1 开始

测试 LaTeX

$a^2 + b^2 = c^2$

$$
\begin{aligned}
|P - P_n| &= \left| a \sum_{k=n}^{\infty} \left(ae^{\frac{\pi}{4}i} \right)^k \right| = \left| a \left(ae^{\frac{\pi}{4}i} \right)^n \sum_{k=0}^{\infty} \left(ae^{\frac{\pi}{4}i} \right)^k \right| \\
&= \left| (2-\sqrt{2})((\sqrt{2}-1)(1+i))^n \right| \cdot |P| \\
&= (2-\sqrt{2})((\sqrt{2}-1)\sqrt{2})^n \frac{\sqrt{6}}{3} \\
&= \frac{\sqrt{6}}{3}(2-\sqrt{2})^{n+1}
\end{aligned}
$$

1.1 例子

量子效应[?]

$2\,\mathrm{H_2} + \mathrm{O_2} = 2\,\mathrm{H_2O}$

character	a	b	c	d	e
$f(c)$	45	13	12	16	9
Huffman Code	0	101	100	111	1101

$$P\left(\frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \leq x \right) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt$$

$$\cos\theta = \frac{\vec{\mathrm{n}_1} \cdot \vec{\mathrm{n}_2}}{|\vec{\mathrm{n}_1}| \cdot |\vec{\mathrm{n}_2}|} = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2}\sqrt{x_2^2 + y_2^2 + z_2^2}}$$

2 人工转变

$$
\begin{cases}
{}^4_2\mathrm{He} + {}^{14}_{7}\mathrm{N} \to {}^{17}_{8}\mathrm{O} + {}^1_1\mathrm{H}(\mathrm{Rutherford}) \\
{}^4_2\mathrm{He} + {}^{8}_{4}\mathrm{Be} \to {}^{11}_{6}\mathrm{C} + {}^1_0\mathrm{n}(\mathrm{Chadwick}) \\
{}^4_2\mathrm{He} + {}^{27}_{13}\mathrm{Al} \to {}^{30}_{15}\mathrm{P} + {}^1_0\mathrm{n}, {}^{30}_{15}\mathrm{P} \to {}^{30}_{14}\mathrm{Si} + {}^0_1\mathrm{e}(\mathrm{Curie})
\end{cases}
$$

$$C_xH_yO_z + (x + \frac{y}{4} - \frac{z}{2})O_2 \to xCO_2 + \frac{y}{2}H_2O$$

3 知乎回答

关于e^{n+1}中超球面特征的一些结果 [?]

$$\text{Proof:}\quad e^{\sqrt{x_1\cdot x_2}}<\frac{e^{x_2}-e^{x_1}}{x_2-x_1}\quad (x_2>x_1)$$

$$e^{\sqrt{x_1\cdot x_2}}<\frac{e^{x_2}-e^{x_1}}{x_2-x_1}\quad (x_2>x_1)\Leftrightarrow \frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{x_2-x_1}>1$$

$$\begin{aligned}\frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{x_2-x_1}&=\frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{(x_2-\sqrt{x_1\cdot x_2})-(x_1-\sqrt{x_1\cdot x_2})}\\&=\frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{\ln(e^{(x_2-\sqrt{x_1\cdot x_2})})-\ln(e^{(x_1-\sqrt{x_1\cdot x_2})})}\\&>\sqrt{e^{x_2-\sqrt{x_1\cdot x_2}+x_1-\sqrt{x_1\cdot x_2}}}\\&=\sqrt{e^{(\sqrt{x_2}-\sqrt{x_1})^2}}>\sqrt{e^0}=1\end{aligned}$$

Rt$\triangle ABC$中，$\angle A=\frac{\pi}{2}$， AM为中线，AD为角平分线
以$\vec{AB}$为x轴，$\vec{AC}$为y轴建系，不妨设$AB=a$，$AC=b$
由中点公式

$$\begin{cases}x_M=\dfrac{x_B+x_C}{2}\\ y_M=\dfrac{y_B+y_C}{2}\end{cases}\Rightarrow M\left(\frac{a}{2},\frac{b}{2}\right)$$

