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Data 102

# Homework 3

October 24, 2019

## Problem 2

1. If $i < k$, the probability that i is the max serial number in the sample set is 0, because the minimum possible serial number is at least k.
   If $k \leq i \leq N$,
2. $E[Y_{(k)}] = Y_{(k)} * Pr(Y_{(k} = i)$
   $\Rightarrow \sum_{i=k}^{N} \binom{i}{k} * Pr(Y_{(k} = i)$
3. Part (D)
   $Pr(N|Y_{(k)}) = \pi(N) * Pr(Y_{(k)}|N)$
   $\Rightarrow \prod_{i=k}^{N} Pr(Y_{(k)} = i) * \sum_{n \in N} \pi(N)$

# Problem 3

1. Let $X = I_1 + ... + I_n$ be the sum of independent indicator variables
   $\mu = E[X] = E[I_1 + ... + I_n] = E[I_1] + ... + E[I_n]$
   $\mu = Pr(I_1) + ... + Pr(I_n) = p_1 + ... + p_n$
   $\mu = \sum_{j=1}^{n} p_j$

   $\sigma^2 = Var(X) = Var(I_1 + ... + I_n) = Var(I_1) + ... + Var(I_n)$
   $\sigma^2 = Pr(I_1) * (1 - Pr(I_1)) + ... + Pr(I_n) * (1 - Pr(I_n))$
   $\sigma^2 = p_1 * (1 - p_1) + ... + p_n * (1 - p_n)$
   $\sigma^2 = \sum_{j=1}^{n} p_j * (1 - p_j)$

2. Markov's Bound: $Pr(X \geq \mu(1 + c)) \leq \frac{1}{1+c}$

   $Pr(X \geq c) \leq \frac{\mu}{c} \Rightarrow$ Let $c = c * \mu \Rightarrow Pr(X \geq c * \mu) \leq \frac{1}{c}$
   $\Rightarrow \hat{c} = \mu * (1 + c) \Rightarrow Pr(X \geq \mu * (1 + c)) \leq \frac{1}{1+c}$

3. Chebyshev's Bound:

   $P(X \geq \mu(1 + c)) = Pr(X \geq \mu + \mu * c) = Pr(X - \mu \geq \mu * c)$
   Definition of Chebyshev: $Pr(|X - \mu| \geq k) \leq \frac{\sigma^2}{k^2}$
   $\Rightarrow$ Let $k = \mu * c \Rightarrow$
   $Pr(|X - \mu| \geq \mu * c) \leq \frac{\sigma^2}{\mu^2 c^2}$

4. From part (c): $Pr(|X - \mu| \geq \mu * c) \leq \frac{\sigma^2}{\mu^2 c^2}$
   $\Rightarrow Pr(|X - np| \geq npc) \leq \frac{n^2 p^2 (1-p)^2}{n^2 p^2 c^2} = \frac{(1-p)^2}{c^2}$

   From part (b): $Pr(X \geq \mu(1 + c)) \leq \frac{1}{1+c}$
   $Pr(X \geq np(1 + c)) \leq \frac{1}{1+c}$

5. $M_{I_j}(t) = 1 + p_j(e^t - 1)$
   $M_{I_j}(t) = E[e^{t*I_j}] = e^{t*0} * Pr(I_j = 0) + e^{t*1} * Pr(I_j = 1)$
   $M_{I_j}(t) = (1 - p_j) + p_j * e^t = 1 + p_j(e^t - 1)$

6. $M_X(t) \leq e^{\mu(e^t - 1)}$
   $\Rightarrow M_X(t) = M_{I_1}(t) * ... * M_{I_n}(t) = (1 + p_j(e^t - 1))^n$
   $\Rightarrow (1 + x) \leq e^x$ and $\mu = p_j$ because $I_j$ is an indicator random variable
   $(1 + \mu(e^t - 1))^n \leq e^{\mu(e^t - 1)}$ where $x = \mu(e^t - 1)$

7.

```
In [1]: import random
        import math
        import pandas as pd
        import numpy as np
        from scipy.stats import norm
        from scipy.stats import binom
        from scipy.special import comb
        import matplotlib.pyplot as plt
        %matplotlib inline
```

```
In [2]: serial_numbers_1=[331]
        serial_numbers_2=[331,134, 306,  53, 272,  97, 100, 255,   3, 298]
        serial_numbers_3=[111, 228, 139, 216,  36, 213, 189,  71, 184, 331, 49,
        224, 173, 311,305, 208, 231, 285, 142,  22, 168, 263, 135, 149, 155]
```

# 2c: Unbiased Frequentist Estimator

Fill out the following function with the unbiased estimator of maximum number of tanks, $N$, that you have derived.

```
In [3]: def frequentist_estimator(serial_numbers):
            """ Returns the frequentist estimator for N, the total number of tan
        ks,
            given a sample of k tanks with maximum serial number Y_k

            Parameters
            ----------
                serial_numbers : list of observed tank serial numbers

            Returns: estimate N_hat
            """
            y_k = max(serial_numbers)
            sample_size = len(serial_numbers)
            N_hat = y_k * (sample_size**(-1) + 1) - 1
            return N_hat
```

# 2e: Bayesian Posterior

Fill out the following functions with the posterior using the given uniform prior on $[100, 1000]$. Then use this function to find the credible interval for $N$. The function $comb(n, k)$ returns $n$ choose $k$.

