}^{\infty}\beta^{t}u\left(\left(1+\omega^{lev}\right)\left(1+\omega^{unc}\right)c_{t}^{A}\right). \textbf{QED}
$$

Floden (2001) contains a more general proof of additive decomposition for preferences which also depend on leisure, and for the notion of ex-ante welfare which also includes a third component, what we called the egalitarian effect.

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