Methods in Panel Data Models with Heterogeneous Time Trends

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1 Introduction

Panel data models offer the ability to manage unobserved endogeneity and allow for broader forms of heterogeneity (**?**).

? propose estimating the time-varying individual effects nonparametrically. This is achieved through a two-stage procedure involving spline smoothing and principal component analysis.

2 The Model

We assume balanced panel data with n cross-sectional units and T time periods. We aim to model the variation of an independent variable y_{it} for $i \in \{1,...,n\}$ and $t \in \{1,...,T\}$ in dependent explanatory variables $x_{it} \in \mathbb{R}^P$. We consider the following panel data model:

$$y_{it} = \sum_{j=1}^{P} x_{itj}\beta_j + \nu_{it} + \varepsilon_{it} \tag{1}$$

Where x_{itj} is the jth element of the vector of independent variables, ε_{it} are idiosyncratic errors, and $\nu_{it} \in \mathbb{R}$ are unobserved non-constant individual effects. Note that whenever x_{it} includes an intercept, identifiability requires v_{it} to be centred around zero. Otherwise, the non-constant individual effects are centred around the overall mean.

Our main goal is to estimate and analyze v_{it}. However, the estimation of β remains of interest to us. We assume that ν_{it} has a factor structure which can be parametrized in terms of d common factors as follows:

$$\nu_{it} = \begin{cases} v_{it} = \sum_{l=1}^{d} \lambda_{il} f_{lt} \\ v_i(t) = \sum_{l=1}^{d} \lambda_{il} f_l(t) \end{cases} \tag{2}$$

Here λ_{il} are the individual loading parameters, f_{lt} are the common factors of the general model of **?**, and $f_l(t)$ are the common factors for the model of **?**.

? treats both λ_{il} and f_{lt} as fixed-effects parameters to be jointly estimated with β. Heteroskedasticity and dependency across time and cross-sectional units are allowed. Further, f_{lt} is modelled as an integrated or stationary process, which is allowed to have a non-zero mean. Both the individual loading parameters and the common factors parameters are allowed to be correlated with the regressors x_{it}.

In contrast, **?** model the time-varying individual effects as linear combinations of a small number of unknown basis functions ($f_l(t)$), where the individual loading parameters set the weight of every basis function for each cross-sectional unit. This translates into smooth, slowly varying local trends. The author's setup consequently allows for strongly correlated stationary and non-stationary factors.

This work will centre around the discussion of the model and estimation methods proposed by **?**, and its comparison to the general model synthesized by **?** and some of the estimation methods applicable to that setup.

2.1 Interactive and additive fixed-effects

The specification in (1) includes the classic panel data models with additive fixed-effects model as a special case. Indeed, for $d=2$, a first common factor $f_{1t}=1, \forall t$ with individual loading parameters λ_{i1} and a second common factor of the form f_{2t} with identical loading parameters $\lambda_{i2}=1, \forall i$ we get the classical two-way error component model:

$$y_{it} = \sum_{j=1}^{\rho} x_{itj}\beta_j + \lambda_{i1} + f_{2t} + \varepsilon_{it}$$

Nonetheless, unlike the case of classical additive effects panel models, well-known estimation methods, such as the within-transformation, are generally inadequate. To see this, consider the case where $d=1$, $y_{it} = \sum_{j=1}^{\rho} x_{itj}\beta_j + \lambda_{i1} f_{1t} + \varepsilon_{it}$. Then, the within-transformation $\dot{y}_{it} = y_{it} - \bar{y}_i = \sum_{j=1}^{\rho}(x_{it} - \bar{x}_i)\beta_j + \lambda_i(f_{1t} - \bar{f}) + \varepsilon_{it} - \bar{\varepsilon}_i$ is unable to eliminate the interactive effects since, generally, $f_{1t} \neq \bar{f}$. Hence, the within estimator is inconsistent for the model as the potential endogeneity between regressors and unobservables can't be addressed (**?**).

It is worth noting that since interactive-effects models encompass additive-effects models as a specific case, a consistent estimator for the former will also be consistent for the latter, albeit less efficient than the classical estimator (**?**). To prevent inefficiency, it is possible to enhance model (1) by explicitly incorporating the classical additive effects,

$$y_{it} = \sum_{j=1}^{\rho} x_{itj}\beta_j + \nu_{it} + \alpha_i + \xi_t + \varepsilon_{it}, \tag{3}$$

where α_i and ξ_t denote the unit- and time-specific fixed effects. This specification also allows for further interpretability. Model (3) can be estimated via augmented versions of the methods proposed by both **?** and **?**. This is discussed in detail **?** and **?**.

2.2 Factor modelling in economics

Model (1) can fit a wide range of economic phenomena where unobservables might be present as common factors. A few examples might elucidate the usefulness of the approach discussed in this paper.

Macroeconomics Let y_{it} be the growth rate for a country i in period t and x_{it} be a series of inputs, such as labour and capital. The factors f_{lt} could represent common macroeconomic shocks such as technological change and financial crises, and the individual loading parameters λ_{il} the heterogeneous impacts of such shocks to the countries' growth rates.

Microeconomics Consider a setup where y_{it} represents the wage for individual.

3 Identification

In the absence of any further constraints, the problem of the non-uniqueness of common factors leads to indeterminacy. This, and the ensuing normalizations, are easier understood when using matrix notation. Let $Y_i = (y_{i1}, \ldots, y_{iT})'$, $X_i = (x_{i1}, \ldots, x_{iT})$, $F = (F_1, \ldots, F_T)'$,$\lambda_i = (\lambda_{i1}, \ldots, \lambda_{id})'$ and $\varepsilon_i = (\varepsilon_{i1}, \ldots, \varepsilon_{iT})'$, with $F_t = (f_1(t), \ldots, f_d(t))'$. We then write the model as:

$$Y_i = X_i\beta + F\lambda_i' + \varepsilon_i \tag{4}$$

Now, since, for any invertible $d \times d$ matrix A, it holds that $FAA^{-1}\lambda_i' = F\lambda_i'$, the model, with factors FA and loading parameters $\lambda_i(A^{-1})'$ is also true. Hence, factors are only identifiable up to linear transformations of the form presented above. Sine matrix A has $d \times d$ free elements, we require d^2 restrictions on the model for identifiability.

Consider $\Lambda = (\lambda_1, \ldots, \lambda_n)'$. The usual normalizations are then given by:

(a) $F'F/T = I_d$

(b) $\Lambda'\Lambda = \text{diag}(\sum_{i=1}^n \lambda_{i1}^2, \ldots, \sum_{i=1}^n \lambda_{id}^2)$

Where the former yields $\frac{d(d+1)}{2}$ restrictions, and the latter provides the additional $\frac{d(d-1)}{2}$ restrictions. Conditions (a) and (b) ensure identifiability up to sign changes since, e.g. $-F$ and $-\lambda$ also satisfy these restrictions (**??**).

The above normalizations lead to orthogonal vectors F_t and empirically uncorrelated coefficients λ_{il}. Further, under these restrictions, the problem of estimating factors F_t becomes that of principal component analysis (**?**).