In [4]:
```
def uniform_prior(n,Nmin=100,Nmax=1000):
    """ Returns the probability mass function of a uniform distribution
 on the integers from Nmin, Nmax

    Parameters
    ----------
        n : the value at which the prior is being evaluated.
        Nmin : lower bound on support of prior
        Nmax : upper bound on support of prior
    """
    assert Nmax>Nmin
    if n<=Nmax and n>=Nmin:
        return 1.0/(Nmax-Nmin)
    else:
        return 0.0
```

In [5]:
```
def posterior_distribution_with_uniform_prior(n,serial_numbers,Nmin=100,
Nmax=1000):
    """ Returns the frequentist estimator for N, the total number of tan
ks,
    given a sample of k tanks with maximum serial number Y_k

    Parameters
    ----------
        n : value of N
        serial_numbers : list of observed tank serial numbers
        Nmin : lower bound on support of prior
        Nmax : upper bound on support of prior

    Returns: P(N=n |serial_numbers)
    """
    y_k = max(serial_numbers)
    k = len(serial_numbers)
    likelihood = 1
    for i in serial_numbers:
        likelihood = comb(i-1, k-1)/comb(n,k)
    posterior = uniform_prior(n=n, Nmin=Nmin, Nmax=Nmax) * likelihood
    return posterior
```

In [6]:
```
def plot_posteriors(serial_numbers,Nmin=100,Nmax=1000):
    """ Plots the uniform prior and the posterior P(N |Y_k)

    Parameters
    ----------
    serial_numbers : list of observed tank serial numbers
    Nmin: lower bound on support of prior
    Nmax: upper bound on support of prior
    """

    support=np.arange(Nmin-10,Nmax+10)
    uniform=[]
    posterior=[]
    for i in support:
        uniform.append(uniform_prior(i,Nmin,Nmax))
        posterior.append(posterior_distribution_with_uniform_prior(i,ser
ial_numbers,Nmin,Nmax))


    plt.plot([0, 1], [0, 0], color='white', lw=1)
    plt.stem(support, uniform, linefmt='darkblue', label='Uniform prior'
)
    plt.stem(support, posterior, linefmt='green', lw=1, label='Posterior
- w/uniform prior')
    plt.legend()
    return
```

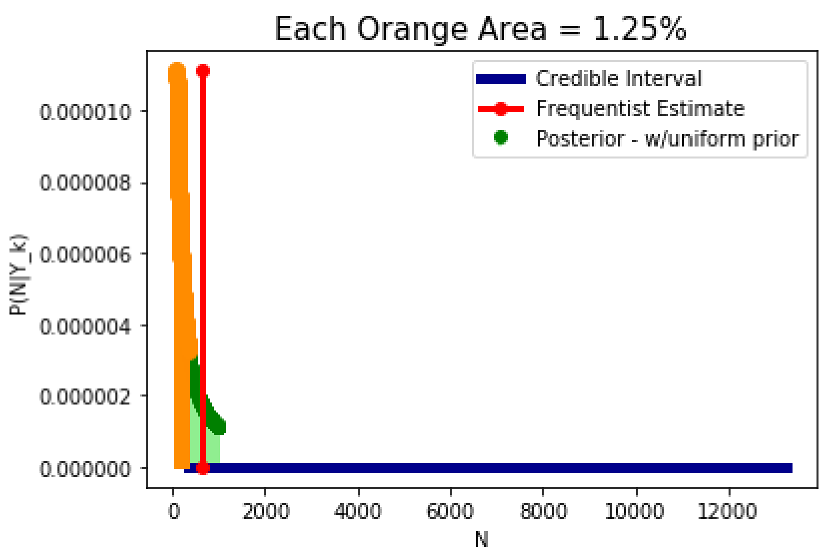
In [7]:
```
def credible_interval(alpha,serial_numbers,Nmin=100,Nmax=1000):
    """ returns the credible interval at the level alpha

    Parameters
    ----------
    alpha: amount of probability mass encompassed by credible interval
    serial_numbers : list of observed tank serial numbers
    Nmin : lower bound on support of prior
    Nmax : upper bound on support of prior

    Returns: a list with the cutoffs for the credible intervals for N in
the form
                interval=[lower,upper]

    """
    y_k = max(serial_numbers)
    k = len(serial_numbers)
    return [y_k/(1-alpha)**(1/k), y_k/(alpha**(1/k))]
```

