1 开始

测试 LaTeX

$a^2 + b^2 = c^2$

$$
\begin{aligned}
|P - P_n| &= \left| a \sum_{k=n}^{\infty} \left(a e^{\frac{\pi}{4} i} \right)^k \right| = \left| a \left(a e^{\frac{\pi}{4} i} \right)^n \sum_{k=0}^{\infty} \left(a e^{\frac{\pi}{4} i} \right)^k \right| \\
&= \left| (2-\sqrt{2})((\sqrt{2}-1)(1+i))^n \right| \cdot |P| \\
&= (2-\sqrt{2})((\sqrt{2}-1)\sqrt{2})^n \frac{\sqrt{6}}{3} \\
&= \frac{\sqrt{6}}{3}(2-\sqrt{2})^{n+1}
\end{aligned}
$$

1.1 例子

量子效应[2]

$2\,\mathrm{H_2} + \mathrm{O_2} = 2\,\mathrm{H_2O}$

character	a	b	c	d	e
$f(c)$	45	13	12	16	9
Huffman Code	0	101	100	111	1101

$$
P\left(\frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma\sqrt{n}} \le x \right) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt
$$

$$
\cos\theta = \frac{\vec{\mathrm{n_1}} \cdot \vec{\mathrm{n_2}}}{|\vec{\mathrm{n_1}}| \cdot |\vec{\mathrm{n_2}}|} = \frac{x_1x_2 + y_1y_2 + z_1z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2}\sqrt{x_2^2 + y_2^2 + z_2^2}}
$$

2 人工转变

$$
\begin{cases}
{}_2^4\mathrm{He} + {}_7^{14}\mathrm{N} \to {}_8^{17}\mathrm{O} + {}_1^1\mathrm{H}(\mathrm{Rutherford}) \\
{}_2^4\mathrm{He} + {}_4^{8}\mathrm{Be} \to {}_6^{11}\mathrm{C} + {}_0^1\mathrm{n}(\mathrm{Chadwick}) \\
{}_2^4\mathrm{He} + {}_{13}^{27}\mathrm{Al} \to {}_{15}^{30}\mathrm{P} + {}_0^1\mathrm{n}, {}_{15}^{30}\mathrm{P} \to {}_{14}^{30}\mathrm{Si} + {}_1^0\mathrm{e}(\mathrm{Curie})
\end{cases}
$$

$$
C_xH_yO_z + (x + \frac{y}{4} - \frac{z}{2})O_2 \to xCO_2 + \frac{y}{2}H_2O
$$

3 Part 2

关于e^{n+1}中超球面特征的一些结果 [1]

$$\text{Proof:}\quad e^{\sqrt{x_1\cdot x_2}} < \frac{e^{x_2}-e^{x_1}}{x_2-x_1}\quad (x_2>x_1)$$

$$e^{\sqrt{x_1\cdot x_2}} < \frac{e^{x_2}-e^{x_1}}{x_2-x_1}\quad (x_2>x_1) \Leftrightarrow \frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{x_2-x_1} > 1$$

$$\begin{aligned}
\frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{x_2-x_1} &= \frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{(x_2-\sqrt{x_1\cdot x_2})-(x_1-\sqrt{x_1\cdot x_2})}\\
&= \frac{e^{x_2-\sqrt{x_1\cdot x_2}}-e^{x_1-\sqrt{x_1\cdot x_2}}}{\ln(e^{(x_2-\sqrt{x_1\cdot x_2})})-\ln(e^{(x_1-\sqrt{x_1\cdot x_2})})}\\
&> \sqrt{e^{x_2-\sqrt{x_1\cdot x_2}+x_1-\sqrt{x_1\cdot x_2}}}\\
&= \sqrt{e^{(\sqrt{x_2}-\sqrt{x_1})^2}} > \sqrt{e^0} = 1
\end{aligned}$$

Rt$\triangle ABC$中，$\angle A=\frac{\pi}{2}$， AM为中线，AD为角平分线

以$\vec{AB}$为x轴，$\vec{AC}$为y轴建系，不妨设$AB=a$，$AC=b$

由中点公式

$$\begin{cases} x_M = \dfrac{x_B+x_C}{2} \\ y_M = \dfrac{y_B+y_C}{2} \end{cases} \Rightarrow M\left(\frac{a}{2},\frac{b}{2}\right)$$

BC的截距式方程为$\frac{x}{a}+\frac{y}{b}=1$，与$y=x$联立得

$$D\left(\frac{ab}{a+b},\frac{ab}{a+b}\right)$$

$$\begin{aligned}
AM^2 &= \frac{a^2}{4}+\frac{b^2}{4}\\
&= \sqrt{\frac{\left(\frac{a}{\sqrt{2}}\right)^2+\left(\frac{b}{\sqrt{2}}\right)^2}{2}}^{\,2}\\
AD^2 &= 2\left(\frac{ab}{a+b}\right)^2\\
&= \left(\frac{2}{\frac{\sqrt{2}}{a}+\frac{\sqrt{2}}{b}}\right)^2
\end{aligned}$$