4 Estimation

4.1 Method by ?

The estimation approach developed by **?** involves a two-step procedure. First, estimates for the common slopes $\hat{\beta}_j$ and initial estimates for the time-varying individual effects $\tilde{v}_i(t)$ are obtained via least squares, where a roughness penalty κ controls the smoothness of the latter. This first step of the estimation relies on the use of an auxiliary function $\vartheta_i(t)$ defined on the interval $[1,T]$, so that $\hat{\vartheta}_i(t) := \tilde{v}_i(t)$.

Second, principal component analysis is used to estimate the common factors $f_l(t)$ and produce a final and more efficient estimate $\hat{v}_i(t)$ for the non-constant individual effects.

In what follows, each step will be discussed in detail.

Step 1: For a given $\kappa > 0$, the unobserved paramters β_j and $v_i(t)$ are estimated by the minimization of

$$\sum_{i=1}^{n} \frac{1}{T} \sum_{t=1}^{T} \left(y_{it} - \sum_{j=1}^{P} x_{itj}\beta_j - \vartheta_i(t) \right)^2 + \sum_{i=1}^{n} \kappa \int_1^T \frac{1}{T} \left(\vartheta_i^{(m)}(s) \right)^2 ds \tag{5}$$

over all $\beta_j \in \mathbb{R}$ and all functions $\vartheta_i(t)$ of class C^m, where $\vartheta_i^{(m)}(t)$ denotes the mth derivative of $\vartheta_i(t)$. Spline theory implies that any solution $\hat{\vartheta}_i(t)$ has an expansion in terms of a natural spline basis $z_1(t), \ldots, z_T(t)$ of order $2m$ such that $\hat{\vartheta}_i(t) = \sum_{s=1}^{T} \hat{\zeta}_{is} z_s(t)$. For a treatment of spline theory and spline smoothing see e.g. **?**, **?**.

Using the model in matrix notation (4) and the expansion of the time-varying individual effects, we can rewrite the objective function in (5) as:

$$S(\beta, \zeta) = \sum_{i=1}^{n} \left(\|Y_i - X_i\beta - Z\zeta_i\|^2 + \kappa \zeta_i' R \zeta_i \right) \tag{6}$$

here $\zeta_i = (\zeta_{i1}, \ldots, \zeta_{iT})'$, Z and R are $T \times T$ matrices with elements $\{z_s(t)\}_{s,t=1,\ldots,T}$ and $\{\int z_s^{(m)}(t) z_k^{(m)}(t), dt\}_{s,k=1,\ldots,T}$ respectively. Further, $\|\cdot\|$ denotes the eculidean norm in $\mathbb{R}^T$. Estimators $\hat{\beta}, \hat{\zeta}_i$ and $\tilde{v}_i$ are hence obtained by minimizing (6) over all $\beta \in \mathbb{R}^\rho$ and $\zeta \in \mathbb{R}^{T \times n}$. With $\mathcal{Z}_\kappa = Z\left(Z'Z + \kappa R\right)^{-1} Z'$, the solutions are given by:

$$\begin{aligned}\hat{\beta} &= \left(\sum_{i=1}^{n} X_i'\left(I-\mathcal{Z}_\kappa\right)X_i\right)^{-1}\left(\sum_{i=1}^{n} X_i'\left(I-\mathcal{Z}_\kappa\right)Y_i\right)\\ \hat{\zeta}_i &= \left(Z'Z+\kappa R\right)^{-1}Z'\left(Y_i - X_i\hat{\beta}\right), \text{ and}\\ \tilde{v}_i &= \mathcal{Z}_\kappa\left(Y_i - X_i\hat{\beta}\right),\end{aligned} \tag{7}$$

Step 2: The common factors are obtained as the principal components of the sample $\tilde{v}_1, \ldots, \tilde{v}_n$. More precisely, let

$$\hat{\Sigma} = \frac{1}{n}\sum_{i=1}^{n}\tilde{v}_i\tilde{v}_i' \tag{8}$$

denote the empirical covariance matrix of $\tilde{v}_1, \ldots, \tilde{v}_n$. Let $\hat{\rho}_1 \geq, \ldots, \geq \hat{\rho}_T$ and $\hat{\gamma}_1, \ldots, \hat{\gamma}_T$ denote the eigenvalues and corresponding eigenvectors of (8). Then, the estimator of the common factor $f_l(t)$ is given by the lth scaled eigenvector

$$\hat{f}_l(t) = \sqrt{T}\hat{\gamma}_{lt}, \text{for all}\, l = \{1, \ldots, d\}, t = \{t = 1, \ldots T\} \tag{9}$$

where $\hat{\gamma}_{lt}$ is the tth element of the eigenvector $\hat{\gamma}_l$. The scaling factor $\sqrt{T}$ yields that (9) fullfils condition (a) in Section 3. The estimates $\hat{\lambda}_{il}$ for the individual loading paramters are obtianed via least squares of $(Y_i - X_i\hat{\beta})$ on $\hat{f}_l = (\hat{f}_l(1), \ldots, \hat{f}_l(T))'$.

A crucial part of the method proposed by **?** involves re-estimating the time-varying individual effects $v_i(t)$ in Step 2 by $\hat{v}_i(t) := \sum_{l=1}^{d}\hat{\lambda}_{il}\hat{f}_l(t)$, where the factor dimension d is determined by a stuiable dimensionality criterion. 6

4.2 Method by ?

? proposes to estimate parameters β, F and λ_i in model (4), given a known factor dimension d, by minimizing the least squares objective function defined as:

$$S\left(\beta, F, \lambda_i\right) = \sum_{i=1}^{n}\left\|Y_i - X_i\beta - F\lambda_i'\right\|^2 \tag{10}$$

subject to conditions (a) and (b) in Section 3. Given the projection matrix $\mathcal{P}_d = I_T - F(F'F)^{-1}F' = I_T - FF'/T$, the least square estimator for β, given F is given by:

$$\hat{\beta}(F) = \left(\sum_{i=1}^{n} X_i'\mathcal{P}_d X_i\right)^{-1}\left(\sum_{i=1}^{n} X_i'\mathcal{P}_d Y_i\right) \tag{11}$$

Now, given β, an estimator for F is given, once again by the first d eigenvectors

$\hat{\gamma} = (\hat{\gamma}_1, \ldots, \hat{\gamma}_d)$ of the empirical covariance matrix $\hat{\Sigma} = (nT)^{-1} w_i w_i'$, where $w_i = Y_i - X_i\beta$. More concretely:

$$\hat{F}(\beta) = \sqrt{T}\hat{\gamma} \tag{12}$$

The solution for $\hat{\beta}$ and $\hat{F}$ can then be obtained via iteration. Estimates for the individual loading parameters can then be obtained as follows:

$$\hat{\lambda}_i = \frac{1}{T}\hat{F}'(Y_i - X_i\hat{\beta}) \tag{13}$$

? propose agumenting the method by **?**

5 Asymptotics

Assumption 1. *For some fixed $d \in \{0,1,2,\ldots\}, d < T$ there exists an D- dimensional subspace $\mathcal{D}_T$ of $\mathbb{R}^T$ such that $v_i \in \mathcal{D}_T$ holds with probability 1.*