In [8]:
```
def plot_frequentist_estimate_and_credible_int(alpha,serial_numbers,Nmin
=100,Nmax=1000):
    """ plots the posterior P(N| Y_k), the credible interval, and the fr
equentist estimate of N.

    Parameters
    ----------
    alpha : amount of probability mass encompassed by credible interval
    serial_numbers : list of observed tank serial numbers
    Nmin : lower bound on support of prior
    Nmax : upper bound on support of prior

    """

    #potential support of N
    support=np.arange(Nmin,Nmax)

    #potential support of N
    posterior=[]
    low_interval_probs=[]
    low_interval=[]
    upper_interval_probs=[]
    upper_interval=[]

    #gets credible_interval
    left_end, right_end=credible_interval(alpha,serial_numbers,Nmin,Nmax
)

    #gets frequentist estimate
    frequentist_estimate=frequentist_estimator(serial_numbers)

    #print out information
    print("Credible Interval: "+str(left_end)+'-'+str(right_end))
    print("Frequentist Estimate: "+str(int(frequentist_estimate)))

    #keeps track of which elements in the support are in which intervals
    for i in support:
        prob=posterior_distribution_with_uniform_prior(i,serial_numbers,
Nmin,Nmax)
        posterior.append(prob)
        if i<=left_end:
            low_interval_probs.append(prob)
            low_interval.append(i)
        if i>=right_end:
            upper_interval_probs.append(prob)
            upper_interval.append(i)

    #plot posterior
    plt.stem(support, posterior, linefmt='lightgreen',markerfmt='go',bas
efmt='None',label='Posterior - w/uniform prior')


    #plot credible interval
    plt.plot([left_end, right_end], [0, 0], color='darkblue', lw=5, labe
l='Credible Interval')
```

```
    #plot the lower interval
    if len(low_interval)>0:
        markerline, stemlines, baseline = plt.stem(low_interval,low_inte
rval_probs,markerfmt='o',linefmt='darkorange',  basefmt='None')
        markerline.set_markerfacecolor('darkorange')
        markerline.set_markersize(8)
        markerline.set_color('darkorange')

    #plot the higher interval
    if len(upper_interval)>0:
        markerline, stemlines, baseline=plt.stem(upper_interval, upper_i
nterval_probs,markerfmt='o',linefmt='darkorange', basefmt='None')
        markerline.set_markerfacecolor('darkorange')
        markerline.set_markersize(8)
        markerline.set_color('darkorange')

    #plot the frequentist estimate
    plt.plot([frequentist_estimate,frequentist_estimate], [0,max(posteri
or)] ,'o-',color='red', lw=3, label='Frequentist Estimate')

    #Axes labels
    plt.title('Each Orange Area = '+str(alpha*50.0)+'%', fontsize=15)
    plt.xlabel('N')
    plt.ylabel('P(N|Y_k)')
    plt.legend(bbox_to_anchor=[1.0,1.0])
    return
```

In [9]:
```
plot_frequentist_estimate_and_credible_int(alpha=0.025,serial_numbers=se
rial_numbers_1,Nmin=100,Nmax=1000)
```

```
Credible Interval: 339.4871794871795-13240.0
Frequentist Estimate: 661

/usr/local/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:
45: UserWarning: In Matplotlib 3.3 individual lines on a stem plot will
be added as a LineCollection instead of individual lines. This signific
antly improves the performance of a stem plot. To remove this warning a
nd switch to the new behaviour, set the "use_line_collection" keyword a
rgument to True.
/usr/local/anaconda3/lib/python3.7/site-packages/ipykernel_launcher.py:
53: UserWarning: In Matplotlib 3.3 individual lines on a stem plot will
be added as a LineCollection instead of individual lines. This signific
antly improves the performance of a stem plot. To remove this warning a
nd switch to the new behaviour, set the "use_line_collection" keyword a
rgument to True.
```



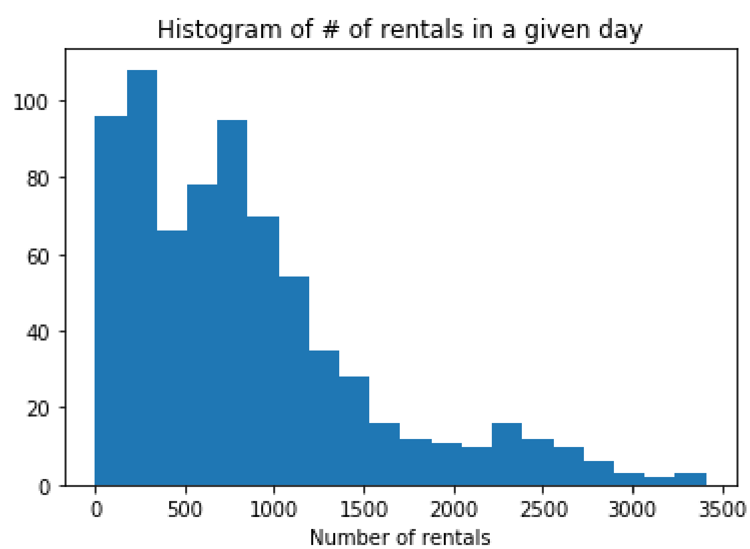

In [ ]:

In [1]:
```
%pylab inline

import matplotlib.pyplot as plt
import numpy as np
import pandas as pd
import scipy.stats
import seaborn as sns
```

```
Populating the interactive namespace from numpy and matplotlib
```

