**For the following questions, follow your S18S632_01-11-18.pdf lecture notes**

1. We have shown (or will show) in class that

$$bias(\hat{y}|\mathbf{x}_i) = (\mathbf{x}_{1,i}^\top(\mathbf{X}_1^\top\mathbf{X}_1)^{-1}\mathbf{X}_1^\top\mathbf{X}_2 - \mathbf{x}_{2,i}^\top)\boldsymbol{\beta}_2.$$

   Show that

$$\sum_{i=1}^{n}\{bias(\hat{y}|\mathbf{x}_i)\}^2 = (\mathbf{X}_2\boldsymbol{\beta}_2)^\top(\mathbf{I} - \mathbf{H}_1)(\mathbf{X}_2\boldsymbol{\beta}_2)$$

2. Let $\mathbf{X} = (\mathbf{X}_1|\mathbf{X}_2)$, $\mathbf{H}$ the hat matrix for $\mathbf{X}$, and $\mathbf{H_1}$ the hat matrix for $\mathbf{X}_1$.

   a) Show that $\mathbf{H} - \mathbf{H}_1$ is positive semidefinite, i.e., show that for any non-zero vector $\mathbf{z}$,

$$\mathbf{z}^\top(\mathbf{H} - \mathbf{H}_1)\mathbf{z} \geq 0$$

   Hint: Recall that $\mathbf{a}^\top\mathbf{a} = \sum a_i^2$

   b) Find the $n \times 1$ vector $\mathbf{z}$, such that using $\mathbf{z}^\top(\mathbf{H} - \mathbf{H}_1)\mathbf{z}$ you can show that

$$h_{ii}^F \geq h_{ii}^R$$

   for all $i = 1, \ldots, n$, where $h_{ii}^F$ and $h_{ii}^R$ are the $i$th diagonal elements of $\mathbf{H}$ and $\mathbf{H}_1$, respectively.

   Hint: It should be a very simple non-zero vector