BC的截距式方程为$\frac{x}{a}+\frac{y}{b}=1$，与$y=x$联立得

$$D\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$$

$$\begin{aligned}AM^2&=\frac{a^2}{4}+\frac{b^2}{4}\\&=\sqrt{\frac{\left(\frac{a}{\sqrt{2}}\right)^2+\left(\frac{b}{\sqrt{2}}\right)^2}{2}}^{\,2}\end{aligned}$$

$$\begin{aligned}AD^2&=2\left(\frac{ab}{a+b}\right)^2\\&=\left(\frac{2}{\frac{\sqrt{2}}{a}+\frac{\sqrt{2}}{b}}\right)^2\end{aligned}$$

已知

$$H_n = \frac{n}{\sum\limits_{i=1}^{n} \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}$$

$$Q_n = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{\frac{x_1^2+x_2^2+\cdots+x_n^2}{n}}$$

$$H_n \le Q_n$$

由$H_2 \le Q_2$, 得

$$AM \ge AD$$

等号当且仅当$AB = AC$时成立

另解，由中线定理

$$(2AM)^2 + BC^2 = 2(AB^2 + AC^2$$

$$AM = \frac{2(a^2+b^2)-(a^2+b^2)}{4} = \frac{a^2}{4} + \frac{b^2}{4}$$

由$S_{\triangle ABC} = S_{\triangle ABD} + S_{\triangle ACD}$

$$\frac{1}{2}AB \cdot AC = \frac{1}{2}AB \cdot AD \cdot \sin\frac{\pi}{4} + \frac{1}{2}AC \cdot AD \cdot \sin\frac{\pi}{4}$$

$$AD = \frac{\sqrt{2}AB \cdot AC}{AB + AC} = \frac{\sqrt{2}ab}{a+b}$$

以F_1为极点，垂直于左准线方向为极轴建系

$$L = \rho_\theta + \rho_{\pi-\theta} = \left|\frac{2ep}{1-e^2\cos^2\theta}\right|$$

$$\begin{aligned} S_{PMQN} &= \frac{1}{2}|PQ| \cdot |MN| \\ &= \frac{1}{2} \cdot \frac{2ep}{1-e^2\cos^2\theta} \cdot \frac{2ep}{1-e^2\cos^2(\pi-\theta)} \\ &= \frac{8e^2p^2}{4-4e^2+e^4\sin^2 2\theta} \end{aligned}$$

$$e = \frac{c}{a}, p = \frac{a^2}{c} - c = \frac{b^2}{c}$$

$$S_{\min} = \frac{8e^2p^2}{4-4e^2+e^4} = \frac{8a^2b^4}{(a^2+b^2)^2}$$

$$S_{\max} = \frac{2e^2p^2}{1-e^2} = 2b^2$$

另解

设PQ方程为$x=my-c$，联立

$$\begin{cases} \frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \\ x=my-c \end{cases} \Rightarrow (a^2+b^2m^2)y^2-2mcb^2y-b^2c^2=0$$

$$|PQ|=\sqrt{1+m^2}\cdot|y_1-y_2|=\frac{2ab^2(m^2+1)}{a^2+b^2m^2}$$

用$-\frac{1}{m}$替换m

$$|MN|=\frac{2ab^2\left[\left(-\frac{1}{m}\right)^2+1\right]}{a^2+b^2(-\frac{1}{m})^2}=\frac{2ab^2\left(m^2+1\right)}{a^2m^2+b^2}$$

令$t=m^2$

$$\begin{aligned} S_{PMQN}&=\frac{1}{2}|PQ|\cdot|MN| \\ &=\frac{2a^2b^4(t+1)^2}{(a^2+b^2t)(a^2t+b^2)} \\ &=\frac{2a^2b^4(t^2+2t+1)}{a^2b^2t^2+(a^4+b^4)t+a^2b^2} \end{aligned}$$