# (a) Download the data and plot a histogram.

In [2]:
```
# Load bikeshare.csv, you might want to look into the read_csv function
 in pandas.
# The columns of the data are:
# sunny: 1 if the day was sunny, 0 otherwise.
# working_day: 1 if the day was a working day, 0 otherwise.
# month: the month the day was in, where the first month is 1.
# num_rentals: the number of people that rented a bike on the day.
data = pd.read_csv('bikeshare.csv') # TODO

# Plot a histogram of the number of rentals in a given day with 20 bins.
# TODO
plt.figure()
plt.hist(data['num_rentals'], bins=20)
plt.xlabel('Number of rentals')
plt.title('Histogram of # of rentals in a given day')
plt.show()
```




# (b) Use Gibb's sampling to simulate from the graphical model in the problem statement.

### (i) Implement `get_subgroup_statistics`.

In [3]:
```
def get_subgroup_statistics(data):
    """
    Given bikesharing data, returns the means, standard deviations, and
 counts
    split by categories sunny and working_day.

    Parameters
    ----------
    data : dataframe
        A dataframe of bikesharing data with 0/1 categories `sunny` and
 `working_day`
        and numerical value `num_rentals`.

    Returns
    -------
    means : 2x2 array of floats
        An array where mean[i, j] corresponds to the empirical mean of r
entals for
        all days with working_day=i and sunny=j.
    stds : 2x2 array of floats
        An array where stds[i, j] corresponds to the empirical standard
 deviation
        of rentals for all days with working_day=i and sunny=j.
    counts : 2x2 array of floats
        An array where mean[i, j] corresponds to the total number of ren
tals for
        all days with working_day=i and sunny=j.
    """
    num_categories = 2
    means = np.zeros((num_categories, num_categories))
    stds = np.zeros((num_categories, num_categories))
    counts = np.zeros((num_categories, num_categories))

    # Iterate through all possible combinations of working_day and sunn
y.
    for working_day in range(2):
        for sunny in range(2):
            filter1 = data['working_day'] == working_day
            filter2 = data['sunny'] == sunny
            counts[working_day, sunny] = data.where(filter1 & filter2)[
'num_rentals'].sum() # TODO
            means[working_day, sunny] = data.where(filter1 & filter2)['n
um_rentals'].mean() # TODO
            stds[working_day, sunny] = data.where(filter1 & filter2)['nu
m_rentals'].std()# TODO

    return means, stds, counts
```

Now that we've implemented `get_subgroup_statistics` , let's see what one of its output looks like.

In [4]:
```
means, stds, counts = get_subgroup_statistics(data)
print("counts:")
print("         rainy   sunny")
print("weekend  {0}    {1}".format(counts[0, 0], counts[0, 1]))
print("workday  {0}   {1}".format(counts[1, 0], counts[1, 1]))
```

```
counts:
         rainy   sunny
weekend  85263.0    231469.0
workday  88408.0   214877.0
```

### (ii) Fill in the following code:

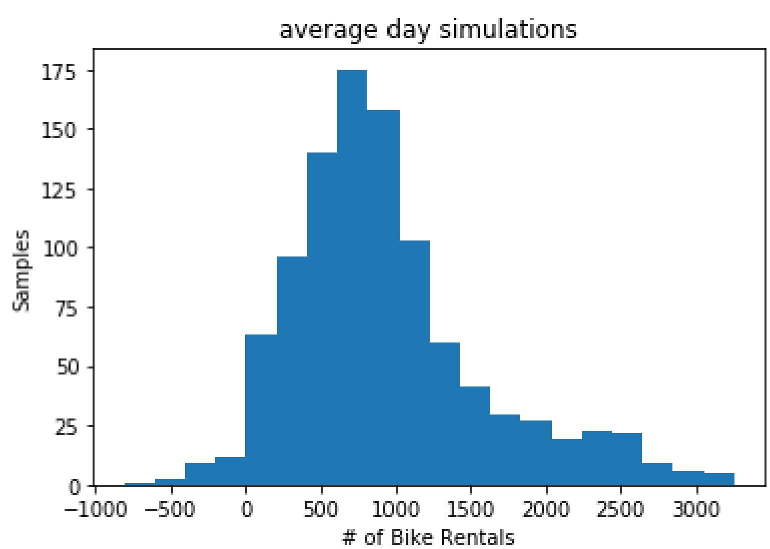
In [5]:
```
def sample_num_rentals(workday, sunny, means, stds):
    """
    Randomly sample the number of bike rentals from one of four Gaussian
s depending
    on the given daily conditions.

    Parameters
    ----------
    workday : int
        1 if the day we wish to sample for is a workday, 0 if it's a wee
kend.
    sunny : int
        1 if the day we wish to sample for is sunny, 0 if it's rainy.
    means : 2x2 array of floats
        An array where mean[i, j] corresponds to the mean of the Gaussia
n
        we sample from when working_day=i and sunny=j.
    stds : 2x2 array of floats
        An array where mean[i, j] corresponds to the standard deviation
 of the Gaussian
        we sample from when working_day=i and sunny=j.

    Returns
    -------
    count : float
        A Gaussian sampled count.
    """
    mu, sigma = means[workday, sunny], stds[workday, sunny]
    return np.random.normal(loc=mu, scale=sigma, size=1) # TODO


def simulate_rentals(num_samples, p_workday, p_sunny, means, stds):
    """
    Simulate `num_samples` days by using the sampling procedure defined
    in the homework sheet.

    Parameters
    ----------
    num_samples : int
        The number of days to simulate.
    p_workday : float
        The probability that any given day will be a workday.
    p_sunny : float
        The probability that any given day will be sunny.
    means : 2x2 array of floats
        An array where mean[i, j] corresponds to the mean of the Gaussia
n
        we sample from for the number of rentals when working_day=i and
 sunny=j.
    stds : 2x2 array of floats
        An array where stds[i, j] corresponds to the standard deviation
 of the Gaussian
        we sample from for the number of rentals when working_day=i and
 sunny=j.

    Returns
```

```
    -------
    samples : num_samples x 3 array of floats
        The generated samples where samples[i, 0] is 1 if day i was a wo
rkday,
        samples[i, 1] is 1 if day i was sunny, and samples[i, 2] is the
 number
        of rentals that happened on that day.
    """
    samples = np.zeros((num_samples, 3))
    for t in range(num_samples):
        x_t = np.zeros(3)
        x_t[0] = np.random.binomial(n=1, p=p_workday) # TODO: This shoul
d be a bernouili sample that repesents workday.
        x_t[1] = np.random.binomial(n=1, p=p_sunny) # TODO: This should
 be a bernouilli sample that represents sunny.
        x_t[2] = sample_num_rentals(workday=int(x_t[0]), sunny=int(x_t[1
]), means=means, stds=stds) # TODO: The sampled number of rentals. You m
ight want to cast x_t[0], x_t[1] to ints.
        samples[t] = x_t

    return samples
```

