

1. Using your class notes, matrix algebra handout, and textbook material, show the following equalities:

a) $\sum_{i=1}^n (y_i - \mathbf{x}_i^\top \hat{\boldsymbol{\beta}})^2 = (\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}})^\top (\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}})$

b) \mathbf{H} is symmetric and idempotent (Hint: Recall that $\mathbf{H} \equiv \mathbf{X}(\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top$).

c) $\mathbf{I} - \mathbf{H}$ is symmetric and idempotent. (Hint: Use part b)

d) $\mathbf{H}\mathbf{X} = \mathbf{X}$.

e) $(\mathbf{I} - \mathbf{H})(\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}) = (\mathbf{I} - \mathbf{H})\mathbf{Y}$ (Hint: Use part d)

f) $(\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}})^\top (\mathbf{I} - \mathbf{H})(\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}) = \mathbf{Y}^\top (\mathbf{I} - \mathbf{H})\mathbf{Y}$ (Hint: Use parts c and e)

g) $RSS(\hat{\boldsymbol{\beta}}) = \mathbf{Y}^\top (\mathbf{I} - \mathbf{H})\mathbf{Y}$ (Hint: Use parts a and c and recall that $\mathbf{X}\hat{\boldsymbol{\beta}} = \mathbf{H}\mathbf{Y}$)