

Recall that

1. From ALR 5.14,
2. Using your class notes and/or textbook, show the following equalities:

a) $\mathbf{H}\mathbf{H}_R = \mathbf{H}_R$ and $\mathbf{H}\mathbf{X}_1 = \mathbf{H}_R\mathbf{X}_1 = \mathbf{X}_1$
(Recall that $\mathbf{X} = (\mathbf{X}_1|\mathbf{X}_2)$ and \mathbf{H} and \mathbf{H}_R are the hat matrices for \mathbf{X} and \mathbf{X}_1 , respectively).

b) $\mathbf{H} - \mathbf{H}_R$ is symmetric and idempotent.

c) Under H_0 , and only under H_0 ,

$$\frac{1}{\sigma^2}SSreg \sim \chi_q^2$$

with $SSreg = RSS_R - RSS_F$.

(Hint: Under the null $\beta_2 = \mathbf{0}$ and $E(\mathbf{Y}|\mathbf{X}) = \mathbf{X}_1\beta_1$, or $E(\mathbf{Y} - \mathbf{X}_1\beta_1) = \mathbf{0}$).

d) $SSreg$ and $\hat{\sigma}^2$ are independent.

e) Use all previous parts to show that

$$\frac{\frac{SSreg}{q}}{\frac{RSS}{n-p'}} \sim Fq, n - p'$$

3. From ALR 6.4. In addition, using the full model, perform the test

$$H_0 : \beta_{02} - \beta_{03} = 14 \text{ and } \beta_{12} + \beta_{13} = .2$$

with H_A : at least one equality doesn't hold. Show your work. In addition, how could you interpret this test?