## (c) Draw 1000 samples; plot a histogram of the resulting draws for the number of bikes in a given day.

In [6]:
```
# Assume a 5 day workweek; ignore holidays.
p_workday = 5.0 / 7.0

# Use the fraction of sunny days from your dataset.
frac_sunny_days = data.where(data['sunny'] == 1).shape[0] / data.shape[0
] # TODO
p_sunny = frac_sunny_days

# Get the data statistics.
means, stds, counts = get_subgroup_statistics(data)

T = 1000
samples = simulate_rentals(num_samples=T,
                           p_workday=p_workday,
                           p_sunny=p_sunny,
                           means=means,
                           stds=stds)

# Plot the histogram.
plt.hist(samples[:, 2], bins=20)
plt.xlabel("# of Bike Rentals");
plt.ylabel("Samples");
plt.title("average day simulations")
```

Out[6]:
```
Text(0.5, 1.0, 'average day simulations')
```

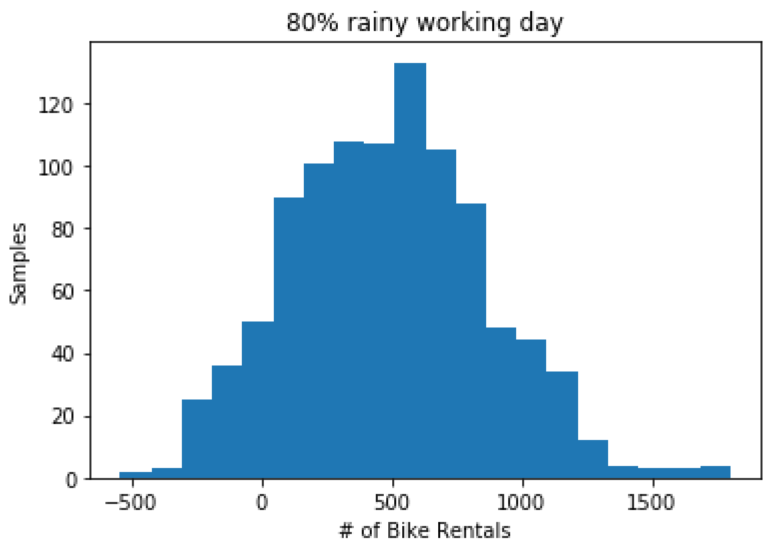



## (d) Run 1000 simulations given a forecast of 80% rain tomorrow; plot a histogram of the resulting draws for the number of bikes in a given day.

### (i) if tomorrow is a weekday

In [7]:
```
p_workday = 1 # TODO
p_sunny = 0.2 # TODO

T = 1000 # TODO
samples = simulate_rentals(num_samples=T,
                           p_workday=p_workday,
                           p_sunny=p_sunny,
                           means=means,
                           stds=stds)

plt.hist(samples[:, 2], bins=20)
plt.xlabel("# of Bike Rentals");
plt.ylabel("Samples");
plt.title("80% rainy working day")
```