Assumption 2. *There exist a nondecreasing function $c(T)$ of T such that for all $l, k = 1, \ldots, d, l \neq k$:*

1. $E(\frac{1}{T}\sum_{t=1}^{T} v_i(t)^2) = O(c(T))$,
2. $\frac{1}{n}\sum_{i=1}^{n}\lambda_{il}^2 = O_P(c(T))$
3. $c(T) = 0_P(\sum_{i=1}^{n}\lambda_{il}^2)$
4. $\sum_{i=1}^{n}\lambda_{il}^2 = O_P(c(T)^2)$
5. $c(T) = O_P(|\sum_{i=1}^{n}\lambda_{il}^2 - \sum_{i=k}^{n}\lambda_{il}^2|)$

Assumption 3. *There exists a nonincreasing function $b(T)$ sucht that as $n, T \to \infty$, the second-order differences of $v_i(t)$ satisfy:*

$$E\left(\frac{1}{T}\sum_{t=2}^{T-1} v_i(t-1) - 2v_i(t) + v_i(t+1)^2\right) = O(b(T))$$

Assumption 4. *There exists a nondecreasing function $d(T) \leq c(T)$ with $d(T) = o(T)$ such that $n, T \to \infty E(\frac{1}{T}\sum x_{itj}^2 = O(d(T))$ holds for all $j = 1, \ldots, \rho$. Furthermore, there is a constant $C_0 < \infty$ such that for all $\kappa \geq 1$:*

$$E\left(\kappa_{\max}\left(\left[\sum_{i=1}^{n}(X_i'(I - \mathcal{Z}_k)X_i\right]^{-1}\right)\right) \leq C_0\frac{1}{nT}$$

Assumption 5. *The error terms* ε_{it} *are i.i.d with* $E(\varepsilon_{it}) = 0, \mathrm{Var}(\varepsilon_{it}) = \sigma^2 > 0,$ *and* $E(\varepsilon_{it}^4) < \infty$. *Moreover* ε_{it} *is independent form* $v_i(s)$ *and* x_{isj} *for all* t, s, j.

Theorem 1. *Under the above assumptions, it holds that as* $n, T \to \infty$*:*

1.

6 Finite Sample Properties via Simulations

The finite sample properties of the estimators presended above are studied via Monte Carlo simulations. In addition to the KSS and Eup methods, we also consider the classical time-invariant fixed effects estimator. The setup of the simulation study is taken form **?**.

The panel-data model is given by.

$$y_{it} = \sum_{j=1}^{P} x_{itj}\beta_j + v_i(t) + \varepsilon_{it} \quad i = 1, \ldots, n; \quad t = 1, \ldots, T \tag{14}$$

we and simulate samlpes of size $n = 30, 100, 300$ with $T = 12, 30$ in a model with $P = 2$ regressors. The slope parameters' true values given by $\beta_1 = \beta_2 = 0.5$. The regressors $X_{it} = (x_{it1}, x_{it2})'$ are generated acording to a bivariate vetor autoregression model:

$$X_{it} = RX_{i,t-1} + \eta_{it} \quad \text{with} \quad R = \begin{pmatrix} 0.4 & 0.05 \\ 0.05 & 0.4 \end{pmatrix} \quad \text{and} \quad \eta_{it} \sim N(0, I_2) \tag{15}$$

To intitalize the simulation, we set $X_{i1} \sim N(0, (I_2 - R^2)^{-1})$ and generate the rest of the sample according to (15). Thereafter, the n regressor-series $(x_{1i1}, x_{2i1})', \ldots, (x_{1iT}, x_{2iT})'$ are additionally shifted such that there are three different mean-value-clusters, fixed at $\mu_1 = (5,5)', \mu_2 = (7.5, 7.5)'$, and $\mu_3 = (10, 10)'$. This is to get a reasonable cloud of points for the regressors (**?**).

We generate time-varying individual effects following five different data generating processes:

DGP1: $v_i(t) = \theta_{i0} + \theta_{i1}\frac{t}{T} + \theta_{i2}\left(\frac{t}{T}\right)^2,$

DGP2: $v_i(t) = \phi_i r_t,$

DGP3: $v_i(t) = v_{i1}\sin(\pi t/4) + v_{i2}\cos(\pi t/4),$

DGP4: $v_i(t) = \xi_i,$

Table 1: DGP 1, homoskedastic errors

MSE of Effects

n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.4760	0.7438	2.0010	2.9270
	30	0.2075	0.2995	2.0240	2.4150
100	12	0.3118	0.3579	2.0000	2.4850
	30	0.1561	0.1700	2.0070	2.2590
300	12	0.2593	0.2639	2.0000	2.2240
	30	0.1430	0.1407	2.0000	2.2220

MSE, Bias, Variance for Coefficients

	$T=12$			$T=30$		
	KSS	Eup	Within	KSS	Eup	Within
$n=30$						
MSE	0.00375	0.00540	0.02982	0.00119	0.00144	0.01021
BIAS1	-0.00029	0.00052	0.00278	-0.00082	-0.00071	0.00067
BIAS2	0.00144	0.00140	0.08113	0.00051	0.00082	0.07531
VAR1	0.06651	0.06282	0.12900	0.03638	0.03396	0.07339
VAR2	0.01181	0.01174	0.02390	0.00699	0.00687	0.01428
$n=100$						
MSE	0.00123	0.00137	0.00939	0.00036	4e-04	0.00377
BIAS1	-0.00012	-0.00042	-0.00386	-0.00099	-0.00093	-0.00555
BIAS2	0.00098	0.00062	0.07835	0.00049	0.00077	0.07291
VAR1	0.03521	0.03391	0.06941	0.01943	0.01860	0.03977
VAR2	0.00615	0.00601	0.01283	0.00367	0.00359	0.00770
$n=300$						
MSE	0.00038	0.00037	0.00326	0.00013	0.00015	0.00113
BIAS1	-0.00229	-0.0019	-0.00694	0.00092	0.00095	-0.00276
BIAS2	0.00105	0.00097	0.07717	0.00044	0.00075	0.07261
VAR1	0.02031	0.01949	0.04073	0.01118	0.01078	0.02337
VAR2	0.00350	0.00341	0.00739	0.00209	0.00205	0.00445

DGP5: $v_i(t) = v_{i1}e^{-\frac{t}{4}}\sin(\pi t/4)$

where $\theta_{ij}(j=0,1,2) \sim$ i.i.d. $5N(0,1), \xi_i, v_{ij}(j=1,2) \sim$ i.i.d. $3N(0,1)$, and $r_{t+1} = r_t + \delta_t$, with $\delta_t, r_1 \sim$ i.i.d. $3N(0,1)$.

The second regressor x_{it2} is allowed to be endogenous with a correlation with $v_i(t)$ of $\rho = 0.5$. Let w_{it} be the endogenous part of the regressor, so that $w_{it} = \rho v_i(t) + \sigma_v\sqrt{1-\rho^2}\epsilon_{it}$[1], where σ_v is the standard deviation of $v_i(t)$ and $\epsilon_{it} \sim N(0,1)$. We then define the endogenous regressor as $\tilde{x}_{it2} = x_{it2} + w_{it}$.

