$\eta_{ij} = \log(P_{ij}/P_{i1})$. Show that $P_{ij} = \dfrac{e^{\eta_{ij}}}{1+\sum_{k=2}^{J} e^{\eta_{ik}}}$

Question 3

$\eta_{ij} = \log(P_{ij}/P_{i1})$

$e^{\eta_{ij}} = P_{ij}/P_{i1}$

$P_{ij} = P_{i1} e^{\eta_{ij}}$

Then, recall that $\sum_{k=1}^{K} P_{ik} = \sum_{k=1}^{K} P_{i1} e^{\eta_{ik}}$

Also, recall that $\sum_{k=1}^{K} P_{ik} = 1$

So,

$$1 = \sum_{k=1}^{K} P_{i1} e^{\eta_{ik}}$$

$$1 = P_{i1} \sum_{k=1}^{K} e^{\eta_{ik}}$$

$$P_{i1} = 1/\sum_{k=1}^{K} e^{\eta_{ik}}$$

Then, recall $P_{i1} = \dfrac{P_{ij}}{e^{\eta_{ij}}}$

So,

$$\frac{P_{ij}}{e^{\eta_{ij}}} = \frac{1}{\sum_{k=1}^{K} e^{\eta_{ik}}}$$

$$P_{ij} = \frac{e^{\eta_{ij}}}{e^{\eta_{i1}} + \sum_{k=2}^{K} e^{\eta_{ik}}}$$

And, recall that $\eta_{i1} = \emptyset$

So, $$P_{ij} = \frac{e^{\eta_{ij}}}{1+\sum_{k=2}^{K} e^{\eta_{ik}}}$$ ✓