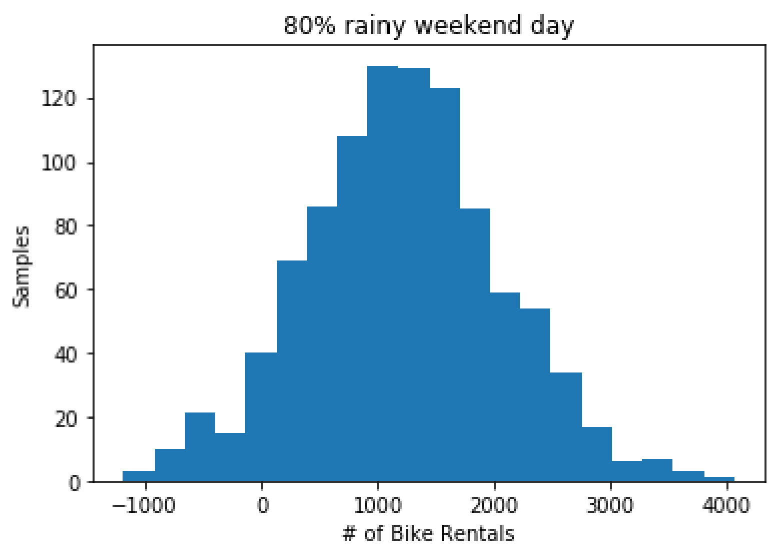
Out[7]:
```
Text(0.5, 1.0, '80% rainy working day')
```




### (ii) if tomorrow is a weekend

In [8]:
```
p_workday = 0 # TODO
p_sunny = 0.2 # TODO

T = 1000 # TODO
samples = simulate_rentals(num_samples=T,
                           p_workday=p_workday,
                           p_sunny=p_sunny,
                           means=means,
                           stds=stds)

plt.hist(samples[:, 2], bins=20)
plt.xlabel("# of Bike Rentals");
plt.ylabel("Samples");
plt.title("80% rainy weekend day")
```

Out[8]:
```
Text(0.5, 1.0, '80% rainy weekend day')
```




# (e) Can you run the procedure to its completion using only the first ten entries of data?

In [9]:
```
data_smaller = data.head(10) # TODO

means, stds, counts = get_subgroup_statistics(data_smaller)
print("counts:")
print("          rainy sunny")
print("weekend  {0}   {1}".format(counts[0,0], counts[0,1]))
print("workday  {0}   {1}".format(counts[1,0], counts[1,1]))
frac_sunny_days = data['sunny'].mean()
print('fraction sunny days: {0:.2f}'.format(frac_sunny_days))
```

```
counts:
         rainy sunny
weekend  0.0   6042.0
workday  484.0   2278.0
fraction sunny days: 0.63
```

**TODO: Fill this in, can you run the procedure to completion?**

Yes, it can be run to completion. Although, I believe that the samples produced will not be as accurate.