Table 2: DGP 2, homoskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	28.0242	0.4301	1.0970	2.6140
	30	14.6777	0.0732	1.0410	1.0200
100	12	22.5974	0.0950	1.0450	1.0010
	30	13.4557	0.0440	1.0060	1.0000
300	12	20.8949	0.0874	1.0080	1.0000
	30	12.6196	0.0366	1.0000	1.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.15096	0.00068	0.79135	0.06411	1e-04	0.62111
BIAS1	-0.00537	-2e-05	0.01034	-0.00627	0.00014	-0.01656
BIAS2	0.10071	0.00026	0.28647	0.04526	-3e-05	0.28446
VAR1	0.29218	0.01437	0.61181	0.16205	0.00730	0.55454
VAR2	0.01956	0.00445	0.03695	0.00707	0.00171	0.02276
$n = 100$						
MSE	0.04572	5e-05	0.20256	0.02031	2e-05	0.22889
BIAS1	-0.00703	-0.00013	0.00115	-0.0024	-0.00026	-0.03364
BIAS2	0.10011	-5e-05	0.27875	0.04465	1e-05	0.27166
VAR1	0.15527	0.00673	0.32295	0.08580	0.00401	0.30113
VAR2	0.01038	0.00222	0.01992	0.00369	0.00088	0.01215
$n = 300$						
MSE	0.01606	2e-05	0.06997	0.00734	1e-05	0.07304
BIAS1	-0.00628	0.00013	0.00583	-0.00462	-4e-05	-0.01025
BIAS2	0.09846	3e-05	0.27707	0.04177	-4e-05	0.27273
VAR1	0.08818	0.00389	0.19778	0.04839	0.00226	0.17906
VAR2	0.00569	0.00122	0.01144	0.00201	5e-04	0.00706

Table 3: DGP 3, homoskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	1.1257	0.3859	2.5650	2.8740
	30	21.9290	0.1364	0.9990	2.0460
100	12	0.6088	0.1907	2.5300	2.0410
	30	21.2968	0.0859	1.0000	2.0000
300	12	0.4827	0.1722	2.6220	2.0000
	30	21.3619	0.0730	1.0000	2.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00952	0.00061	0.03172	0.01324	0.00018	0.01181
BIAS1	0.00751	-5e-05	0.00015	-0.01363	0.00084	-0.0172
BIAS2	0.05446	0.00031	0.45741	0.46035	-0.00057	0.45038
VAR1	0.09374	0.02110	0.15299	0.09570	0.01248	0.09049
VAR2	0.02824	0.01993	0.04961	0.03147	0.01187	0.03039
$n = 100$						
MSE	0.00275	0.00013	0.00947	0.00432	4e-05	0.00423
BIAS1	0.00256	0.00092	-0.01959	-0.01869	0.00012	-0.02833
BIAS2	0.05339	-7e-04	0.45910	0.46390	-2e-05	0.45317
VAR1	0.04960	0.01081	0.08244	0.05176	0.00660	0.04909
VAR2	0.01501	0.01030	0.02709	0.01700	0.00629	0.01660
$n = 300$						
MSE	9e-04	4e-05	0.00349	0.00148	2e-05	0.00142
BIAS1	0.00182	0.00036	-0.00956	-0.01371	0.00011	-0.01759
BIAS2	0.05352	-0.00028	0.46077	0.46381	-9e-05	0.45380
VAR1	0.02838	0.00616	0.04830	0.02985	0.00379	0.02864
VAR2	0.00854	0.00587	0.01565	0.00978	0.00361	0.00959

Table 4: DGP 4, homoskedastic errors

MSE of Effects

n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.3437	0.7783	1.0000	2.6290
	30	0.1144	0.1612	1.0000	1.0080
100	12	0.1513	0.1545	1.0000	1.0040
	30	0.0560	0.0653	1.0000	1.0000
300	12	0.1090	0.1070	1.0000	1.0000
	30	0.0414	0.0441	1.0000	1.0000

MSE, Bias, Variance for Coefficients

	$T=12$			$T=30$		
	KSS	Eup	Within	KSS	Eup	Within
$n=30$						
MSE	0.00387	0.00551	0.00306	0.00116	0.00131	0.00106
BIAS1	0.00283	0.00414	0.00173	5e-05	0.00092	0.00065
BIAS2	0.00112	0.00126	9e-04	0.00013	0.00031	2e-04
VAR1	0.06024	0.05148	0.05332	0.03368	0.02959	0.03178
VAR2	0.02191	0.02128	0.02035	0.01290	0.01247	0.01242
$n=100$						
MSE	0.00099	0.00088	0.00083	0.00031	0.00031	0.00028
BIAS1	-0.0015	-0.0011	-0.00109	8e-05	0.00029	0.00013
BIAS2	2e-05	3e-05	5e-05	0.00017	0.00022	0.00018
VAR1	0.03226	0.02808	0.02875	0.01812	0.01680	0.01718
VAR2	0.01154	0.01086	0.01085	0.00684	0.00667	0.00666
$n=300$						
MSE	0.00036	0.00029	0.00028	0.00012	0.00011	0.00011
BIAS1	0.00035	0.00055	6e-04	-5e-05	6e-05	4e-05
BIAS2	0.00041	0.00036	0.00036	-0.00026	-0.00024	-0.00025
VAR1	0.01868	0.01671	0.01685	0.01046	0.00995	0.01003
VAR2	0.00662	0.00625	0.00625	0.00392	0.00384	0.00384

Table 5: DGP 5, homoskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	1.0420	0.4061	1.5200	2.6140
	30	0.4770	0.0794	1.4850	1.0070
100	12	0.7104	0.0961	1.6550	1.0050
	30	0.3656	0.0466	1.7220	1.0000
300	12	0.6238	0.0874	1.7840	1.0000
	30	0.3269	0.0379	1.9380	1.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00461	0.00183	0.00523	0.00148	0.00046	0.00137
BIAS1	0.00261	-0.00011	-0.00151	-8e-05	-0.0010	-0.00325
BIAS2	0.07052	0.00058	0.18376	0.05202	9e-04	0.09410
VAR1	0.07130	0.03465	0.06717	0.03863	0.02182	0.03553
VAR2	0.05572	0.03446	0.05288	0.03470	0.02179	0.03200
$n = 100$						
MSE	0.00153	0.00039	0.00169	0.00046	0.00014	0.00046
BIAS1	0.00035	-0.00042	-0.00719	-3e-04	-2e-04	-0.00545
BIAS2	0.07454	0.00044	0.18431	0.05370	0.00022	0.09321
VAR1	0.03767	0.01788	0.03632	0.02077	0.01181	0.01926
VAR2	0.02938	0.01780	0.02863	0.01864	0.01179	0.01736
$n = 300$						
MSE	0.00049	0.00012	0.00056	0.00014	5e-05	0.00014
BIAS1	-0.00031	5e-05	-0.00422	0.00017	0.00054	-0.00317
BIAS2	0.07495	0.00012	0.18718	0.05248	-0.00054	0.09379
VAR1	0.02167	0.01027	0.02123	0.01195	0.00679	0.01124
VAR2	0.01686	0.01022	0.01664	0.01070	0.00678	0.01011

