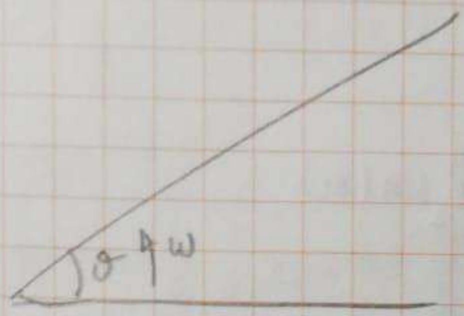
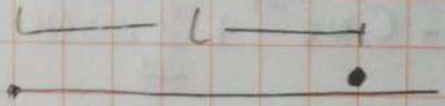
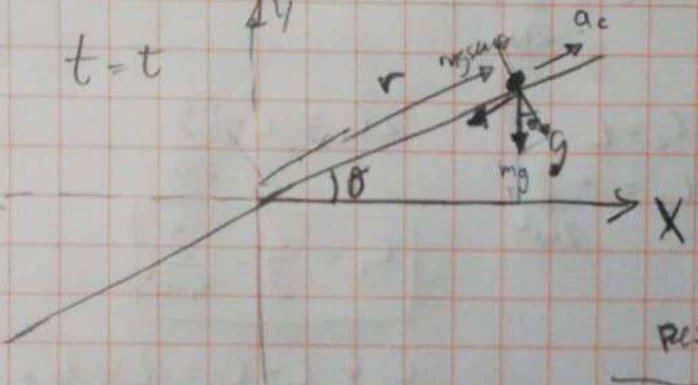
$\theta(t) = \omega t$



$t=0$



$t=t$



r variable

$$\cancel{m}\ddot{r} = \overbrace{-\cancel{m}g\sin\theta}^{\text{peso}} + \overbrace{\cancel{m}\omega^2 r}^{\text{centripeta}}$$

$$\ddot{r} = -g\sin(\omega t) + \omega^2 r$$

$$\ddot{r} - \omega^2 r + g\sin(\omega t) = 0$$

Caracteristica: $\ddot{r}_c - \omega^2 r_c = 0$ $\Rightarrow$ $\boxed{r_c(t) = C_1 e^{\omega t} + C_2 e^{-\omega t}}$

$$\lambda^2 - \omega^2 = 0$$
$$\lambda = \pm\,\omega$$

Particular: $\ddot{r}_p - \omega^2 r_p + g\sin(\omega t) = 0$

$$r_p(t) = C_3 \sin(\omega t)$$

$$\ddot{r}_p(t) = -\omega^2 C_3 \sin(\omega t)$$

$$\Rightarrow -\omega^2 C_3 \cancel{\sin(\omega t)} - \omega^2 C_3 \cancel{\sin(\omega t)} + g\cancel{\sin(\omega t)} = 0$$

$$+2\omega^2 C_3 = g \qquad \therefore \boxed{r_p(t) = \frac{g}{2\omega^2}\sin(\omega t)}$$

$$C_3 = \frac{g}{2\omega^2}$$

$$r(t) = r_p(t) + r_c(t)$$

$$r(t) = C_1 e^{\omega t} + C_2 e^{-\omega t} + \frac{g}{2\omega^2} \sin(\omega t)$$

$$r(t=0) = C_1 + C_2 = L$$

$$r'(t=0) = C_1 \omega\, e^{\omega t} - C_2 \omega\, e^{-\omega t} + \frac{g}{2\omega} \cos\omega t \quad (t \to 0)$$

$$= \omega C_1 - \omega C_2 + \frac{g}{2\omega} = 0$$

$$\Rightarrow \quad C_1 + C_2 = L \qquad 2C_1 = L - \frac{g}{2\omega}$$

$$C_1 - C_2 = -\frac{g}{2\omega^2}$$

$$\boxed{C_1 = \frac{L}{2} - \frac{g}{4\omega^2}}$$

$$C_2 = L - C_1 = L - \frac{L}{2} + \frac{g}{4\omega}$$

$$\boxed{C_2 = \frac{L}{2} + \frac{g}{4\omega^2}}$$

Solución:

$$\boxed{r(t) = \left(\frac{L}{2} - \frac{g}{4\omega^2}\right) e^{\omega t} + \left(\frac{L}{2} + \frac{g}{4\omega^2}\right) e^{-\omega t} + \frac{g}{2\omega^2} \sin(\omega t)}$$

$$\theta = \omega t = \frac{\pi}{2} \Rightarrow \quad t = \frac{\pi}{2\omega}$$