已知

$$H_n = \frac{n}{\sum\limits_{i=1}^{n} \frac{1}{x_i}} = \frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}$$

$$Q_n = \sqrt{\sum_{i=1}^{n} x_i^2} = \sqrt{\frac{x_1^2+x_2^2+\cdots+x_n^2}{n}}$$

$$H_n \le Q_n$$

由$H_2 \le Q_2$, 得

$$AM \ge AD$$

等号当且仅当$AB = AC$时成立

另解，由中线定理

$$(2AM)^2 + BC^2 = 2(AB^2 + AC^2$$

$$AM = \frac{2(a^2+b^2)-(a^2+b^2)}{4} = \frac{a^2}{4} + \frac{b^2}{4}$$

由$S_{\triangle ABC} = S_{\triangle ABD} + S_{\triangle ACD}$

$$\frac{1}{2}AB \cdot AC = \frac{1}{2}AB \cdot AD \cdot \sin\frac{\pi}{4} + \frac{1}{2}AC \cdot AD \cdot \sin\frac{\pi}{4}$$

$$AD = \frac{\sqrt{2}AB \cdot AC}{AB + AC} = \frac{\sqrt{2}ab}{a+b}$$

以F_1为极点，垂直于左准线方向为极轴建系

$$L = \rho_\theta + \rho_{\pi-\theta} = \left|\frac{2ep}{1-e^2\cos^2\theta}\right|$$

$$\begin{aligned}
S_{PMQN} &= \frac{1}{2}|PQ| \cdot |MN| \\
&= \frac{1}{2} \cdot \frac{2ep}{1-e^2\cos^2\theta} \cdot \frac{2ep}{1-e^2\cos^2(\pi-\theta)} \\
&= \frac{8e^2p^2}{4-4e^2+e^4\sin^2 2\theta}
\end{aligned}$$

$$e = \frac{c}{a}, p = \frac{a^2}{c} - c = \frac{b^2}{c}$$

$$S_{\min} = \frac{8e^2p^2}{4-4e^2+e^4} = \frac{8a^2b^4}{(a^2+b^2)^2}$$

$$S_{\max} = \frac{2e^2p^2}{1-e^2} = 2b^2$$

另解

设PQ方程为$x=my-c$，联立

$$\begin{cases}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\\ x=my-c\end{cases}\Rightarrow (a^2+b^2m^2)y^2-2mcb^2y-b^2c^2=0$$

$$|PQ|=\sqrt{1+m^2}\cdot|y_1-y_2|=\frac{2ab^2(m^2+1)}{a^2+b^2m^2}$$

用$-\frac{1}{m}$替换m

$$|MN|=\frac{2ab^2\left[\left(-\frac{1}{m}\right)^2+1\right]}{a^2+b^2(-\frac{1}{m})^2}=\frac{2ab^2\left(m^2+1\right)}{a^2m^2+b^2}$$

令$t=m^2$

$$\begin{aligned}S_{PMQN}&=\frac{1}{2}|PQ|\cdot|MN|\\&=\frac{2a^2b^4(t+1)^2}{(a^2+b^2t)(a^2t+b^2)}\\&=\frac{2a^2b^4(t^2+2t+1)}{a^2b^2t^2+(a^4+b^4)t+a^2b^2}\end{aligned}$$

若$t=0\Rightarrow S=2b^2$

若$t\neq 0$，分离常数

$$\begin{aligned}S_{PMQN}&=2a^2b^4\cdot\left[\frac{\left(2-\frac{a^4+b^4}{a^2b^2}\right)t}{a^2b^2t^2+(a^4+b^4)t+a^2b^2}+\frac{1}{a^2b^2}\right]\\&=2b^2-\frac{2(a^2-b^2)b^2}{a^2b^2\left(t+\frac{1}{t}\right)+a^4+b^4}\\&\geq 2b^2-\frac{2(a^2-b^2)b^2}{(a^2+b^2)^2}\\&=\frac{8a^2b^4}{(a^2+b^2)^2}\end{aligned}$$

当且仅当$t=m^2=1,k=\pm1$ 时取等

$$\lim_{t\to 0/+\infty}\frac{2(a^2-b^2)b^2}{a^2b^2\left(t+\frac{1}{t}\right)+a^4+b^4}=0$$

$$S_{\min}=\frac{8a^2b^4}{(a^2+b^2)^2}$$

$$S_{\max} = 2b^2$$

注

$$\frac{2a^2b^4(t+1)^2}{(a^2+b^2t)(a^2t+b^2)} \geq \frac{2a^2b^4(t+1)^2}{\frac{1}{4}\cdot[(a^2+b^2)t+(a^2+b^2)]^2} = \frac{8a^2b^4}{(a^2+b^2)^2}$$

参考文献

[1] 闻家君 陈抚良胡名成. 关于e^{n+1}中超球面特征的一些结果. 江西科学, 24(6):400–402, 2006.

[2] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys Rev*, 47(10):696–702, 1935.