Table 6: DGP 1, heteroskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	57.0611	110.5374	1.0100	2.9330
	30	20.1377	54.3103	1.0390	2.0280
100	12	28.6135	31.1248	1.0020	1.1570
	30	12.2606	14.8909	1.0110	1.0000
300	12	20.5047	19.8192	1.0060	1.0000
	30	9.7311	10.2803	1.0000	1.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.62852	0.47255	0.52730	0.18555	0.27007	0.18209
BIAS1	0.01764	-0.2943	0.02406	-0.00199	-0.31411	-0.00916
BIAS2	0.00215	0.08429	0.08419	0.00179	0.06344	0.07546
VAR1	0.77857	0.13640	0.69299	0.43312	0.09164	0.40988
VAR2	0.14083	0.08211	0.12931	0.08434	0.06315	0.08004
$n = 100$						
MSE	0.18401	0.10493	0.15098	0.05824	0.06526	0.05866
BIAS1	-0.01846	-0.14205	-0.02519	0.00353	-0.16503	-0.00286
BIAS2	0.00292	0.06678	0.08058	0.00279	0.05494	0.07409
VAR1	0.41923	0.07403	0.37487	0.23347	0.05286	0.22202
VAR2	0.07450	0.04863	0.06938	0.04473	0.03465	0.04304
$n = 300$						
MSE	0.05573	0.02764	0.04843	0.01731	0.01546	0.01663
BIAS1	0.00776	-0.06562	0.00801	0.00136	-0.07466	0.00143
BIAS2	0.00406	0.05759	0.07932	0.00412	0.05269	0.07410
VAR1	0.24191	0.04723	0.21890	0.13492	0.03514	0.12981
VAR2	0.04247	0.02790	0.03972	0.02558	0.02024	0.02471

Table 7: DGP 2, heteroskedastic errors

MSE of Effects

n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	262.1845	326.3663	1.0440	2.9310
	30	234.3432	323.7860	1.1280	1.9400
100	12	148.2636	131.3483	1.0170	1.4810
	30	157.6732	79.6151	1.0360	1.0100
300	12	117.6485	67.8770	0.9940	1.0180
	30	138.0031	60.8781	1.0100	1.0000

MSE, Bias, Variance for Coefficients

	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	2.48822	0.20856	2.58247	1.74984	0.12441	2.13680
BIAS1	-0.04624	-0.04232	-0.06031	0.04446	-0.05006	-0.00289
BIAS2	0.15999	0.00690	0.27926	0.11237	0.00232	0.27522
VAR1	1.46153	0.13144	1.37026	1.19766	0.13200	1.20275
VAR2	0.08884	0.06177	0.08427	0.04694	0.04795	0.04671
$n = 100$						
MSE	0.76397	0.03782	0.77390	0.50429	0.02605	0.62673
BIAS1	-0.0420	-0.01202	-0.05248	0.00587	-0.01459	-0.00038
BIAS2	0.15692	0.00448	0.28455	0.10934	-0.00081	0.27538
VAR1	0.79463	0.07793	0.75118	0.64468	0.07457	0.65242
VAR2	0.04714	0.04007	0.04549	0.02529	0.02872	0.02554
$n = 300$						
MSE	0.23800	0.01302	0.22304	0.16342	0.01004	0.18119
BIAS1	-0.03609	-0.0059	-0.02566	-0.00075	-0.00402	0.00021
BIAS2	0.15333	-0.00206	0.27695	0.10635	-0.00142	0.26285
VAR1	0.45201	0.04382	0.43368	0.37018	0.04417	0.37816
VAR2	0.02700	0.02379	0.02623	0.01447	0.01654	0.01467

Table 8: DGP 3, heteroskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	67.8142	72.1473	0.7710	2.8160
	30	37.2463	19.8953	0.8350	0.7100
100	12	41.8852	9.6719	0.8150	0.0400
	30	28.0734	8.9661	0.9370	0.0000
300	12	36.0379	8.9588	0.9120	0.0000
	30	25.9276	8.9699	1.0000	0.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.52781	0.22845	0.41759	0.16613	0.15740	0.15649
BIAS1	0.03267	-0.34558	0.02024	-0.01135	-0.36663	-0.01424
BIAS2	0.47459	0.32114	0.45964	0.47240	0.37129	0.46527
VAR1	0.59655	0.11359	0.63227	0.35356	0.07995	0.37664
VAR2	0.21294	0.11258	0.20716	0.12921	0.08342	0.12703
$n = 100$						
MSE	0.14288	0.16306	0.12161	0.04639	0.15729	0.03964
BIAS1	-0.00666	-0.38891	-0.02027	-0.01889	-0.39083	-0.02365
BIAS2	0.48182	0.39625	0.46373	0.45878	0.39609	0.44996
VAR1	0.33237	0.05917	0.34088	0.20544	0.04240	0.20347
VAR2	0.11490	0.06269	0.11218	0.07026	0.04483	0.06903
$n = 300$						
MSE	0.05246	0.15834	0.04188	0.01398	0.15469	0.01352
BIAS1	-0.01124	-0.3928	-0.0190	-0.02397	-0.39153	-0.02713
BIAS2	0.48727	0.40301	0.46496	0.46256	0.39875	0.45158
VAR1	0.20706	0.03522	0.19948	0.12384	0.02492	0.11880
VAR2	0.06706	0.03734	0.06487	0.04059	0.02629	0.03982

Table 9: DGP 4,heteroskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	43.8641	81.5951	0.9390	2.8810
	30	15.0669	33.7164	0.9960	1.5570
100	12	21.0356	15.3963	0.9960	0.3900
	30	7.3341	10.6590	1.0000	0.4540
300	12	14.4503	10.0594	1.0000	0.1040
	30	5.3102	8.6405	1.0000	0.3680

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.47333	0.28342	0.38989	0.15610	0.16529	0.14492
BIAS1	0.02637	-0.30957	0.05343	0.00768	-0.28908	0.01454
BIAS2	0.02037	0.15203	0.00494	0.00513	0.18288	0.00372
VAR1	0.66735	0.13374	0.61825	0.38758	0.09112	0.36649
VAR2	0.24894	0.12192	0.23537	0.14961	0.08743	0.14415
$n = 100$						
MSE	0.14360	0.15110	0.11745	0.04344	0.11362	0.03901
BIAS1	-0.01736	-0.30349	-0.0189	0.00128	-0.27938	0.00092
BIAS2	-0.00328	0.27670	-0.00584	-0.00874	0.20710	-0.00894
VAR1	0.37128	0.06677	0.33248	0.20886	0.04875	0.19802
VAR2	0.13345	0.06534	0.12568	0.07900	0.04719	0.07689
$n = 300$						
MSE	0.04684	0.14835	0.03717	0.01267	0.10763	0.01198
BIAS1	0.01079	-0.35718	0.00973	0.00783	-0.27351	0.00911
BIAS2	5e-04	0.35418	0.00153	-0.00058	0.24673	-0.00089
VAR1	0.21561	0.03720	0.19444	0.12060	0.02892	0.11568
VAR2	0.07638	0.03720	0.07220	0.04524	0.02661	0.04423