## (f) Implement the missing code below.

In [10]:
```
def sample_mu_by_category(prior_means, prior_sigma, working_day, sunny):
    """
    Sample mu from a Gaussian prior that depends on the properties of th
e day.

    Parameters
    ----------
    prior_means : 2x2 array of floats
        The prior means where prior_means[i, j] is the prior mean for da
ys
        with working_day=i and sunny=j.
    prior_sigma : float
        The standard deviation of the prior from which we are sampling.
    working_day : int
        1 if we are sampling the mean for a day that is a workday and 0
 for
        a day that is a weekend.
    sunny : int
        1 if we are sampling the mean for a day that is sunny and 0 for
        a day that is rainy.

    Returns
    -------
    mu : float
        The sampled mean.
    """
    prior_mean = prior_means[working_day, sunny] # TODO
    return np.random.normal(prior_mean, prior_sigma)


def prior_prob_by_category(observed_mu, prior_means, prior_sigma, workin
g_day, sunny):
    """
    Compute the probability that we observed mu given specific propertie
s of the day.

    Parameters
    ----------
    observed_mu : float
        The value of mu for which we wish to compute the prior probabili
ty.
    prior_means : 2x2 array of floats
        The prior means where prior_means[i, j] is the prior mean for da
ys
        with working_day=i and sunny=j.
    prior_sigma : float
        The standard deviation of the prior from which we are sampling.
    working_day : int
        1 if we are sampling the mean for a day that is a workday and 0
 for
        a day that is a weekend.
    sunny : int
        1 if we are sampling the mean for a day that is sunny and 0 for
        a day that is rainy.

    Returns
```

```
    -------
    prob : float
        The computed probability.
    """
    prior_mean = prior_means[working_day, sunny] # TODO
    return prior_mean # TODO


def compute_likelihood(data, likelihood_means, likelihood_sigma):
    """
    Compute the likelihood that we observed the data given the
    means across the four daily categories.

    Parameters
    ----------
    data : dataframe
        A dataframe of bikesharing data with 0/1 categories `sunny` and
 `working_day`
        and numerical value `num_rentals`.
    likelihood_means : 2x2 array of floats
        The likelihood means where sampled_means[i, j] is the likelihood
mean when
        work_day=i and sunny=j.
    likelihood_sigma : float
        The standard deviations of the Gaussians from which the data is
 drawn.

    Returns
    -------
    prob : float
        The likelihood of the data, may be scaled by a fixed constant.
    """
    sunny = data["sunny"].values
    working_day = data["working_day"].values
    num_rentals = data["num_rentals"].values

    likelihood = 1
    # Assume draws are i.i.d. so that likelihoods of each datapoint mult
iply.
    for i in range(len(data)):
        x = num_rentals[i]
        # mu_this_category is the sampled mean for the sunny/workday com
bo of row i.
        mu_this_category = likelihood_means[working_day[i], sunny[i]]

        # Premultiply by 1e4 to keep numerical stability,
        # because data is the same length every time this will be ok.
#         n = scipy.stats.norm(loc=mu_this_category, scale=likelihood_si
gma[working_day[i], sunny[i]])
#         likelihood *= 1e4 * x * np.random.normal(mu_this_category, lik
elihood_sigma) # TODO
        likelihood *= 1e4 * scipy.stats.norm.pdf(x=x, loc=mu_this_catego
ry, scale=likelihood_sigma)
    return likelihood


def gibbs_sampling_posterior(data, num_samples, prior_means, prior_sigma
```

```
, likelihood_sigma):
    """
    Sample likelihood parameters (the means) along with their posterior
 probability.

    Parameters
    ----------
    num_samples : int
        The number of samples to draw.
    prior_means : 2x2 array of floats
        The prior means associated to the likelihood means, where prior_
means[i, j]
        is the prior mean for working_day=i and sunny=j.
    prior_sigma : float
        The standard deviation of all the prior Gaussians.
    likelihood_sigma : float
        The standard deviation of all the likelihood Gaussians.

    Returns
    -------
    samples : num_samples x 5 array of floats
        The array of all generated samples where samples[i] is the subar
ray with
        [mu_sunny_workday, mu_rainy_workday, mu_sunny_weekend, mu_rainy_
weekend, c*p(theta, x)]
        where the last element is the posterior scaled by any arbitrary
 positive constant.
    """
    samples = np.zeros((num_samples, 5))
    for t in range(num_samples):
        # Sample the likelihood parameters.
        mu_sunny_workday = sample_mu_by_category(prior_means, prior_sigm
a, working_day=1, sunny=1) # TODO
        mu_sunny_weekend = sample_mu_by_category(prior_means, prior_sigm
a, working_day=1, sunny=0) # TODO
        mu_rainy_workday = sample_mu_by_category(prior_means, prior_sigm
a, working_day=0, sunny=1) # TODO
        mu_rainy_weekend = sample_mu_by_category(prior_means, prior_sigm
a, working_day=0, sunny=0) # TODO

        # Now compute the likelihood.
        theta = np.array([[mu_rainy_weekend, mu_sunny_weekend],
                          [mu_rainy_workday, mu_sunny_workday]])
        likelihood = compute_likelihood(data, theta, likelihood_sigma)

        # Compute P(theta).
        prior_sunny_work = prior_prob_by_category(mu_sunny_workday, prio
r_means, prior_sigma, working_day=1, sunny=1)
        prior_sunny_weekend = prior_prob_by_category(mu_sunny_weekend, p
rior_means, prior_sigma, working_day=0, sunny=1)
        prior_rainy_work = prior_prob_by_category(mu_rainy_workday, prio
r_means, prior_sigma, working_day=1, sunny=0)
        prior_rainy_weekend = prior_prob_by_category(mu_rainy_weekend, p
rior_means, prior_sigma, working_day=0, sunny=0)

        p_theta = prior_sunny_work * prior_sunny_weekend * prior_rainy_w
ork * prior_rainy_weekend # TODO
```

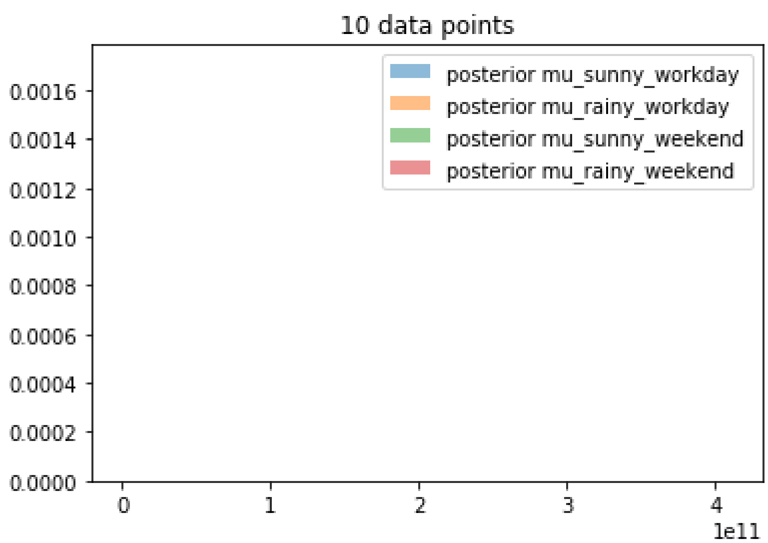
```
        # Save the sample.
        x_t = np.zeros(5)
        x_t[0] = mu_sunny_workday
        x_t[1] = mu_rainy_workday
        x_t[2] = mu_sunny_weekend
        x_t[3] = mu_rainy_weekend
        x_t[4] = likelihood * p_theta # TODO: This should be the posteri
or probability that this sample was drawn.
        samples[t] = x_t

    return np.array(samples)
```

## (g) Plot the estimated distributions of the posterior marginals $p(\mu_{00})$, $p(\mu_{01})$, $p(\mu_{10})$, and $p(\mu_{11})$ from your sample. To do so, plot a histogram of each draw for all means in your sample, weighted by the calculated posterior density associated with that draw.