若$t=0\Rightarrow S=2b^2$

若$t\neq 0$，分离常数

$$\begin{aligned} S_{PMQN}&=2a^2b^4\cdot\left[\frac{\left(2-\frac{a^4+b^4}{a^2b^2}\right)t}{a^2b^2t^2+(a^4+b^4)t+a^2b^2}+\frac{1}{a^2b^2}\right] \\ &=2b^2-\frac{2(a^2-b^2)b^2}{a^2b^2\left(t+\frac{1}{t}\right)+a^4+b^4} \\ &\geq 2b^2-\frac{2(a^2-b^2)b^2}{(a^2+b^2)^2} \\ &=\frac{8a^2b^4}{(a^2+b^2)^2} \end{aligned}$$

当且仅当$t=m^2=1,k=\pm1$ 时取等

$$\lim_{t\to 0/+\infty}\frac{2(a^2-b^2)b^2}{a^2b^2\left(t+\frac{1}{t}\right)+a^4+b^4}=0$$

$$S_{\min}=\frac{8a^2b^4}{(a^2+b^2)^2}$$

$$S_{\max} = 2b^2$$

注

$$\frac{2a^2b^4(t+1)^2}{(a^2+b^2t)(a^2t+b^2)} \ge \frac{2a^2b^4(t+1)^2}{\frac{1}{4}\cdot[(a^2+b^2)t+(a^2+b^2)]^2} = \frac{8a^2b^4}{(a^2+b^2)^2}$$

4 2021 高考英语短文改错

Last week our teacher asked us to fill in a questionnaire. One of the questions $\underset{\text{is}}{\underline{\text{are}}}$： Who will you go $\underset{\text{to}}{\wedge}$ in times of trouble? Here are the results. Many students say they will talk to their $\underset{\text{friends}}{\underline{\text{friend}}}$ or classmates because they're of the same age and can understand each other. Some will turn ~~out~~ to their parents or teachers for help. Only a $\underset{\text{few}}{\underline{\text{little}}}$ choose to deal with the problems on $\underset{\text{their}}{\underline{\text{our}}}$ own. Their answers also show that they dislike $\underset{\text{talking}}{\underline{\text{talk}}}$ to others. They $\underset{\text{keep}}{\underline{\text{kept}}}$ very much to themselves. In my opinion, $\underset{\text{whenever}}{\underline{\text{where}}}$ in trouble, we should seek help from those we trust $\underset{\text{most}}{\underline{\text{mostly}}}$.

5 化学反应原理

$$a\,\mathrm{A} + b\,\mathrm{B} \rightleftharpoons \mathrm{pC} + \mathrm{qD}$$

5.1 速率常数

$v_f = k_f \cdot c^a(\mathrm{A}) \cdot c^b(\mathrm{B})$

$v_b = k_b \cdot c^p(\mathrm{C}) \cdot c^q(\mathrm{D})$

当达平衡,$v_f = v_b$

$\mathrm{K} = \frac{c^p(\mathrm{C})\cdot c^q(\mathrm{D})}{c^a(\mathrm{A})\cdot c^b(\mathrm{B})} = \frac{k_f}{k_b}$

5.2 分压平衡常数

$\mathrm{K}_p = \frac{p^p(\mathrm{C})\cdot p^q(\mathrm{D})}{p^a(\mathrm{A})\cdot p^b(\mathrm{B})}$

恒压时 $\mathrm{K}_p = \frac{\varphi^p(\mathrm{C})\cdot \varphi^q(\mathrm{D})}{\varphi^a(\mathrm{A})\cdot \varphi^b(\mathrm{B})} \cdot p_0{}^{\frac{c\cdot d}{a\cdot b}}$

5.3 转化率

$\alpha = \frac{n_{参加反应的物质}}{n_{投入的物质}} \times 100\%$

5.4 常见方程式

$$CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g) \quad \Delta H{=}-41\,kJ/mol$$
$$H^+ + OH^- = H_2O \quad \Delta H{=}-57.3\,kJ/mol$$
$$N_2(g) + 3\,H_2(g) \rightleftharpoons 2\,NH_3(g) \quad \Delta H{=}-92.4\,kJ/mol$$
$$2\,NO + 2\,CO \rightleftharpoons 2\,CO_2 + N_2$$
$$CO_2(g) + 3\,H_2(g) 3\,H_2(g)(l) + H_2O(l) \quad \Delta H{=}-93.8\,kJ/mol$$
$$CO + 2\,H_2 2\,H_2$$