Table 10: DGP 5, heteroskedastic errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	56.3641	66.9815	0.7830	2.8300
	30	18.2381	14.4057	0.8000	0.8590
100	12	22.1774	0.8852	0.7380	0.0080
	30	8.2525	0.2734	0.7580	0.0000
300	12	13.0714	0.6763	0.6700	0.0000
	30	5.2294	0.2702	0.7380	0.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.46420	0.25203	0.38805	0.13472	0.06628	0.12373
BIAS1	0.01746	-0.14949	0.01153	-0.00576	-0.05988	-0.0037
BIAS2	0.15933	0.10339	0.20800	0.08291	0.04917	0.09076
VAR1	0.60235	0.19443	0.59734	0.34681	0.13973	0.35441
VAR2	0.48530	0.19276	0.46947	0.31657	0.14004	0.31915
$n = 100$						
MSE	0.13145	0.05039	0.10850	0.04164	0.02077	0.03851
BIAS1	0.01382	-0.10706	0.00945	-0.00206	-0.0491	-0.00265
BIAS2	0.15523	0.10825	0.18454	0.08804	0.05013	0.09198
VAR1	0.31639	0.10022	0.32272	0.18435	0.07490	0.19202
VAR2	0.25730	0.10089	0.25477	0.16860	0.07514	0.17296
$n = 300$						
MSE	0.04324	0.02507	0.03498	0.01431	0.00902	0.01356
BIAS1	-0.02195	-0.11542	-0.01616	-0.0156	-0.05595	-0.0129
BIAS2	0.15130	0.11975	0.18112	0.08552	0.05563	0.09455
VAR1	0.17560	0.05795	0.18858	0.10554	0.04446	0.11204
VAR2	0.14461	0.05804	0.14804	0.09671	0.04456	0.10075

Table 11: DGP 1, weakly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.5847	0.8074	2.1870	2.9970
	30	0.2374	0.3804	2.5000	2.9090
100	12	0.3092	0.3528	2.0000	2.4760
	30	0.1733	0.2078	2.5400	2.7190
300	12	0.2424	0.2373	2.0000	2.0420
	30	0.1502	0.1447	2.0080	2.4520

MSE, Bias, Variance for Coefficients						
	$T=12$			$T=30$		
	KSS	Eup	Within	KSS	Eup	Within
$n=30$						
MSE	0.00361	0.00482	0.02878	0.00125	0.00167	0.01093
BIAS1	-0.00028	0.0027	0.00038	-0.00193	-0.00112	6e-04
BIAS2	2e-04	0.00038	0.08084	-0.00012	0.00021	0.07660
VAR1	0.06629	0.03973	0.12689	0.03688	0.02774	0.07300
VAR2	0.01136	0.00724	0.02361	0.00695	0.00543	0.01431
$n=100$						
MSE	0.00114	0.00126	0.00931	0.00039	0.00046	0.00362
BIAS1	-0.00078	-0.00044	-0.00336	0.00024	6e-05	-0.00301
BIAS2	0.00127	0.00095	0.07842	0.00023	0.00035	0.07294
VAR1	0.03527	0.02452	0.06967	0.01915	0.01580	0.03981
VAR2	0.00612	0.00441	0.01282	0.00356	0.00294	0.00771
$n=300$						
MSE	0.00043	0.00044	0.00274	0.00012	0.00013	0.00115
BIAS1	9e-04	0.00116	-0.00347	-9e-05	-8e-05	-0.00356
BIAS2	0.00089	0.00079	0.07757	5e-04	0.00065	0.07307
VAR1	0.02043	0.01445	0.04071	0.01117	0.00928	0.02332
VAR2	0.00354	0.00261	0.00738	0.00209	0.00175	0.00445

6.1 Baseline scenario

6.2 Heteroskedastic error terms

6.3 Weakly autocrorrelated error terms

6.4 (

Strongly autocrorrelated error terms

7 Application

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[1] In generating w_{it}, the effects $v_i(t)$ is multiplied by 10 to balance with the magnitude of x_{it2}.

Table 12: DGP 2, weakly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	28.8176	0.4372	1.1090	2.6190
	30	14.6903	0.0837	1.0350	1.0480
100	12	21.8306	0.3339	1.0350	2.2620
	30	12.4957	0.0670	1.0100	1.0010
300	12	21.5824	0.1082	1.0280	1.0000
	30	13.3566	0.0487	1.0020	1.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.16805	0.00076	0.89272	0.06091	9e-05	0.80917
BIAS1	-0.01858	0.00192	-0.00797	0.00185	0.00015	0.02906
BIAS2	0.09459	0.00032	0.27393	0.04554	8e-05	0.27930
VAR1	0.29633	0.01048	0.62208	0.16277	0.00678	0.55561
VAR2	0.01901	0.00300	0.03631	0.00717	0.00148	0.02254
$n = 100$						
MSE	0.04166	2e-04	0.20167	0.01921	3e-05	0.22681
BIAS1	-0.00989	0.00023	-0.02138	0.00577	0.00029	-0.02044
BIAS2	0.10470	0.00016	0.28553	0.04406	-4e-05	0.28084
VAR1	0.15080	0.00672	0.31884	0.08384	0.00403	0.30323
VAR2	0.01043	0.00172	0.02014	0.00363	0.00078	0.01234
$n = 300$						
MSE	0.01549	2e-05	0.07863	0.00690	1e-05	0.06897
BIAS1	-0.00076	-1e-05	-0.0209	0.00184	-1e-05	-0.01544
BIAS2	0.10308	8e-05	0.27216	0.04846	0.00000	0.27573
VAR1	0.08772	0.00326	0.19121	0.04880	0.00220	0.17187
VAR2	0.00592	0.00101	0.01139	0.00219	0.00046	0.00706

Table 13: DGP 3, weakly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	1.2074	0.3738	2.5000	2.9220
	30	21.9435	0.2674	0.9990	2.8850
100	12	0.6996	0.3945	3.6600	2.9450
	30	21.1519	0.0662	1.0000	2.0000
300	12	0.5314	0.1646	2.3860	2.2840
	30	21.3893	0.0490	1.0000	2.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.01100	0.00057	0.03603	0.01238	0.00028	0.01117
BIAS1	0.0025	-0.00037	-0.00945	-0.01525	-0.00046	-0.0182
BIAS2	0.06409	0.00039	0.45688	0.46272	0.00086	0.45254
VAR1	0.09616	0.01470	0.15271	0.09536	0.01053	0.09014
VAR2	0.02937	0.01417	0.04988	0.03130	0.00964	0.03022
$n = 100$						
MSE	0.00194	0.00025	0.01055	0.00443	4e-05	0.00440
BIAS1	0.00013	-2e-05	-0.02168	-0.02143	-7e-05	-0.03172
BIAS2	0.04010	-0.00021	0.45794	0.46391	6e-05	0.45336
VAR1	0.04563	0.00869	0.08204	0.05181	0.00566	0.04914
VAR2	0.01371	0.00750	0.02699	0.01699	0.00548	0.01660
$n = 300$						
MSE	0.00113	4e-05	0.00311	0.00130	2e-05	0.00125
BIAS1	0.00347	-0.00063	-0.00745	-0.01355	0.00018	-0.01732
BIAS2	0.07148	0.00056	0.46011	0.46405	-0.00021	0.45414
VAR1	0.03009	0.00485	0.04821	0.02991	0.00326	0.02870
VAR2	0.00910	0.00469	0.01566	0.00978	0.00317	0.00960