### Do this using the first 10 datapoints, and then 100 datapoints from the total data.

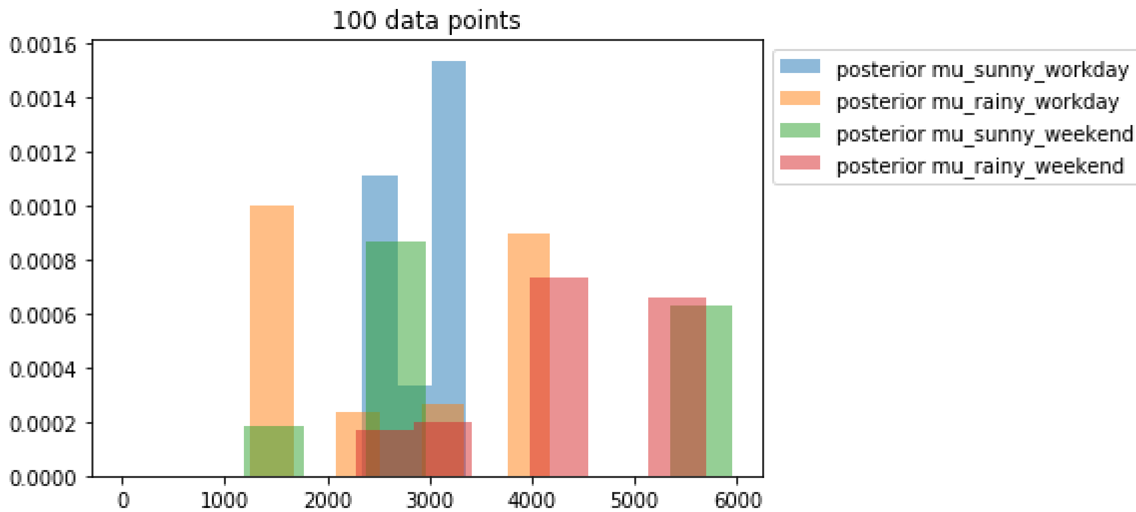
In [13]:
```
mu_rainy_weekend_prior = 4000
mu_rainy_workday_prior = 2000

mu_sunny_weekend_prior = 4000
mu_sunny_workday_prior = 2000

prior_sigma = 1000
likelihood_sigma = 1000

prior_means = np.array([[mu_rainy_weekend_prior, mu_sunny_weekend_prior
],
                        [mu_rainy_workday_prior, mu_sunny_workday_prior
]])

samples = gibbs_sampling_posterior(data[:10],
                                   num_samples=1000,
                                   prior_means=prior_means,
                                   prior_sigma=prior_sigma,
                                   likelihood_sigma=likelihood_sigma)
fig, ax = plt.subplots()

ax.hist(samples[:10][0], weights=samples[:10][4], label="posterior mu_su
nny_workday", density=True, alpha=0.5)
ax.hist(samples[:10][1], weights=samples[:10][4], label="posterior mu_ra
iny_workday", density=True, alpha=0.5)
ax.hist(samples[:10][2], weights=samples[:10][4], label="posterior mu_su
nny_weekend", density=True, alpha=0.5)
ax.hist(samples[:10][3], weights=samples[:10][4], label="posterior mu_ra
iny_weekend", density=True, alpha=0.5)

ax.legend(bbox_to_anchor=(1, 1))

plt.title("10 data points")
```

Out[13]:
```
Text(0.5, 1.0, '10 data points')
```



In [14]:
```
samples = gibbs_sampling_posterior(data[:100],
                                   num_samples=1000,
                                   prior_means=prior_means,
                                   prior_sigma=prior_sigma,
                                   likelihood_sigma=likelihood_sigma)

fig, ax = plt.subplots()

ax.hist(samples[:10][0], weights=samples[:10][4], label="posterior mu_su
nny_workday", density=True, alpha=0.5)
ax.hist(samples[:10][1], weights=samples[:10][4], label="posterior mu_ra
iny_workday", density=True, alpha=0.5)
ax.hist(samples[:10][2], weights=samples[:10][4], label="posterior mu_su
nny_weekend", density=True, alpha=0.5)
ax.hist(samples[:10][3], weights=samples[:10][4], label="posterior mu_ra
iny_weekend", density=True, alpha=0.5)

ax.legend(bbox_to_anchor=(1, 1))

plt.title("100 data points")
```

Out[14]:
```
Text(0.5, 1.0, '100 data points')
```




### (h) Compare and contrast the two motivations above. specifically, address (i) what quantity are you sampling (and thus plotting in the histogram) in each method, and (ii) which approach you would prefer for a small dataset, and why.

TODO: Fill this in