Table 14: DGP 4, weakly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.2980	0.7217	1.0000	2.8090
	30	0.1030	0.1529	1.0000	1.0230
100	12	0.1148	0.3190	1.0000	2.2240
	30	0.0377	0.0474	1.0000	1.0000
300	12	0.1116	0.1123	1.0000	1.0000
	30	0.0389	0.0416	1.0000	1.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00347	0.00417	0.00267	0.00105	0.00125	0.00098
BIAS1	0.00046	0.00061	0.00053	5e-04	0.00047	0.00082
BIAS2	0.00189	0.00225	0.00173	0.00037	0.00021	0.00033
VAR1	0.06092	0.03395	0.05407	0.03379	0.02490	0.03190
VAR2	0.02207	0.01443	0.02051	0.01303	0.01081	0.01254
$n = 100$						
MSE	0.00089	8e-04	0.00067	0.00027	0.00027	0.00024
BIAS1	-0.00029	-7e-04	-0.00028	0.00039	0.00049	0.00036
BIAS2	0.00031	0.00032	0.00026	-0.00012	-8e-05	-0.00011
VAR1	0.03287	0.02009	0.02931	0.01824	0.01370	0.01730
VAR2	0.01178	0.00815	0.01107	0.00689	0.00583	0.00670
$n = 300$						
MSE	0.00032	0.00028	0.00027	1e-04	9e-05	9e-05
BIAS1	0.00038	0.00027	0.00026	-0.00068	-0.00062	-0.00073
BIAS2	-0.00023	-0.00014	-0.00017	0.00023	0.00015	0.00016
VAR1	0.01862	0.01287	0.01678	0.01046	0.00860	0.01003
VAR2	0.00660	0.00485	0.00623	0.00392	0.00337	0.00384

Table 15: DGP 5, weakly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.8148	0.5688	1.9210	2.9590
	30	0.5215	0.0804	1.3210	1.0320
100	12	0.8940	0.1551	1.5220	1.5020
	30	0.3065	0.0496	1.7880	1.0010
300	12	0.7599	0.0729	1.7940	1.0000
	30	0.3792	0.0357	1.8960	1.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00441	0.00239	0.00597	0.00151	0.00052	0.00146
BIAS1	0.00185	8e-05	-0.00405	-0.00371	-0.00104	-0.00489
BIAS2	0.03416	0.00184	0.18339	0.06002	0.00102	0.09416
VAR1	0.07302	0.02546	0.06600	0.03832	0.01890	0.03569
VAR2	0.05654	0.02498	0.05191	0.03429	0.01887	0.03201
$n = 100$						
MSE	0.00159	3e-04	0.00167	0.00044	0.00015	0.00045
BIAS1	0.00098	-5e-05	-0.00688	-0.00049	-0.00046	-0.00621
BIAS2	0.08962	0.00013	0.18334	0.04597	0.00051	0.09380
VAR1	0.03846	0.01418	0.03662	0.02103	0.01073	0.01924
VAR2	0.02994	0.01413	0.02884	0.01884	0.01071	0.01733
$n = 300$						
MSE	0.00054	0.00011	0.00056	0.00014	4e-05	0.00015
BIAS1	0.00092	-0.00067	-0.00411	-0.00138	-0.00015	-0.00417
BIAS2	0.08784	0.00066	0.18679	0.05934	7e-05	0.09467
VAR1	0.02191	0.00841	0.02136	0.01188	0.00606	0.01126
VAR2	0.01706	0.00837	0.01676	0.01064	0.00605	0.01012

Table 16: DGP 1, strongly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.8790	0.9115	3.5460	3.0000
	30	0.8399	0.8227	7.3890	5.0000
100	12	0.7742	0.7510	3.6220	3.0000
	30	0.7032	0.6029	7.8030	5.0000
300	12	0.8460	0.7954	3.9860	3.0000
	30	0.8172	0.6999	8.6520	5.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00137	0.00218	0.03075	0.00042	0.00105	0.01061
BIAS1	-0.00082	0.00072	0.00447	0.0010	0.00266	-0.00571
BIAS2	3e-05	0.00011	0.07956	1e-05	2e-05	0.07665
VAR1	0.04509	0.02697	0.12493	0.02754	0.01911	0.07237
VAR2	0.00720	0.00460	0.02301	0.00447	0.00328	0.01403
$n = 100$						
MSE	0.00053	0.00074	0.00999	0.00022	0.00039	0.00383
BIAS1	-0.00061	-0.00025	-0.00789	-6e-05	-0.00049	-0.00353
BIAS2	1e-05	-6e-05	0.07822	-5e-05	1e-05	0.07378
VAR1	0.02609	0.01673	0.06780	0.01737	0.01306	0.03944
VAR2	0.00412	0.00277	0.01250	0.00280	0.00221	0.00767
$n = 300$						
MSE	0.00011	0.00015	0.00292	5e-05	8e-05	0.00106
BIAS1	6e-04	0.00097	0.00053	-0.00012	-0.00012	-0.0045
BIAS2	5e-05	4e-05	0.07770	-2e-05	2e-05	0.07256
VAR1	0.01229	0.00827	0.03934	0.00778	0.00640	0.02293
VAR2	0.00191	0.00135	0.00712	0.00123	0.00104	0.00436

Table 17: DGP 2, strongly autocorrelated errors

MSE of Effects

n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	29.8443	0.8524	1.0960	3.0000
	30	14.6219	0.6941	1.0410	4.9730
100	12	22.3905	0.6467	1.0330	3.0000
	30	13.2654	0.6727	1.0050	5.0000
300	12	20.1982	0.6357	1.0300	3.0000
	30	12.4592	0.5426	1.0040	5.0000

MSE, Bias, Variance for Coefficients

	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.14386	0.00158	0.67009	0.06133	9e-04	0.71645
BIAS1	4e-05	-0.00062	-0.03949	-0.00383	0.00142	-0.03157
BIAS2	0.10246	-2e-05	0.28056	0.04392	-9e-05	0.27759
VAR1	0.29667	0.01461	0.59358	0.15936	0.00931	0.55479
VAR2	0.01983	0.00196	0.03624	0.00695	0.00108	0.02248
$n = 100$						
MSE	0.04344	0.00034	0.21744	0.01756	2e-04	0.20866
BIAS1	0.00251	-0.00084	0.00639	-0.00203	-0.00039	0.01174
BIAS2	0.09700	8e-05	0.26908	0.04524	1e-05	0.26709
VAR1	0.15119	0.00875	0.32003	0.08467	0.00645	0.29845
VAR2	0.01004	0.00128	0.01962	0.00364	0.00051	0.01207
$n = 300$						
MSE	0.01304	8e-05	0.07598	0.00662	6e-05	0.07634
BIAS1	0.00713	0.00068	0.00164	-0.00229	0.0000	-0.01219
BIAS2	0.09807	-5e-05	0.27847	0.04409	1e-05	0.27567
VAR1	0.08502	0.00547	0.18998	0.04810	0.00410	0.17669
VAR2	0.00568	0.00073	0.01149	0.00207	0.00034	0.00706

Table 18: DGP 3, strongly autocorrelated errors

MSE of Effects

n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	1.0779	0.8383	4.9100	3.0000
	30	21.4318	0.7917	1.3040	5.0000
100	12	0.9907	0.6738	5.7830	3.0000
	30	21.6087	0.6763	1.0290	5.0000
300	12	0.9030	0.5372	5.6840	3.0000
	30	21.6825	0.5802	1.0000	5.0000

MSE, Bias, Variance for Coefficients

	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00188	0.00246	0.03330	0.01267	0.00116	0.01145
BIAS1	0.00178	0.00108	-0.00508	-0.02052	0.00054	-0.02208
BIAS2	0.02214	0.00061	0.45621	0.45217	0.00014	0.45236
VAR1	0.04685	0.01914	0.14830	0.09516	0.01142	0.08935
VAR2	0.01390	0.01013	0.04835	0.03118	0.00646	0.02997
$n = 100$						
MSE	0.00058	0.00065	0.00998	0.00386	0.00034	0.00395
BIAS1	0.00057	-0.00123	-0.02912	-0.0170	0.00046	-0.02778
BIAS2	0.02277	0.00013	0.45894	0.46410	1e-04	0.45279
VAR1	0.02423	0.01210	0.08031	0.05099	0.00771	0.04832
VAR2	0.00716	0.00582	0.02640	0.01676	0.00364	0.01636
$n = 300$						
MSE	0.00028	0.00014	0.00357	0.00139	9e-05	0.00134
BIAS1	3e-04	-0.00046	-0.01523	-0.01135	-0.00085	-0.01518
BIAS2	0.02755	-4e-05	0.46061	0.46422	0.00018	0.45417
VAR1	0.01789	0.00668	0.04761	0.02953	0.00493	0.02831
VAR2	0.00528	0.00377	0.01543	0.00968	0.00230	0.00949

Table 19: DGP 4, strongly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.8588	0.9413	3.2640	3.0000
	30	0.8246	0.8268	7.1850	4.9990
100	12	0.8504	0.8324	3.7170	3.0000
	30	0.8200	0.7289	8.2940	5.0000
300	12	0.8416	0.8039	3.8920	3.0000
	30	0.7082	0.5812	8.1220	5.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00172	0.00269	0.00281	0.00054	0.00127	0.00144
BIAS1	-6e-05	-0.00017	-0.00126	-9e-05	-0.00081	-0.00163
BIAS2	0.00036	0.00061	-0.00018	-2e-05	6e-04	0.00049
VAR1	0.04901	0.02734	0.04462	0.02900	0.01899	0.02887
VAR2	0.01618	0.00991	0.01713	0.00954	0.00679	0.01135
$n = 100$						
MSE	0.00035	0.00048	0.00074	0.00014	0.00026	5e-04
BIAS1	-0.0012	-0.00039	-0.00103	-0.00018	-0.00016	0.00086
BIAS2	-0.00019	-4e-04	-0.00043	8e-05	0.00019	0.00025
VAR1	0.02142	0.01350	0.02213	0.01388	0.01072	0.01541
VAR2	0.00685	0.00452	0.00833	0.00442	0.00357	0.00596
$n = 300$						
MSE	0.00012	0.00017	0.00027	6e-05	0.00011	0.00015
BIAS1	-0.00012	0.00089	0.00058	-0.00013	2e-05	-0.00041
BIAS2	0.00012	9e-05	9e-05	4e-05	0.00013	0.00036
VAR1	0.01221	0.00798	0.01302	0.00958	0.00745	0.00942
VAR2	0.00386	0.00263	0.00481	0.00306	0.00247	0.00359

Table 20: DGP 5, strongly autocorrelated errors

MSE of Effects					
n	T	KSS	Eup	d_{KSS}	d_{Eup}
30	12	0.9804	0.9070	3.6030	3.0000
	30	0.8361	0.7796	7.3200	5.0000
100	12	0.9001	0.8054	3.9960	3.0000
	30	0.7687	0.6270	8.1760	5.0000
300	12	0.8099	0.7024	3.8680	3.0000
	30	0.7074	0.5404	8.2160	5.0000

MSE, Bias, Variance for Coefficients						
	$T = 12$			$T = 30$		
	KSS	Eup	Within	KSS	Eup	Within
$n = 30$						
MSE	0.00187	0.00194	0.00617	0.00066	0.00105	0.00195
BIAS1	-0.00059	-0.00199	-0.00361	-3e-05	-6e-04	-0.00357
BIAS2	0.01985	0.00277	0.18402	0.00996	0.00104	0.09630
VAR1	0.05324	0.02143	0.06045	0.03252	0.01566	0.03345
VAR2	0.04040	0.01963	0.04735	0.02869	0.01530	0.03008
$n = 100$						
MSE	0.00044	0.00041	0.00174	0.00021	0.00031	0.00059
BIAS1	0.00052	-0.00028	-0.00855	9e-04	-0.00099	-0.00454
BIAS2	0.01683	0.00077	0.18401	0.01124	5e-05	0.09358
VAR1	0.02411	0.01144	0.03153	0.01723	0.00960	0.01829
VAR2	0.01829	0.01003	0.02483	0.01523	0.00929	0.01651
$n = 300$						
MSE	0.00022	0.00017	0.00059	7e-05	0.00011	0.00022
BIAS1	0.00155	0.00023	-0.00289	0.00039	-0.00096	-0.00377
BIAS2	0.01834	0.00047	0.18554	0.01084	0.00012	0.09457
VAR1	0.01608	0.00743	0.01927	0.01032	0.00595	0.01076
VAR2	0.01222	0.00664	0.01511	0.00910	0.00573	0.00968

Table 21: Testing the Presence of Interactive Effects - Test of Kneip, Sickles, and Song (2012)

Test-Statistic	p-value	crit.-value	sig.-level
38.49	0.00	2.33	0.01

Table 22: Slope-Coefficients

	Estimate	Std.Err	Z value	Pr(>z)
dem	-0.000210	0.000259	-0.81	0.418
lag(l.d.gdp.a, 1)	0.268000	0.022200	12.00	<2e-16 ***
lag(l.d.gdp.a, 2)	0.021100	0.022700	0.93	0.352
lag(l.d.gdp.a, 3)	-0.019400	0.022200	-0.87	0.384
lag(l.d.gdp.a, 4)	0.021600	0.020800	1.04	0.300

Call:

```
Eup.default(formula = l.d.gdp.a ~ dem + lag(l.d.gdp.a, 1) + lag(l.d.gdp.a,
2) + lag(l.d.gdp.a, 3) + lag(l.d.gdp.a, 4) - 1,
additive.effects = "twoways", dim.criterion = "PC3", error.type = 5)
```

Residuals:

Min	1Q	Median	3Q	Max
-0.025100	-0.001840	0.000138	0.002140	0.019000

Additive Effects Type: twoways

Dimension of the Unobserved Factors: 7

Residual standard error: 0.004821 on 2325 degrees of freedom, **R-squared:** 0.7418